Irreducible gravitational wave emission from a network of cosmic defects

PRL 110, 101302 Figueroa, Hindmarsh, Urrestilla
arXiv: 1909.????? Figueroa, Hindmarsh, JLi, Urrestilla

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Outline

1. Defects and scaling
2. GWs background from scaling sources
   - General theory
   - @ Radiation Domination
   - @ Matter Domination
3. Predictions from lattice simulations.
   - UETCs @ Radiation domination and Matter domination.
   - Comparison with real-time calculation.

Cosmic Defects and scaling

- Cosmological phase transitions: symmetry spontaneously broken

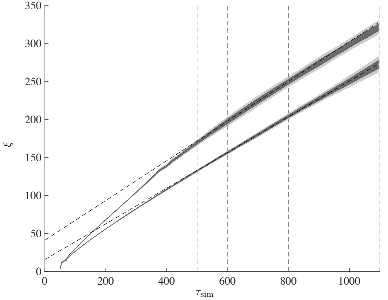
$$V(\phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \eta^2\right)^2 + V_{\rm int}(\Phi, \psi, T)$$

  - Thermal $V_{\rm int} \sim g_T|\Phi|^2 T^2$
  - Field interaction: $V_{\rm int} \sim g|\Phi|^2\psi^2$
- Defect zoo: cosmic strings (local, global, non-Abelian...), monopoles, textures...

Scaling: causality and microphysics.

Correlation length
$\xi(t) = \lambda(t)H^{-1}(t)$

$\lambda \sim$ constant



GWs from scaling sources: general theory
Spectrum of GWs

GW Power Spectrum at sub-horizon scales:

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_{\rm Pl}^2 a^4(t)} \int dt_1 \int dt_2\, a(t_1) a(t_2) \times \cos[k(t_1 - t_2)]\, \Pi^2(k, t_1, t_2)$$

- $a(t')a(t'')$: Expasion of the Universe
- $\Pi^2(k,t,t')$: Unequal Time Correlator of the TT anisotropic-stress $\Pi_{ij} = T_{ij} - p g_{ij}$

$$\langle \Pi_{ij}^{\rm TT}(\mathbf{k}, t_1) \Pi_{ij}^{\rm TT}(\mathbf{k}', t_2) \rangle \equiv (2\pi)^3 \Pi^2(k, t_1, t_2) \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

GWs sourced by any

$$\{T_{ij}\}^{TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{TT}$$

UETCs from scaling

Scaling: invariant number density of defects per comoving Hubble volume.

Dimensional analysis:

$$\Pi^2(k,t,t') = \frac{\eta^4}{\sqrt{t_1 t_2}}\mathcal{U}(x_1,x_2)$$

where,

- η: vev in the broken state.
- $x_1 = kt_1$ and $x_2 = kt_2$.

Substituting,

$$\frac{d\rho_{\mathrm{GW}}}{d\log k} \propto \frac{k^3}{M_{\mathrm{Pl}}^2 a^4(t)} \int dt_1 dt_2 a(t_1) a(t_2) \times \cos(k(t_1 - t_2)) \frac{\eta^4}{\sqrt{t_1 t_2}}\mathcal{U}(kt_1, kt_2)$$

$$\frac{d\rho_{\mathrm{GW}}}{d\log k} \propto \frac{k^2}{Ha^4(t)} \left(\frac{\eta}{M_{\mathrm{Pl}}}\right)^4 \int dx_1 dx_2 \frac{a_1 a_2}{\sqrt{x_1 x_2}} \cos(x_1 - x_2)\mathcal{U}(x_1,x_2)$$

@ Radiation Domination

$a(t) \propto t \Rightarrow a_1 a_2 \propto t_1 t_2$

Radiation domination and scaling:

$$\frac{d\rho_{\rm GW}}{d\log k} \propto \frac{M_{\rm Pl}^2}{a^4(t)}\left(\frac{\eta}{M_{\rm Pl}}\right)^4 \int dx_1 dx_2 \sqrt{x_1 x_2}\cos(x_1 - x_2)\mathcal{U}_{\rm RD}(x_1, x_2)$$

$$\frac{d\rho_{\rm GW}}{d\log k} \propto \frac{M_{\rm Pl}^2}{a^4(t)}\left(\frac{\eta}{M_{\rm Pl}}\right)^4 F_{\rm RD}^{[\mathcal{U}]}(x)$$

Convergence is guaranteed as long as UETCs decay fast enough (k^{-p} with $p > 2$)

$$F_{\rm RD}^{(\infty)} \equiv F_{\rm RD}^{[\mathcal{U}]}(x \to \infty) \to \text{constant.}$$

$$\boxed{h^2\Omega_{\rm GW}^{(0)}(k) = h^2\Omega_{\rm rad}^{(0)}\left(\frac{\eta}{M_{\rm Pl}}\right)^4 F_{\rm RD}^{(\infty)}}$$

Scale invariant spectrum
∀ type of scaling network.

