PULSAR TIMING PROBES OF SMALL SCALE STRUCTURE

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ARXIV:1901.04490
WITH JEFF DROR, TANNER TRICKLE, KATHRYN ZUREK
TO APPEAR
WITH TANNER TRICKLE, KATHRYN ZUREK

NIGHTMARE SCENARIOS FOR 2050

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This is Greenland

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President Kanye West,
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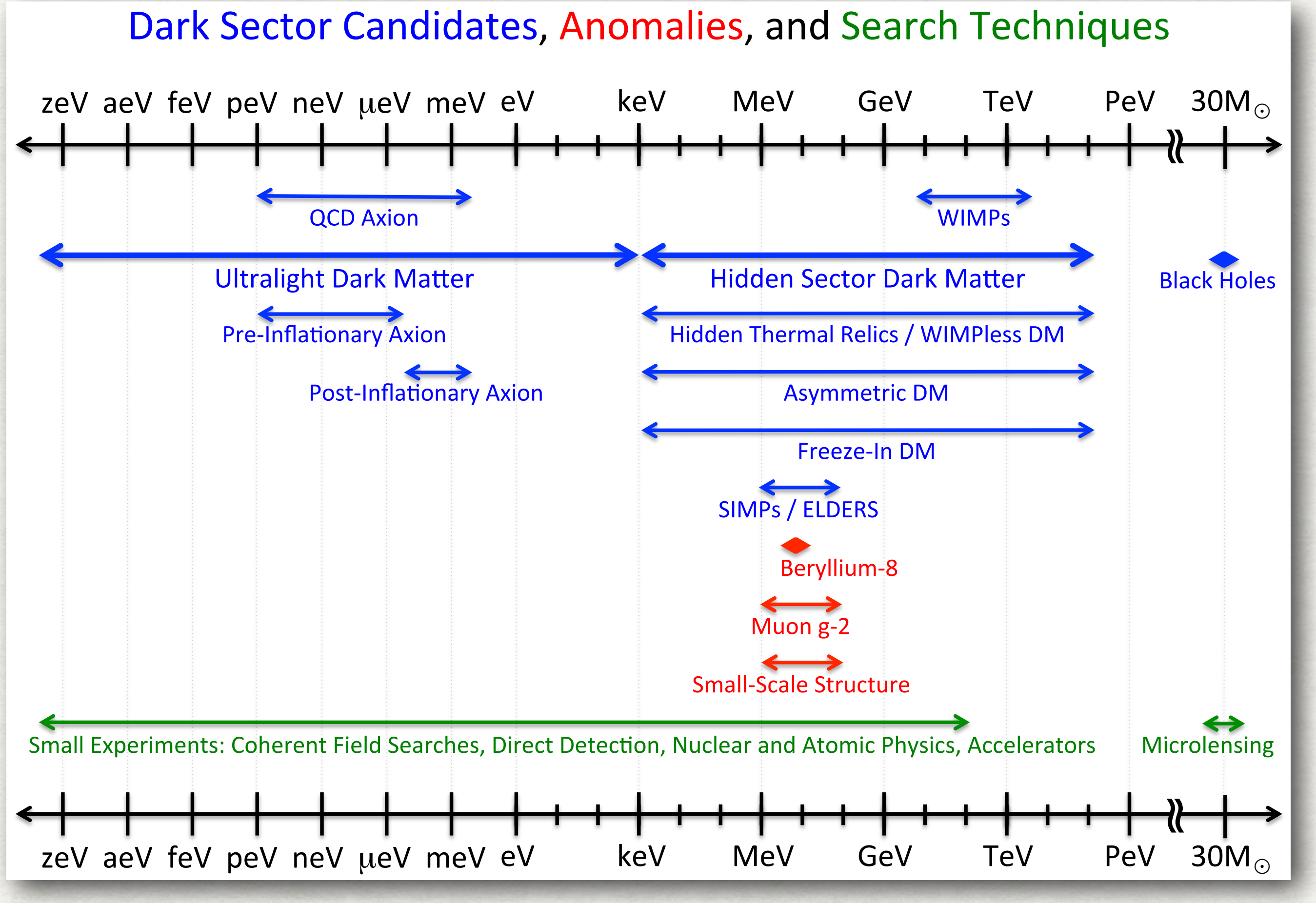
This talk addresses one of these situations

WHAT DO WE KNOW ABOUT DARK MATTER?

- Ample Gravitational Evidence
- No confirmed positive signal in the lamp-post paradigm
- A bevy of promising experiments to probe interactions with SM and several more on the anvil



Cosmic visions

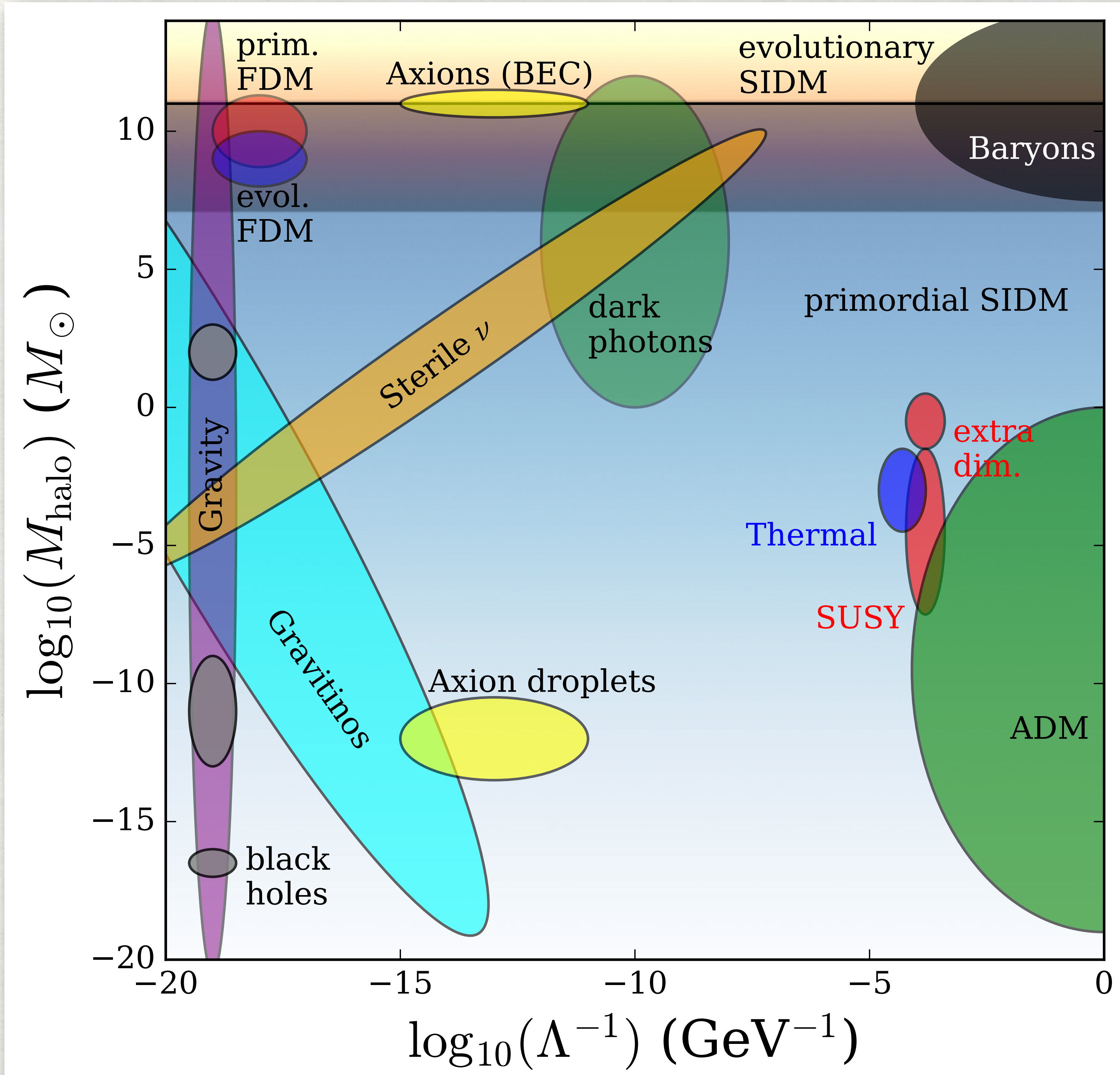
- What about gravitational probes?

GRAVITATIONAL PROBES

Gravitational probes have already provided a wealth of information: sometimes even about particle nature

- Bullet Cluster - self interactions
- Dwarf Galaxies - elementary mass
- Super-radiance, other gravity probes of very light dark matter
- Clues from "small scale" challenges viz. core vs cusp, missing satellites etc.
- How about substructure at even small scales (intra-galactic)?

PROBING SUBSTRUCTURE → PROBING PARTICLE PHYSICS



Source: [Buckley, Peter:1712.06615]

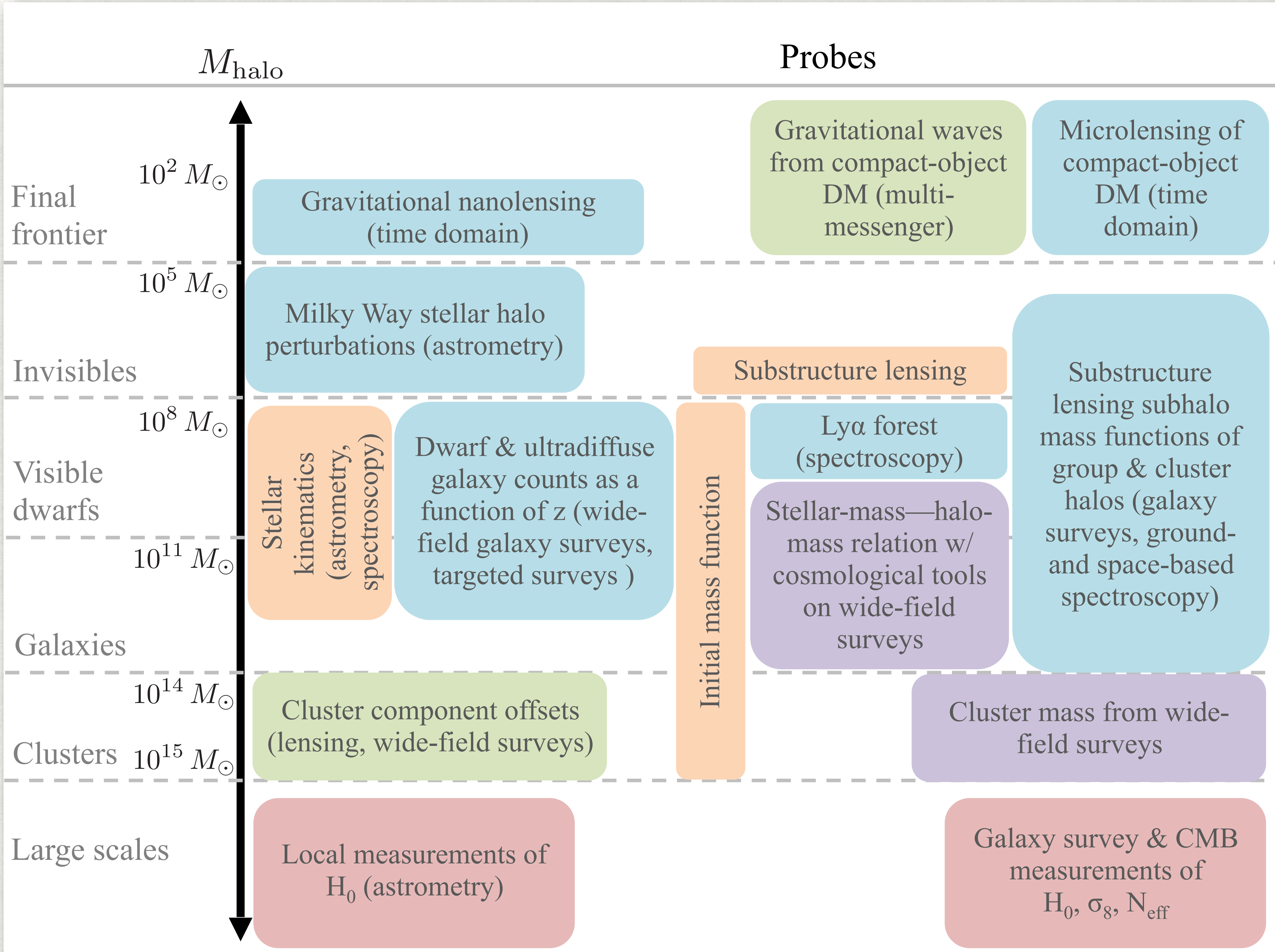
- Most models typically predict an extended halo mass function
- Minimum Halo mass and size set by
- Thermal History
- Collapse Redshift
- Early Matter Domination (See Nikita's talk)
- Inflationary vector production etc
- PQ breaking after inflation

predict enhanced structure in the small scales

SEVERAL UNKNOWNS

- Given an initial power spectrum, what is the substructure today?
- Well posed, hard to solve accurately
- Tidal stripping?
- How much of the DM is still in these miniclusters?
- Answers to these are important for direct detection too.

