

# *Dark Matter - Phonon Scattering*

@ GGI  
09/25/2019

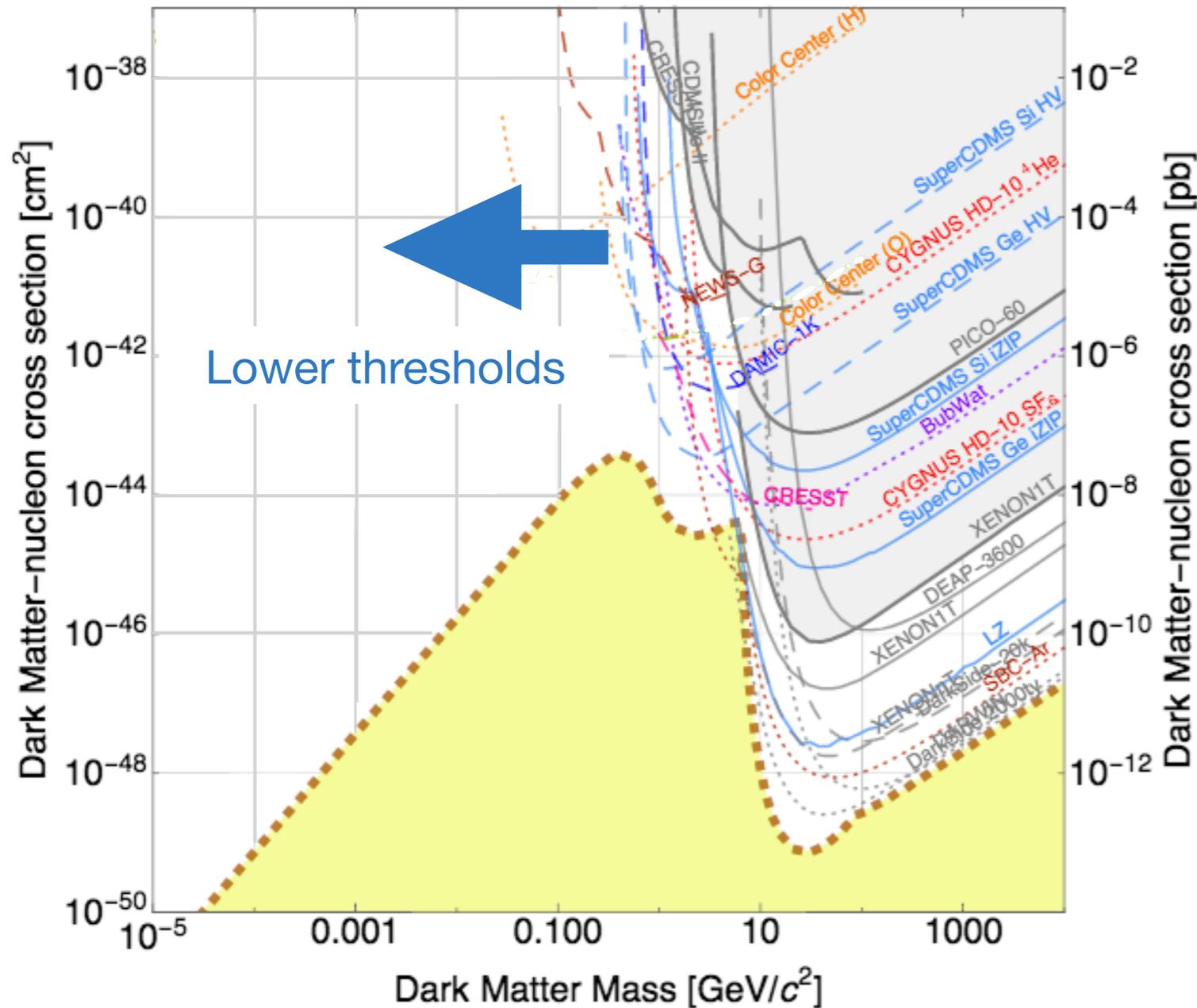
Simon Knapen  
Institute for Advanced Study (IAS)

work with B. Campbell-Deem, P. Cox, S. Griffin, T. Lin, T. Melia,  
M. Pyle & K. Zurek



# Low mass dark matter detection

What do we need?

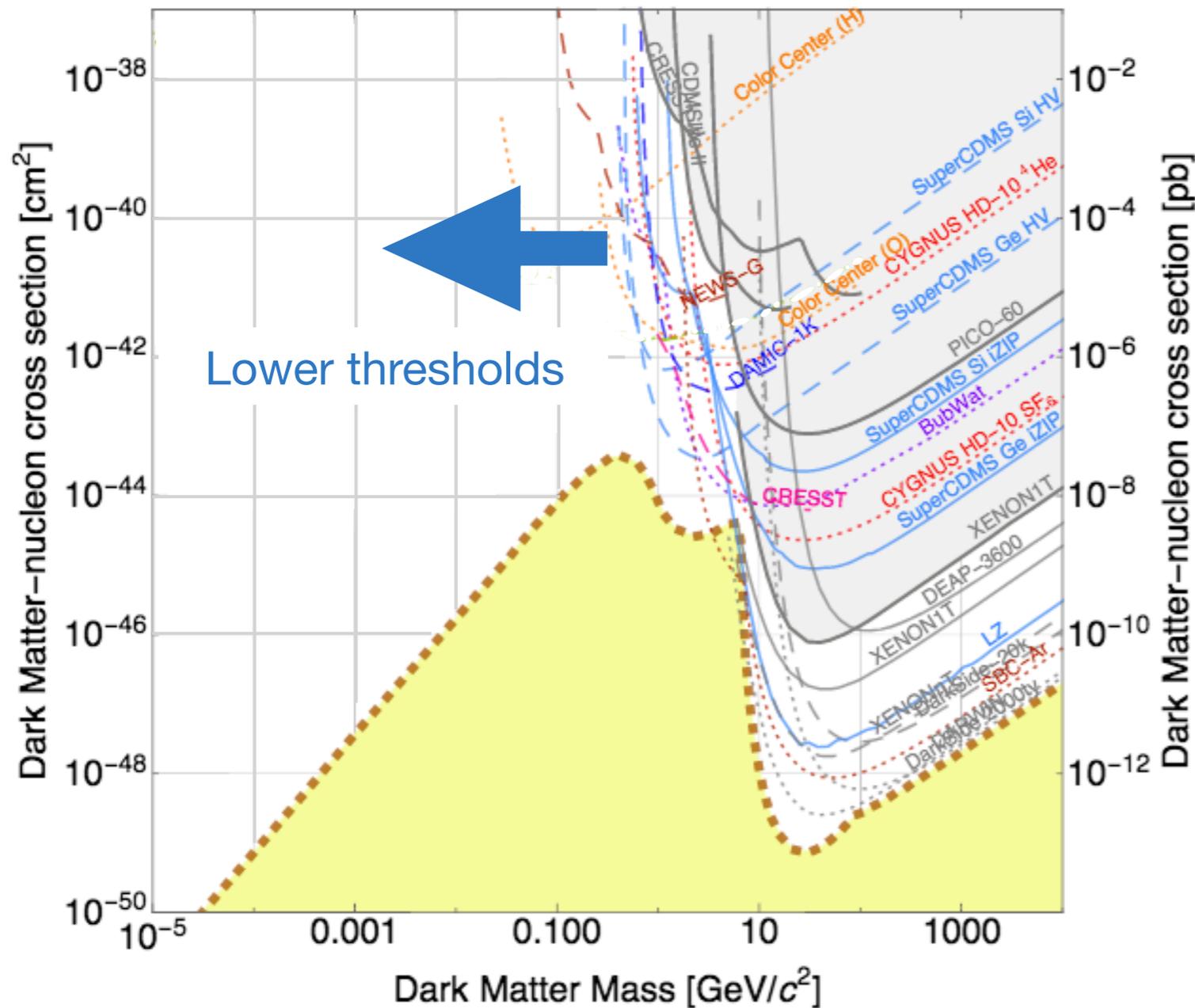


Experiment:

1. Low target mass materials:  
e<sup>-</sup>, He, ...
2. Ultra-sensitive calorimeters / CCD's  
with low dark counts

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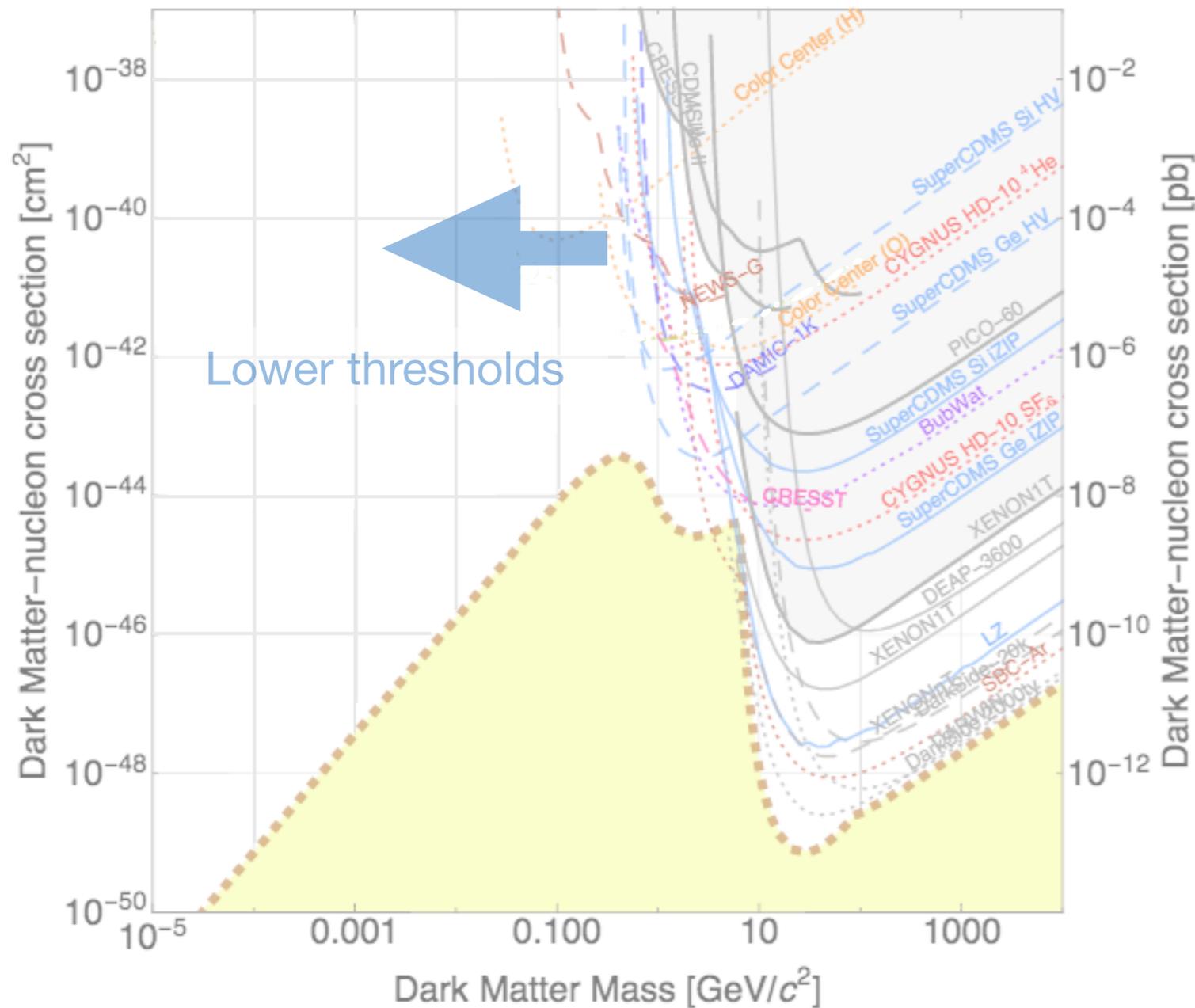
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Theory:

1. The **mediator** is important,  
independent set of constraints  
SK, T. Lin, K. Zurek: 1709.07882
2. Beyond “billiard ball” scattering:  
**structure effects** are critical!

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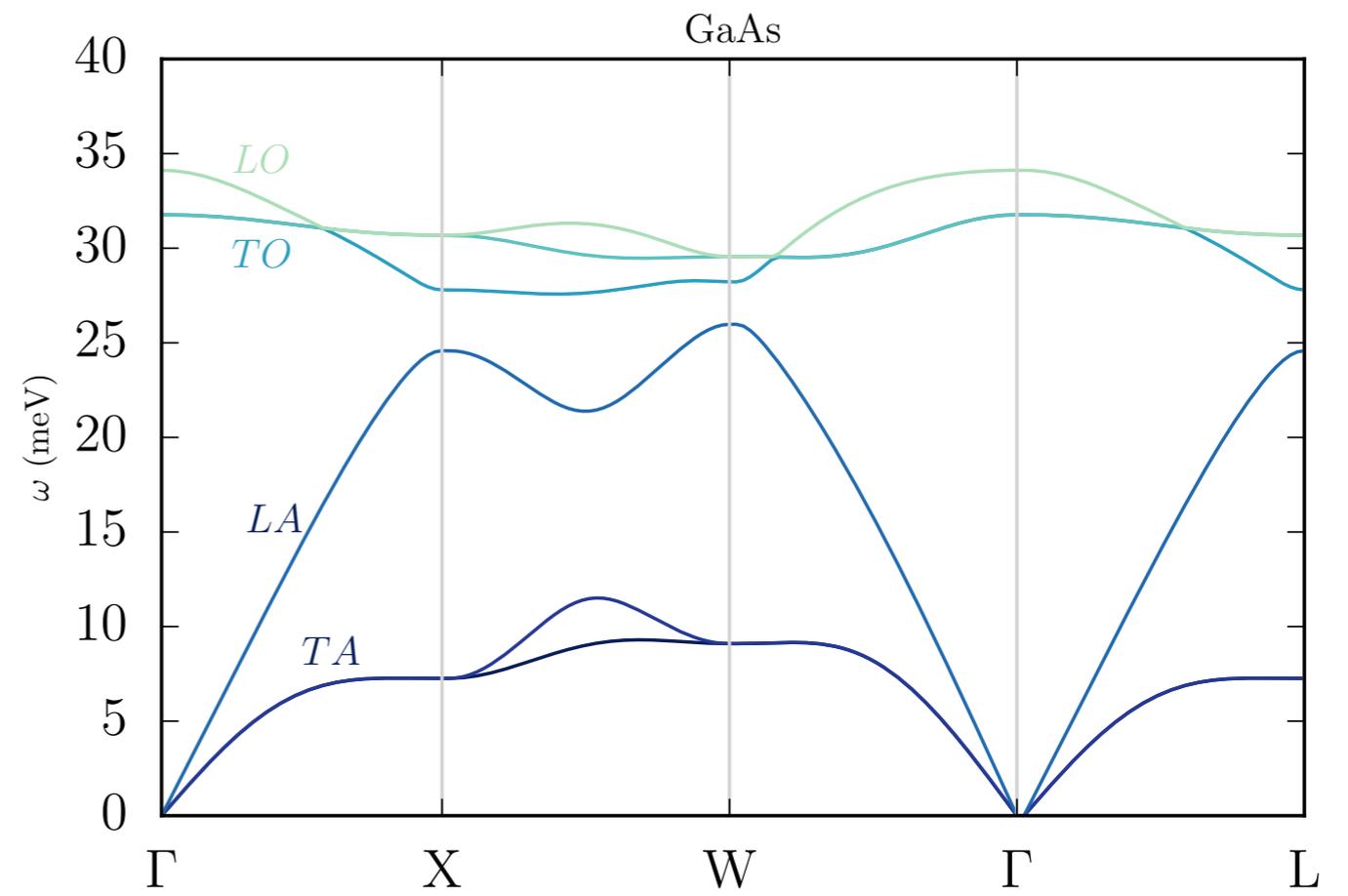
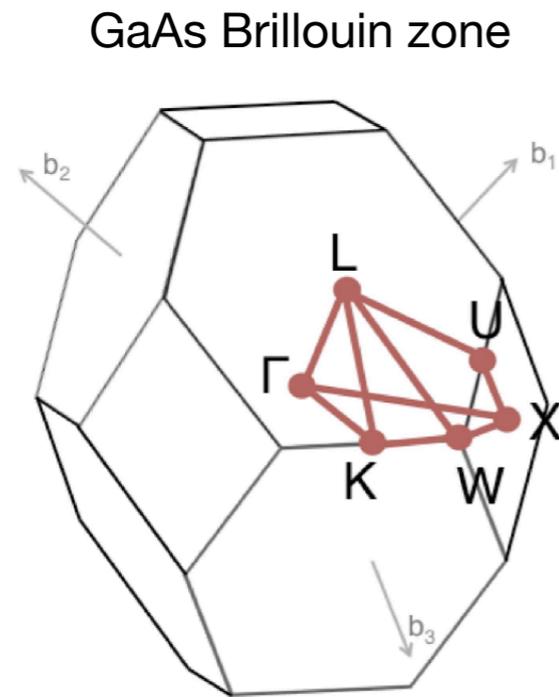
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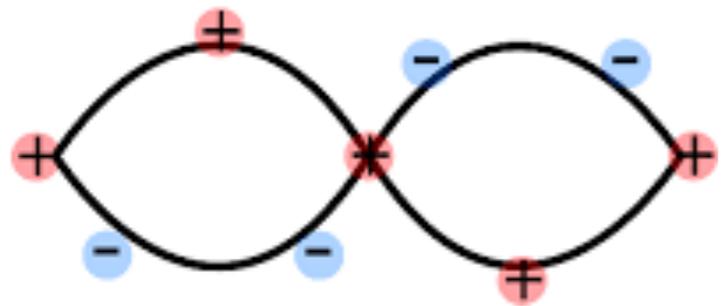
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# Types of phonons

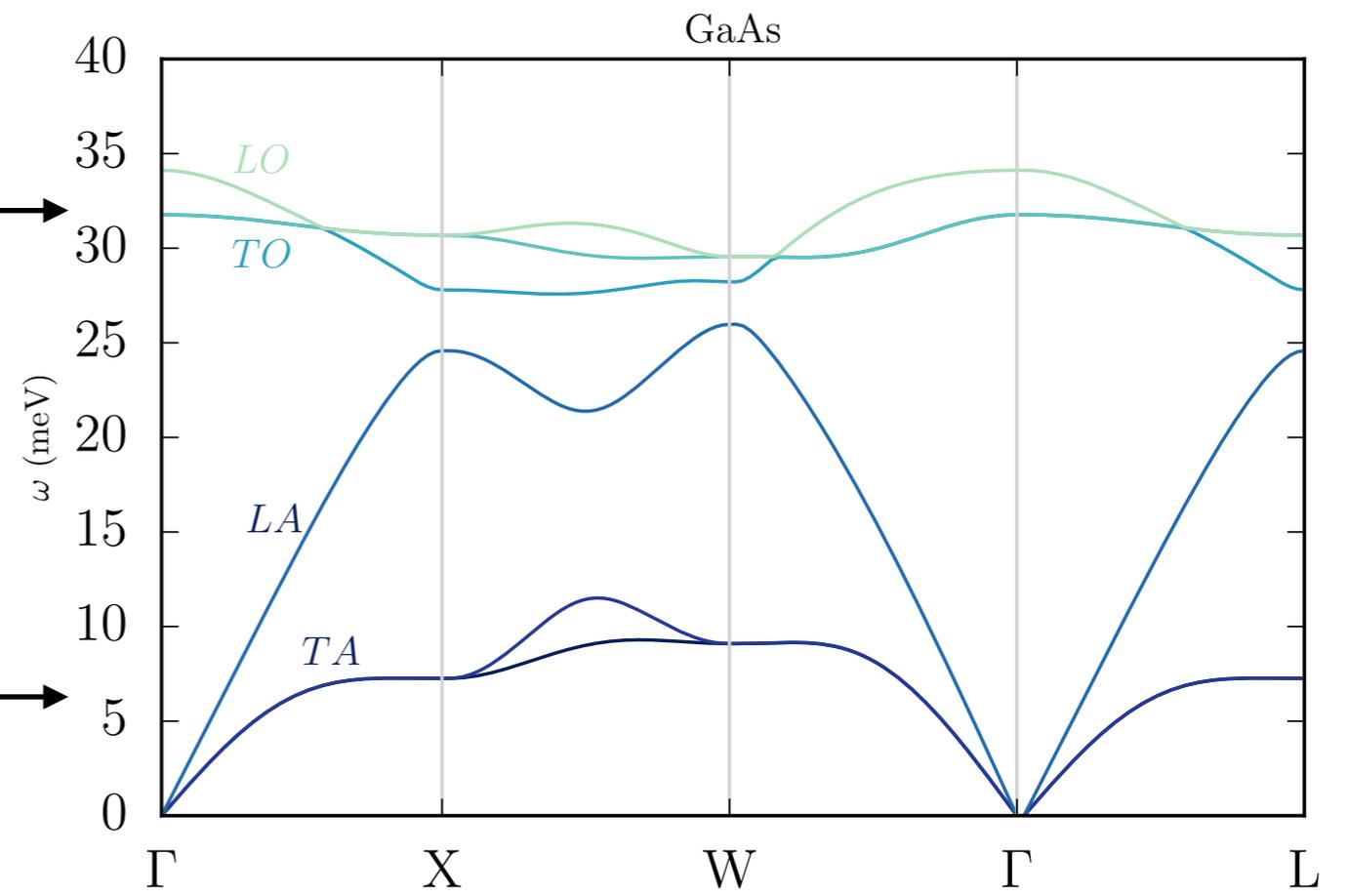
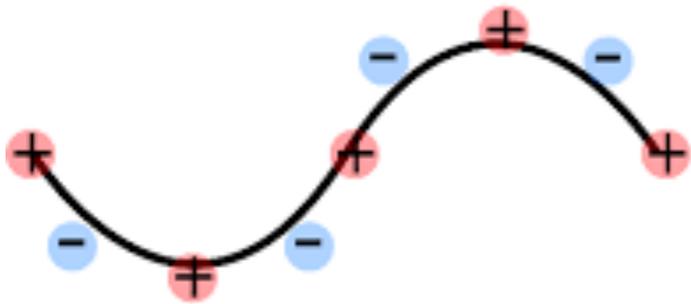


# Types of phonons

Optical phonons

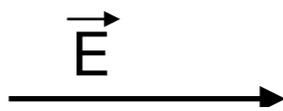
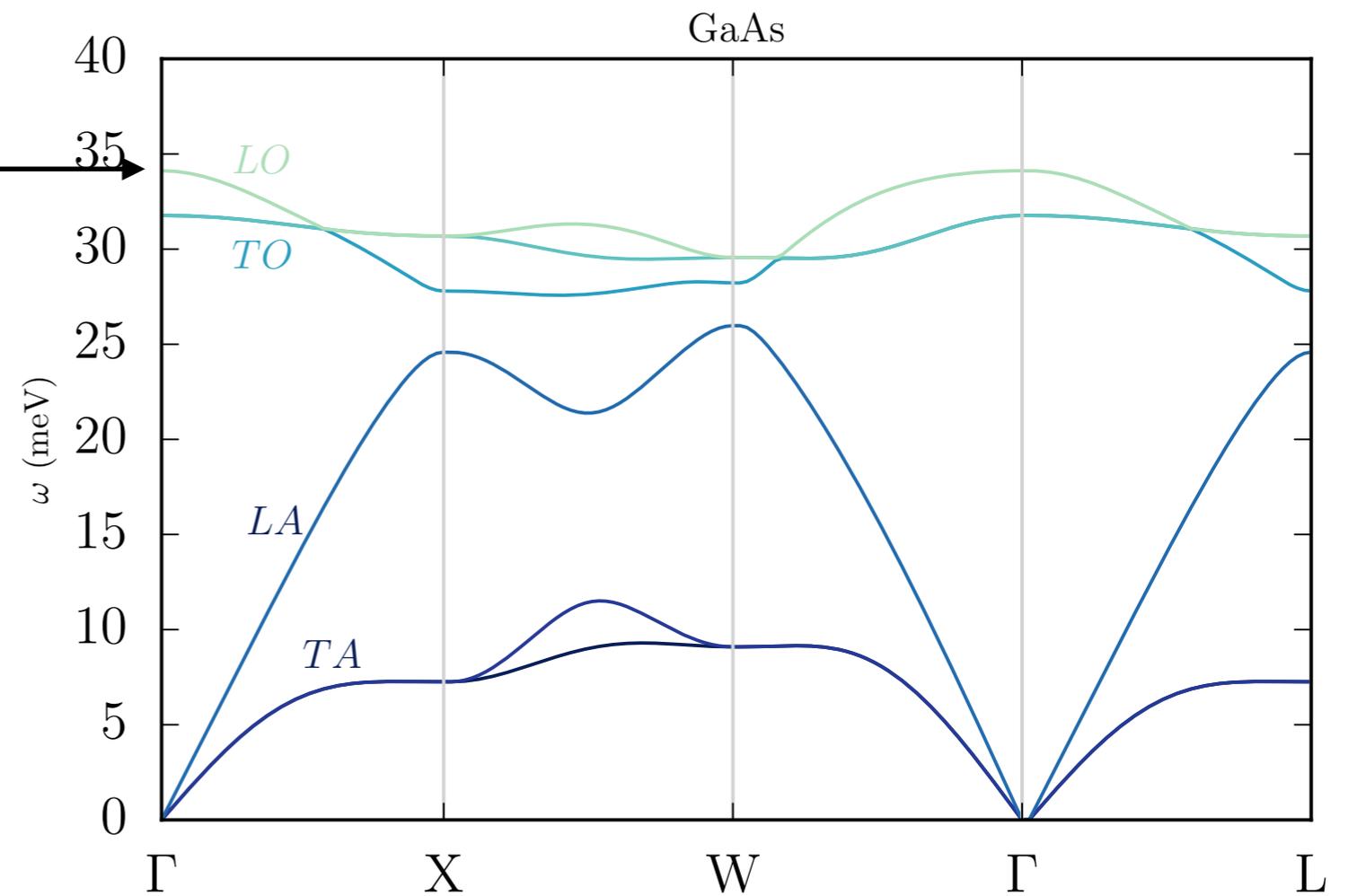
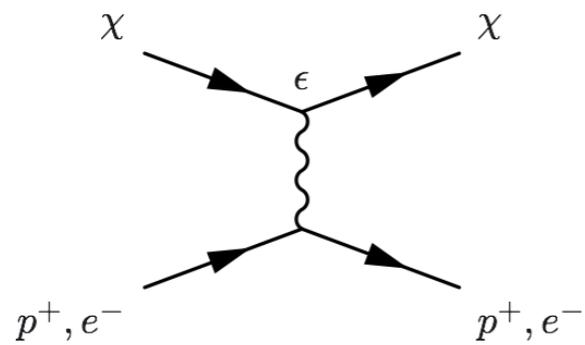