Figueroa, Hindmarsh, Urrestilla 2013

@ Matter Domination

$$a(t) \propto t^2 \Rightarrow a_1 a_2 \propto (t_1 t_2)^2$$

Matter domination and scaling:

$$\frac{d\rho_{\rm GW}}{d\log k} \propto \frac{M_{\rm Pl}^2}{k^2 a^4(t)} \left(\frac{\eta}{M_{\rm Pl}}\right)^4 \int dx_1 dx_2 x_1^{3/2} x_2^{3/2} \cos(x_1 - x_2) \mathcal{U}_{\rm MD}(x_1, x_2)$$

$$\frac{d\rho_{\rm GW}}{d\log k} \propto \frac{M_{\rm Pl}^2}{k^2 a^4(t)} \left(\frac{\eta}{M_{\rm Pl}}\right)^4 F_{\rm MD}^{[\mathcal{U}]}(x)$$

$$\boxed{\Omega_{\rm GW}(x,t) = \Omega_{\rm rad} \left(\frac{\eta}{M_{\rm Pl}}\right)^4 \frac{k_{\rm eq}^2}{k^2} F_{\rm MD}^{\mathcal{U}}}$$

Scale dependent

What about super-horizon modes ($kt \ll 1$)?

$$\mathcal{G}(k,t_1,t_2) = \begin{cases} k(t_1/t_2), & \text{RD} \\ k\left[-1+2(t_1/t_2)\right], & \text{MD} \end{cases}$$

$$+$$

$$\mathcal{U}(x_1 \ll 1, x_2 \ll 1) \to \begin{cases} \simeq \mathcal{U}_{\rm SH} = \text{const}, & x_1 \approx x_2 \\ \ll \mathcal{U}_{\rm SH}, & \text{otherwise} \end{cases}$$

UETCs can be taken out from the integral

$$\boxed{\Omega_{\rm GW}(x \ll 1) \propto \begin{cases} x^3, & \text{RD} \\ \left(\frac{t}{t_{\rm eq}}\right)^2 x^3, & \text{MD} \end{cases}}$$

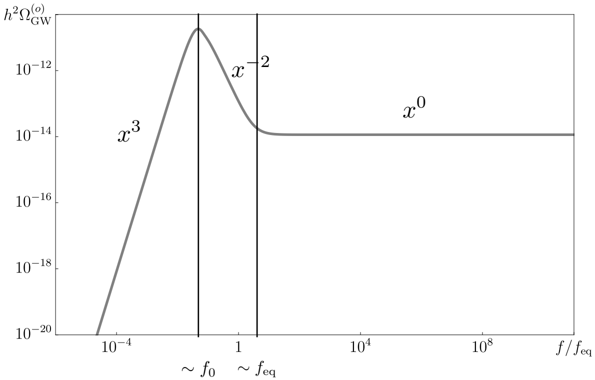
Full spectrum of scaling defect networks

$$h^2\Omega^{(0)}_{\rm GW}(k) = h^2\Omega^{(0)}_{\rm rad}\left(\frac{\eta}{M_{\rm Pl}}\right)^4\left(F^{(\infty)}_{\rm RD}\Theta(x - x_{\rm eq}) + \frac{k^2_{\rm eq}}{k^2}F^{\mathcal{U}}_{\rm MD}(x)\right)$$

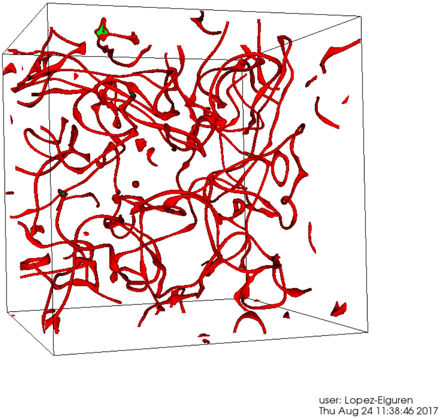
$\Omega_{\rm GW} \propto f^3$, $f < f_0 \sim 10^{-18}$ Hz,