PROBES OF DIFFERENT MASSES



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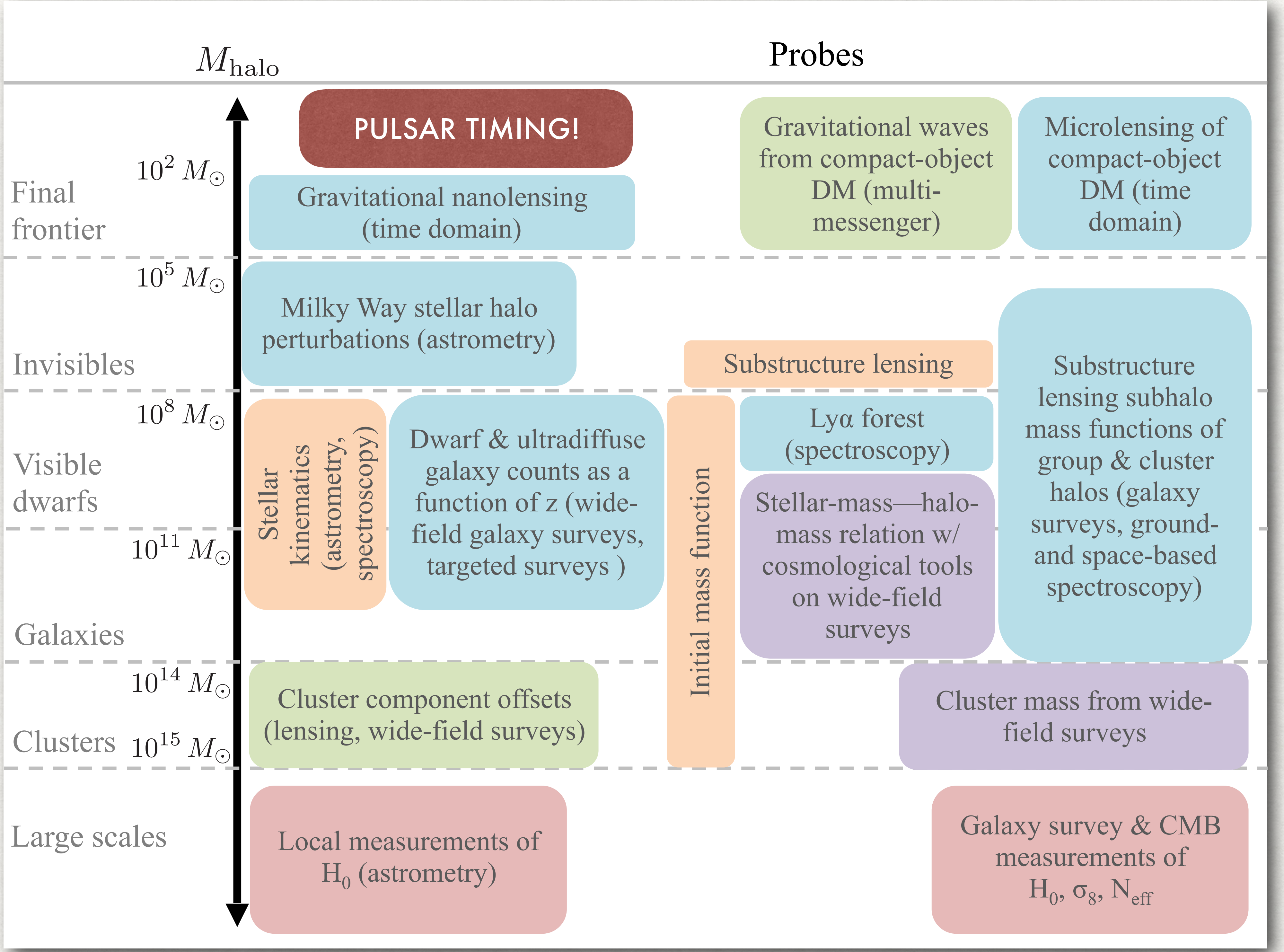
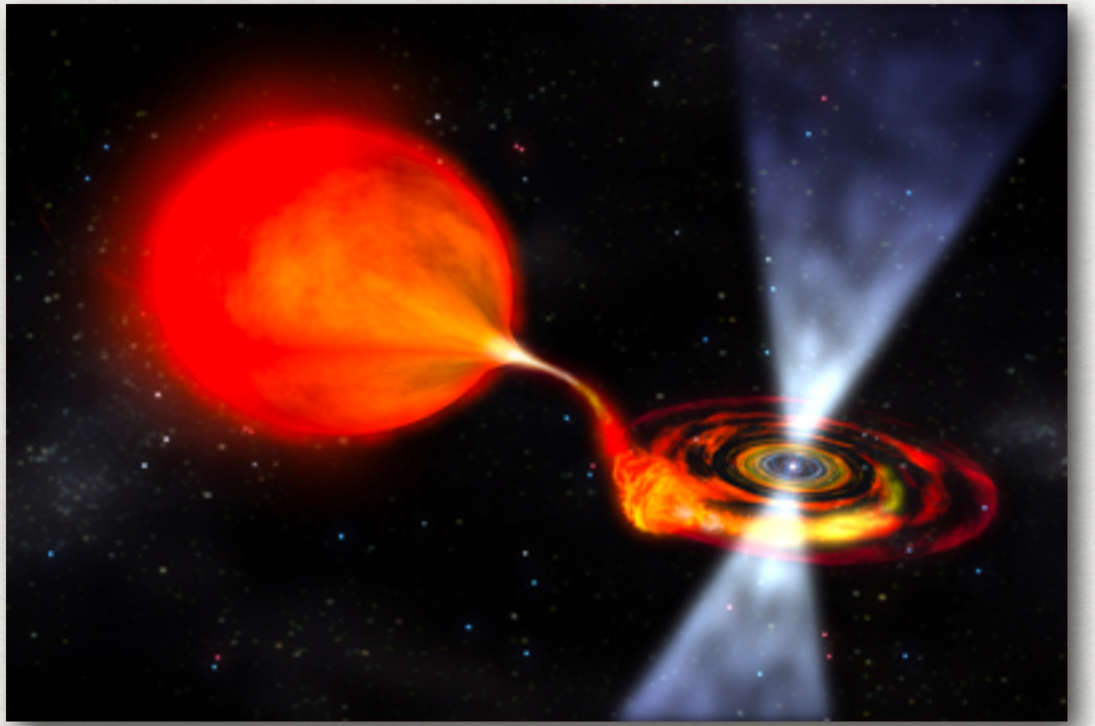


Source: [Buckley, Peter:1712.06615]

MILLI-SECOND PULSARS



- Neutron stars sped up through accretion.
- Fastest rotating pulsars have frequencies of a few kHz.
- Stable over remarkable time-scales (T>20 years)
- Accurate timing models exist

PULSAR TIMING

- Millisecond pulses ~ rotation period
- Phase: $\phi(t) = \phi_0 + \nu\, t + \frac{1}{2}\dot{\nu}\, t^2 + \frac{1}{6}\ddot{\nu}\, t^3 + \ldots$
- ν ~ kHz
- $\dot{\nu}/\nu$ ~ 10^{-23} to 10^{-20} Hz
- $\ddot{\nu}/\nu$ < 10^{-31} Hz^2, not included in fits
- After fitting away the period and derivative, residuals are remarkably small* (and stable).

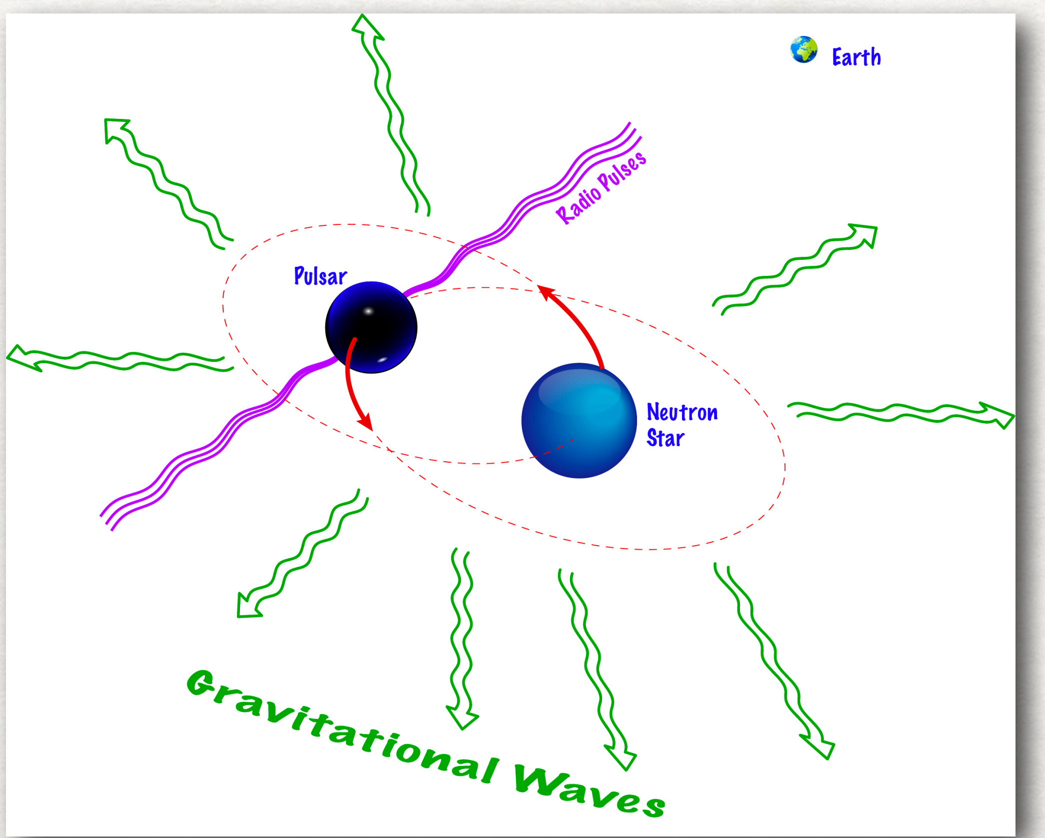
$$t_{\mathrm{RMS}} \equiv \sqrt{\frac{1}{N}\sum_n (t_n^{\mathrm{data}} - t_n^{\mathrm{fit}})^2}$$ ~ 50 ns

*in reality, some other delays, shall describe a relevant few later

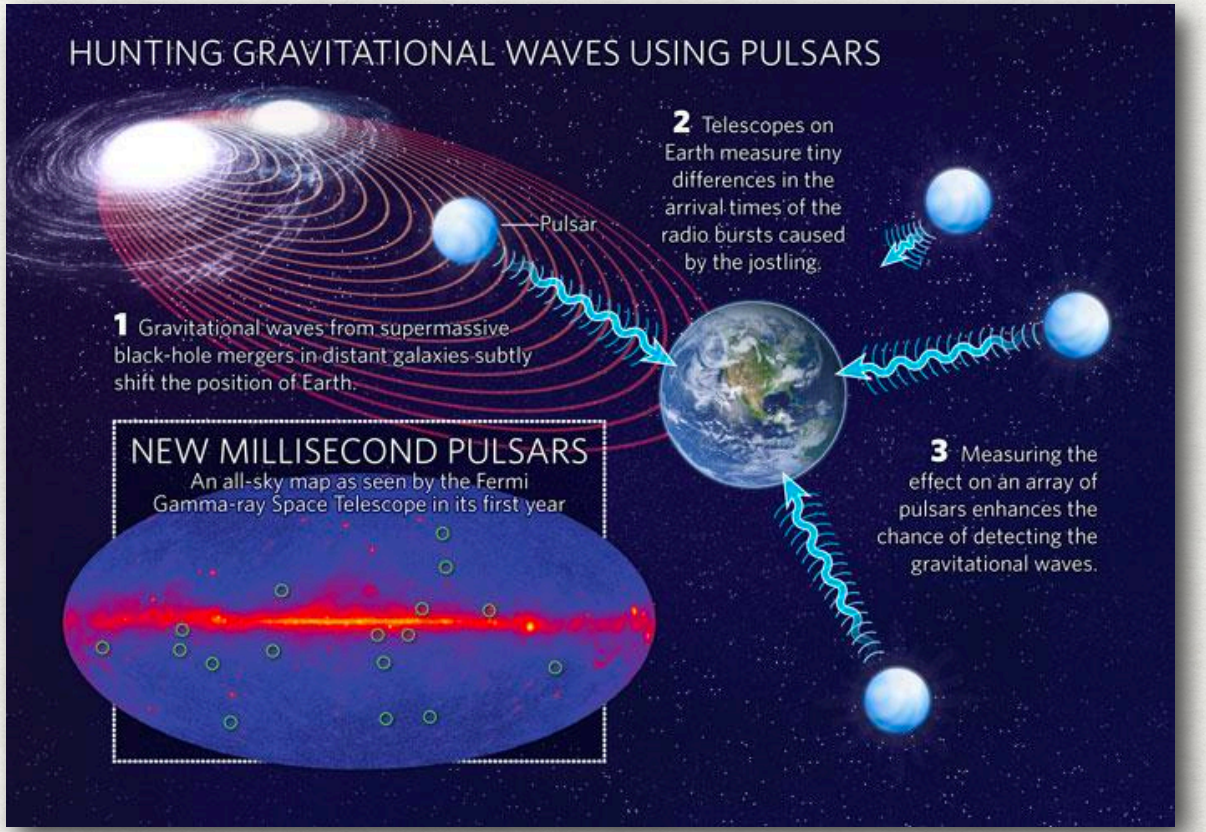
NEW PHYSICS FROM PULSAR TIMING

- Any new physics that predicts time dependent $\delta\phi \equiv \int dt\, \delta\nu(t)$ can possibly be constrained.