Acoustic phonons



# Dark Matter coupling to phonons

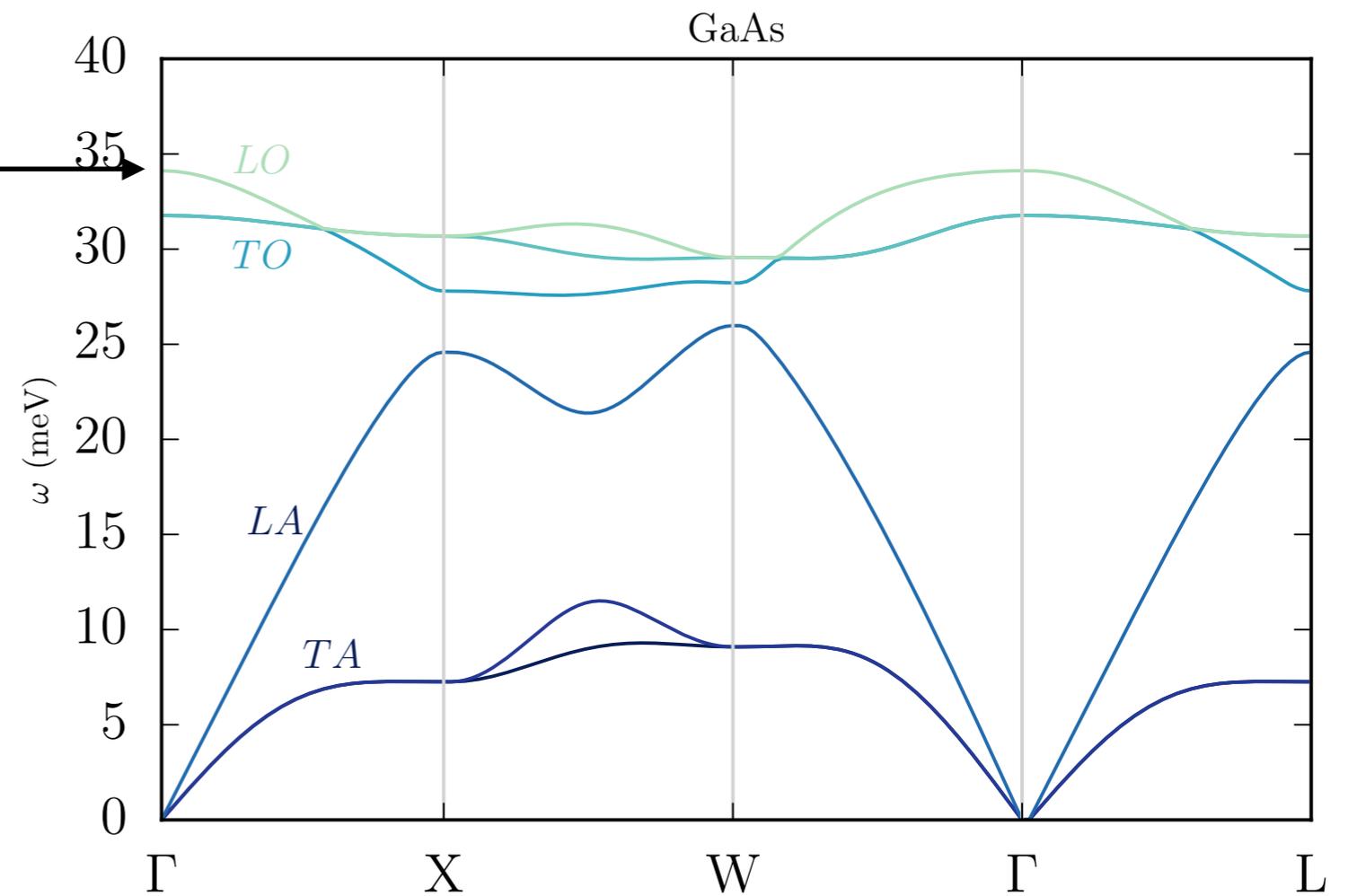
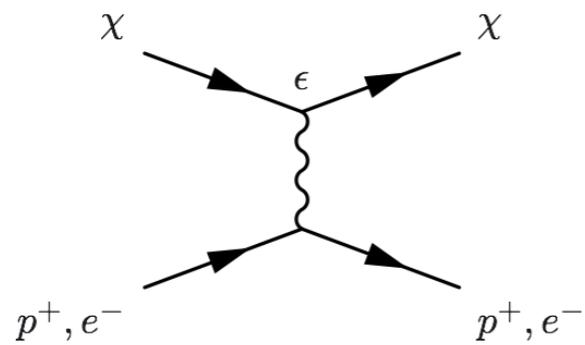
Coupling  
to charge



(in a "Polar" material)

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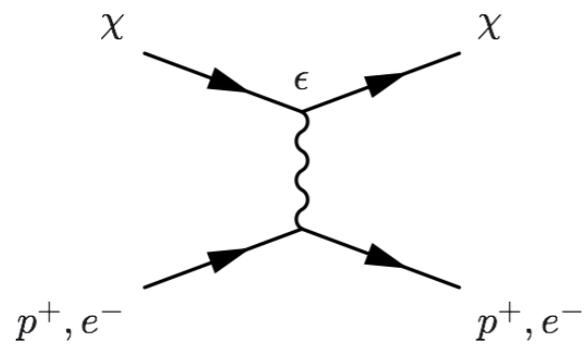
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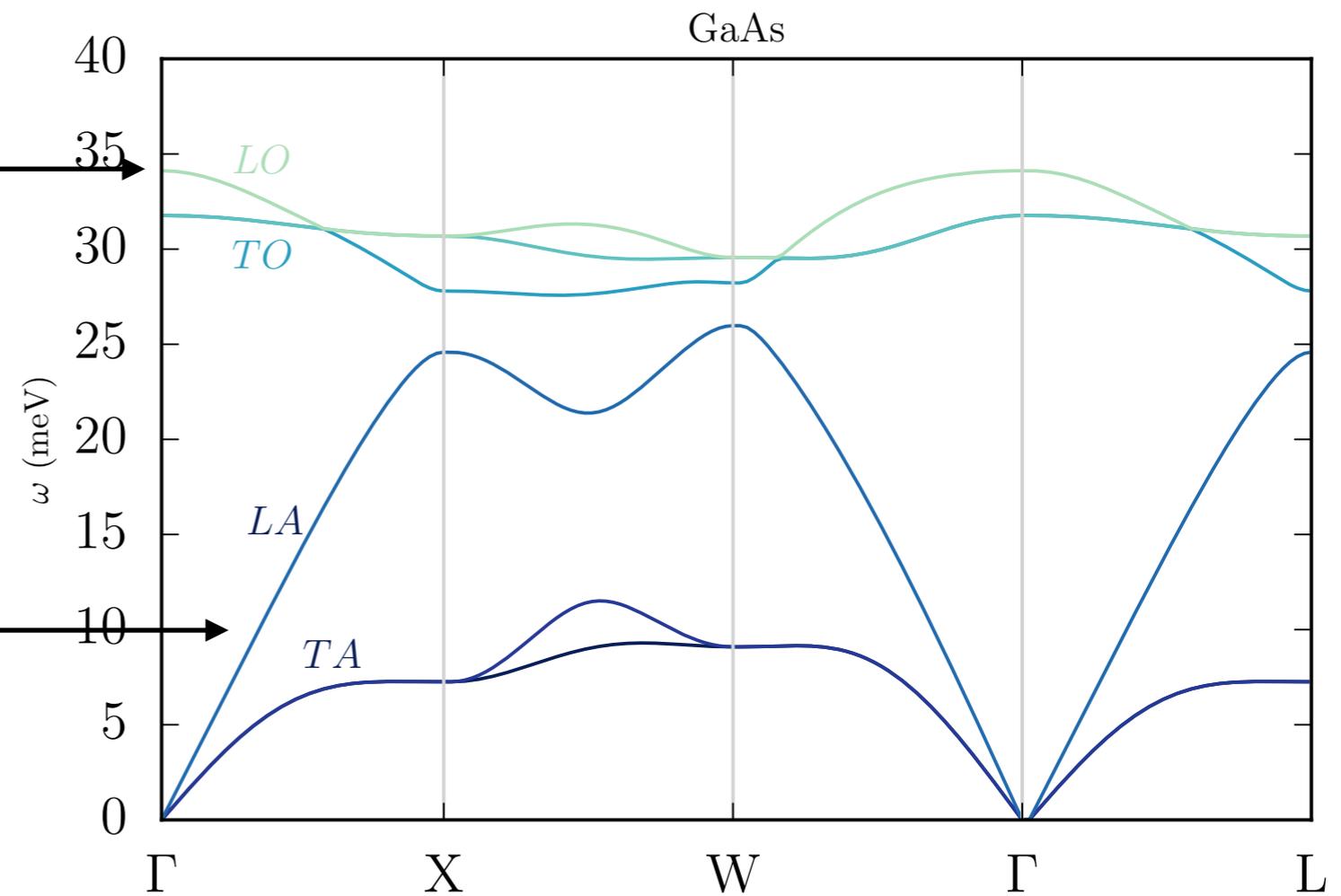
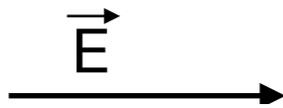
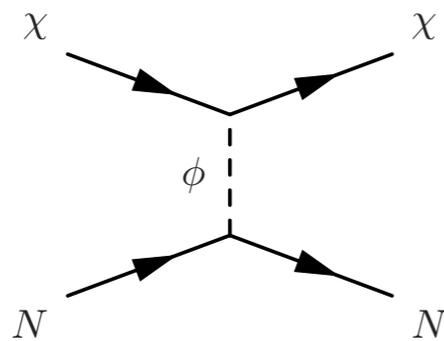
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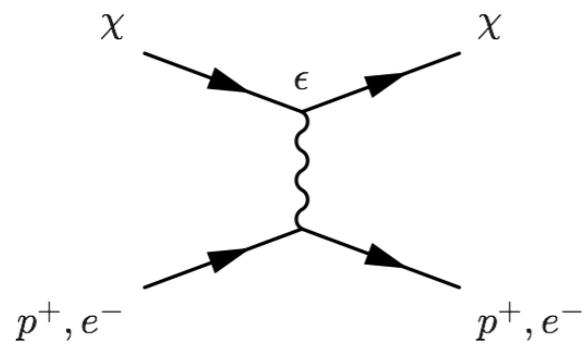
Coupling  
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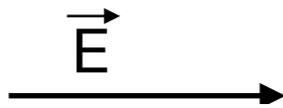
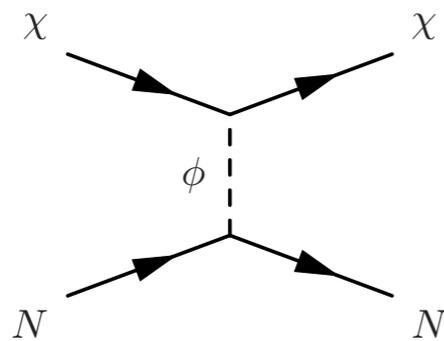
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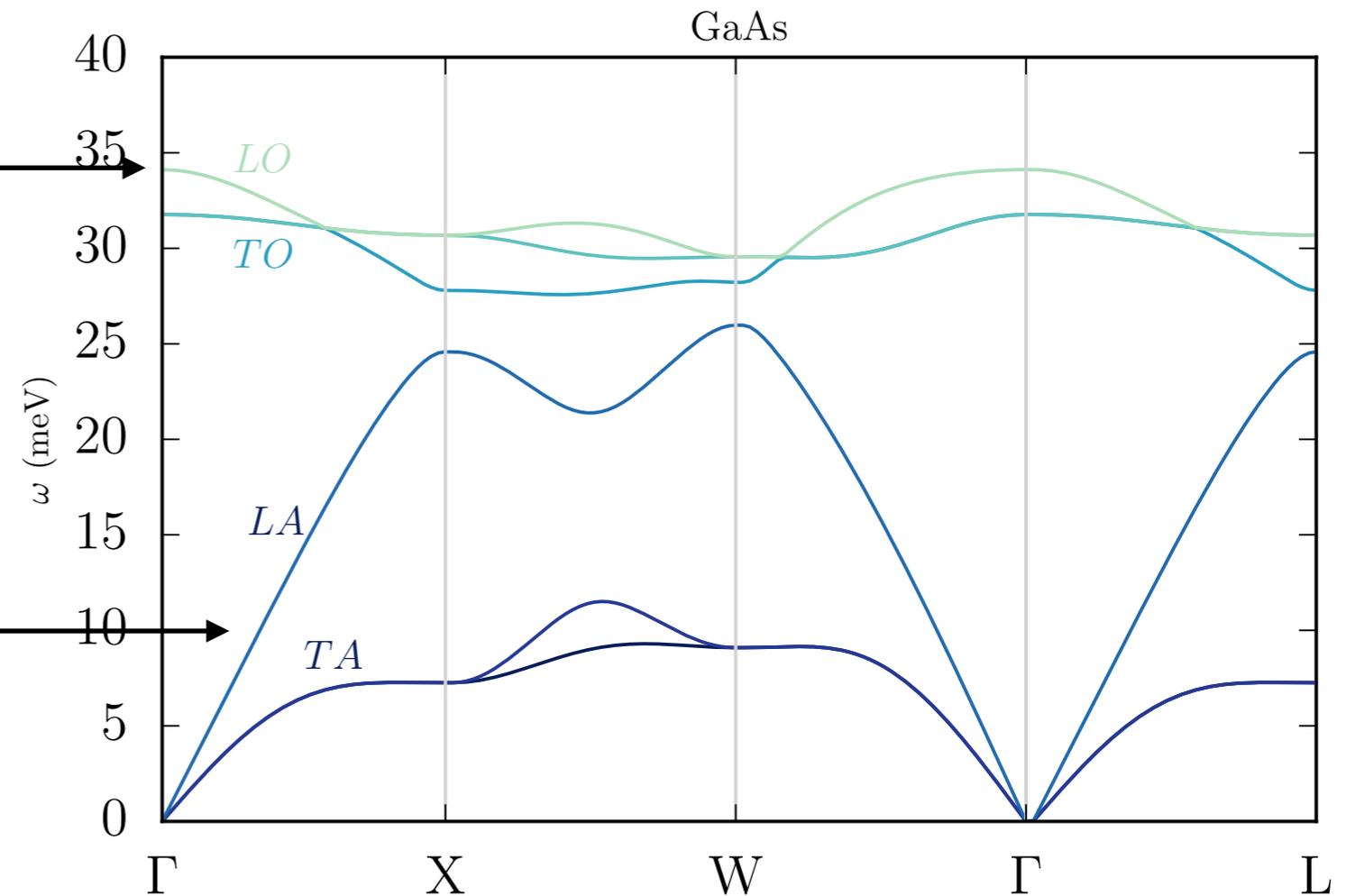
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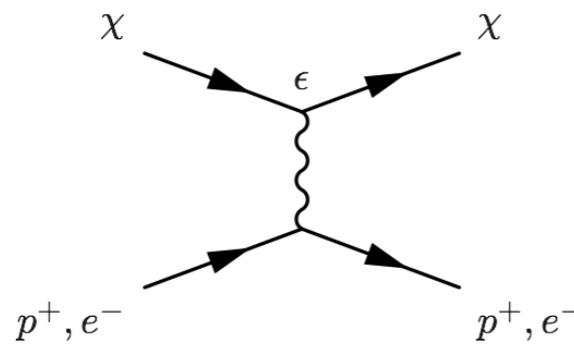
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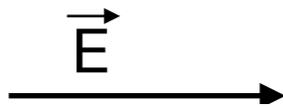
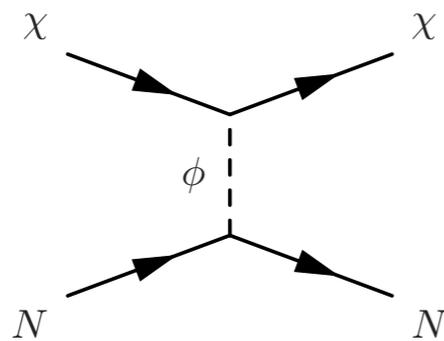
DM-phonon coupling depends strongly on underlying UV physics

# Dark Matter coupling to phonons

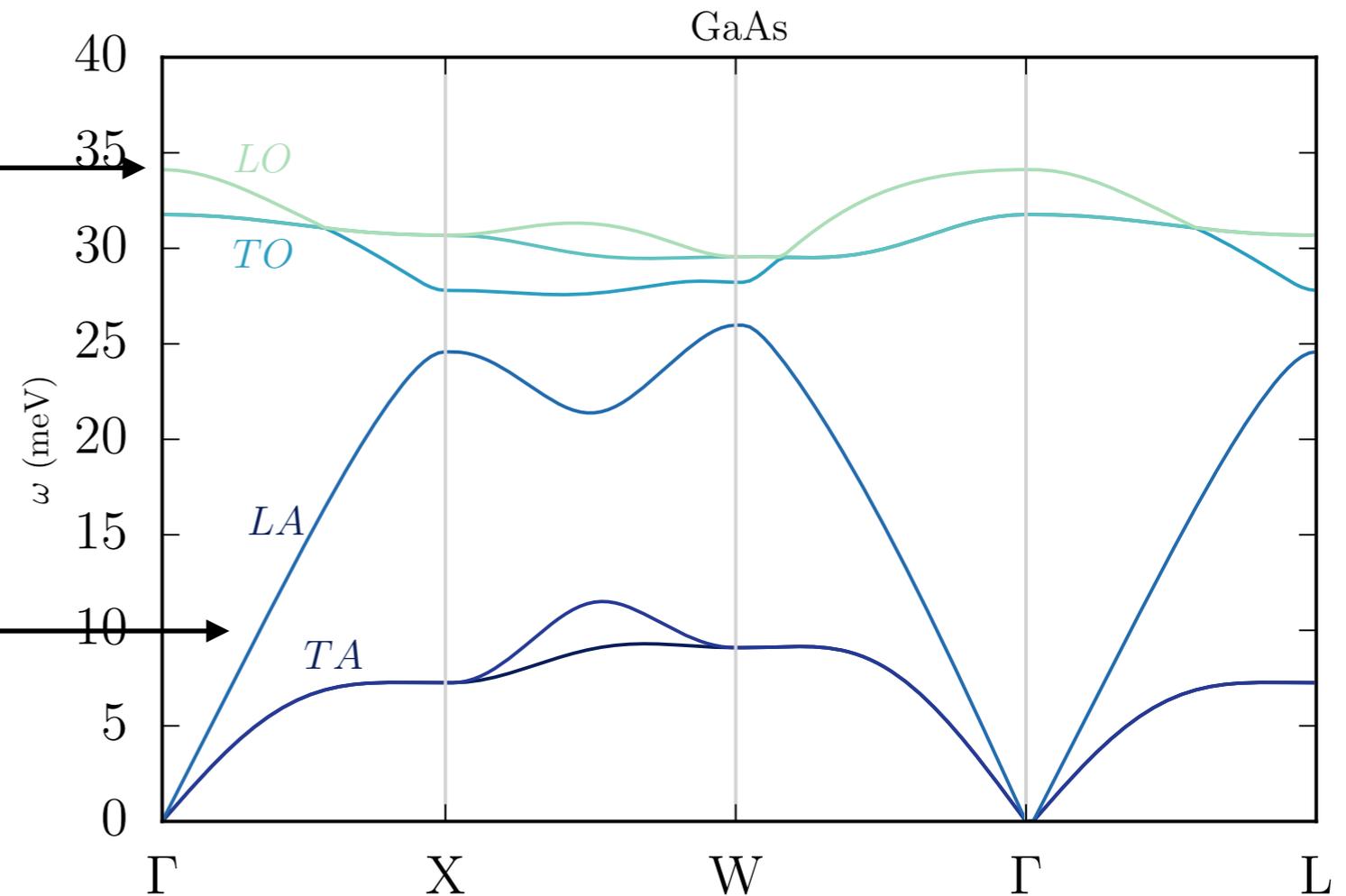
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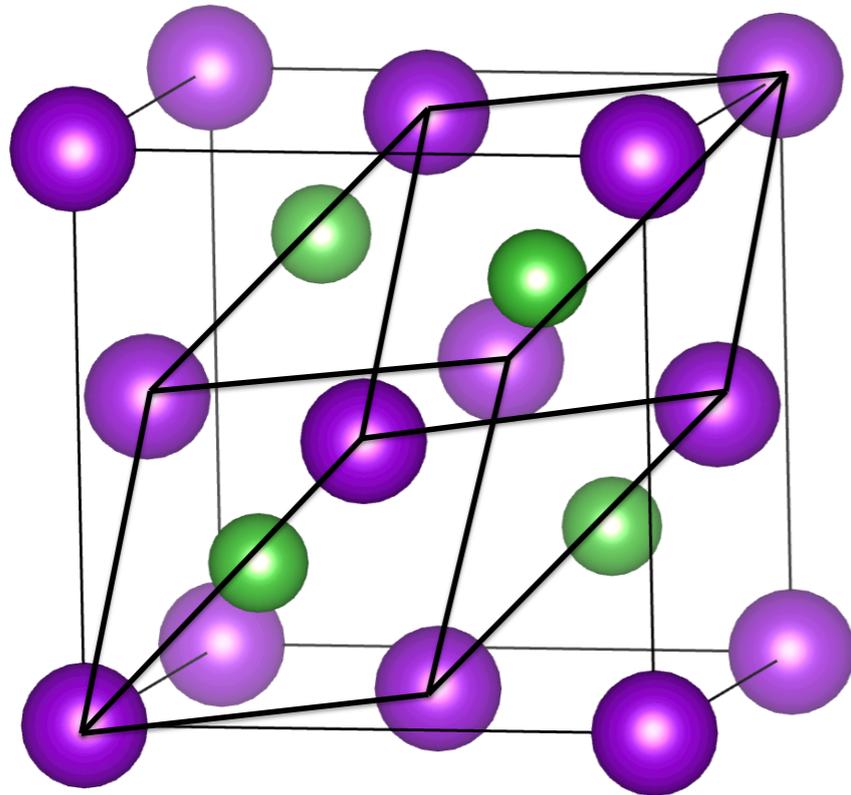
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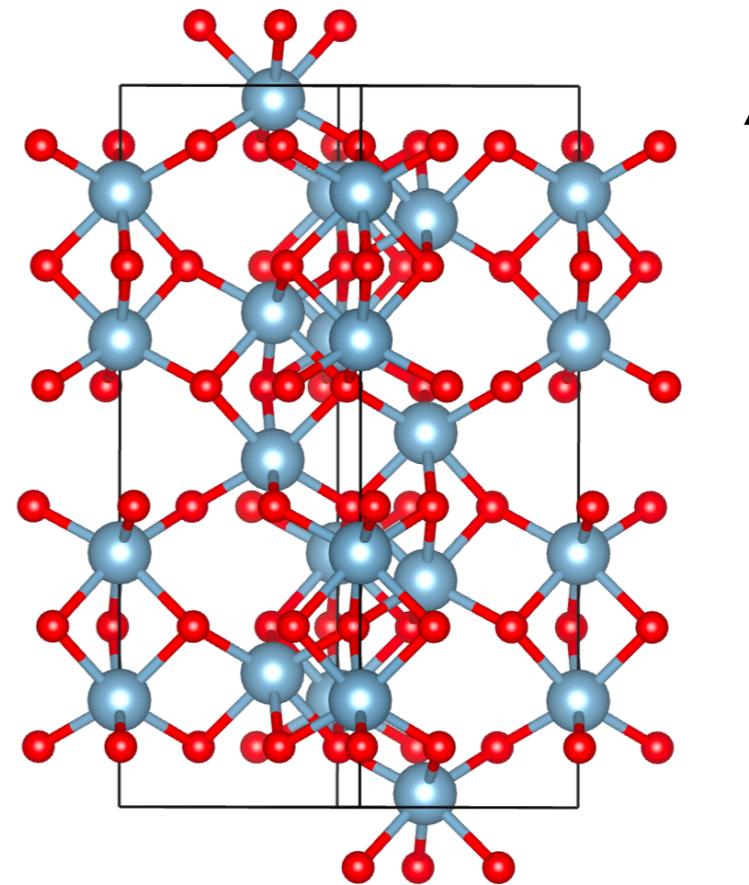
# Examples of Polar Materials

GaAs



2 atoms in primitive cell

Al<sub>2</sub>O<sub>3</sub> (Sapphire)