$\Omega_{\rm GW} \propto f^{-2}$, $f_0 < f < f_{\rm eq} \sim 10^{-15}$ Hz,

$\Omega_{\rm GW} \propto f^0$, $f_{\rm eq} < f < f_* \sim 10^8$ Hz,



Lattice simulations

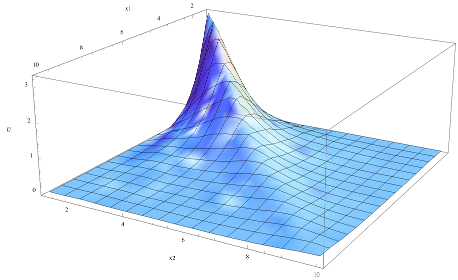


- Field theory simulations
- Solve e.o.m. in Lattices
- Expanding background: Radiation and Mattter domination

1. Extraction of UETCs
2. Real-time calculation of GWs

$O(N)$ Lattice simulations

Extraction of UETCs @ RD and MD



$$F^{\mathcal{U}_N}_{\mathrm{RD,MD}}$$

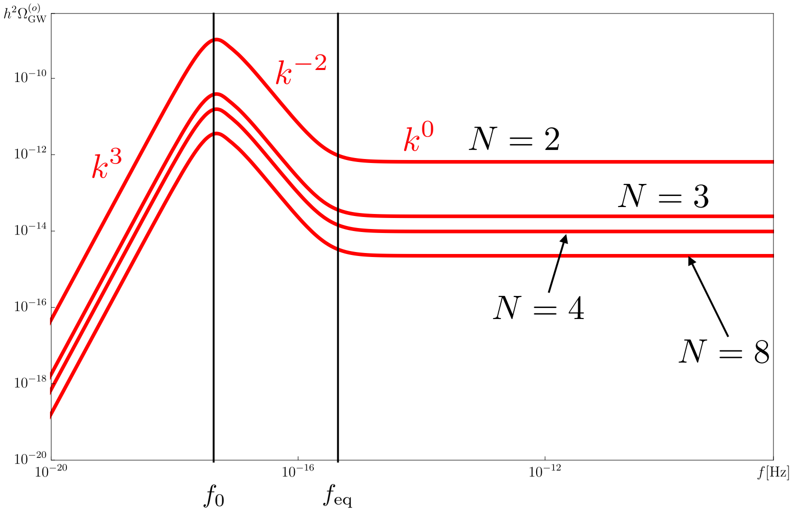
where $N = 2,\ 3,\ 4...$

strings, monopoles, textures...

Simulations with $N = 1024^3,\ 2048^3$ lattice points, and different resolutions $dx = 0.5,\ 1$

Full spectra from UETCs from simulations

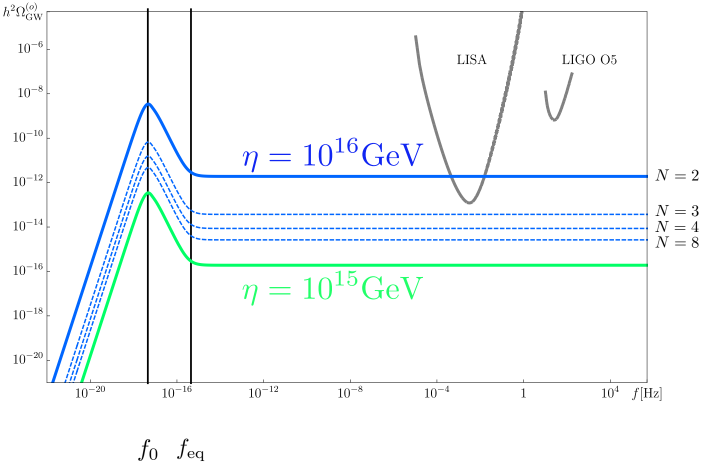
$$h^2\Omega_{\rm GW}^{(0)}(k) = h^2\Omega_{\rm rad}^{(0)}\left(\frac{\eta}{M_{\rm Pl}}\right)^4\left(F_{\rm RD}^{[\infty]} + \frac{k_{\rm eq}^2}{k^2}F_{\rm MD}^{\mathcal{U}}\right)$$