Hulse-Taylor binary used as a test of GR through its contracting orbit



Can be used as an extremely low frequency GW detector



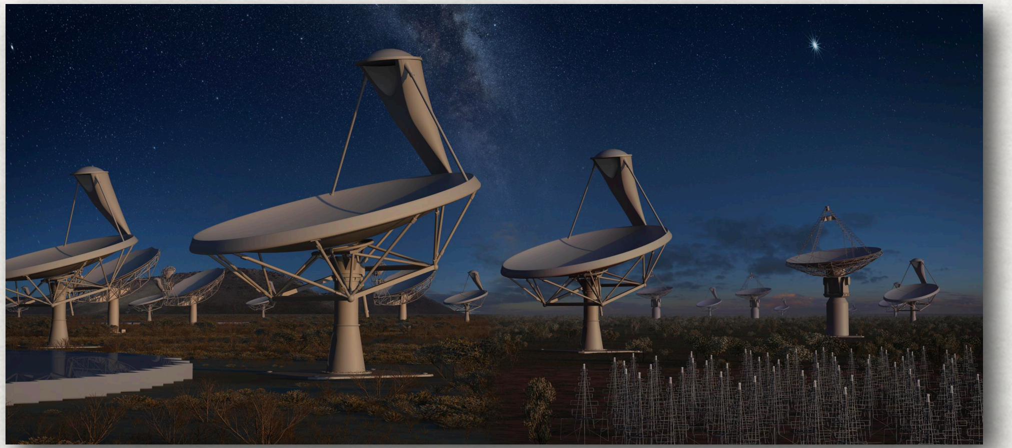
PTA COLLABORATIONS

Today

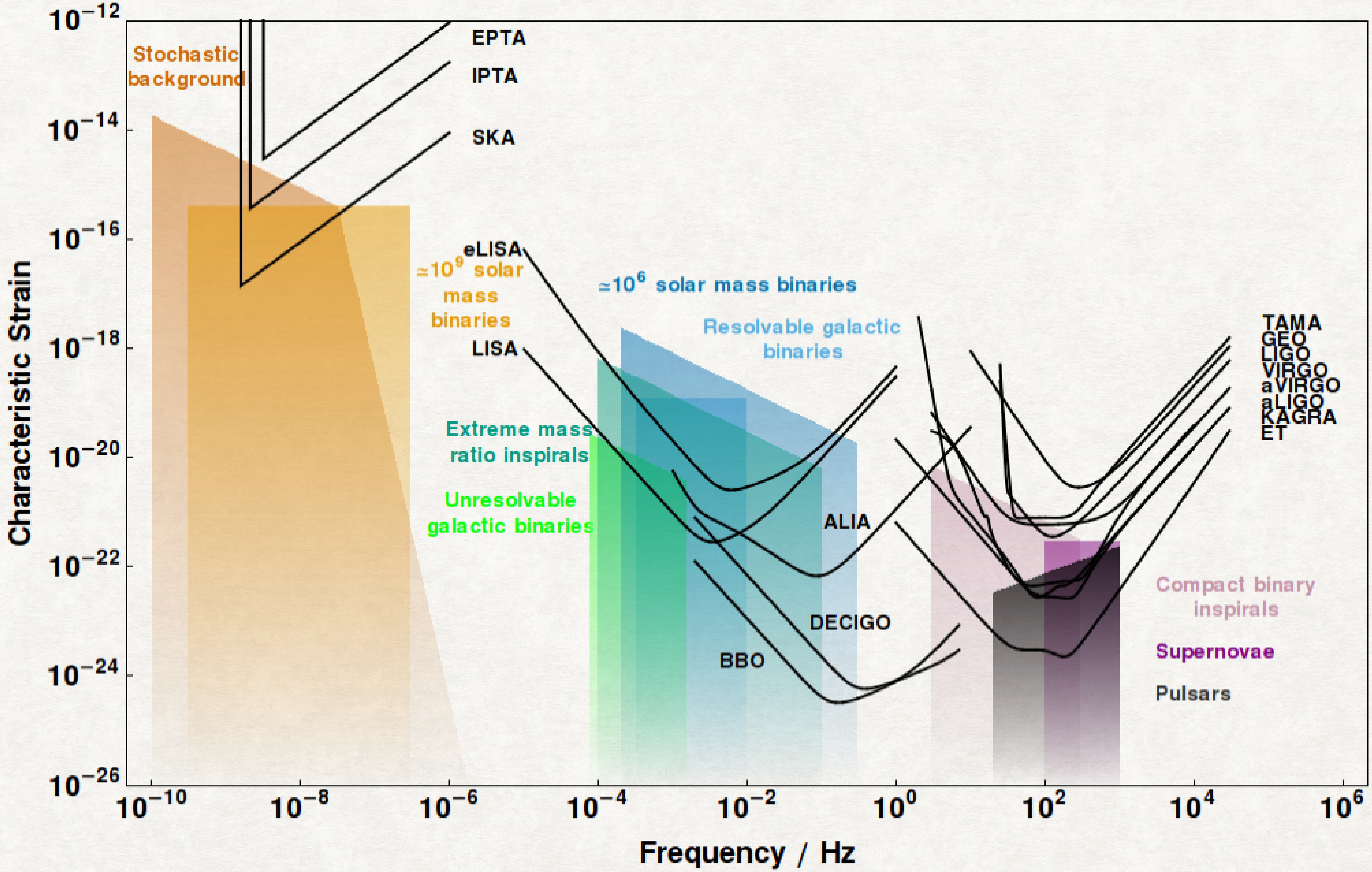
- Np ~ 73
- T =10 to 20 years

Future

- Several precursors currently running
- Np ~ 200-1000
- Projected to start ~ 2030
- T =20+ years



PTA VS OTHER GRAVITY WAVE DETECTORS



Moore, Cole, Berry

PTA FOR DARK MATTER

- Gravitational probes are broadly of two varieties
- Either probe gravitational interaction of DM with some test mass. e.g. Carney, Ghosh, Krnjaic, Taylor. arXiv:1903.00492
- Or probe gravitational interaction between light and DM, e.g. Lensing/ LIGO etc
- PTAs have both kinds of signal (see also 1804.01991 van Tilburg, Taki, Weiner for larger masses with astrometry instead)

PTA FOR DARK MATTER

- Ultralight DM causing GW like delays
- [Khmelnitsky, Rubakov - 1309.5888], [Graham, Kaplan, Mardon, Rajendran, Terrano - 1512.06165]
- PTAs are sensitive accelerometers: Doppler Delay
- [Seto,Corray - astro-ph/0702586] , [Baghram,Afshordi,Zurek - 1101.5487] [Kashiyama, Seto - 1208.4101],[Kazumi, Oguri, Masamune - 1801.07847]
- Gravitational potential wells along the light path: Shapiro Delay
- [Siegel, 0801.3458], [Siegel,Hertzberk,Fry - astro-ph/0702546], [Baghram,Afshordi,Zurek - 1101.5487], [Clark,Lewis,Scott - 1509.02938] ,[Schutz, Liu - 1610.04234]
- This Work:
- Explicit calculations for SNR calculations for Doppler and Shapiro Delays
- Comprehensive analysis of both Doppler/Shapiro for all signal durations
- Extending analysis to diffuse halos

DOPPLER DELAY

- Recognize the ratio $\frac{\delta\nu}{\nu}$ is v_{rel}/c
- Thus sensitive to tiny accelerations

$$\left(\frac{\delta\nu}{\nu}\right)_D = \hat{\mathbf{d}} \cdot \int \nabla\Phi \, dt,$$

- velocity shape for a point object transit looks like:

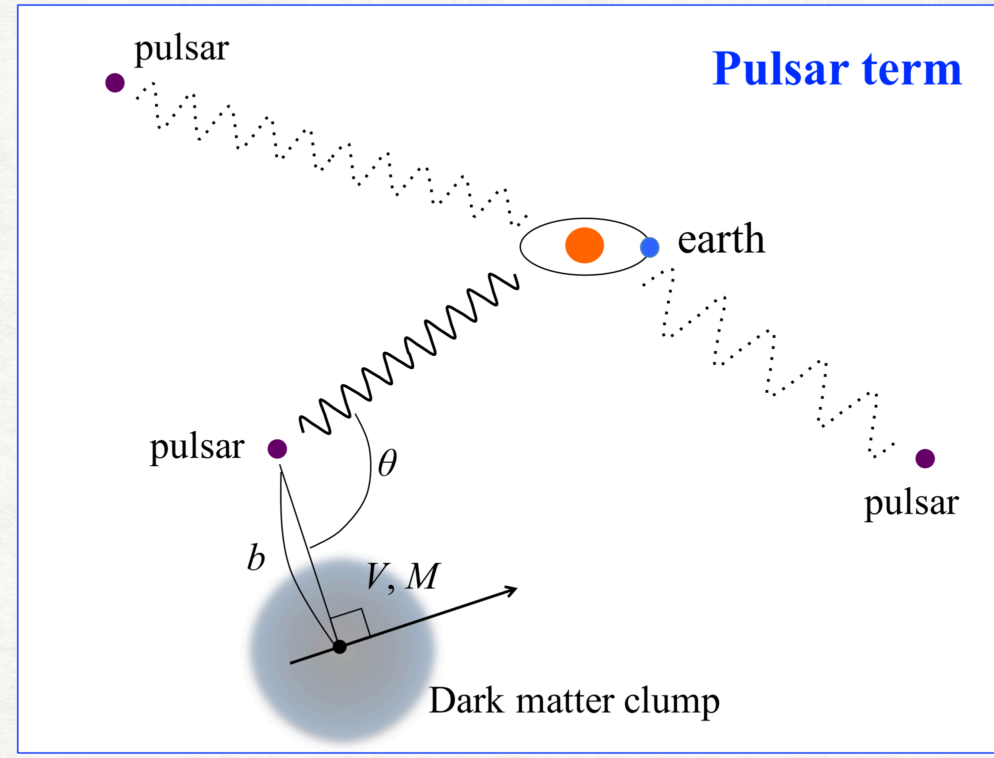
$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{GM}{v^2\tau_D} \frac{1}{\sqrt{1+x_D^2}} \left(x_D\hat{\mathbf{b}} - \hat{\mathbf{v}}\right) \cdot \hat{\mathbf{d}}$$