Primary  
crystal axis

10 atoms in primitive cell

At least two *different* atoms in the unit cell

# Frölich Hamiltonian

H. Frölich, 1954

C. Verdi, F. Giustino, Phys. Rev. Lett. 115, 176401 (2015)

Electric dipole interacting with test charge:

$$H \sim i e \sum_{\mathbf{q}} \frac{\mathbf{q} \cdot \mathbf{P}}{|\mathbf{q}|^2} e^{i\mathbf{q} \cdot \mathbf{r}}$$

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$$H = i \frac{\kappa e^2}{V} \sum_{j, \nu, \mathbf{q}} \sum_{\mathbf{G} \neq \mathbf{q}} \frac{1}{\sqrt{2Nm_j \omega_{\nu, \mathbf{q}}}} \frac{(\mathbf{q} + \mathbf{G}) \cdot \mathbf{Z}_j \cdot \mathbf{e}_{j, \nu}(\mathbf{q})}{(\mathbf{q} + \mathbf{G}) \cdot \epsilon_{\infty} \cdot (\mathbf{q} + \mathbf{G})} e^{i(\mathbf{q} + \mathbf{G}) \cdot (\mathbf{r} + \boldsymbol{\tau}_j)}$$

Born effective charge tensor for each atom

phonon eigenvectors (atomic displacements)

phonon energy

high frequency dielectric tensor

Sum over:

- $j$  atoms in unit cell
- $\nu$  phonon modes
- $\mathbf{q}$  1<sup>st</sup> Brillouin zone
- $\mathbf{G}$  reciprocal lattice

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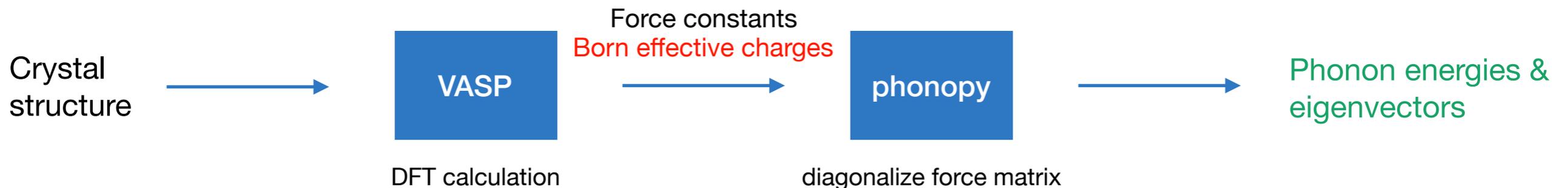
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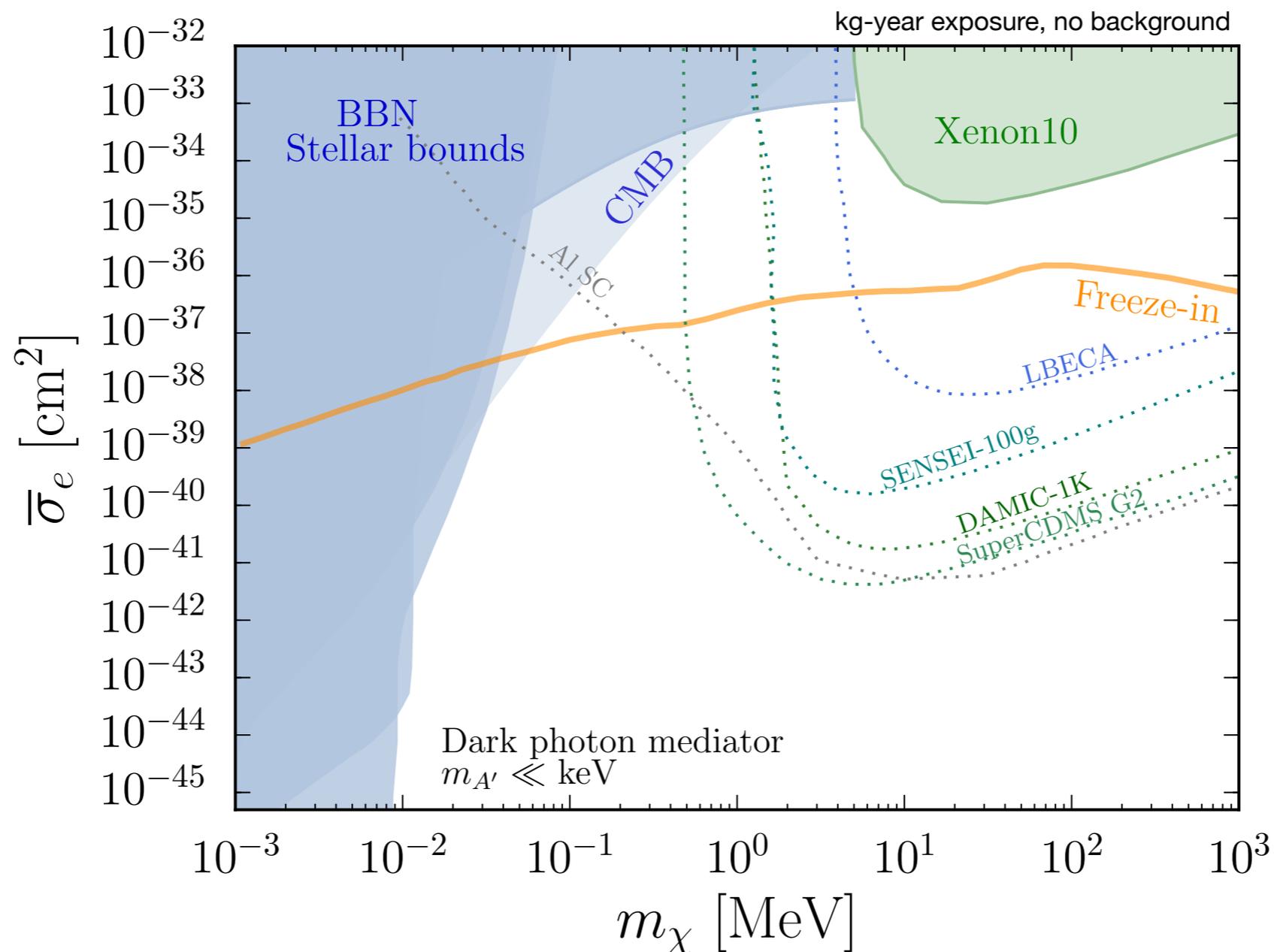
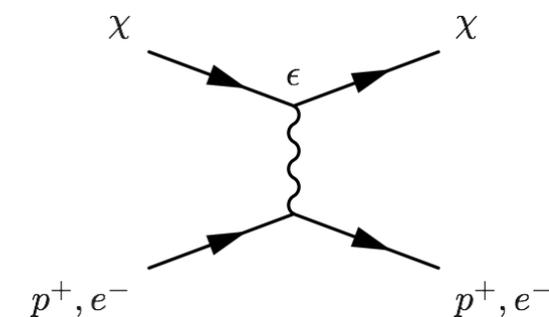
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Calculation overview:



# Reach

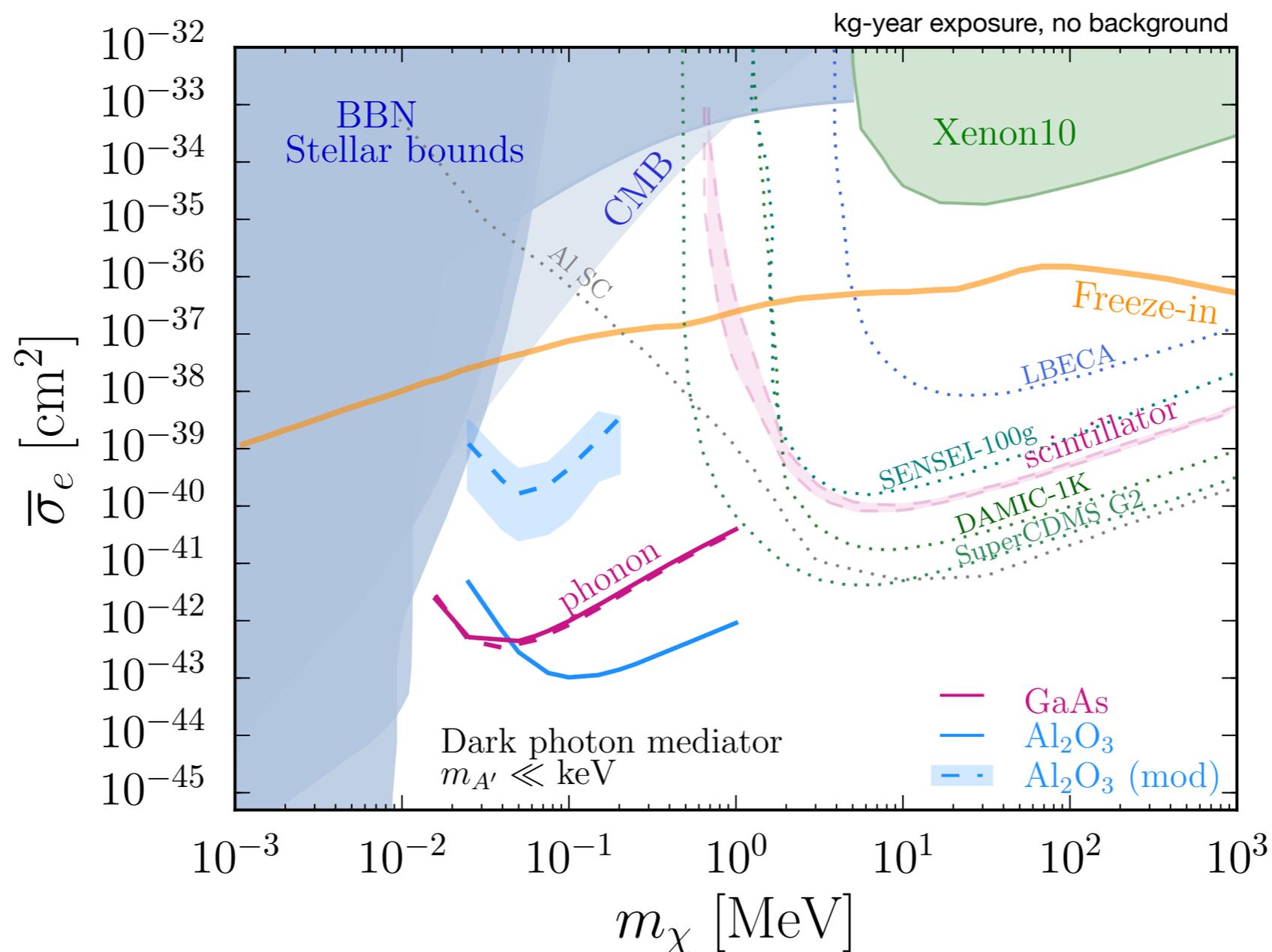
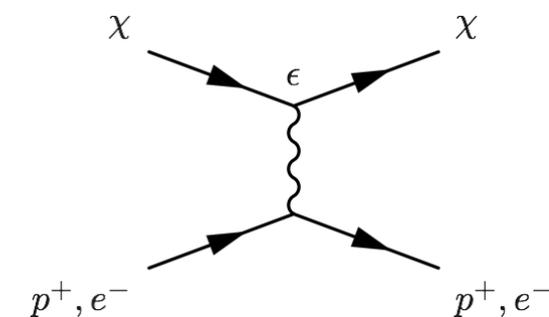
Both GaAs and Sapphire probe Dark Matter masses as low as 10 keV



Probe the new parameter space with milligram-day exposure

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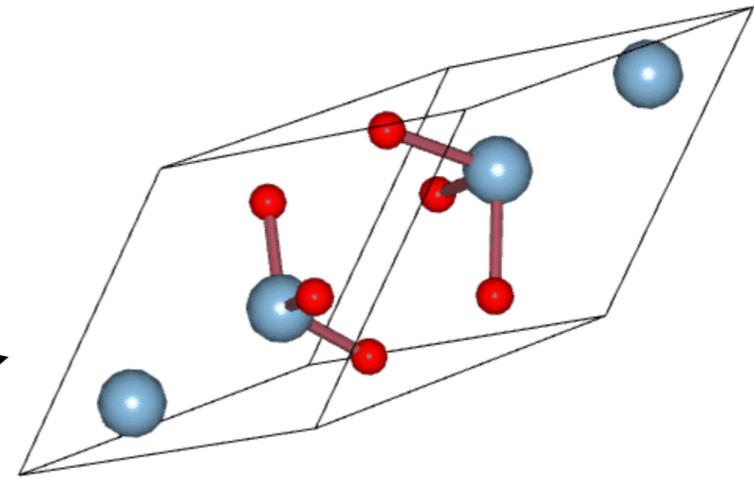
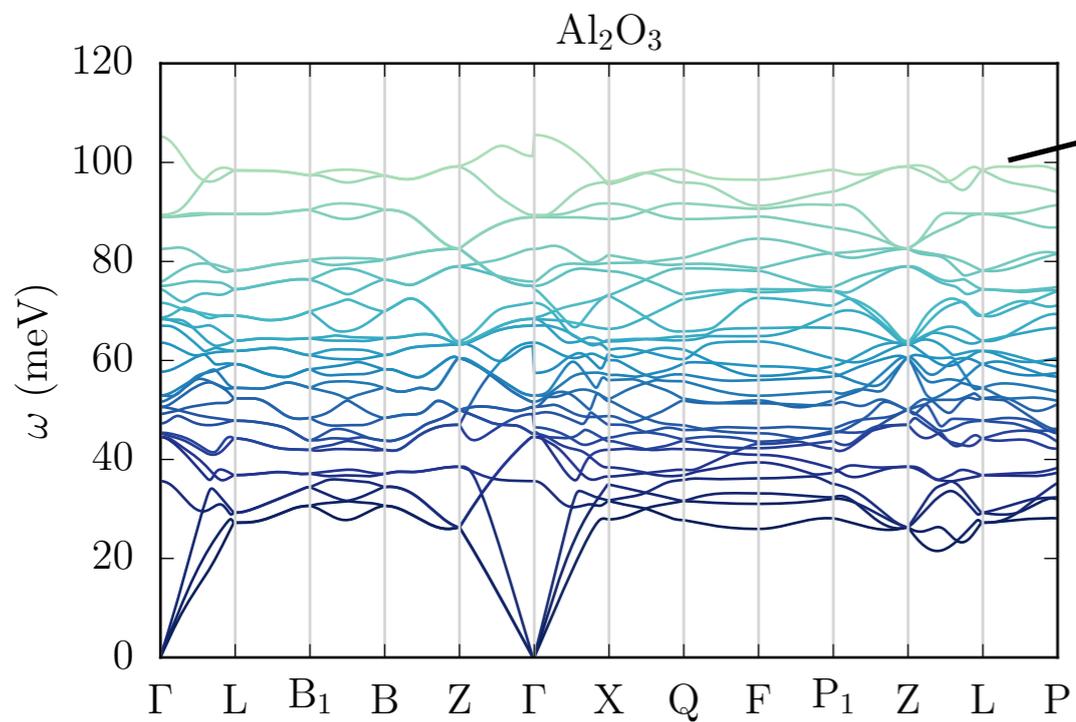
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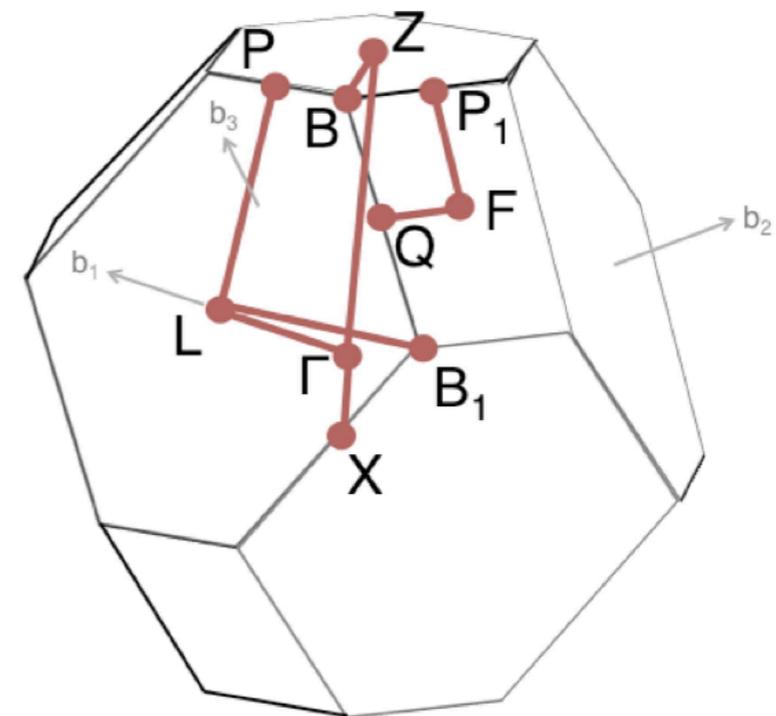
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# Sapphire in more detail

Most energetic mode dominates



Al<sub>2</sub>O<sub>3</sub> Brillouin zone



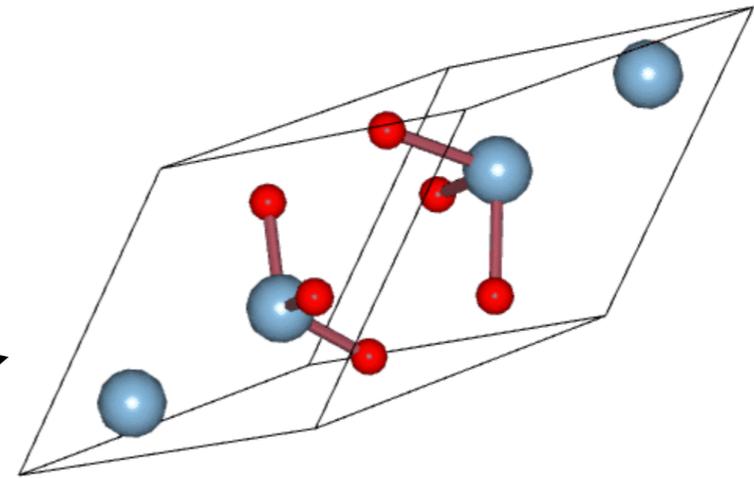
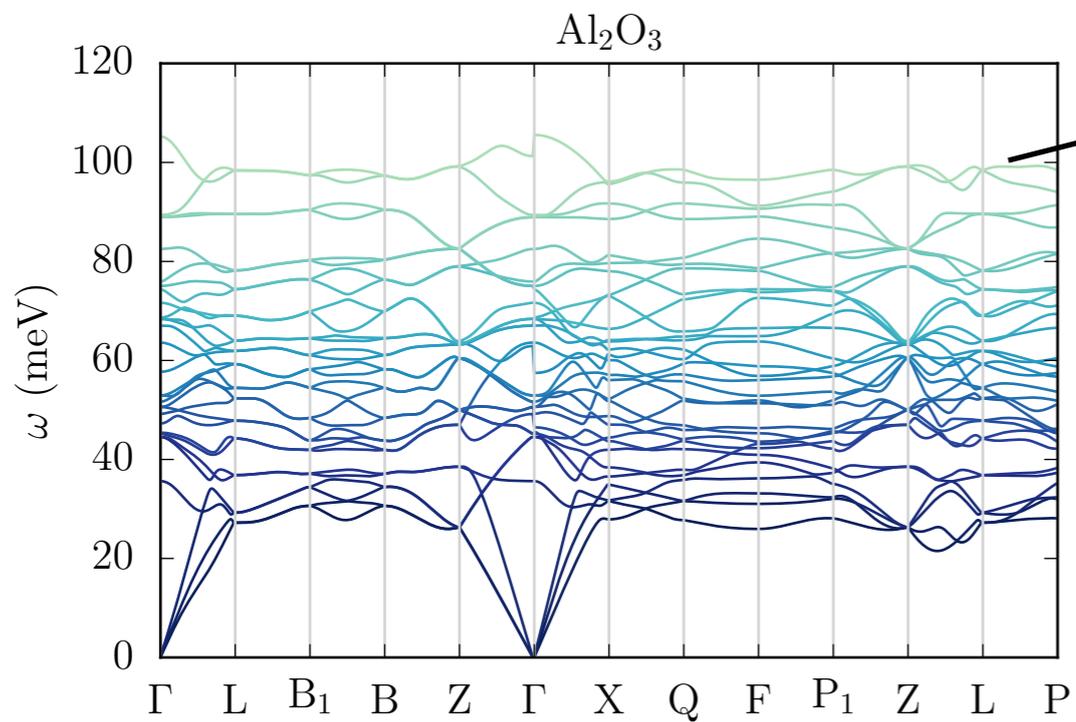
Aluminum atoms move in phase



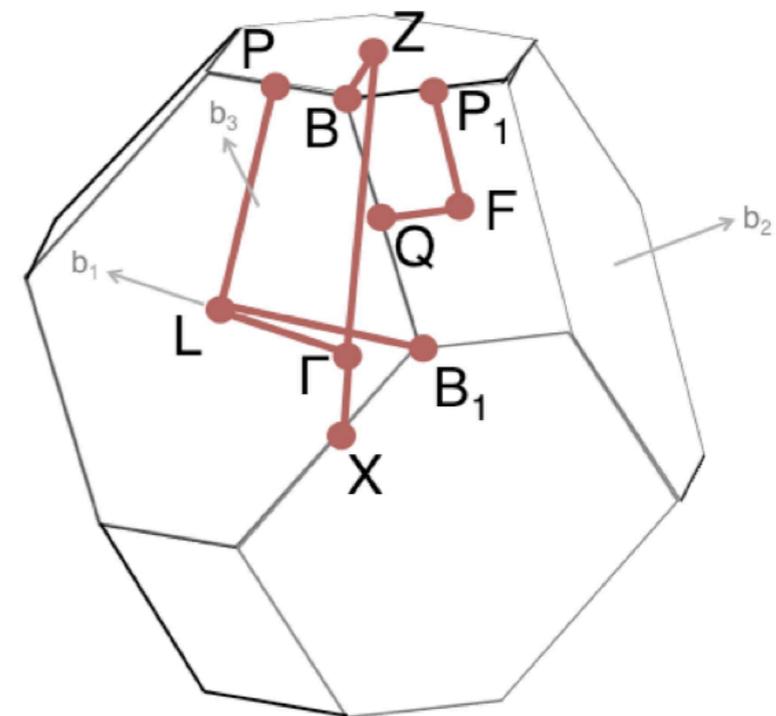
Large dipole

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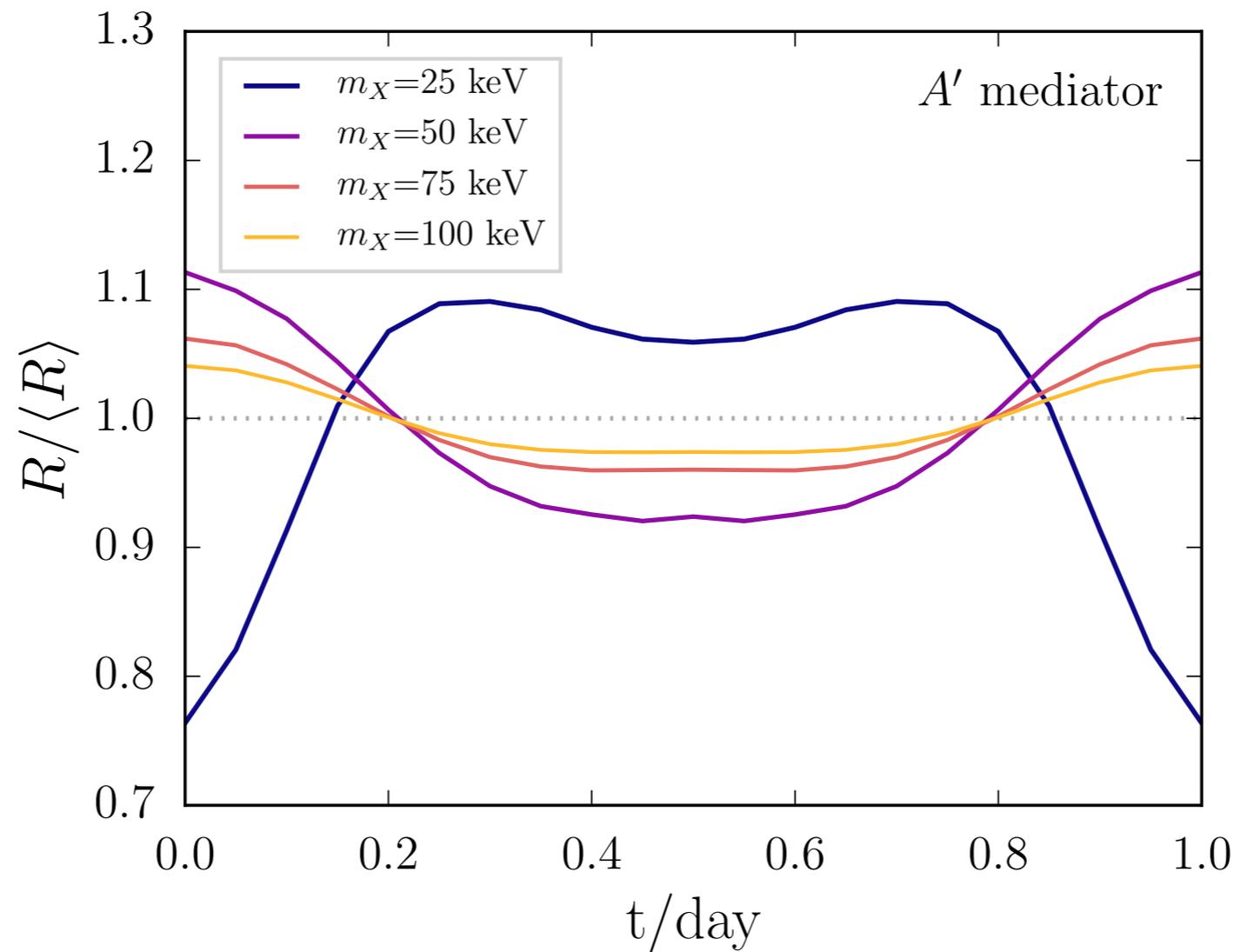
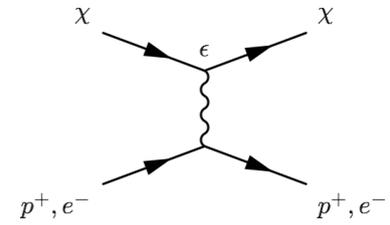