Comparison with LISA and LIGO

$$h^2\Omega_{\rm GW}^{(0)}(k) = h^2\Omega_{\rm rad}^{(0)}\left(\frac{\eta}{M_{\rm Pl}}\right)^4\left(F_{\rm RD}^{[\infty]} + \frac{k_{\rm eq}^2}{k^2}F_{\rm MD}^{\mathcal{U}}\right)$$

$$\frac{\Omega_{\rm GW}\left(N{=}2,\eta{=}5\times10^{15}\right)}{\Omega_{\rm GW}^{(\rm inf)}\left(H_*^{(\rm max)}\right)} \sim 10^3$$



Numerical check: real time computation of GWs

Evolution of GWs e.o.m along with defect dynamics.
Scalar fields,

$$\Box\phi_i = -2\frac{\lambda}{2}a^2(\Phi^2-\eta^2)^2\phi_i$$

GWs,

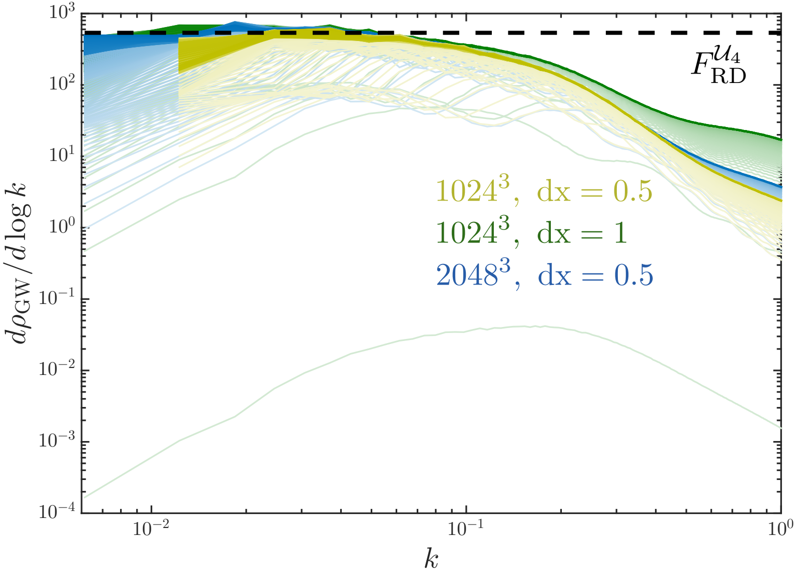
$$\ddot{\bar{h}}_{ij}(\mathbf{x},t) + \left(\nabla^2 + \frac{\ddot{a}(t)}{a(t)}\right)\bar{h}_{ij}(\mathbf{x},t) = 16\pi G a(t)\Pi^{\mathrm{TT}}_{ij}(\mathbf{x},t)$$
$$\Pi^{\mathrm{TT}}_{ij}(\mathbf{x},t) = T^{\mathrm{TT}}_{ij}(\mathbf{x},t) = \partial_i\mathbf{\Phi}^{\mathrm{T}}\partial_j\mathbf{\Phi}$$

Goal: study scaling GW production

$\Downarrow$

Turn on GWs source deep in scaling.

Example: $O(4)$ global textures



Summary

- Irreducible background of GWs
  - For any type of defects (Top. or non-top., $\forall$ PhT...)
  - $\Omega_{\rm GW} \propto \left(\frac{\eta}{M_{\rm Pl}}\right)^4$
  - Frecuency dependence:
    Super-horizon: $\propto f^3$ | MD: $\propto f^{-2}$ | RD: Scale invatiant
- Scale invariance confirmed by real-time calculations
  - Difficult to achieve in lattice simulations
- Observability: below sensibility of LISA & LIGO
  - Future observatories?

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Large-N comparison

Large-N limit comparison @ RD:

Figueroa, Hindmarsh, Urrestilla 2013:

$$\frac{\Omega_{\rm GW}^{\rm Sim}}{\Omega_{\rm GW}^{\rm Analytic}} = \begin{cases} 130 & N=2 \\ 7.3 & N=3 \\ 3.9 & N=4 \\ \ldots \end{cases}$$

New work:

$$\frac{\Omega_{\rm GW}^{\rm Sim}}{\Omega_{\rm GW}^{\rm Analytic}} = \begin{cases} 324.1 & N=2 \\ 9.5 & N=3 \\ 2.9 & N=4 \\ \ldots \end{cases}$$

AH infinite strings?

Subdominant compared to Loops:

$$\Omega_{\rm GW}^{\rm Loops} \propto \left(\frac{\eta}{M_{\rm Pl}}\right)^{2} \quad \text{vs} \quad \Omega_{\rm GW}^{\rm Net} \propto \left(\frac{\eta}{M_{\rm Pl}}\right)^{4}$$