Signal period

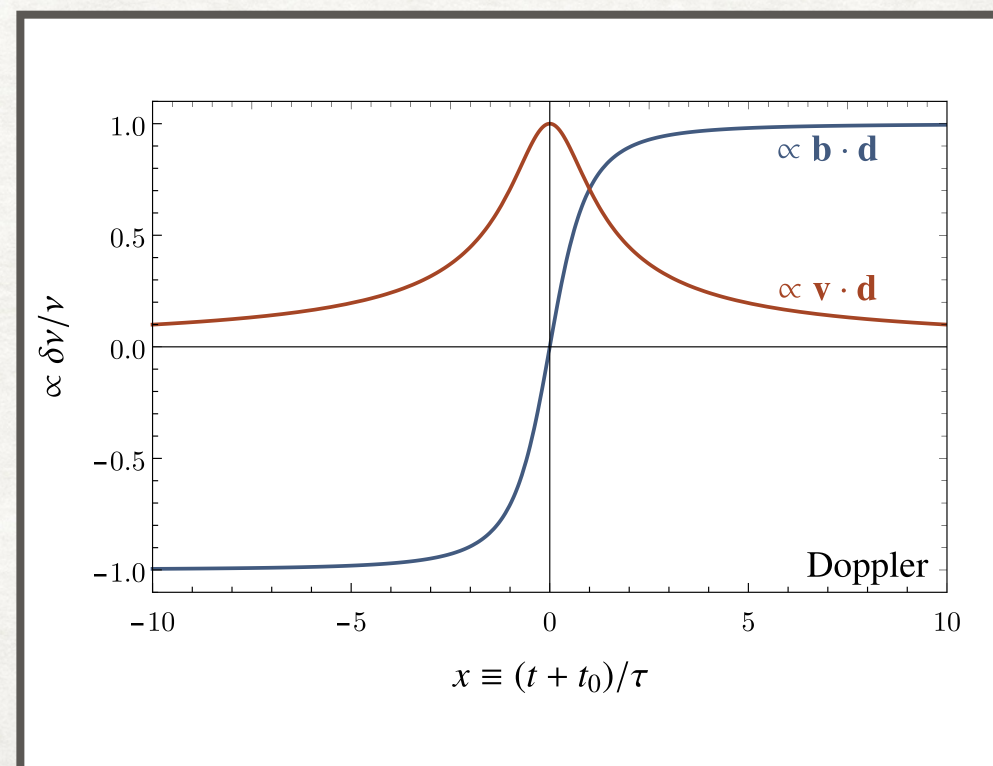
Dimensionless time variable

Impact parameter

$$|\mathbf{b}| = \tau v$$



Source: Kashiyama, Seto - 1208.4101



SHAPIRO DELAY

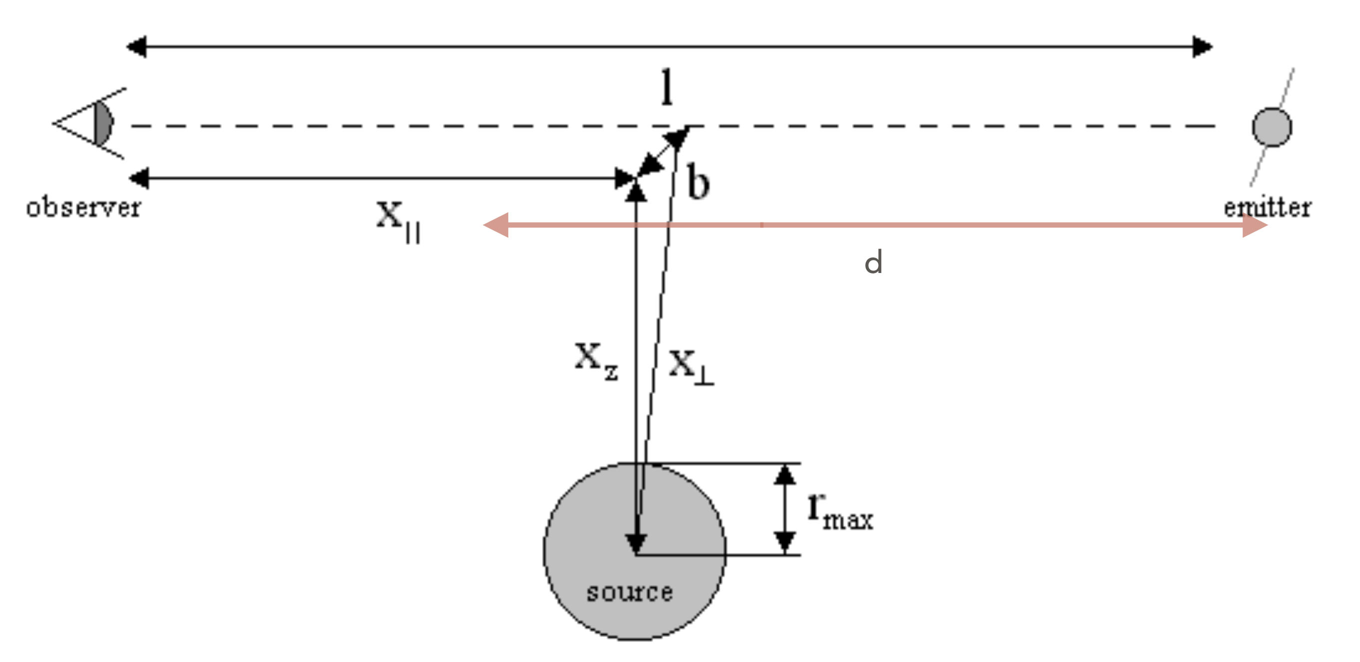
- Similar to Sachs-Wolfe effect
- In frequency domain given by,

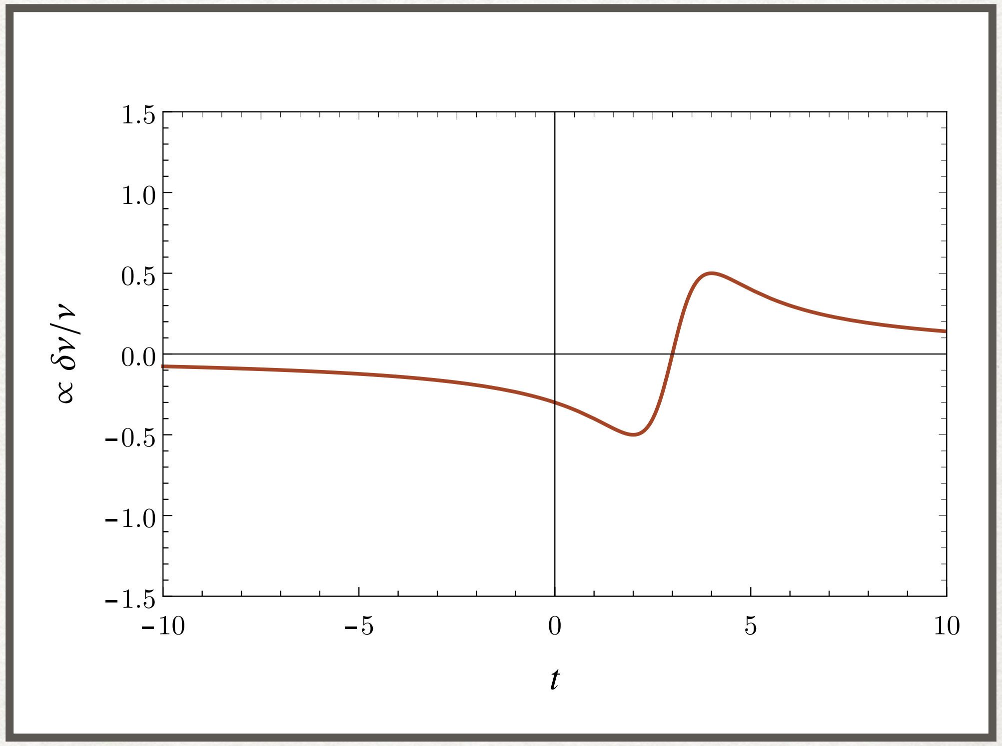
$$\left(\frac{\delta\nu}{\nu}\right)_S = -2\int \mathbf{v}\cdot\nabla\Phi\, dz$$

- For a point object,
- $$\left(\frac{\delta\nu}{\nu}\right)_S = \frac{4GM}{\tau_S}\frac{x_S}{1+x_S^2}$$

Duration of signal

Dimensionless time variable





DYNAMIC VS STATIC

Characteristic signal period

Dynamic if $$\Delta t \ll \tau < t_0 < T - \tau$$

Cadence

Total Time of observation

Static otherwise

$$\tau \gtrsim T$$

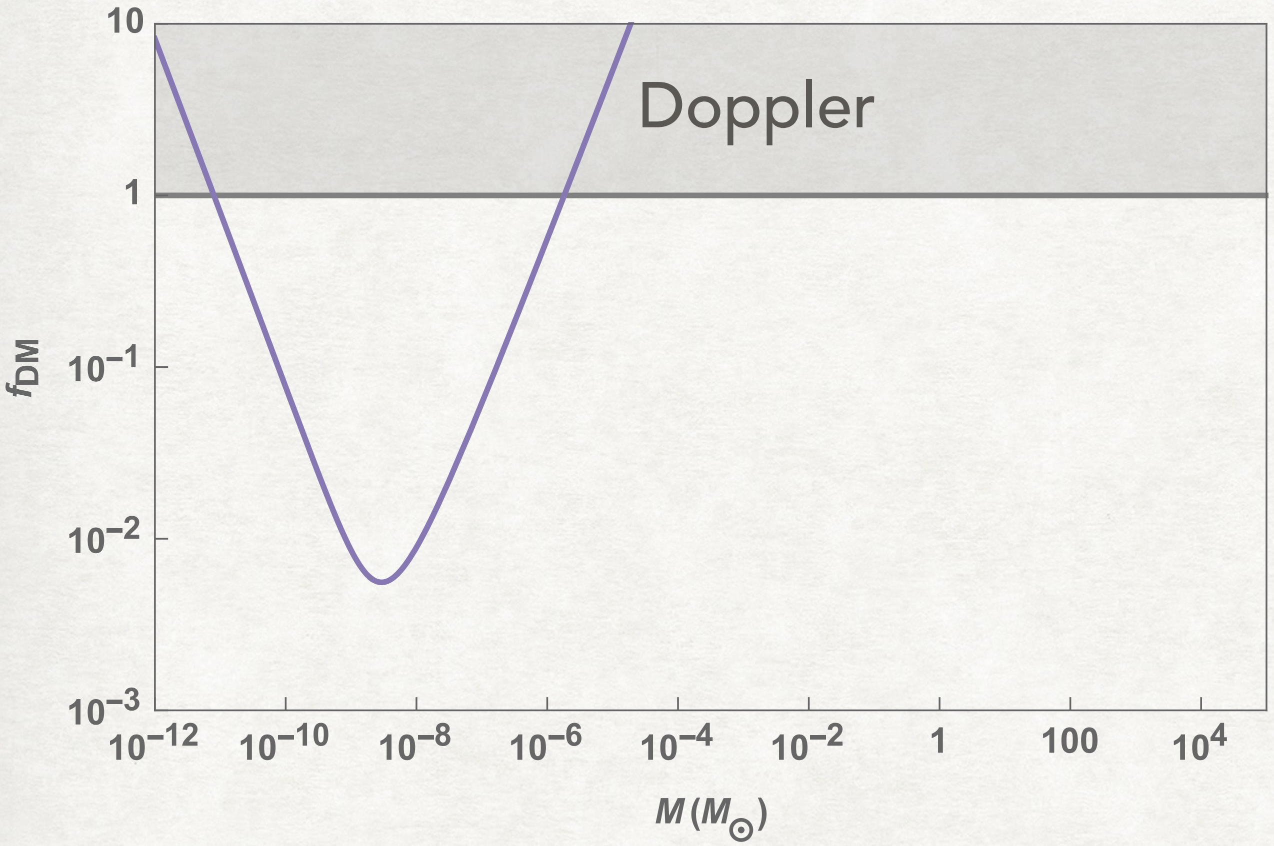
DETECTING SIGNALS

- Dynamic: Similar to a bump hunt / LIGO signal / Microlensing signal
- Single event shape and amplitude, straightforward filtering.
- Static: Works only if a second derivative continues to not be measured.