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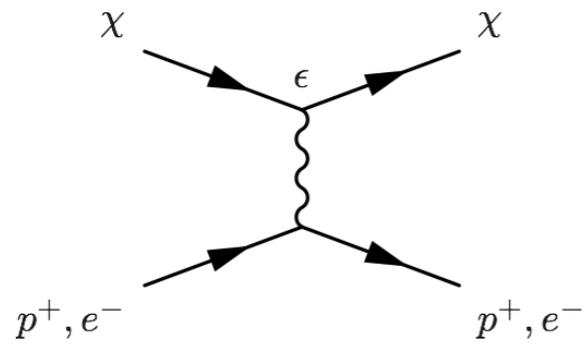
# Daily modulation (dark photon mediator)



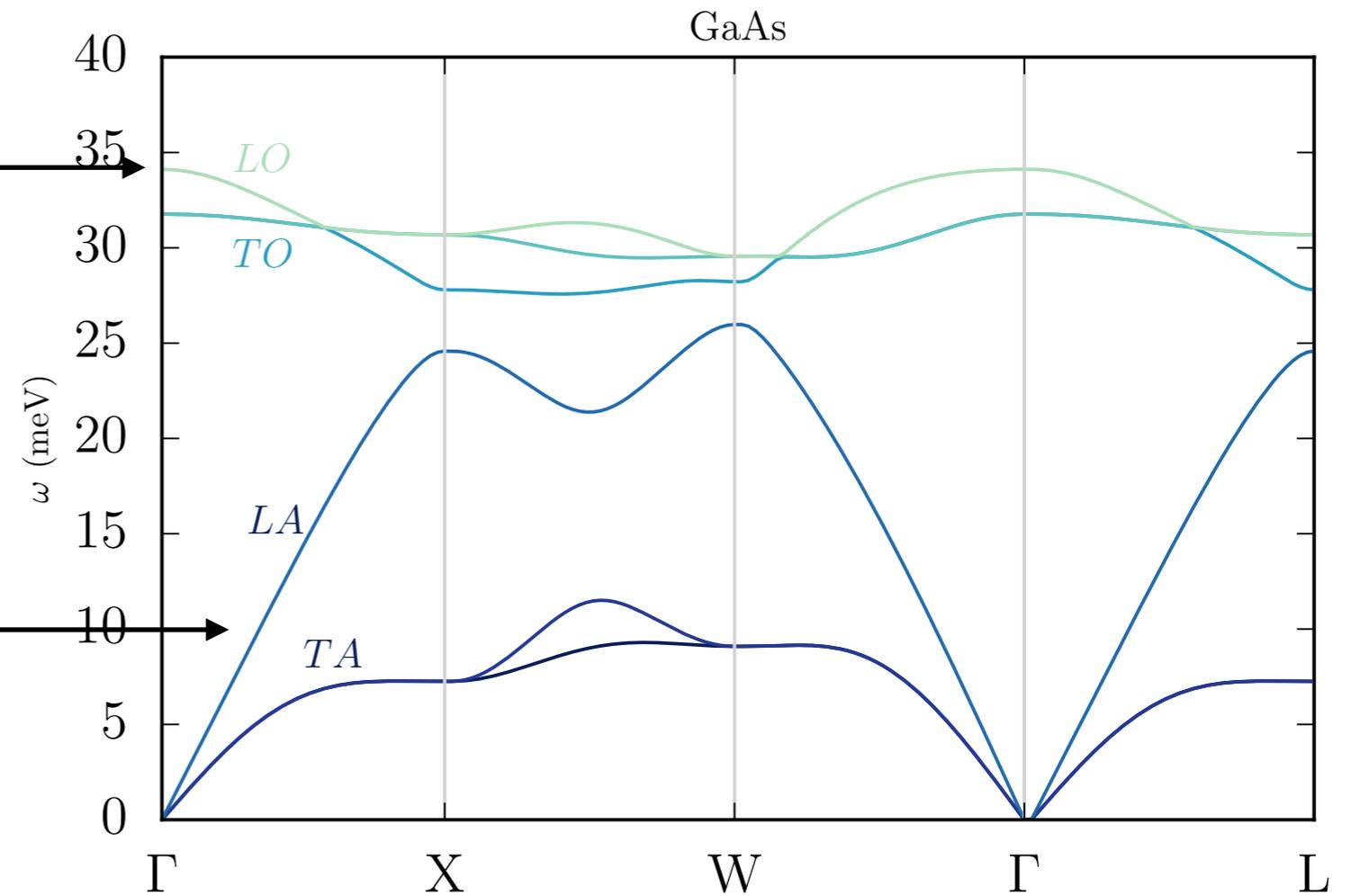
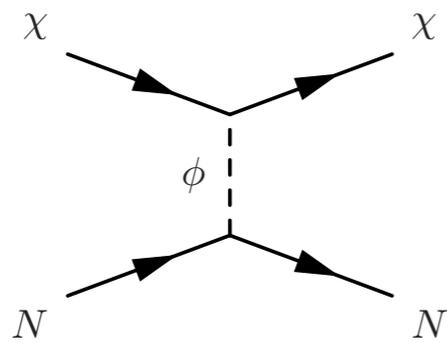
Amplitude and pattern depends on DM mass

# Dark Matter coupling to phonons

Coupling  
to charge

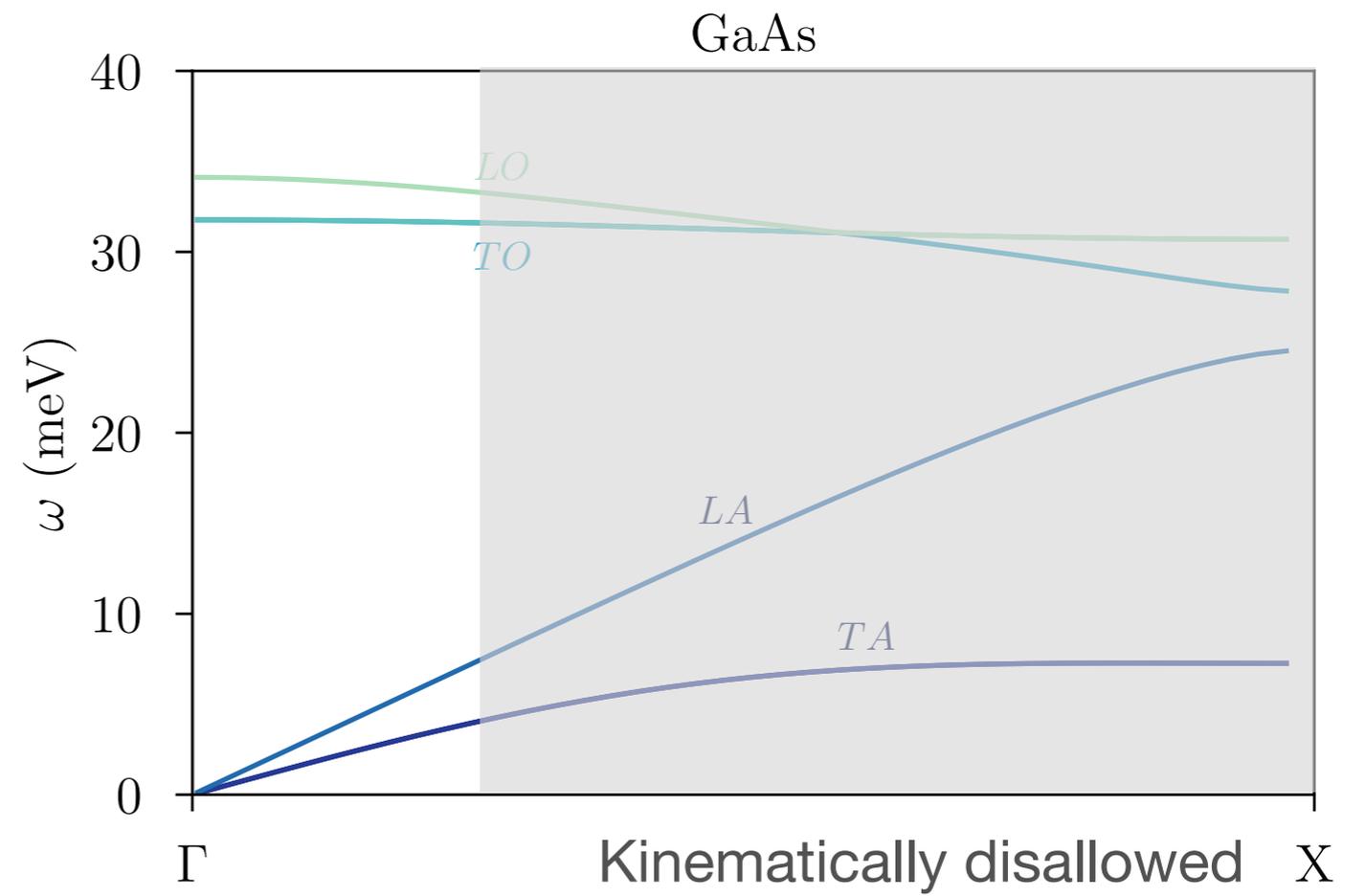
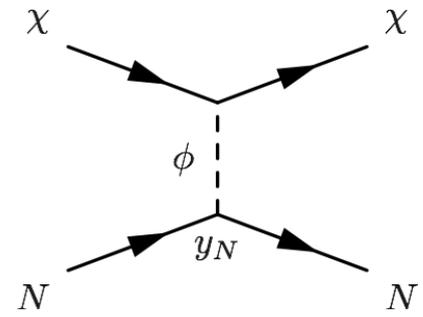


Coupling  
to mass



# Kinematics

Which modes to use?

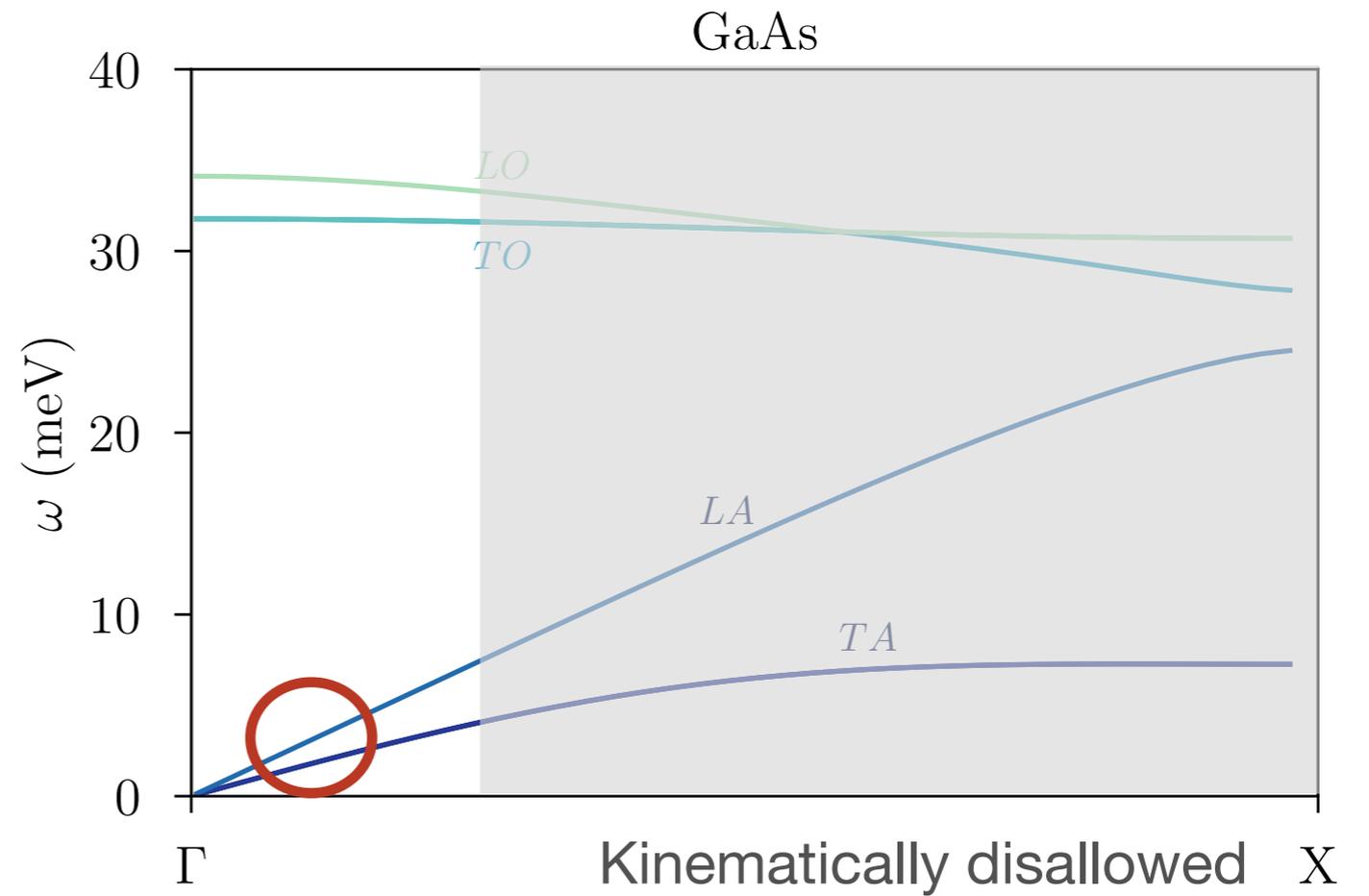
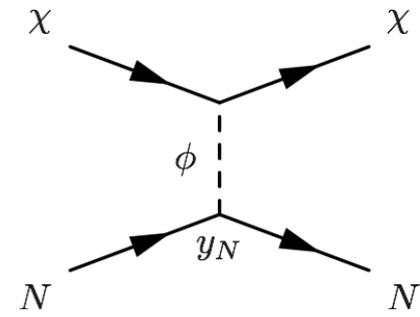


# Kinematics

Which modes to use?

1. **Single acoustic:**

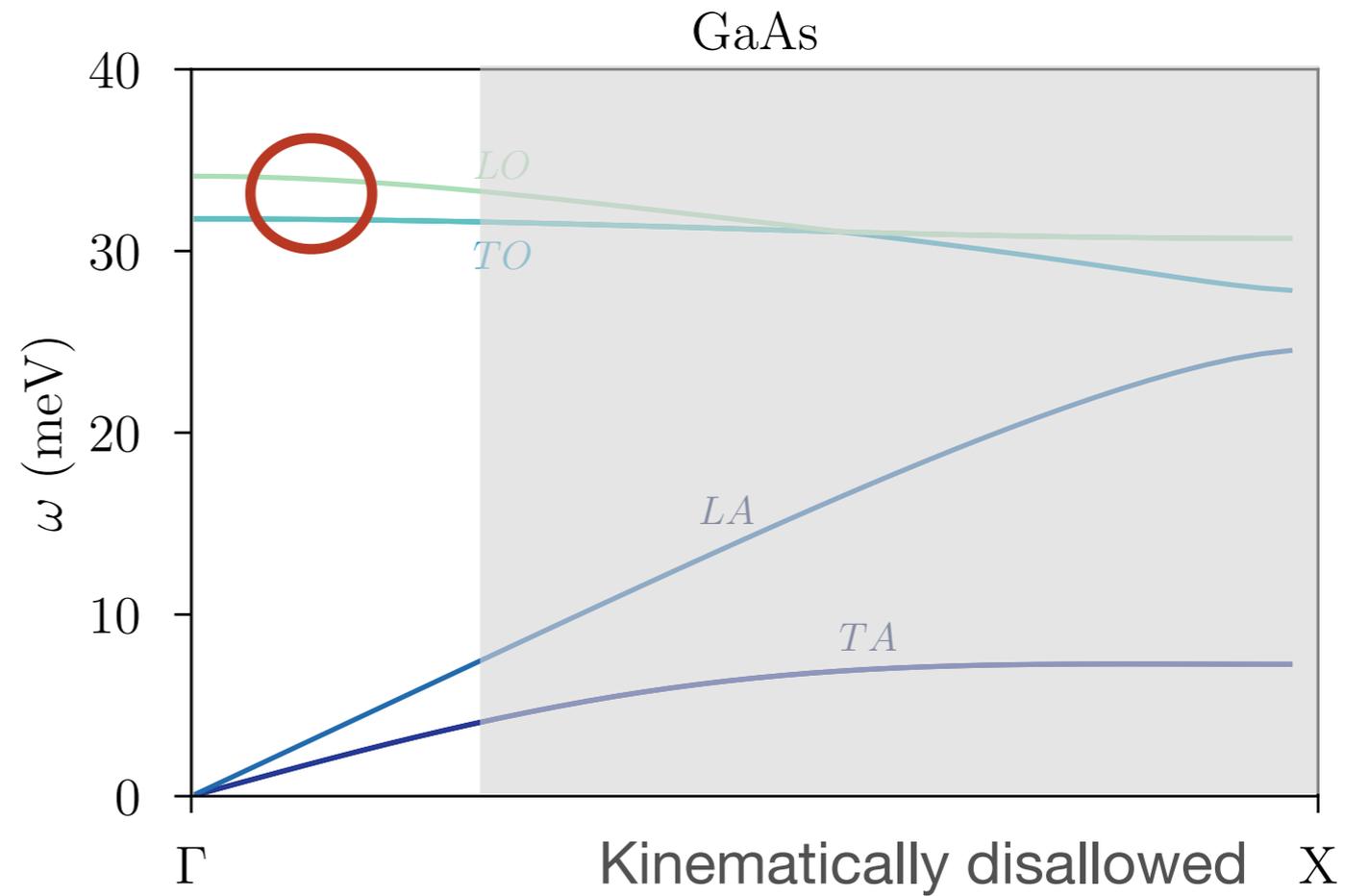
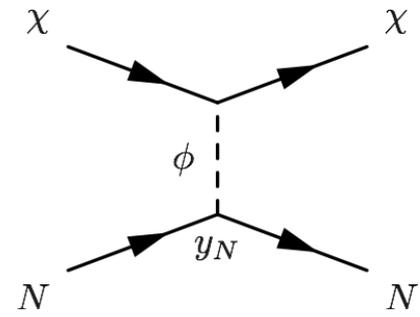
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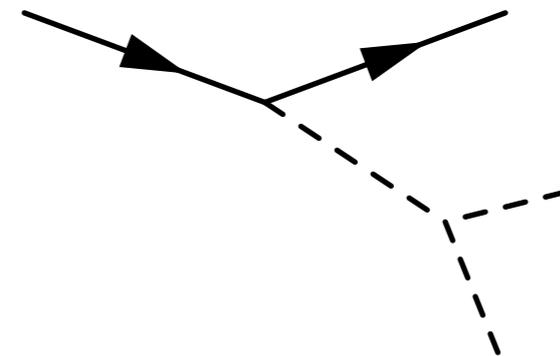
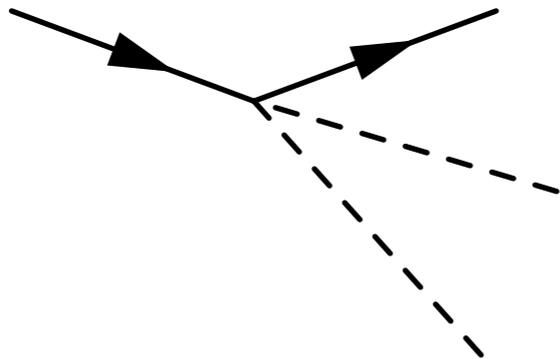
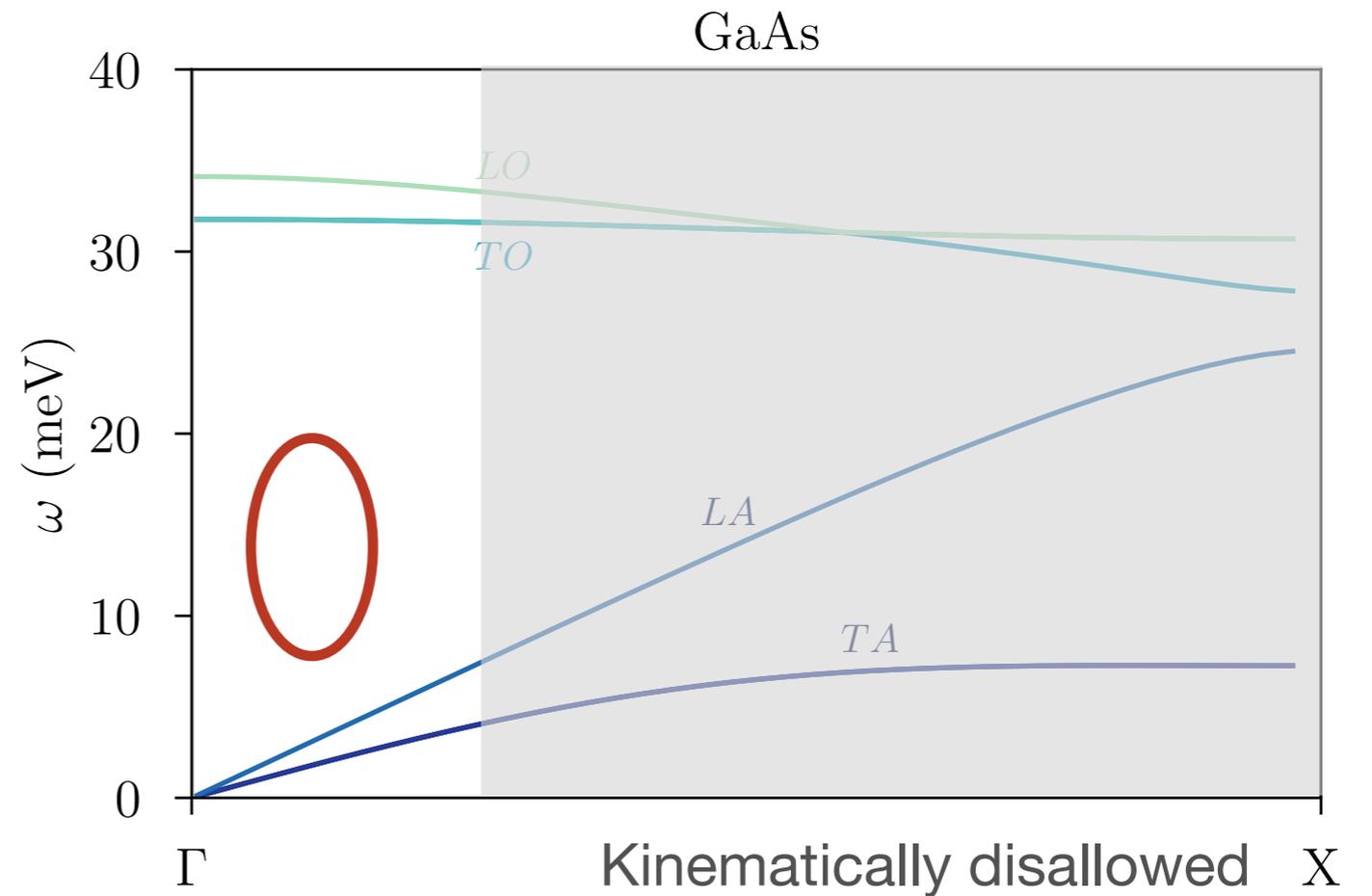
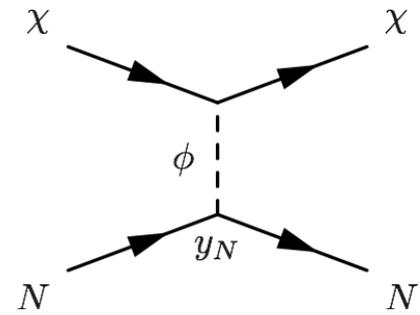
1. **Single acoustic:**  
Experimentally extremely challenging
2. **Single optical:**  
Strong destructive interference



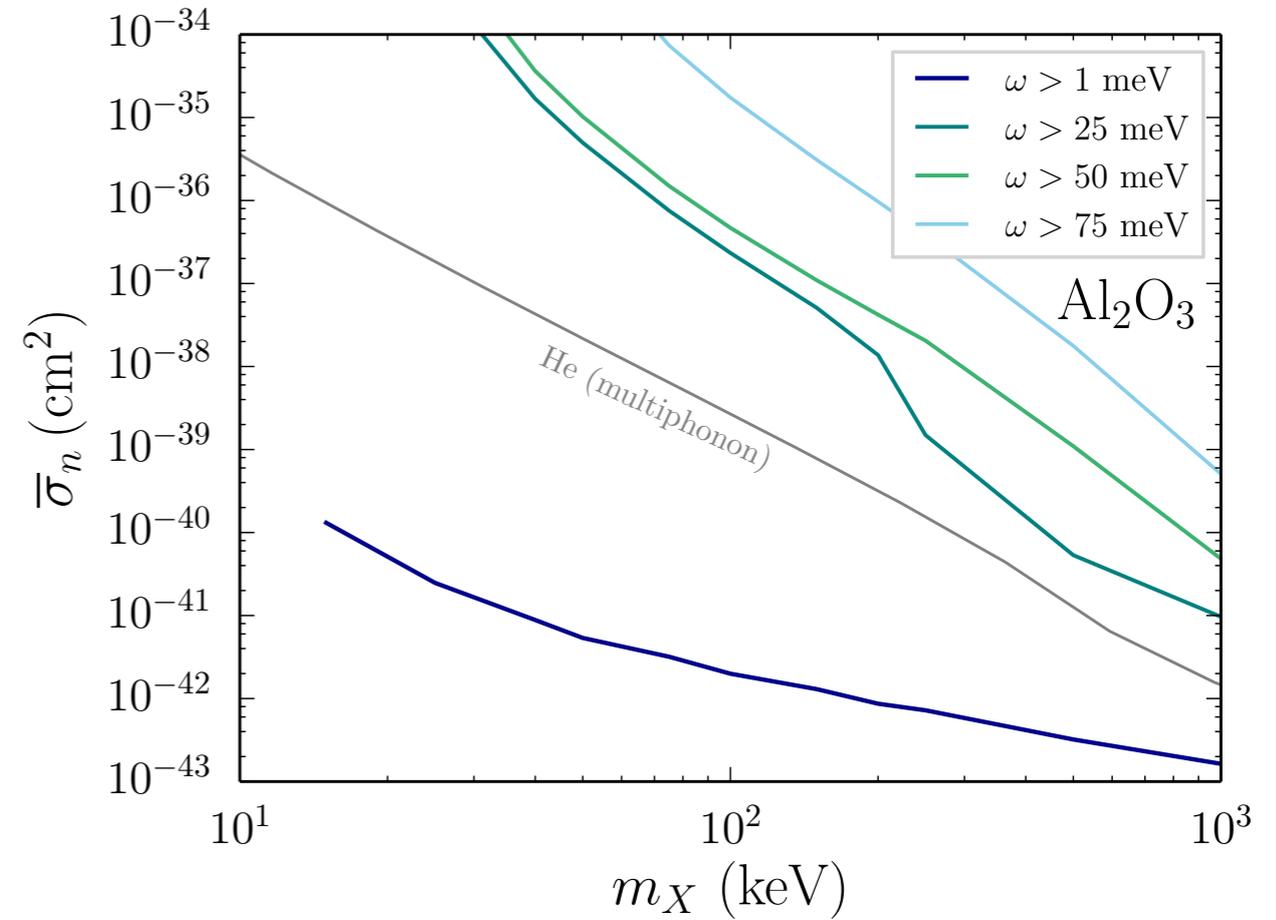
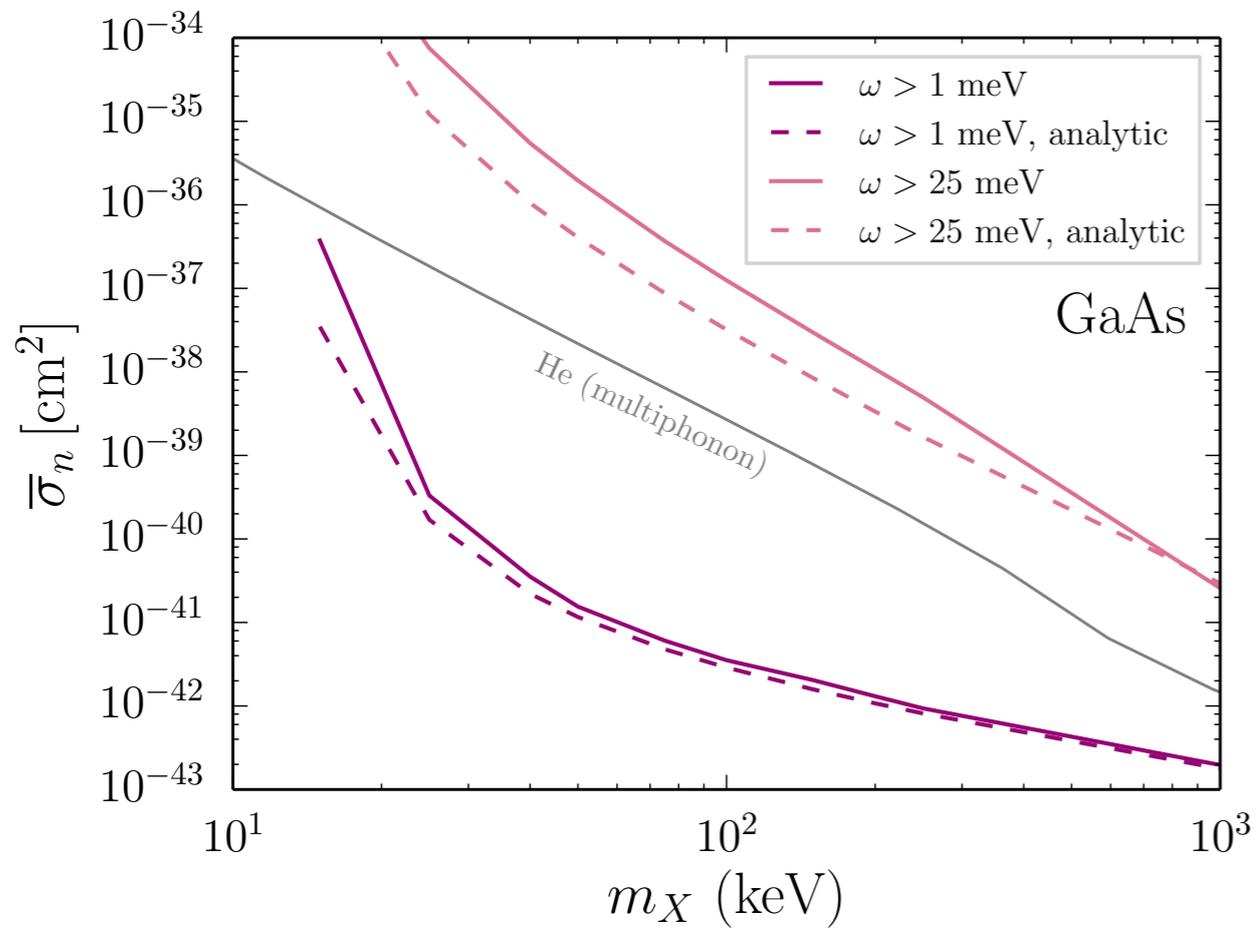
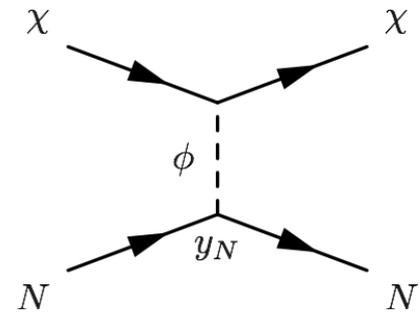
# Kinematics

Which modes to use?

1. **Single acoustic:**  
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2. **Single optical:**  
Strong destructive interference
3. **Double acoustic:**  
Next-to-leading order

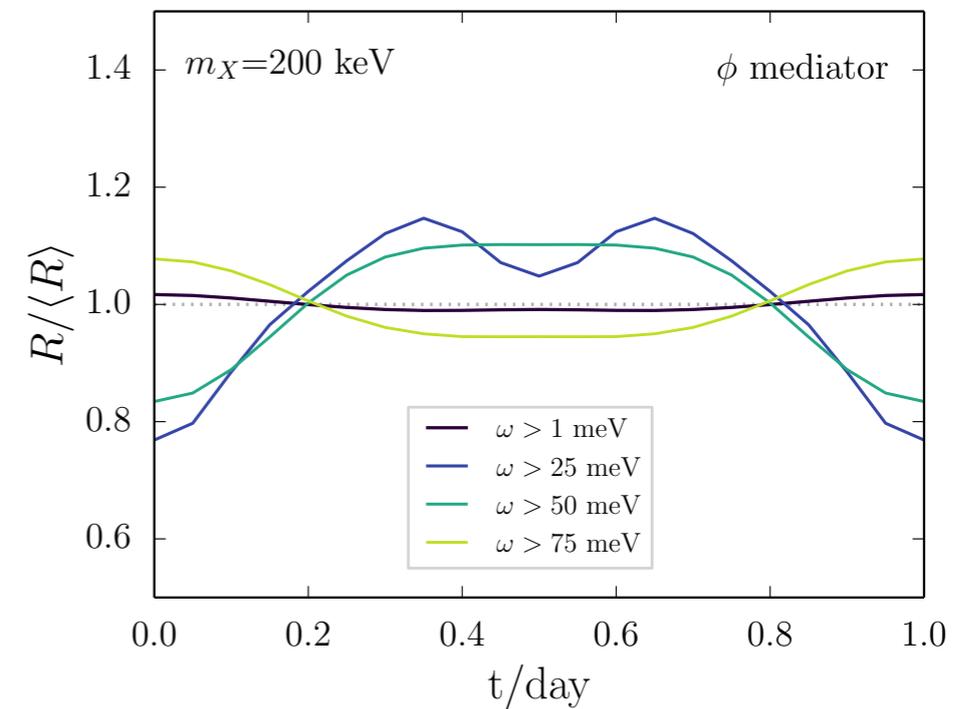
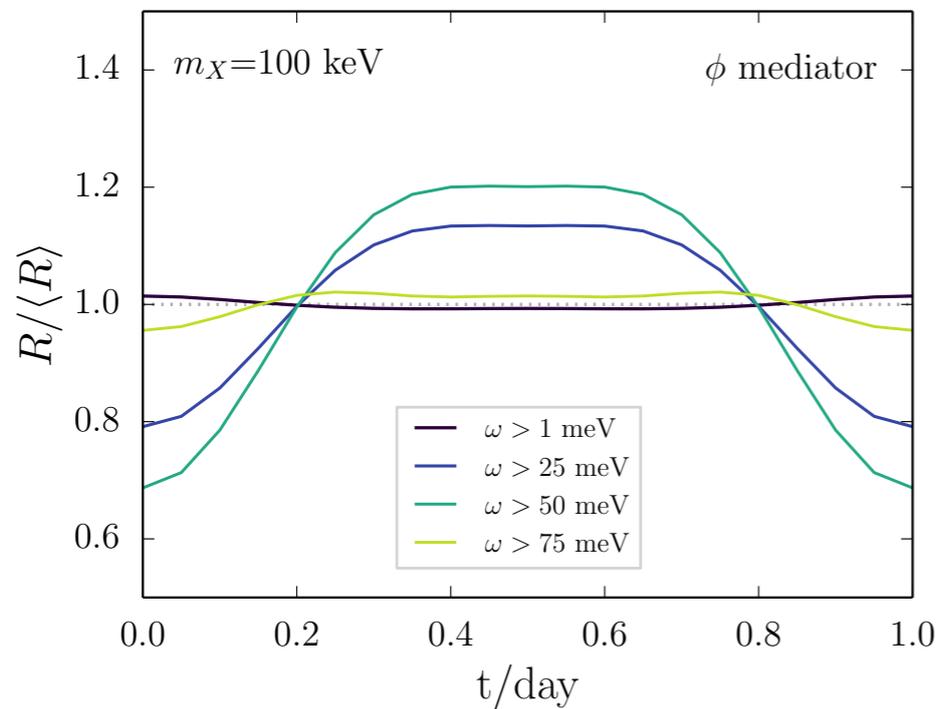
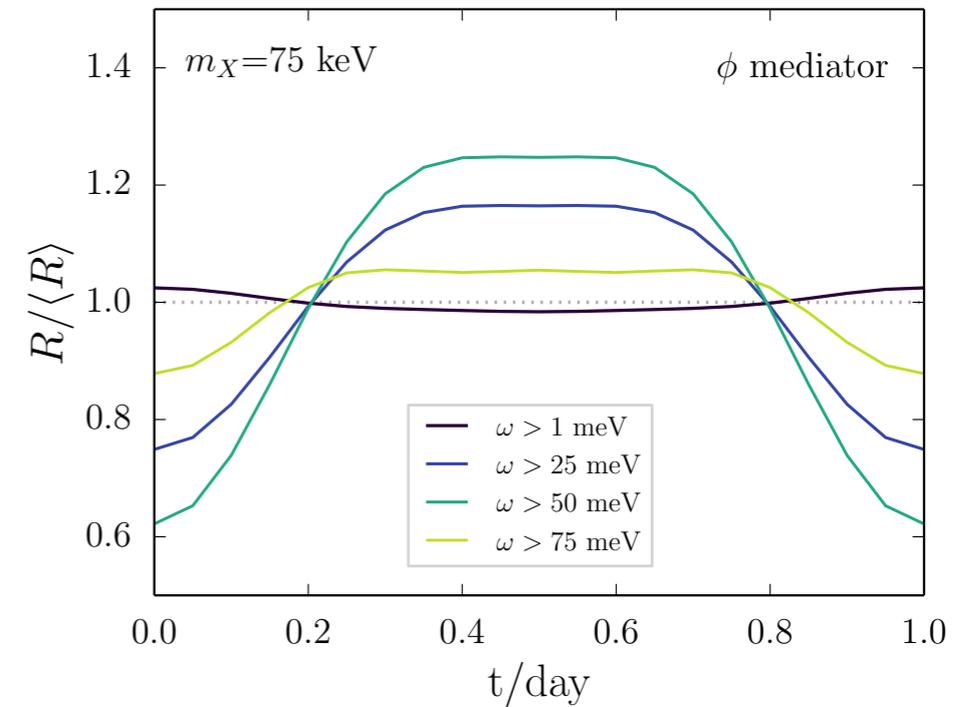
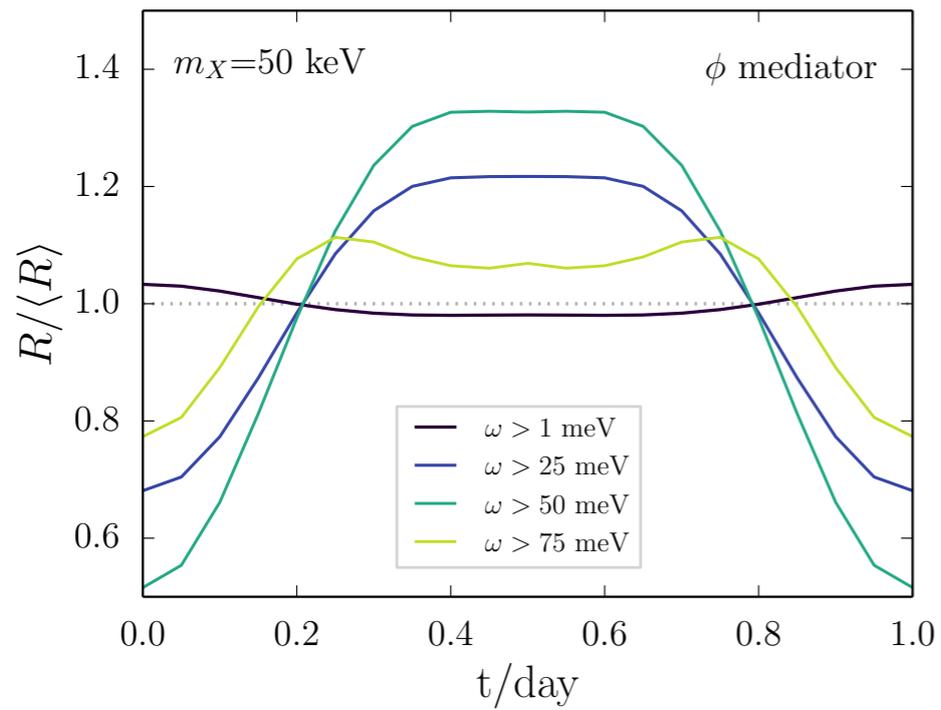
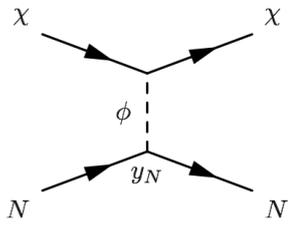


# Single phonon rate



Huge enhancement for the acoustic mode (but needs ultra low threshold)

# Daily modulation (Sapphire)

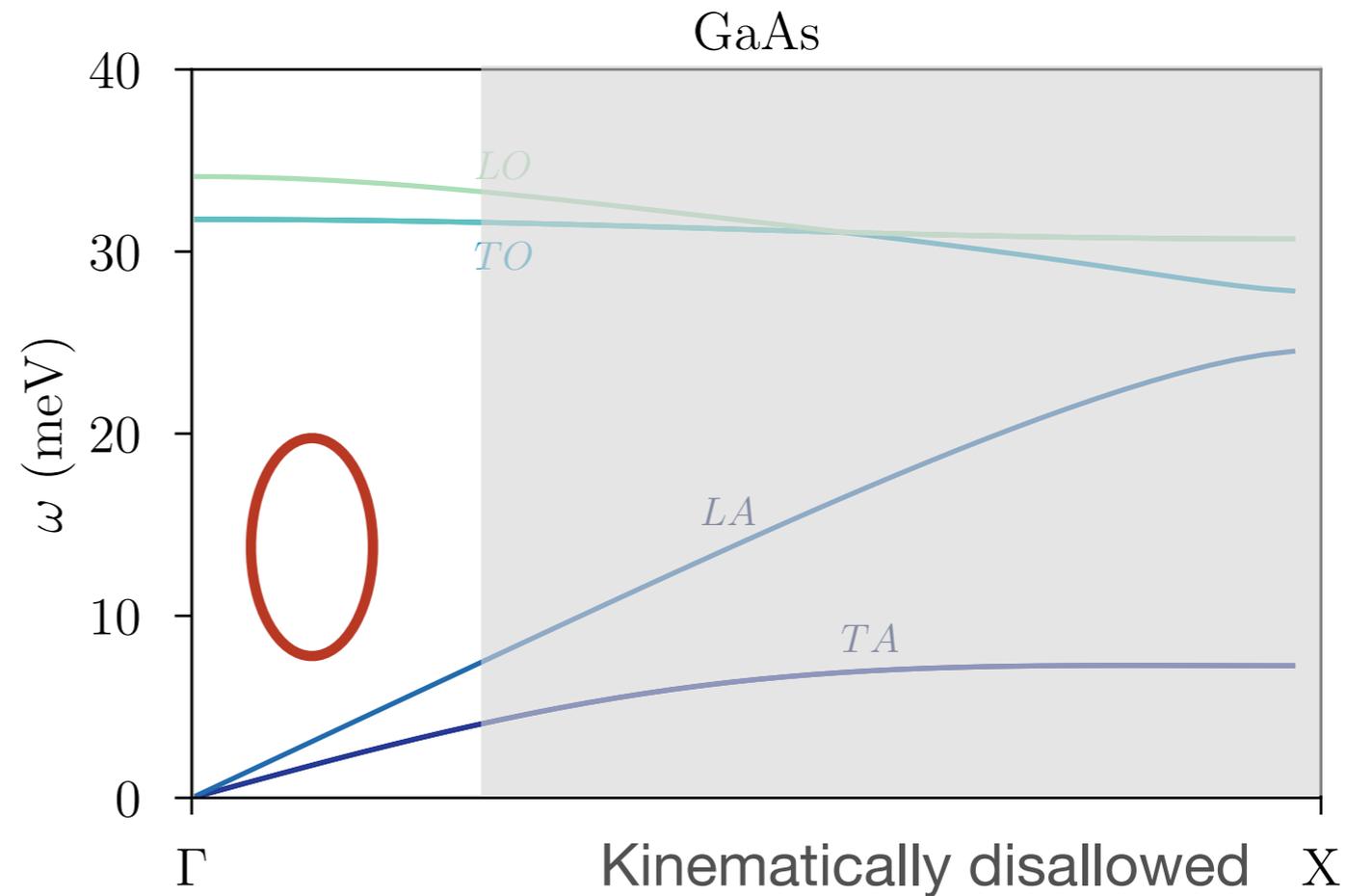
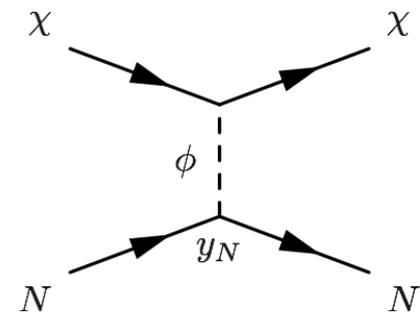
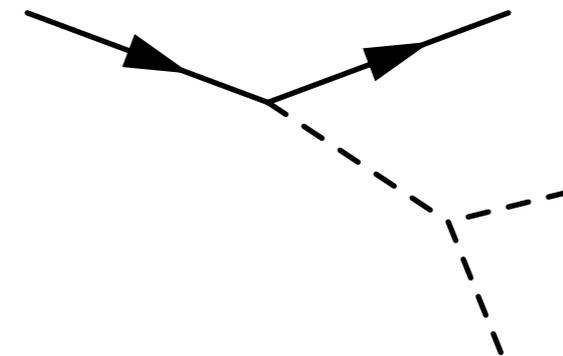
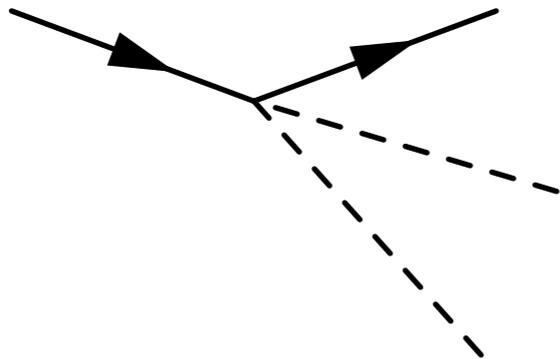


Qualitatively different from dark photon mediator!

# Kinematics

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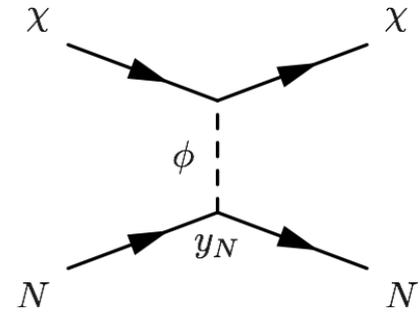
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Next-to-leading order



# Phonon perturbation theory

Point-like scattering centers:

$$V(\mathbf{r}) \sim \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r}) \quad \Rightarrow \quad V(\mathbf{q}) \sim \sum_{\ell,j} A_j e^{i\mathbf{q} \cdot \mathbf{r}_{\ell,j}}$$



Matrix element:

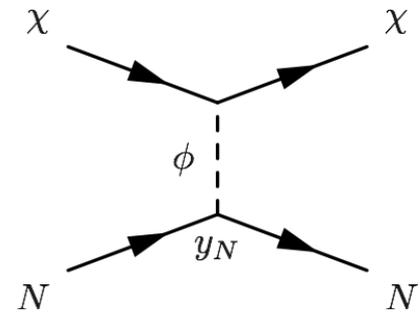
$$|\mathcal{M}|^2 \sim \left| \langle \Phi_f | \sum_{\ell,l} A_j e^{i\mathbf{q} \cdot \mathbf{r}_{\ell,j}} | 0 \rangle \right|^2$$

Double expansion in the phonon self-coupling and momentum transfer

# Phonon perturbation theory

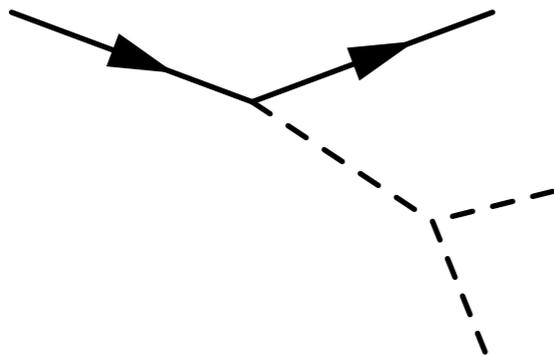
Point-like scattering centers:

$$V(\mathbf{r}) \sim \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r}) \quad \Rightarrow \quad V(\mathbf{q}) \sim \sum_{\ell,j} A_j e^{i\mathbf{q} \cdot \mathbf{r}_{\ell,j}}$$



Matrix element:

$$|\mathcal{M}|^2 \sim \left| \langle \Phi_f | \sum_{\ell,l} A_j e^{i\mathbf{q} \cdot \mathbf{r}_{\ell,j}} | 0 \rangle \right|^2$$



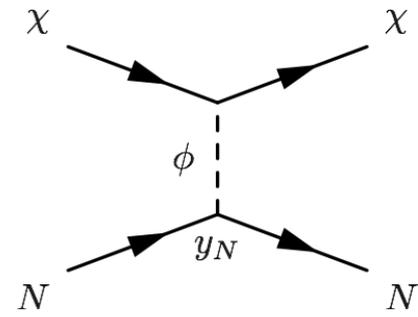
$$\mathcal{O}(\lambda^2 q^2)$$

Double expansion in the phonon self-coupling and momentum transfer

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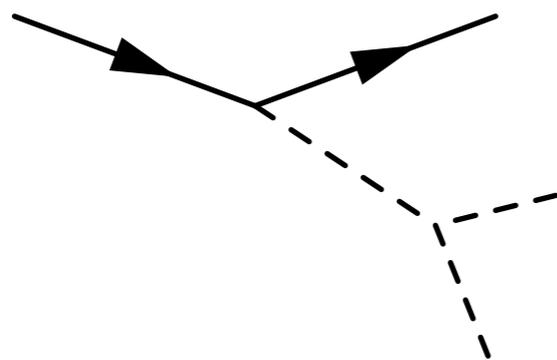
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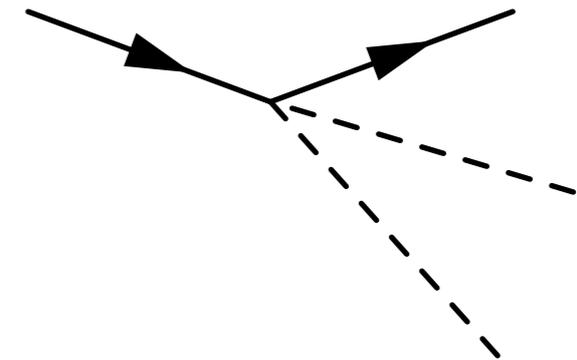


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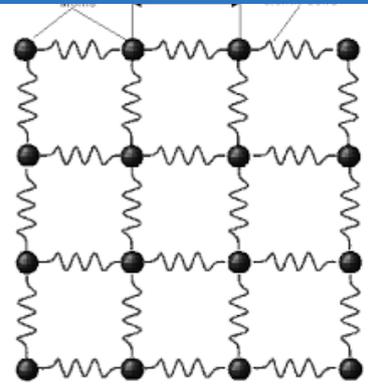
$$\mathcal{O}(\lambda^2 q^2)$$



$$\mathcal{O}(q^4)$$

Double expansion in the phonon self-coupling and momentum transfer

# Phonon self-interactions

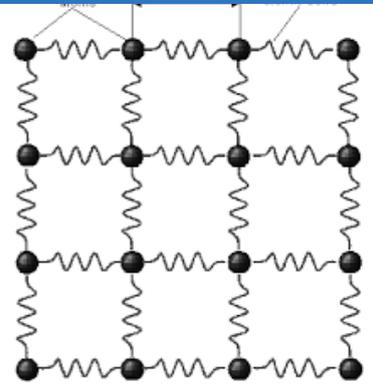


Higher order terms give effective Hamiltonian (in isotropic approximation):

$$\delta H = \int d^3 \mathbf{r} \frac{1}{2} (\beta + \lambda) u_{ii} u_{jk} u_{jk} + (\gamma + \mu) u_{ij} u_{ki} u_{kj} + \frac{\alpha}{3!} u_{ii} u_{jj} u_{kk} + \frac{\beta}{2} u_{ii} u_{jk} u_{kj} + \frac{\gamma}{3} u_{ij} u_{jk} u_{ki}$$

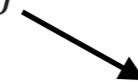
With  $u_{ij} \equiv \partial_i u_j$   Displacement operator: quantize this as harmonic oscillator

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With  $u_{ij} \equiv \partial_i u_j$   Displacement operator: quantize this as harmonic oscillator

Couplings arise from expanding the potential beyond harmonic approximation

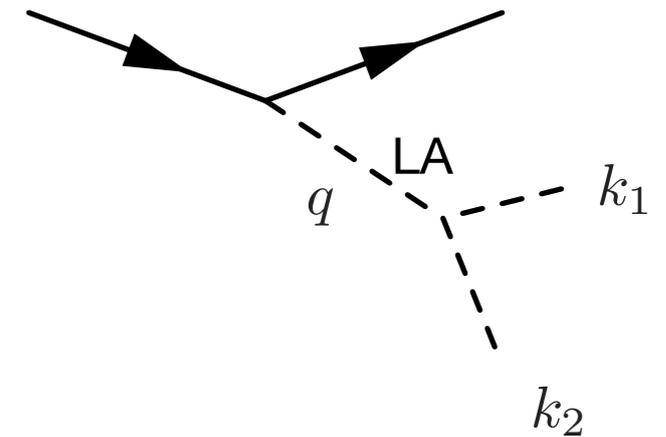
$\lambda, \mu$   Bulk modulus & Young's modulus

$\alpha, \beta, \gamma$   Third order elastic constants

Can be measured or calculated from first principles!