START WITH MONOCHROMATIC POINT MASSES

BOUNDS FROM DYNAMIC SIGNALS

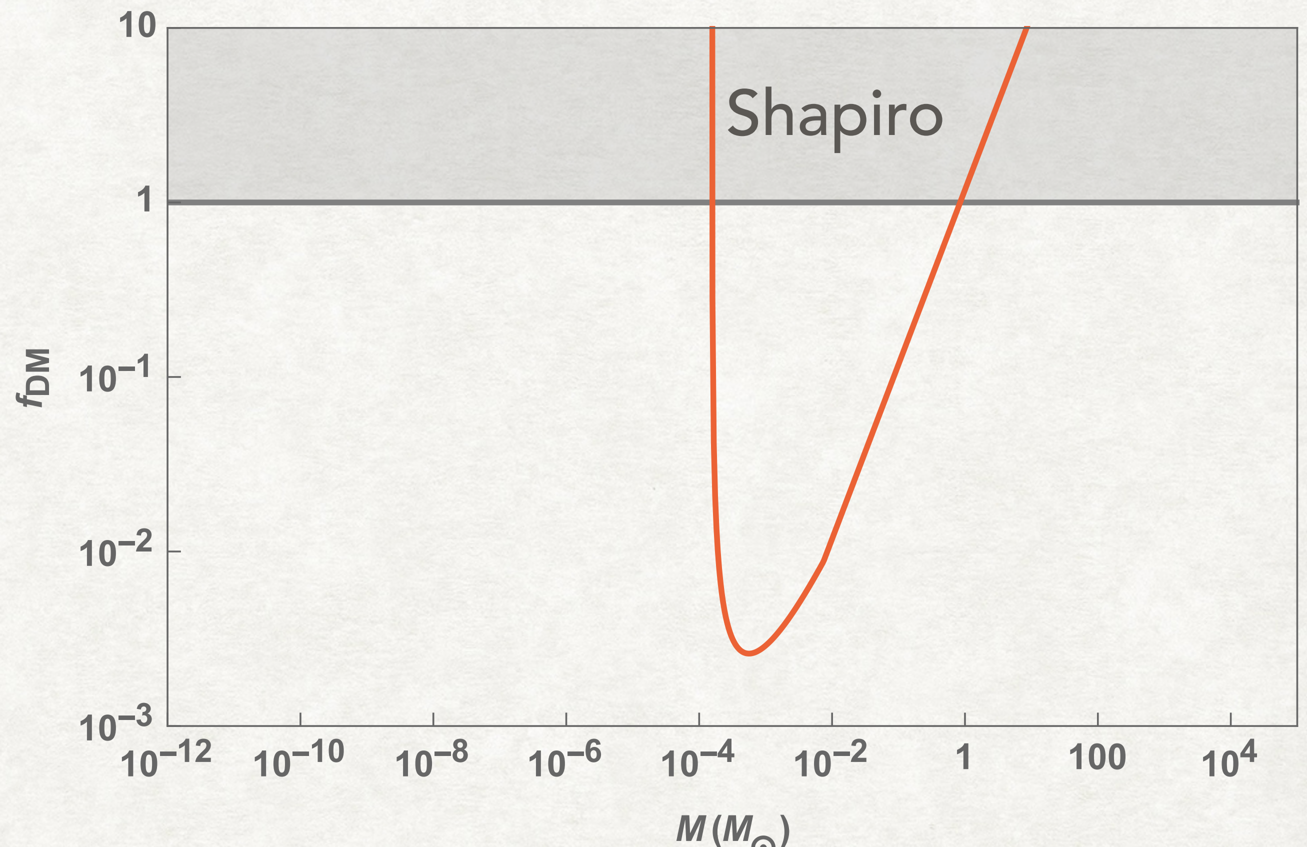
$$\mathrm{SNR}_D = \frac{1}{2\sqrt{3}} \frac{G M T^{\frac{3}{2}}}{c\, t_{\mathrm{rms}} v^2 \sqrt{\Delta t}\, \tau}$$



Small sampling volume
localized around pulsar/earth

- Just like any direct detection experiment
- LHS: closest event below threshold
- RHS: closest event takes too long

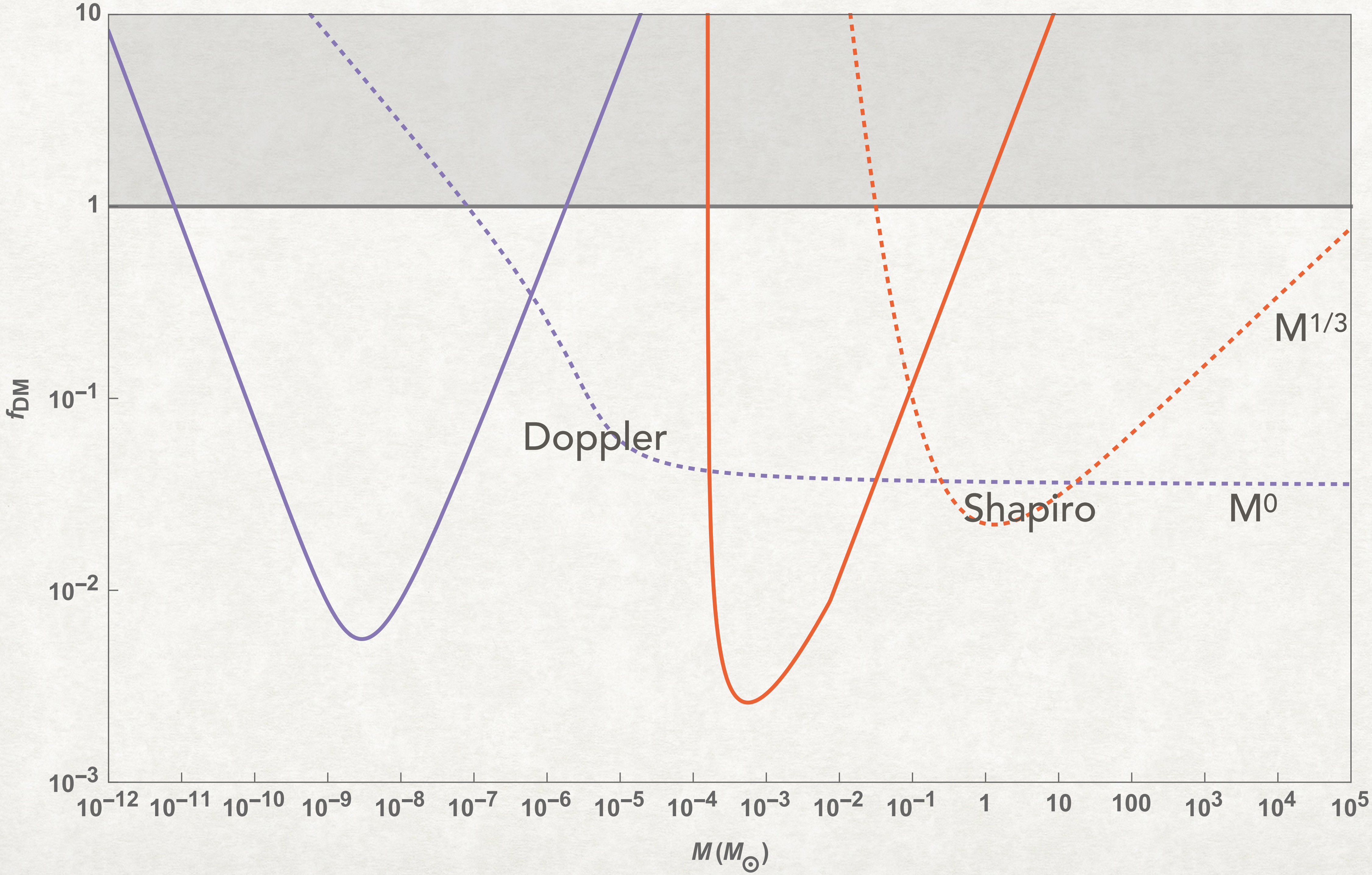
Large sampling volume
Along the whole light path



$$\mathrm{SNR} = 4 \frac{GM}{c^6 t_{\mathrm{rms}}} \sqrt{\frac{T}{\Delta t}}$$

FRACTION VS M SCALING -STATIC

Could also look for a static signal: i.e. a non-zero second derivative



MAJOR BACKGROUNDS

BARYONS

The Cosmic Energy Inventory

Parameter		Components[a]	Totals[a]
Dark sector:			0.954 ± 0.003
Dark energy		0.72 ± 0.03	
Dark matter		0.23 ± 0.03	
Primeval gravitational waves		$\lesssim 10^{-10}$	
Primeval thermal remnants:			0.0010 ± 0.0005
Electromagnetic radiation		$10^{-4.3 \pm 0.0}$	
Neutrinos		$10^{-2.9 \pm 0.1}$	
Prestellar nuclear binding energy		$-10^{-4.1 \pm 0.0}$	
Baryon rest mass:			0.045 ± 0.003
Warm intergalactic plasma		0.040 ± 0.003	
Virialized regions of galaxies	0.024 ± 0.005		
Intergalactic	0.016 ± 0.005		
Intracluster plasma		0.0018 ± 0.0007	
Main-sequence stars: spheroids and bulges		0.0015 ± 0.0004	
Main-sequence stars: disks and irregulars		0.00055 ± 0.00014	
White dwarfs		0.00036 ± 0.00008	
Neutron stars		0.00005 ± 0.00002	
Black holes		0.00007 ± 0.00002	
Substellar objects		0.00014 ± 0.00007	
H I + He I		0.00062 ± 0.00010	
Molecular gas		0.00016 ± 0.00006	
Planets		10^{-6}	
Condensed matter		$10^{-5.6 \pm 0.3}$	
Sequestered in massive black holes		$10^{-5.4}(1 + \epsilon_n)$	

Most of the baryonic component will also be co-rotating with pulsar / earth

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Static

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Static

Dynamic

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OTHER SOURCES OF BACKGROUND

- Glitches: Sudden increase in frequency, followed by a slow relaxation (days-year).

- Dispersion through interstellar medium

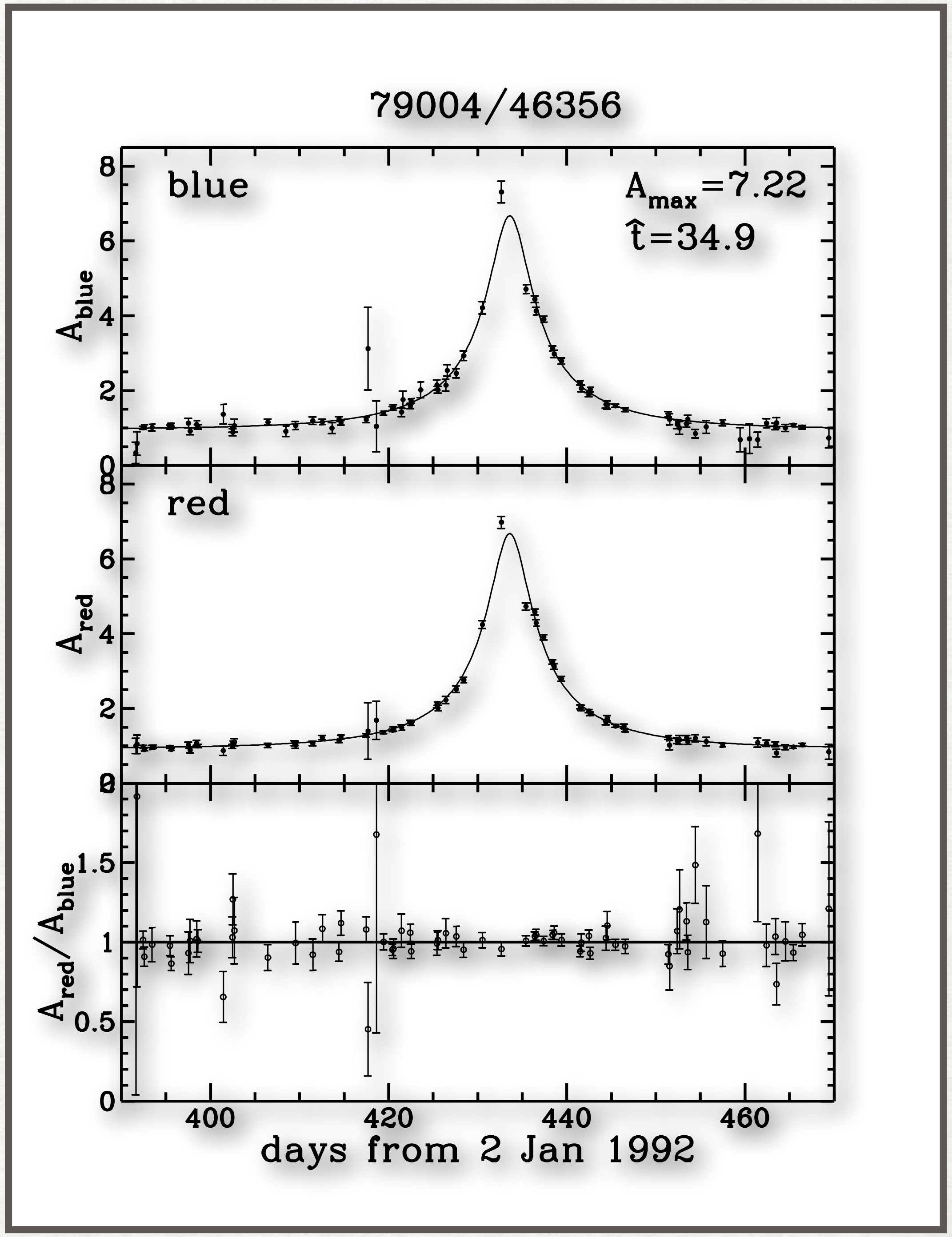
- Pulsars may suffer from intrinsic red noise

- Beyond the scope of a collaboration-independent analysis

- Noise subtractions do not reduce signal when running it through a simulated PTA analysis. (We thank Stephen Taylor of the Nanograv collaboration for this).

DYNAMIC BACKGROUNDS

- Dynamic signal more spectacular than static signal.
- Shape differences could help differentiate from glitches etc.
- Dynamic signals are non-dispersive
- Will not be limited soon by baryonic background.



Dispersion used in Microlensing to differentiate lensing blip from a dispersive blip

PTA TIMELINE

	T [yr]	t_{RMS} [ns]	Δt [wk]	d [kpc]	N_P
Current	$5 - 30$	$50 - 10^4$	$1 - 4$	$0.5 - 5$	73
SKA	20	50	2	5	200
Optimistic	20	25	1	10	1000

With this we consider 4 scenarios:

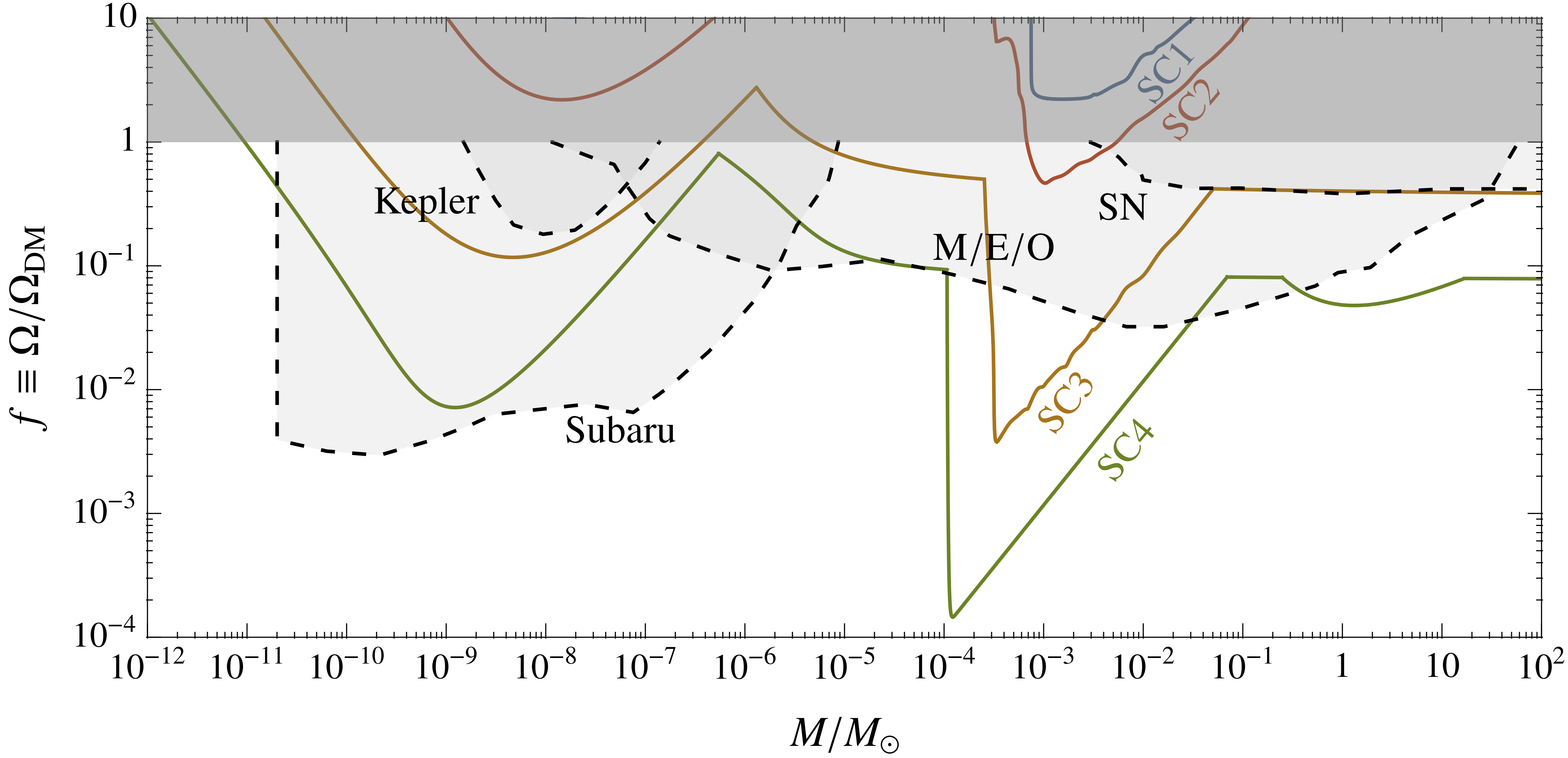
SC1: Estimated limits from current data only

SC2: Current pulsars running for 10 more years

SC3: Assume SKA starts 10 year from now and current pulsars continue

SC4: Optimistic

RESULTS FOR PBH



Lensing constraint from Subaru, Machos, Eros, Ogle (MEO) and SN lensing

IS THIS A SILVER MEDAL?

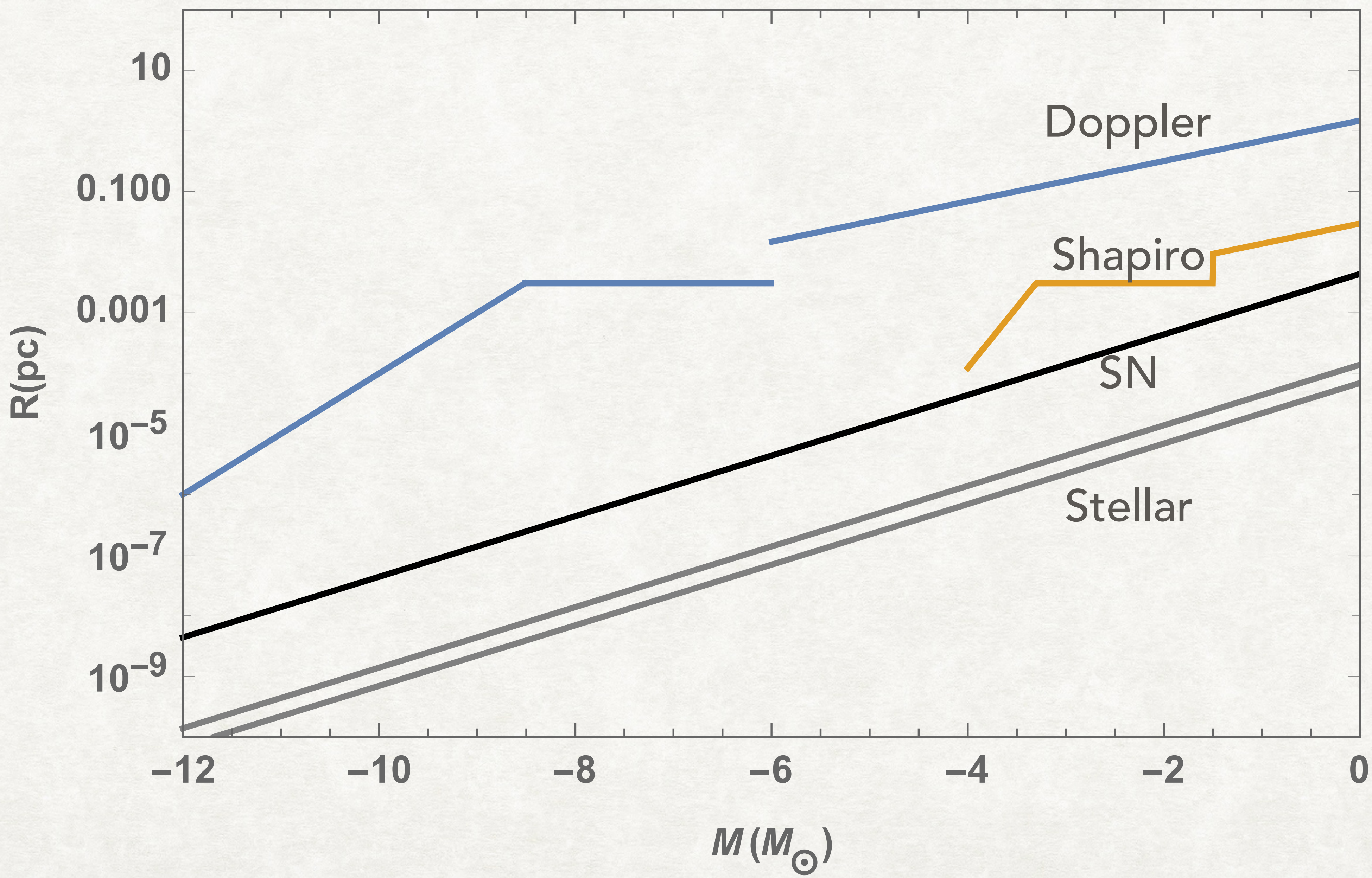
Limits comparable but subdominant to lensing for the most part

MORE DIFFUSE OBJECTS

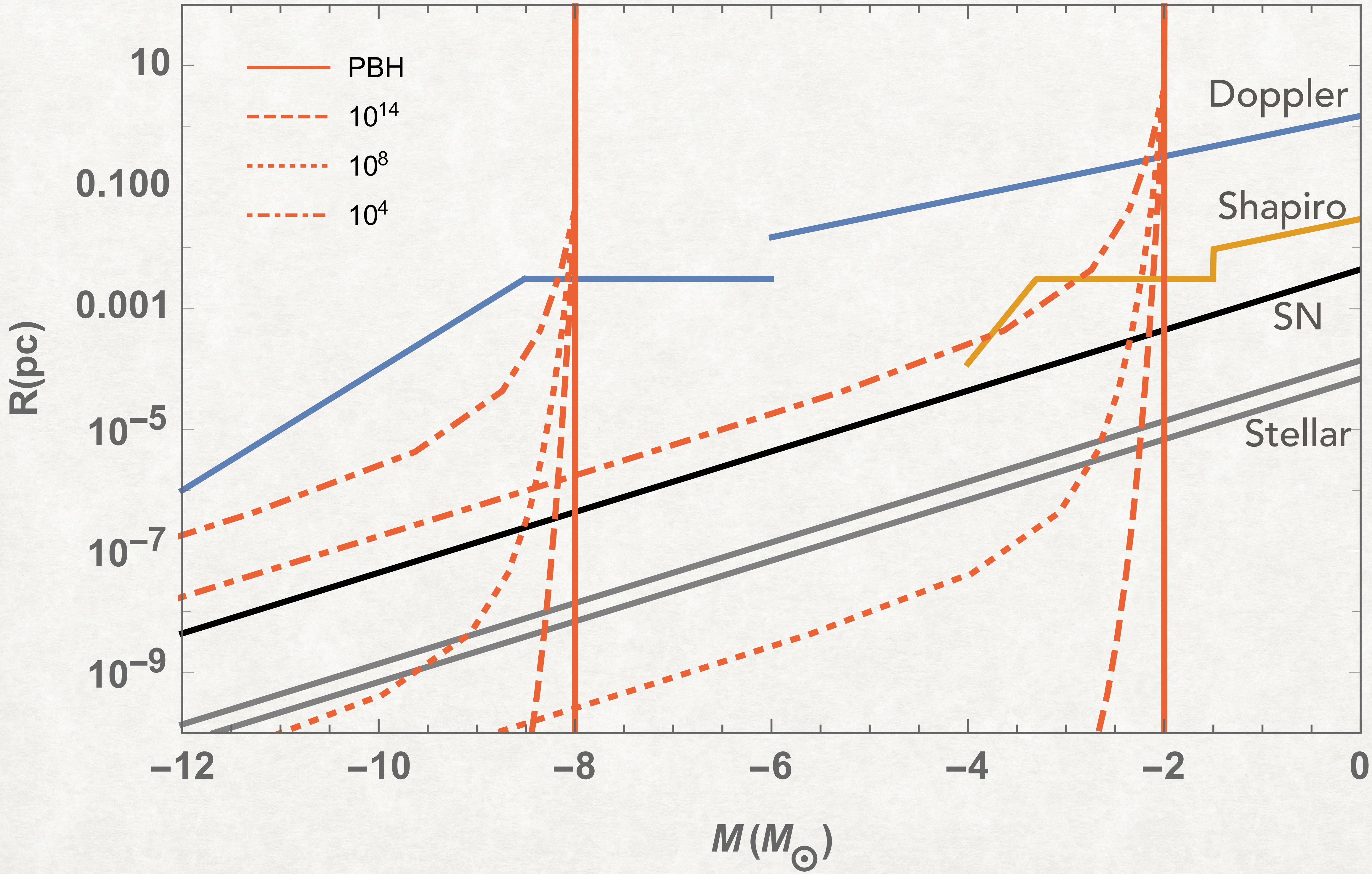
- We have seen point-like objects till now.
- If size of the object < impact parameter, Gauss' law: treat object as point like
- Signal loss if object size > impact parameter.
- Can get conservative estimate with $M_{enc}(b)$.
- For illustrative purposes, NFW halo with clumpiness parametrized by the concentration parameter $c=r_{vir}/r_{c.}$
- retrieve PBH in the large c limit