# Self-interactions matrix element

$$\begin{aligned}
 \widetilde{\mathcal{M}} = & (\beta + \lambda) \left[ (\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{e}_1 \cdot \mathbf{e}_2) + (\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{q} \cdot \mathbf{k}_2)(\mathbf{e} \cdot \mathbf{e}_2) + (\mathbf{k}_2 \cdot \mathbf{e}_2)(\mathbf{k}_1 \cdot \mathbf{q})(\mathbf{e}_1 \cdot \mathbf{e}) \right] \\
 & + (\gamma + \mu) \left[ (\mathbf{q} \cdot \mathbf{k}_2) \left[ (\mathbf{k}_2 \cdot \mathbf{e}_1)(\mathbf{e}_2 \cdot \mathbf{e}) + (\mathbf{k}_2 \cdot \mathbf{e})(\mathbf{e}_2 \cdot \mathbf{e}_1) \right] \right. \\
 & \quad + (\mathbf{k}_2 \cdot \mathbf{k}_1) \left[ (\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{e}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e}_2)(\mathbf{e} \cdot \mathbf{e}_1) \right] \\
 & \quad \left. + (\mathbf{q} \cdot \mathbf{k}_2) \left[ (\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{e}_1 \cdot \mathbf{e}) + (\mathbf{k}_1 \cdot \mathbf{e})(\mathbf{e}_1 \cdot \mathbf{e}_2) \right] \right] \\
 & + \alpha(\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{k}_2 \cdot \mathbf{e}_2) \\
 & + \beta \left[ (\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{q} \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}_1) + (\mathbf{k}_2 \cdot \mathbf{e}_2)(\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e}) \right] \\
 & + \gamma \left[ (\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e})(\mathbf{k}_2 \cdot \mathbf{e}_1) \right]
 \end{aligned}$$

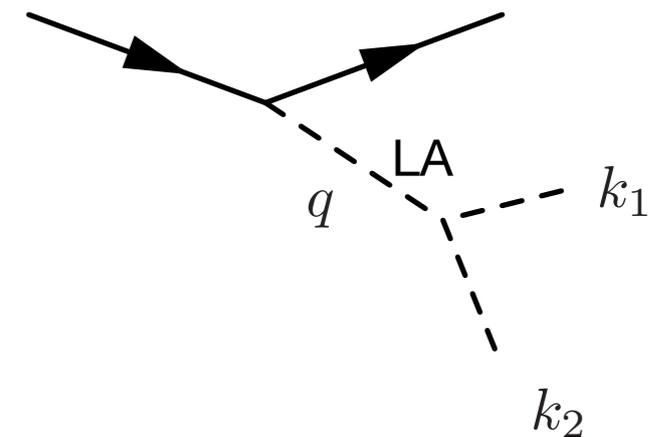


# Self-interactions matrix element

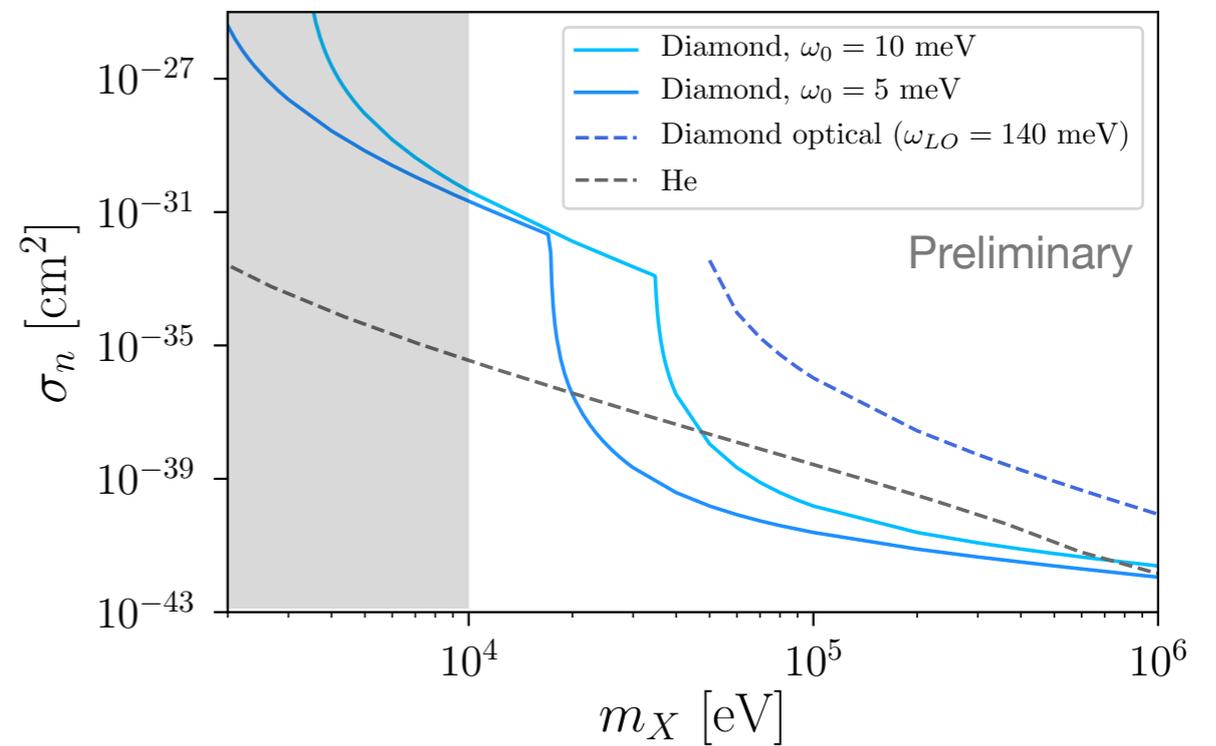
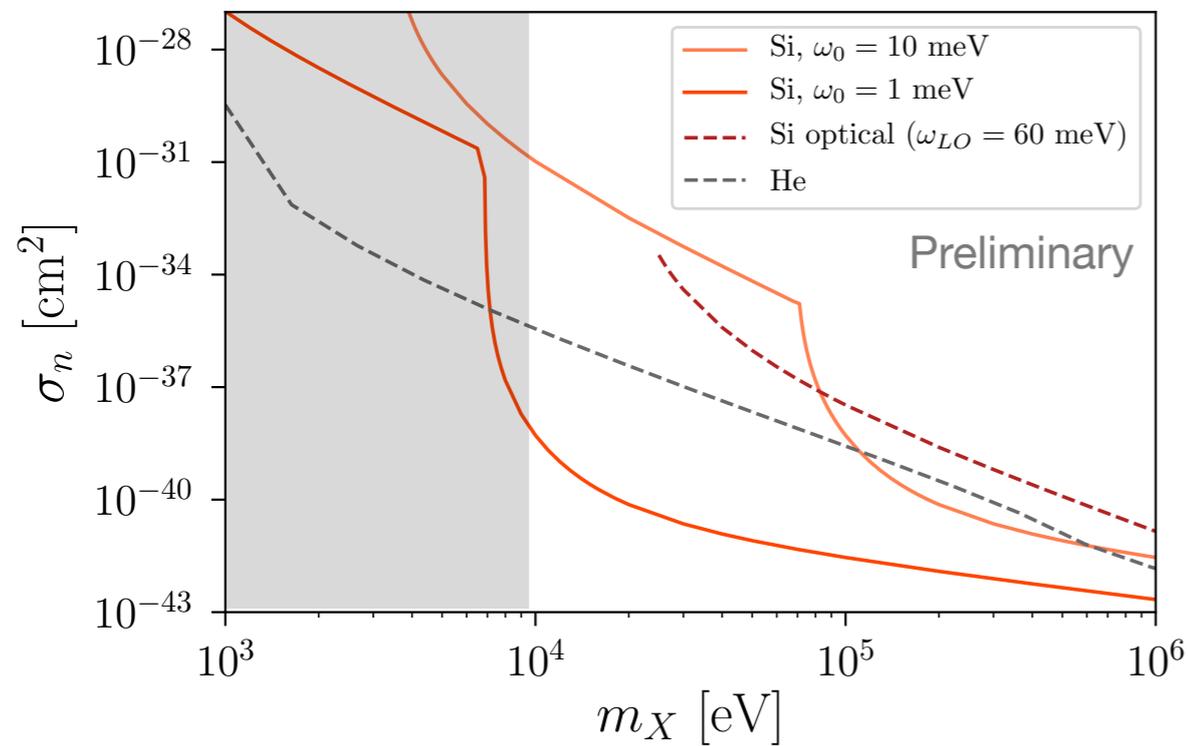
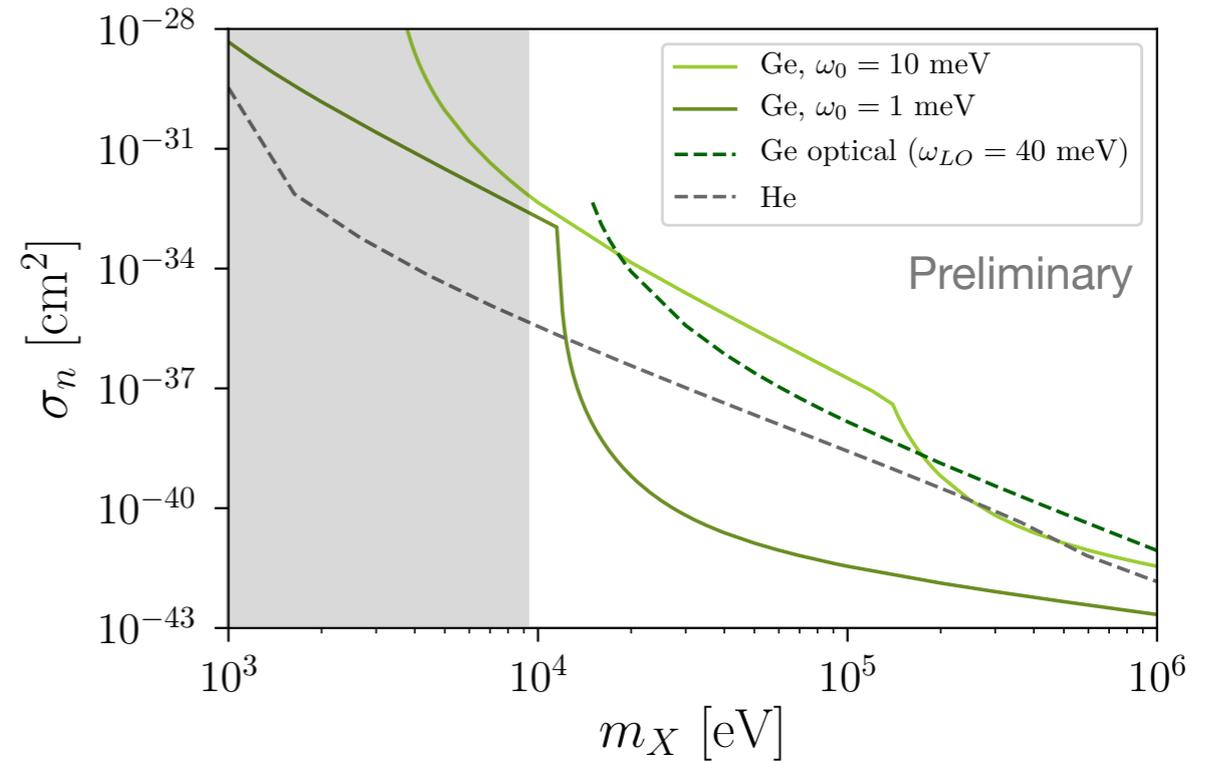
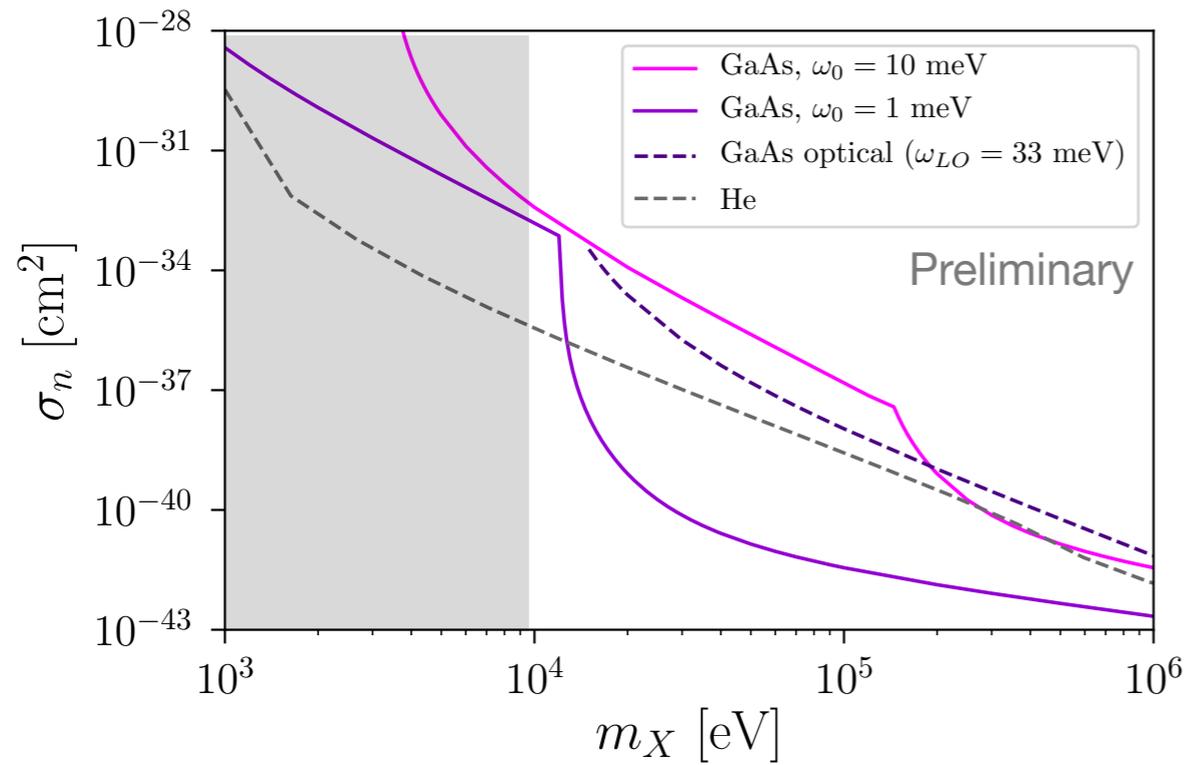
$$\begin{aligned}
 \widetilde{\mathcal{M}} = & (\beta + \lambda) \left[ (\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{e}_1 \cdot \mathbf{e}_2) + (\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{q} \cdot \mathbf{k}_2)(\mathbf{e} \cdot \mathbf{e}_2) + (\mathbf{k}_2 \cdot \mathbf{e}_2)(\mathbf{k}_1 \cdot \mathbf{q})(\mathbf{e}_1 \cdot \mathbf{e}) \right] \\
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 \end{aligned}$$

4 non-vanishing channels:

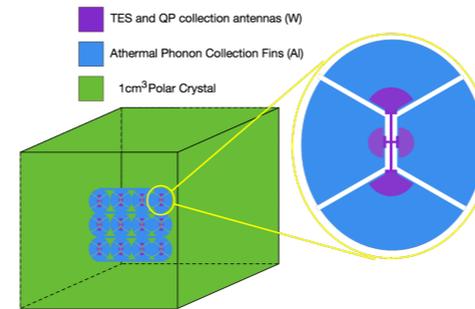
- LA LA
- LA TA
- TA TA (polarized in momentum-plane)
- TA TA (polarized orthogonal to momentum-plane)



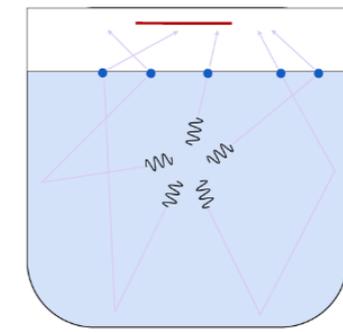
# Preliminary results



# Summary



Crystals



Superfluid Helium

## Scattering

*Coupling to charge*

Single optical  
Single acoustic

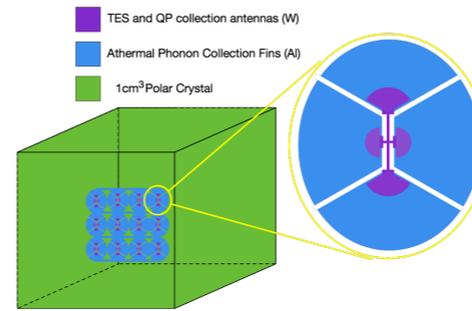
Large [1,2]  
Tiny [1,2]

X  
Tiny [b]

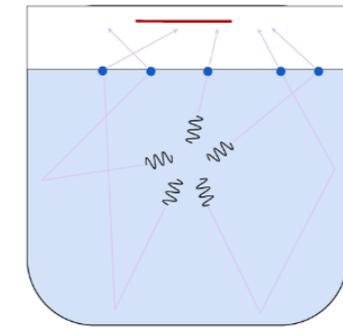
- [1] **SK**, T. Lin, M. Pyle, K. Zurek: 1712.06598  
 [2] S. Griffin, **SK**, T. Lin, M. Pyle, K. Zurek: 1807.10291  
 [3] P. Cox, T. Melia, S. Rajendran: 1905.05575  
 [4] B. Campbell-Deem, P. Cox, T. Melia, **SK**, T. Lin: to appear

- [a] K. Schutz, K. Zurek: 1604.08206  
 [b] **SK**, T. Lin, K. Zurek: 1611.06228  
 [c] F. Acanfora, A. Esposito, A. Polosa: 1902.02361  
 [d] A. Caputo, A. Esposito, A. Polosa: 1907.10635

# Summary



Crystals



Superfluid Helium

## Scattering

<i>Coupling to charge</i>	Single optical	Large [1,2]	X
	Single acoustic	Tiny [1,2]	Tiny [b]
<i>Coupling to mass</i>	Single acoustic	experimentally hard [1,2]	experimentally impossible
	Single optical	small [1,2,3]	X
	multiphonon	small [4]	small [a,b,c,d]

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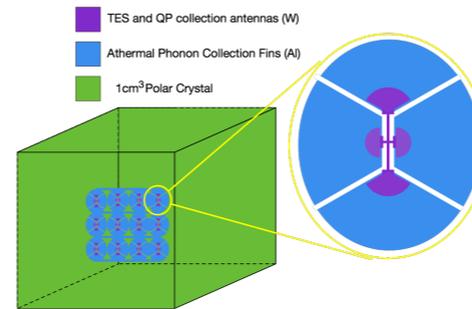
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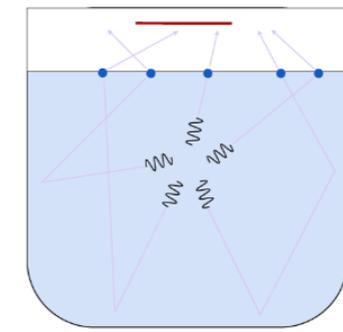
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Crystals



Superfluid Helium

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## Absorption

Dark photon	Large [2]	Tiny [b]
Scalar	Future work	Future work

[1] **SK**, T. Lin, M. Pyle, K. Zurek: 1712.06598

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Thank you!