PROBES VS OBJECT SIZE

- Lensing: (Billion Stars x few hours) small impact parameter
- PTA: (100-1000 pulsars x few years) enormous impact parameter

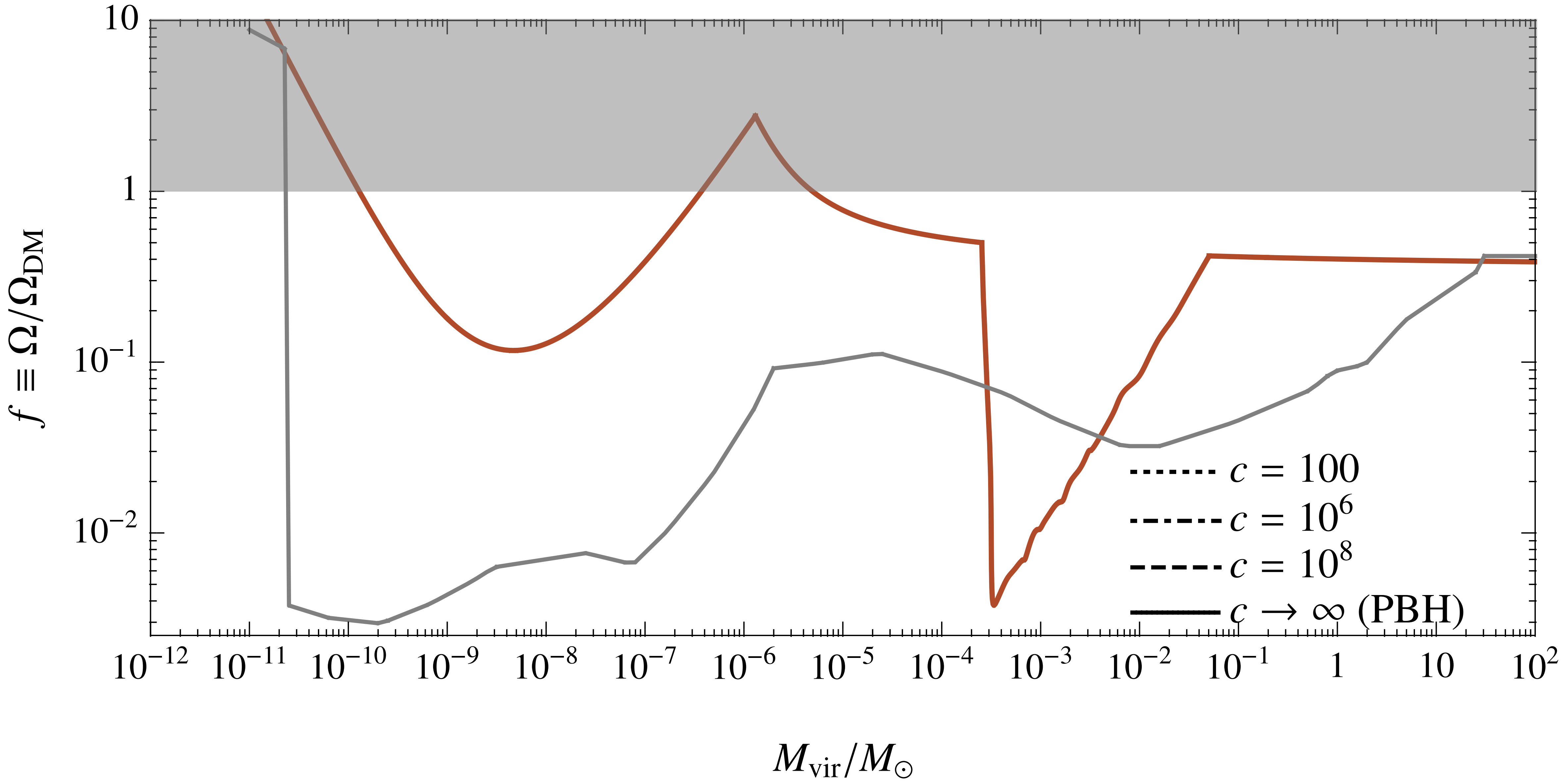


SENSITIVITY TO DIFFUSE HALOS



Limits iff red line intersects a probe radius

LIMITS FOR DIFFUSE OBJECTS



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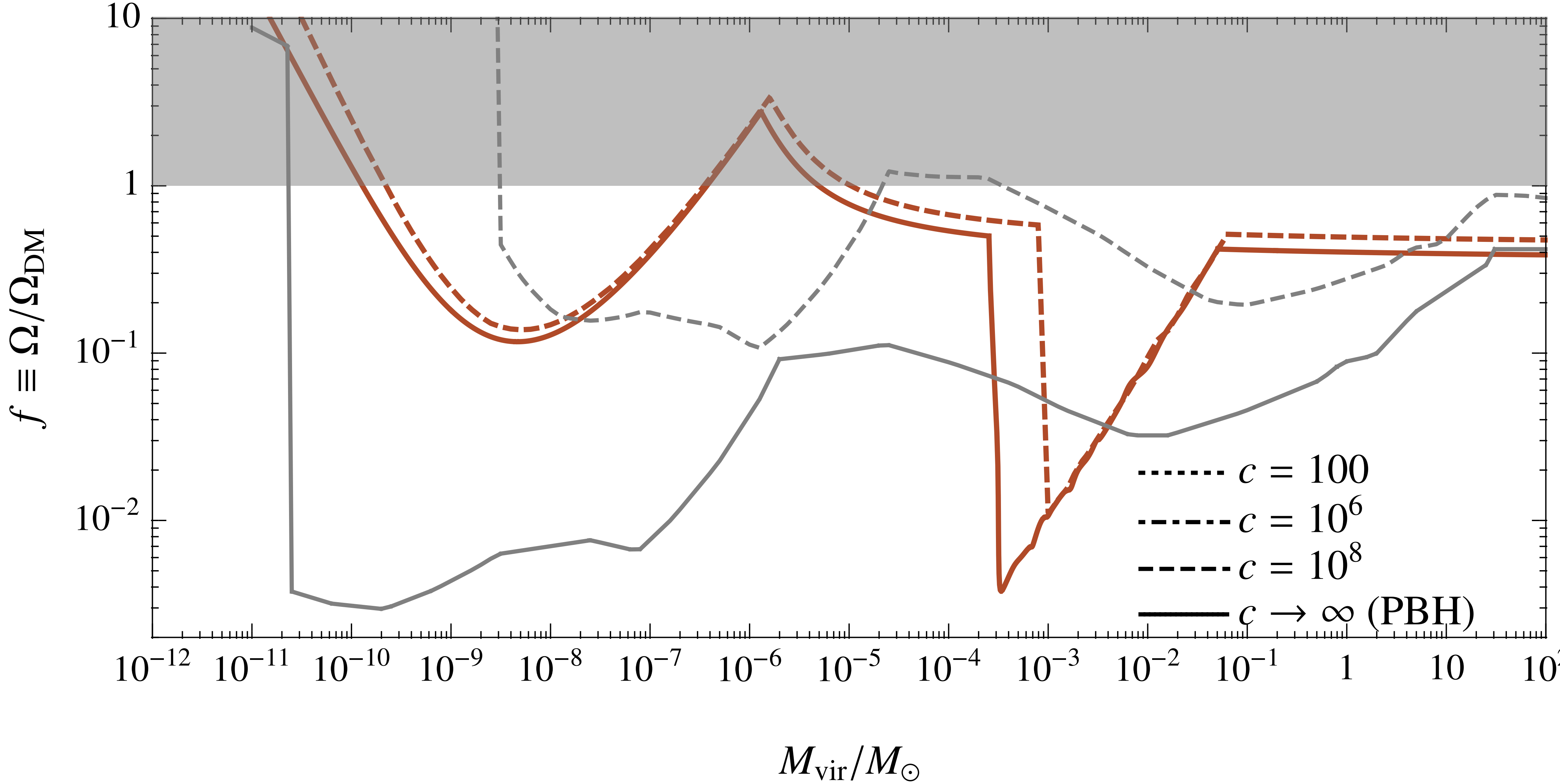

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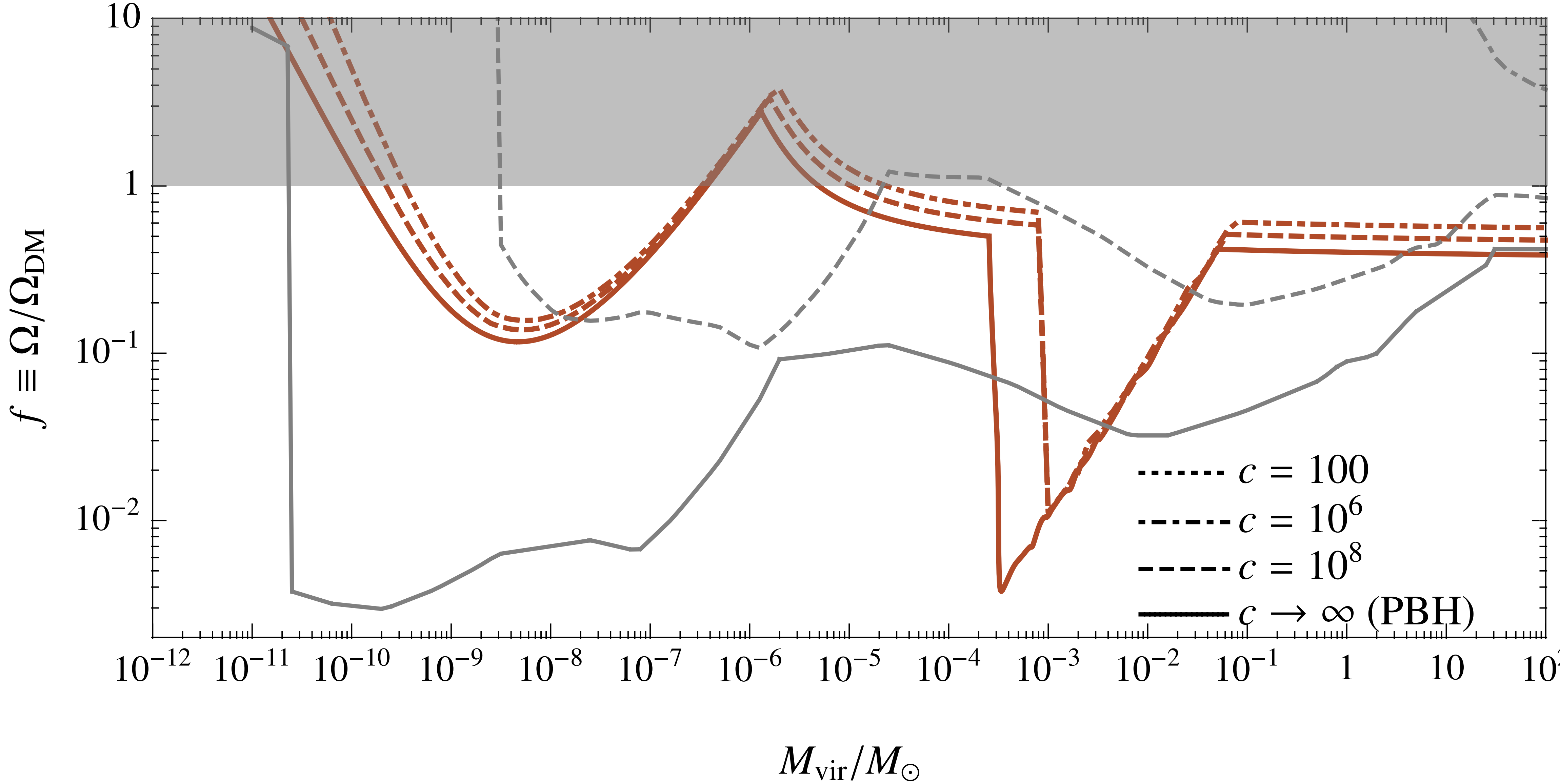