# Dark photon absorption

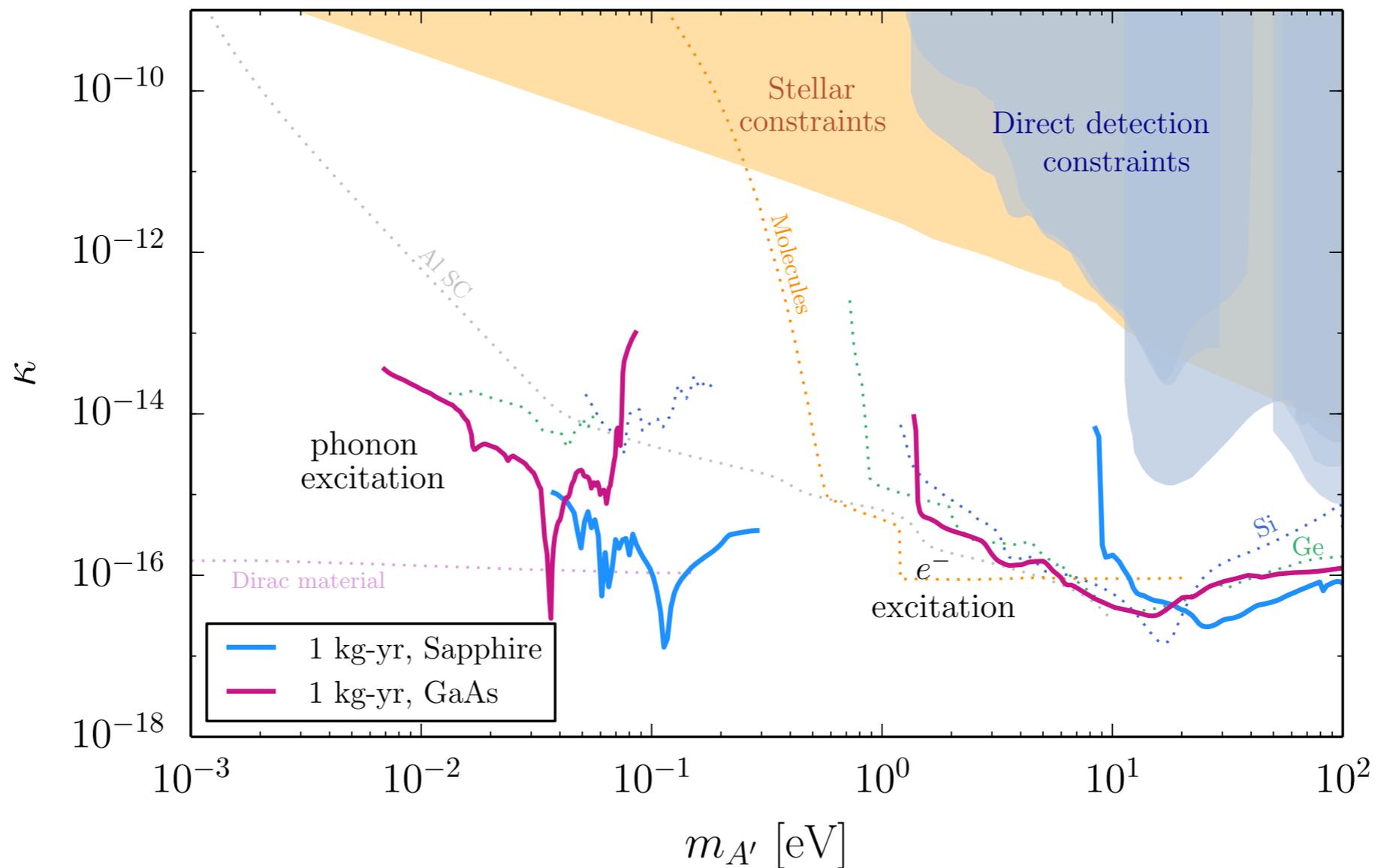
Very light, bosonic dark matter can be **absorbed** on the target

Example: Dark photon dark matter:  $\mathcal{L} \supset -\frac{\kappa}{2} F'_{\mu\nu} F^{\mu\nu}$

# Dark photon absorption

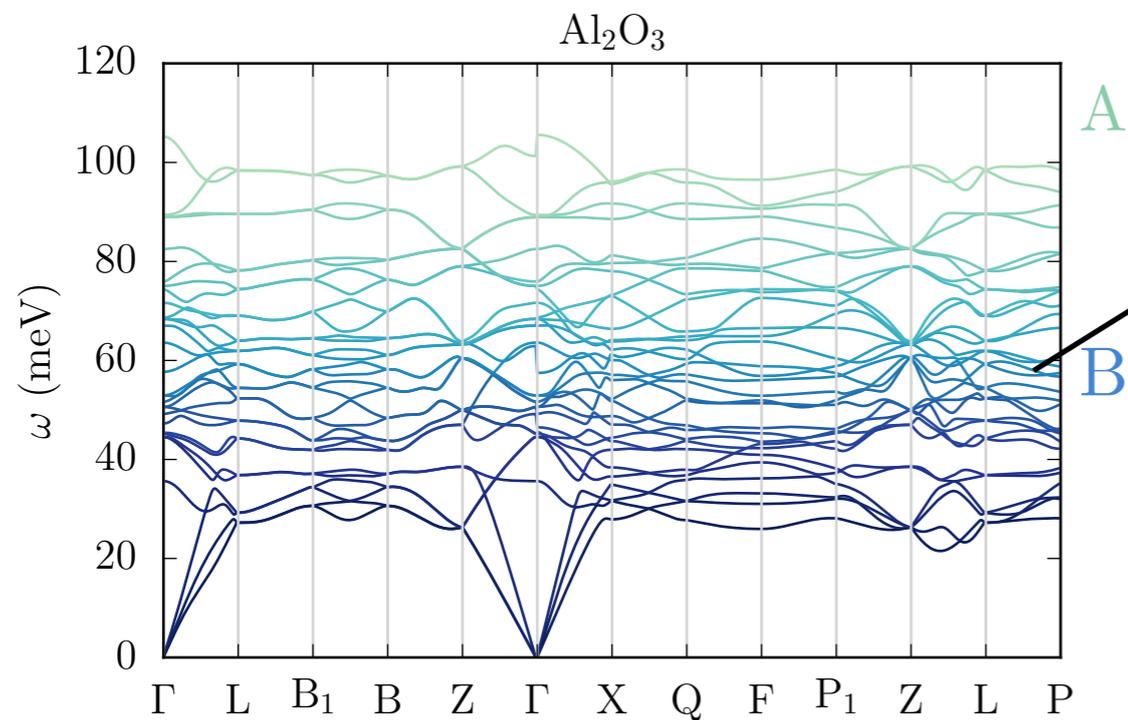
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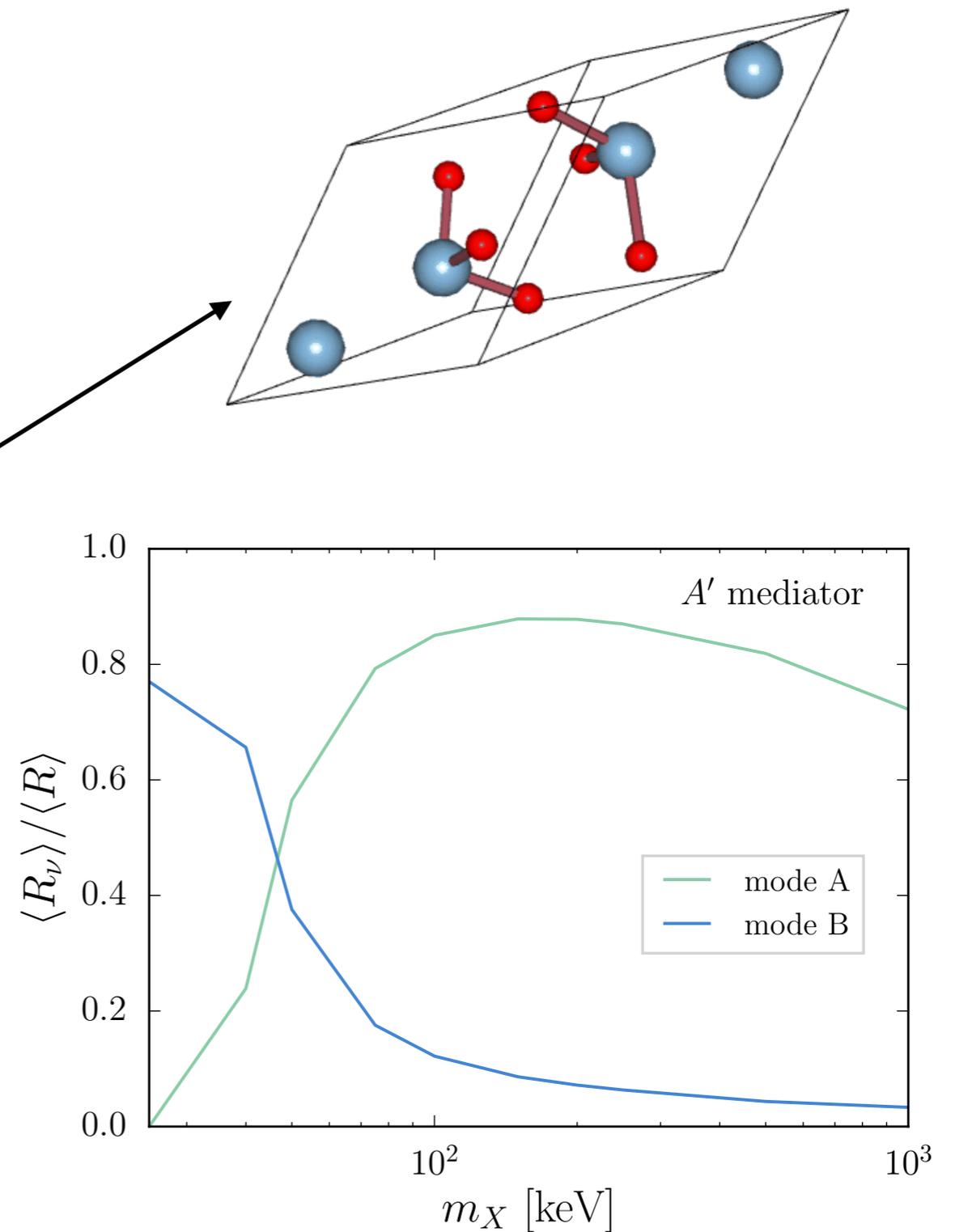


# Sapphire in more detail

For low  $m_X$ , a lower mode dominates

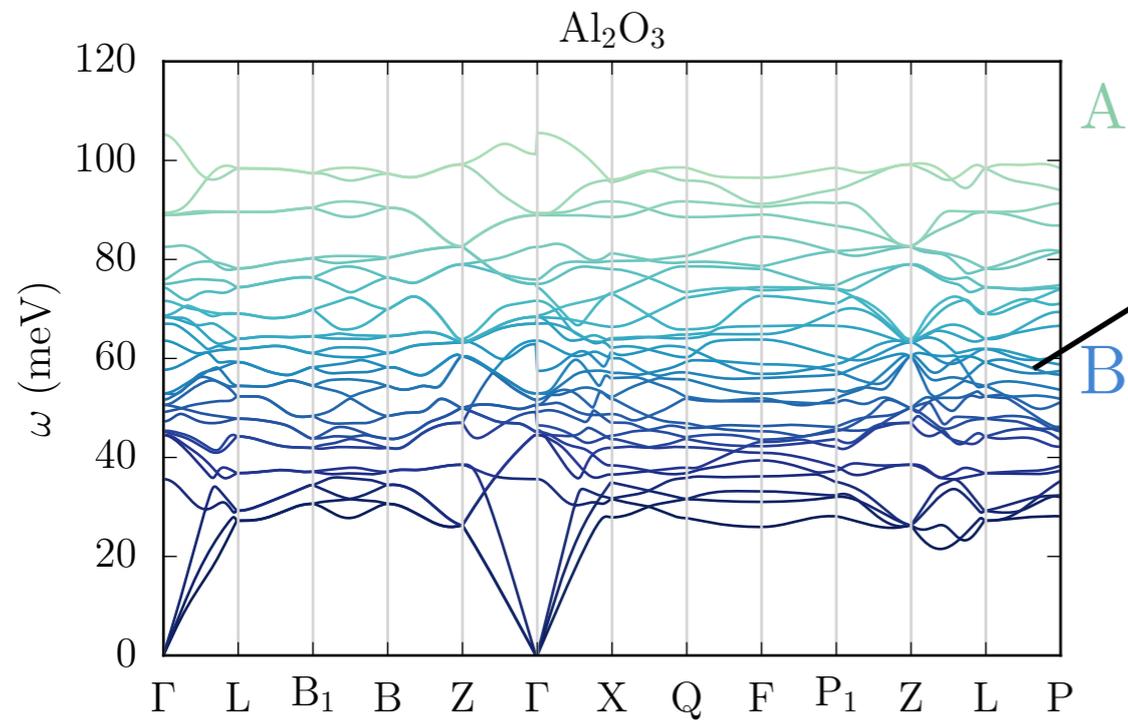


Aluminum atoms still move in phase, but smaller amplitude

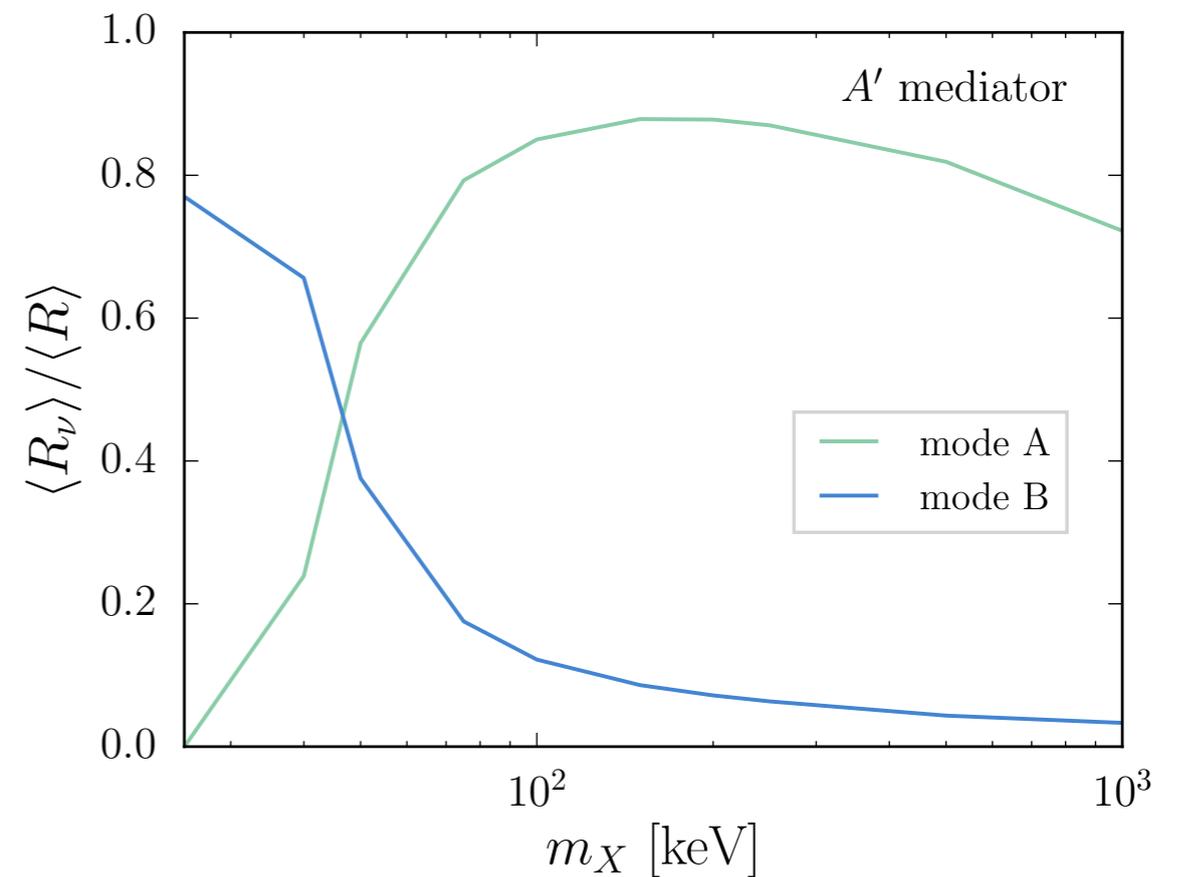
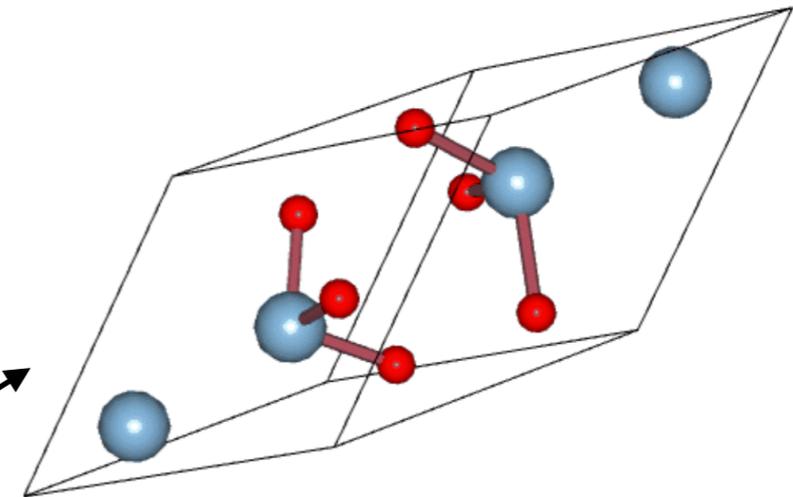


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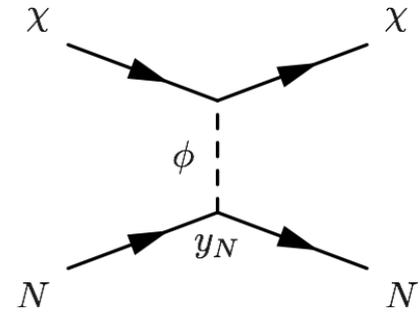
# Scattering formalism

Scattering potential:

$$V(\mathbf{r}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r})$$

Phonon form factor:

$$|F_\nu(\mathbf{q})|^2 = \left| \sum_j \frac{A_j}{\sqrt{m_j}} e^{-W_j(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{\nu,j,\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}_j} \right|^2$$



# Scattering formalism

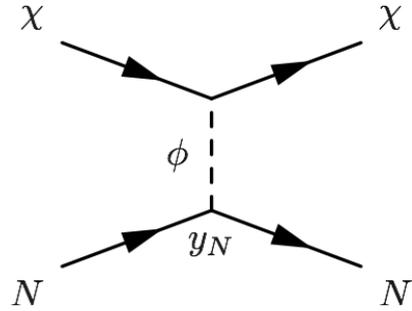
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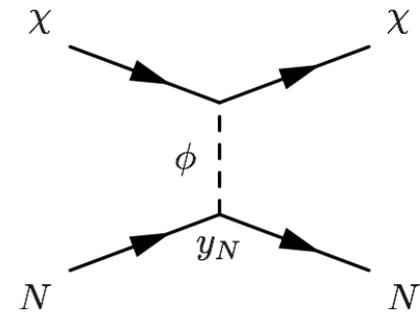
Daily modulation



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Acoustic

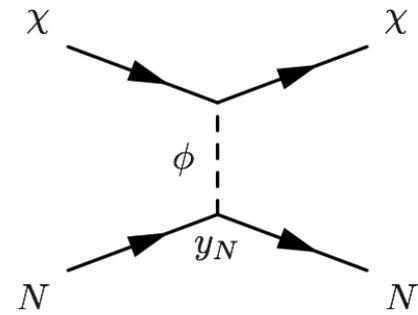
Daily modulation

$$|F_\nu^{(ac)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \left| \sum_j \sqrt{A_j} e^{-i\mathbf{q} \cdot \mathbf{r}_j} \right|^2$$

# Scattering formalism

Scattering potential:

$$V(\mathbf{r}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r})$$



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Daily modulation

Acoustic

Optical

$$|F_\nu^{(ac)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \left| \sum_j \sqrt{A_j} e^{-i\mathbf{q} \cdot \mathbf{r}_j} \right|^2$$

$$|F_\nu^{(opt)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \times \mathcal{O}(q^2 a^2)$$

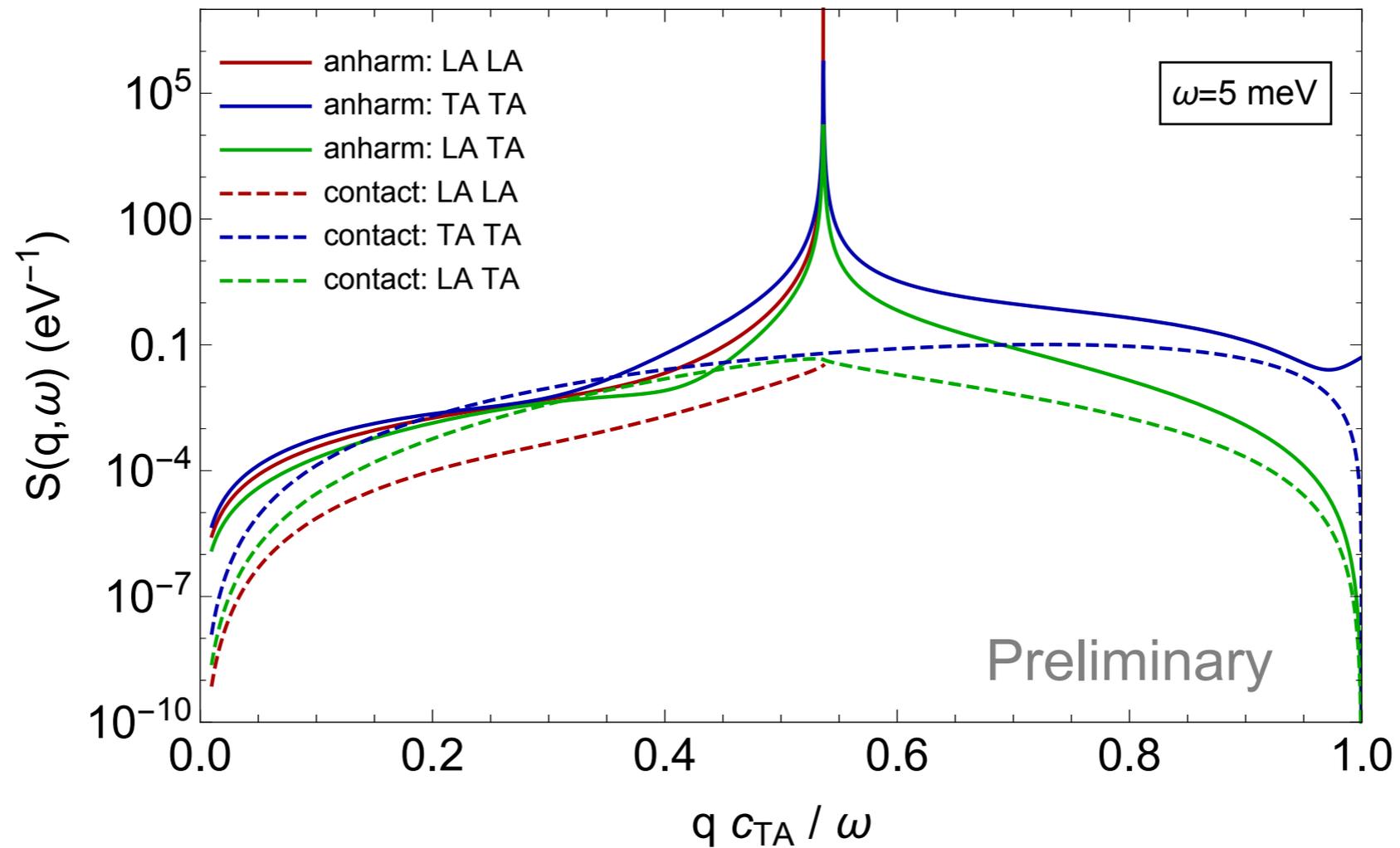
Lattice  
constant

Destructive interference kills leading order piece for optical phonons

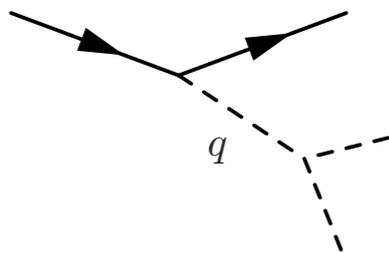
# Comparing different channels

Example GaAs:

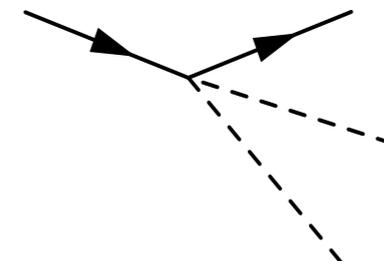
Differential scattering rate



Anharmonic:



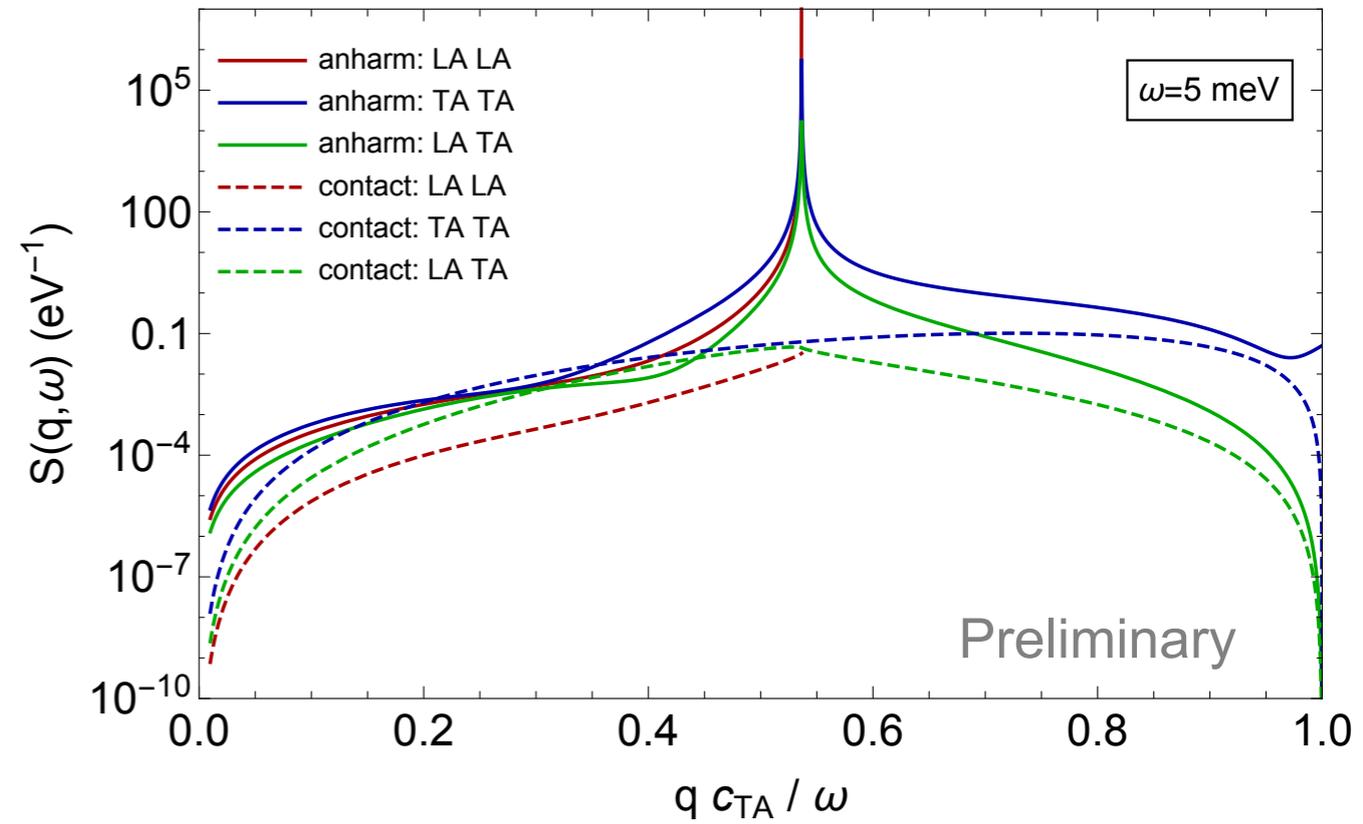
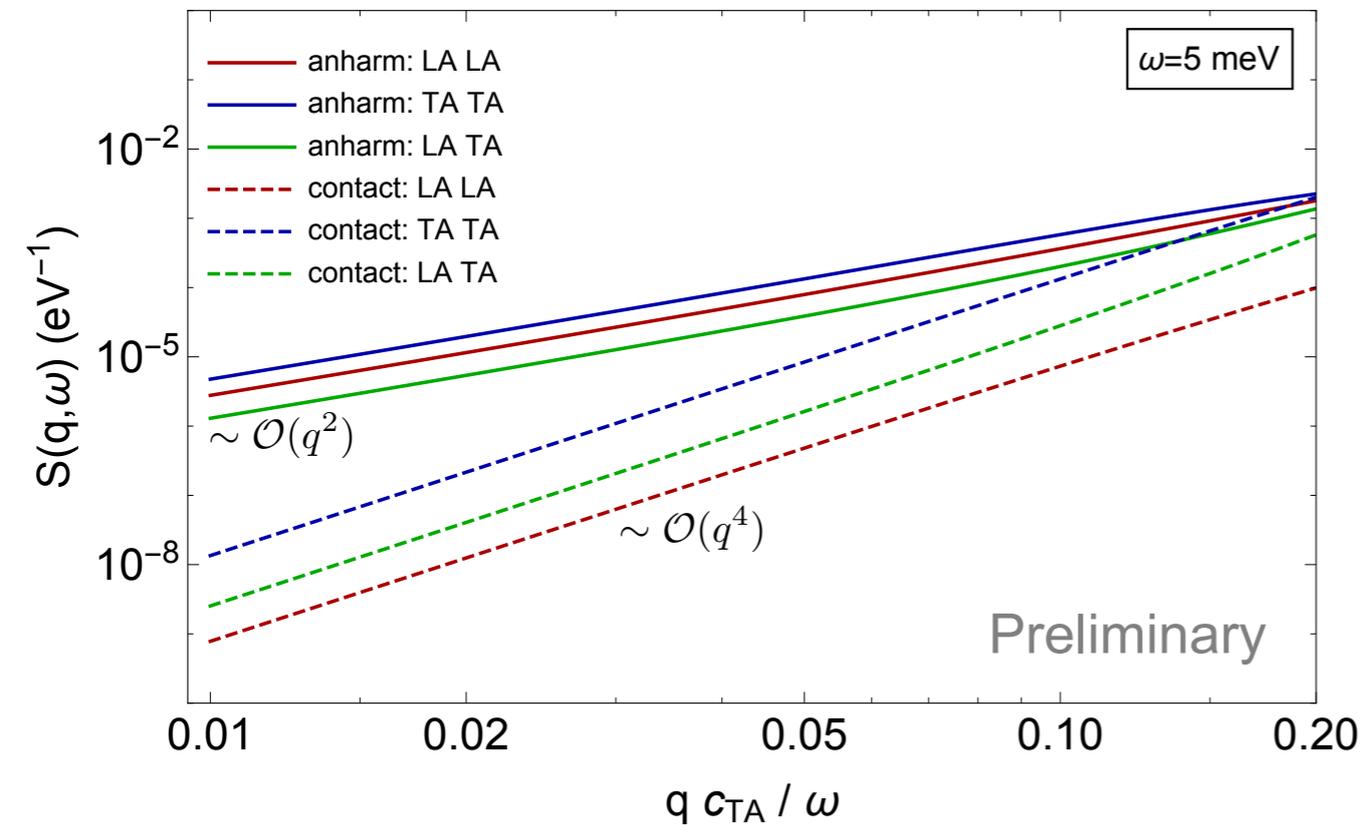
Contact:



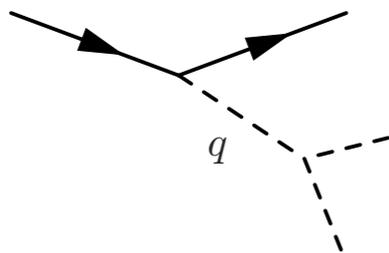
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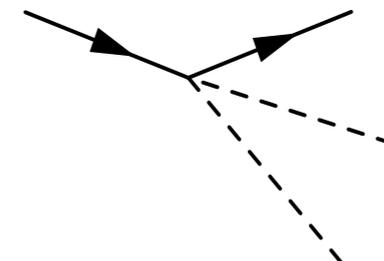
Differential scattering rate



Anharmonic:



Contact:



The anharmonic contribution tends to dominate

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

# Phonon effective theory

Periodic potential (Hooke's law)

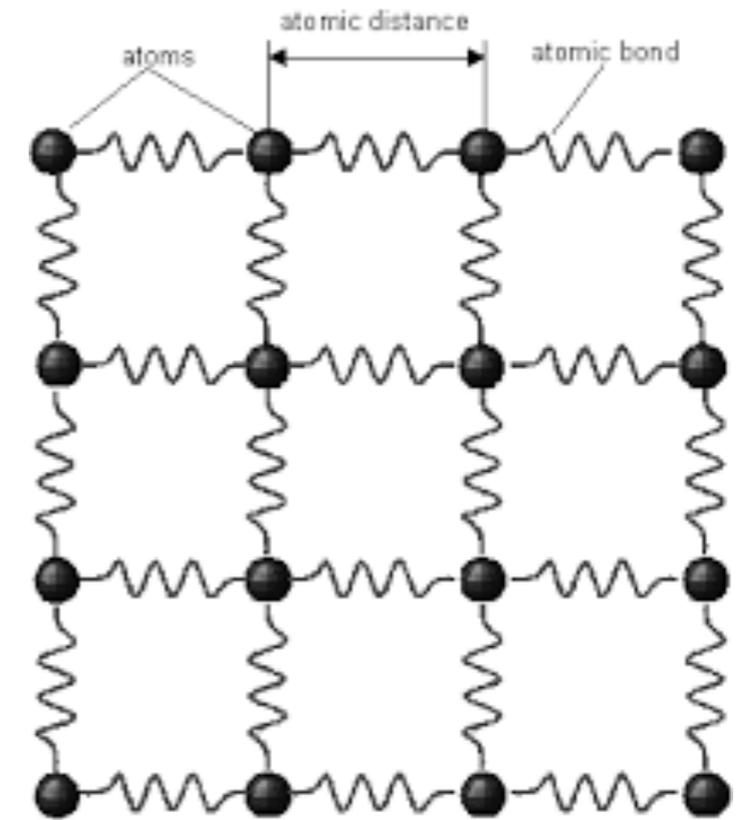
$$\mathcal{V} = \mathcal{V}^{(0)} + \sum_{1,j} \mathcal{V}_{1,j}^{(1)} \cdot \mathbf{u}_{j,1} + \frac{1}{2} \sum_{1,l',j,j'} \mathbf{u}_{j,1} \cdot \mathcal{V}_{1,j,l',j'}^{(2)} \cdot \mathbf{u}_{j',l'} + \dots$$

↙

Atoms in  
unit cell

↘

Lattice  
sites

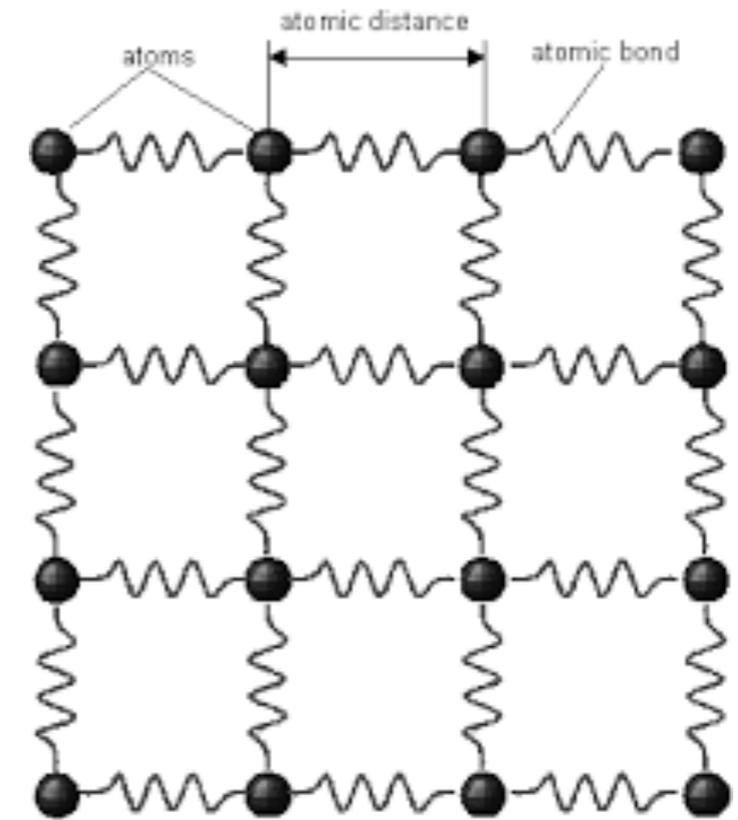


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Periodic potential (Hooke's law)

$$\mathcal{V} = \mathcal{V}^{(0)} + \sum_{1,j} \cancel{\mathcal{V}_{1,j}^{(1)}} \cdot \mathbf{u}_{j,1} + \frac{1}{2} \sum_{1,l',j,j'} \mathbf{u}_{j,1} \cdot \mathcal{V}_{1,j,l',j'}^{(2)} \cdot \mathbf{u}_{j',l'} + \dots$$

$\swarrow$                        $\searrow$   
 Atoms in                  Lattice  
 unit cell                  sites

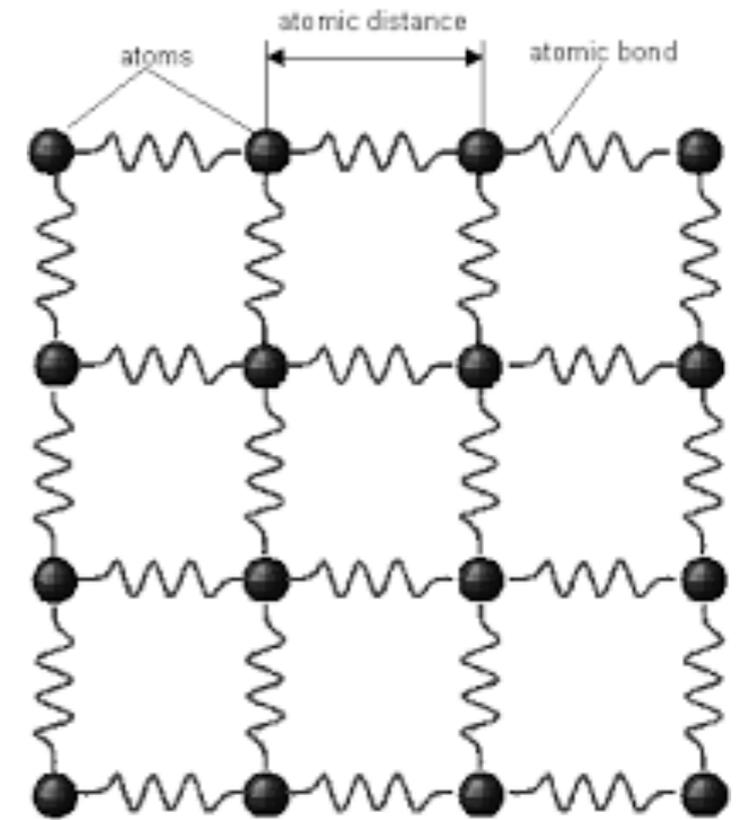


# Phonon effective theory

Periodic potential (Hooke's law)

$$\mathcal{V} = \mathcal{V}^{(0)} + \sum_{\mathbf{l}, j} \cancel{\mathcal{V}_{\mathbf{l}, j}^{(1)}} \cdot \mathbf{u}_{j, \mathbf{l}} + \frac{1}{2} \sum_{\mathbf{l}, \mathbf{l}', j, j'} \mathbf{u}_{j, \mathbf{l}} \cdot \mathcal{V}_{\mathbf{l}, j, \mathbf{l}', j'}^{(2)} \cdot \mathbf{u}_{j', \mathbf{l}'} + \dots$$

$\swarrow$                        $\searrow$   
 Atoms in                  Lattice  
 unit cell                  sites



Displacement operator:

$$\mathbf{u}_{j, \mathbf{l}}(t) = \sum_{\nu} \sum_{\mathbf{q}} \sqrt{\frac{1}{2Nm_j\omega_{\nu, \mathbf{q}}}} \left( \mathbf{e}_{\nu, j, \mathbf{q}} \hat{a}_{\nu, \mathbf{q}} e^{i\mathbf{q} \cdot (\mathbf{l} + \mathbf{r}_j^0) - i\omega_{\nu, \mathbf{q}} t} + \mathbf{e}_{\nu, j, \mathbf{q}}^* \hat{a}_{\nu, \mathbf{q}}^\dagger e^{-i\mathbf{q} \cdot (\mathbf{l} + \mathbf{r}_j^0) + i\omega_{\nu, \mathbf{q}} t} \right)$$

$\nu$  : phonon branches

$\mathbf{q}$  : momentum over Brillouin zone

$j$  : atom in primitive cell

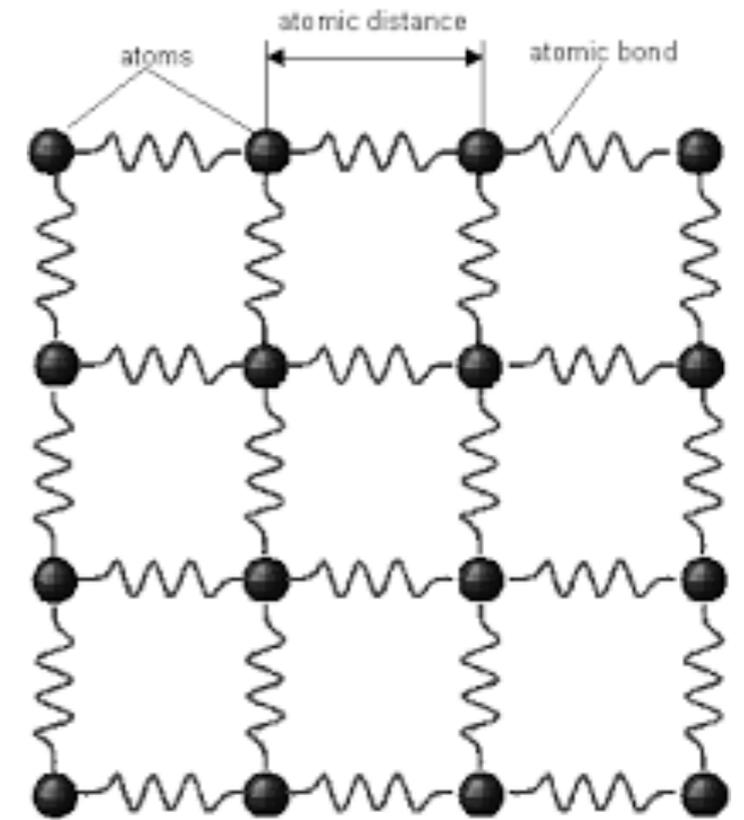
In the harmonic approximation, just quantize as harmonic oscillator

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$\swarrow$                        $\searrow$   
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Polarization tensors  
 $\swarrow$                        $\searrow$   
 creation/annihilation operators

$\nu$  : phonon branches

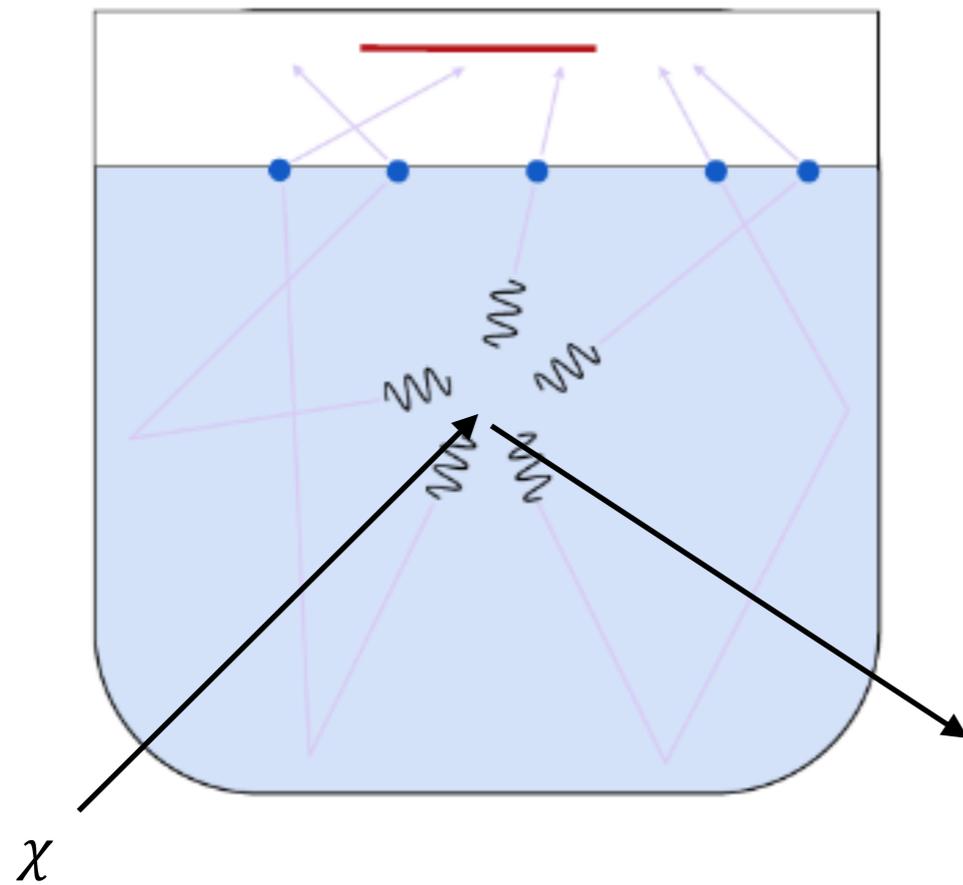
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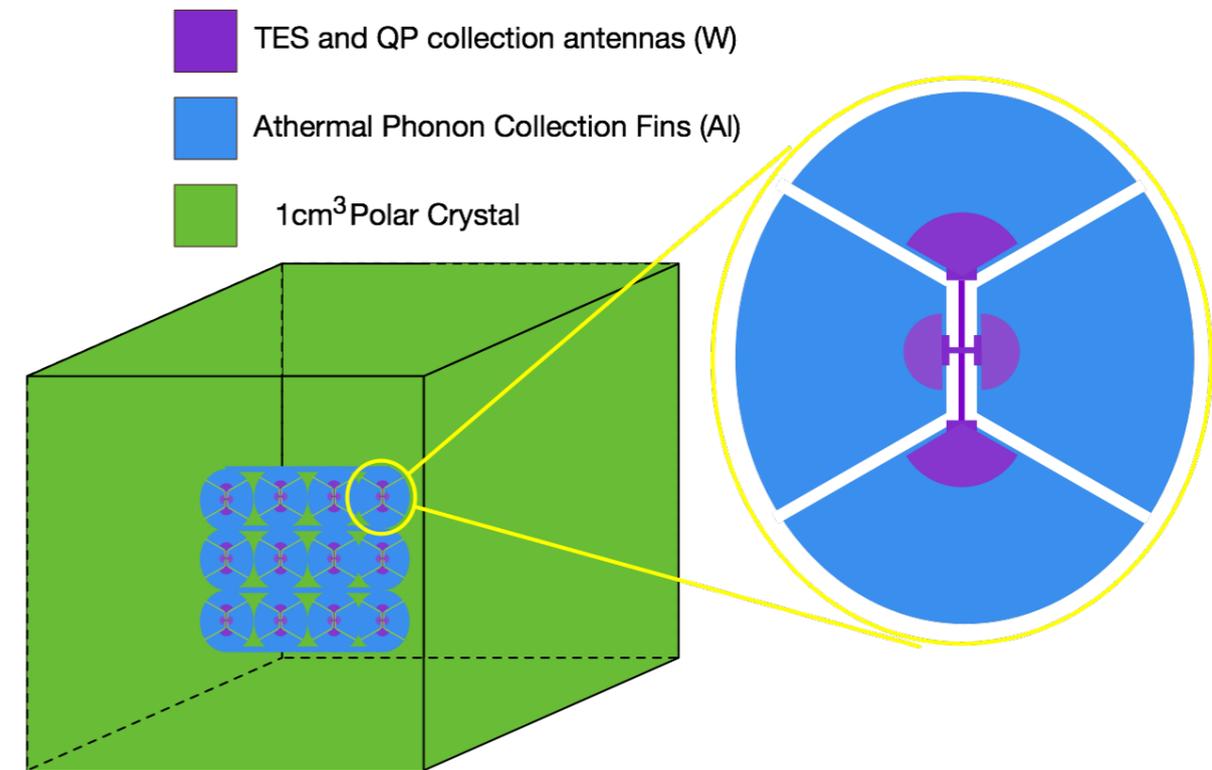
# Experiments under development

HERALD experiment  
(superfluid He)



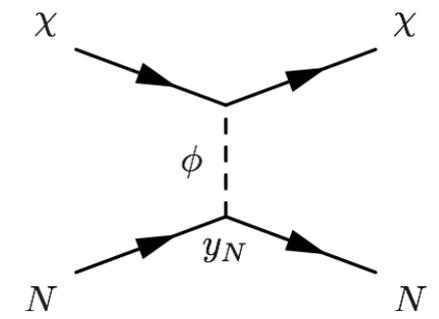
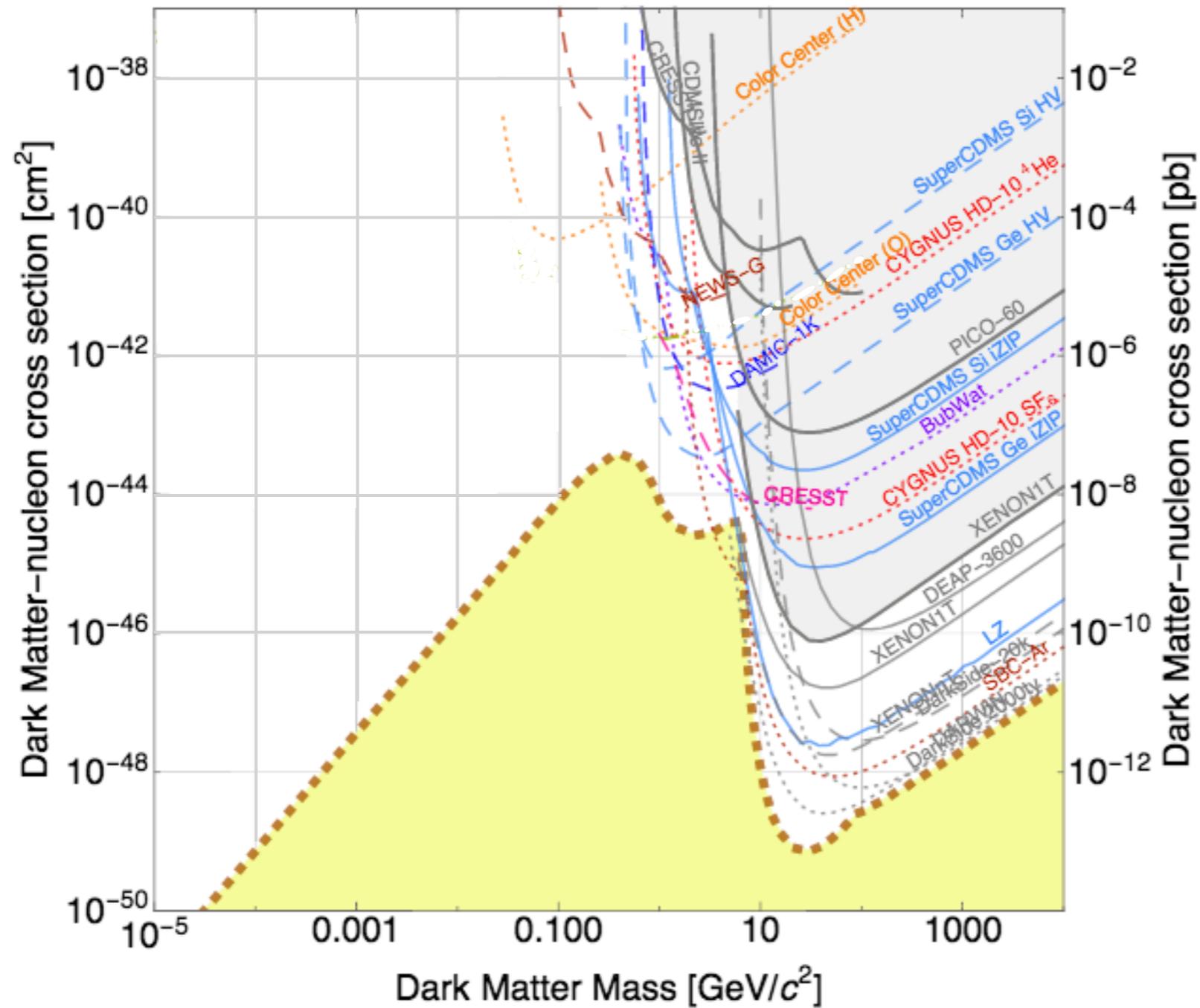
W. Guo, D. McKinsey: 1302.0534

SPICE experiment  
(GaAs, sapphire)



M. Pyle et. al.

# Reach



Superfluid helium can be sensitive down to  $m_\chi \sim 10$  keV