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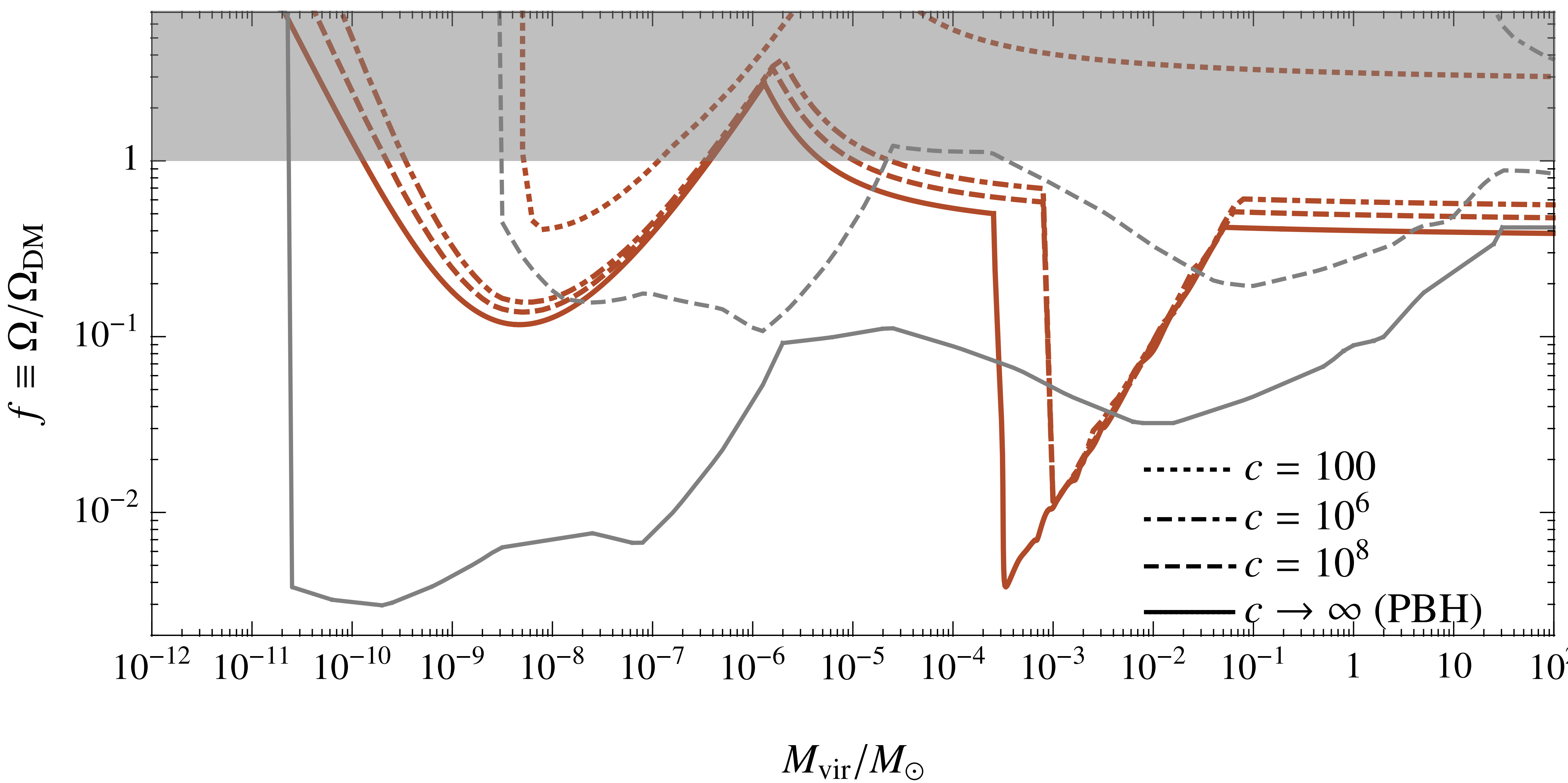
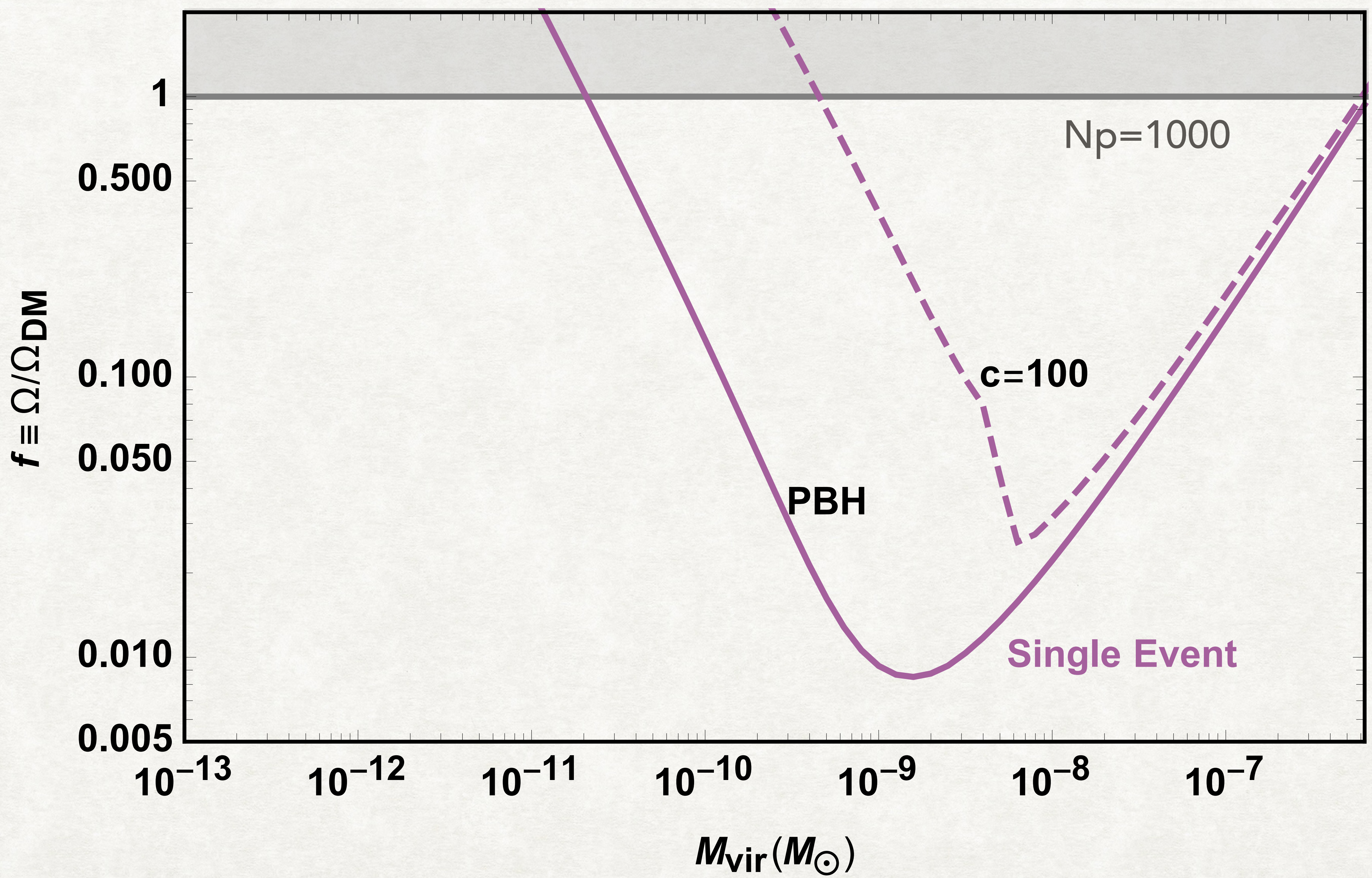


WORK IN PROGRESS

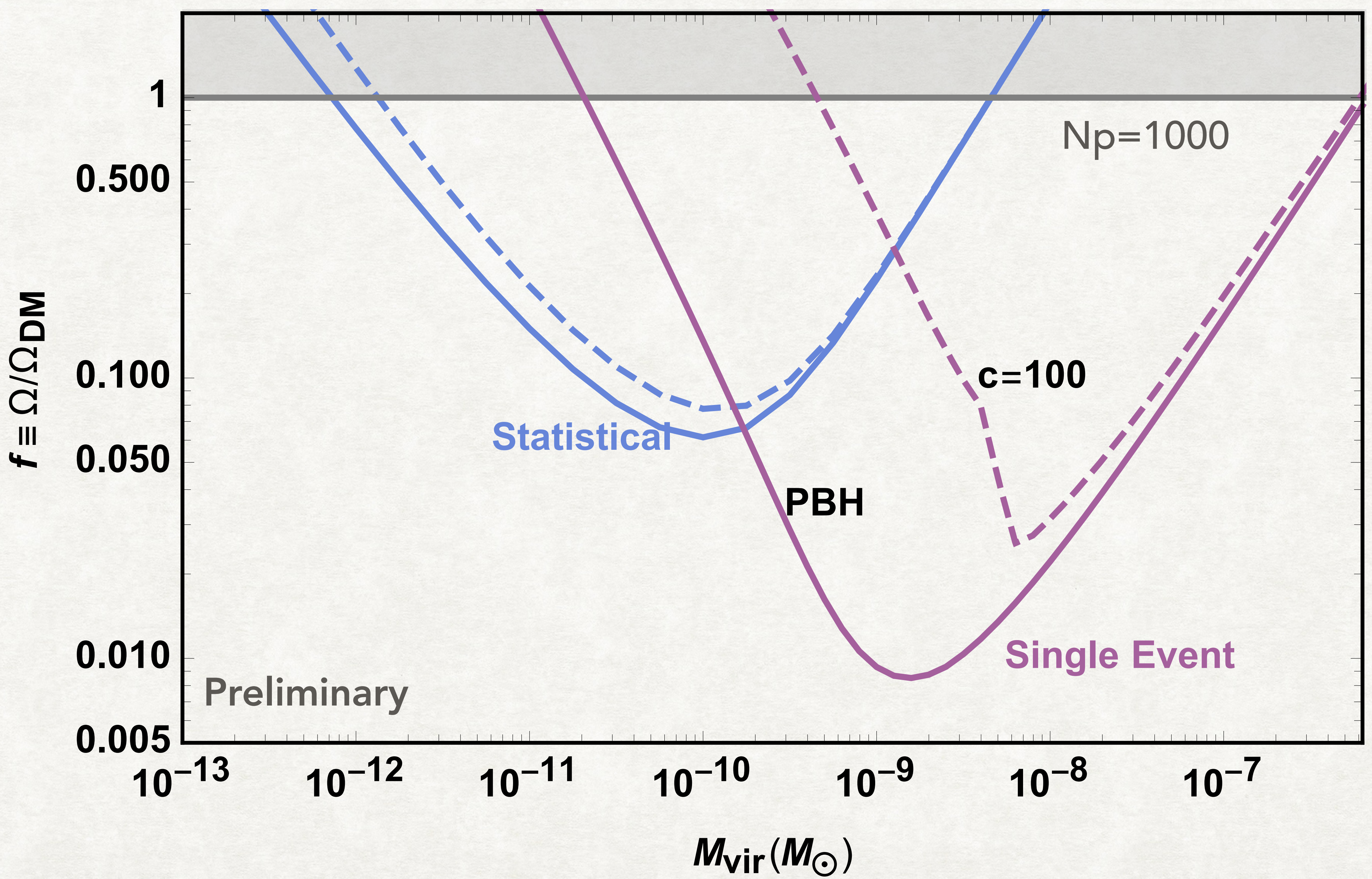
A MULTI EVENT STATISTICAL SIGNAL SEARCH

- Till now only single blip:
- advantage: Shape based filter search with very good sensitivity
- At small masses, can we instead do a statistical search?
- Yes: Compare a Binary merger GW signal to searching for a stochastic signal.
- Doppler: look for a random walk (memory effect)
- Shapiro: Statistics of multiple blips

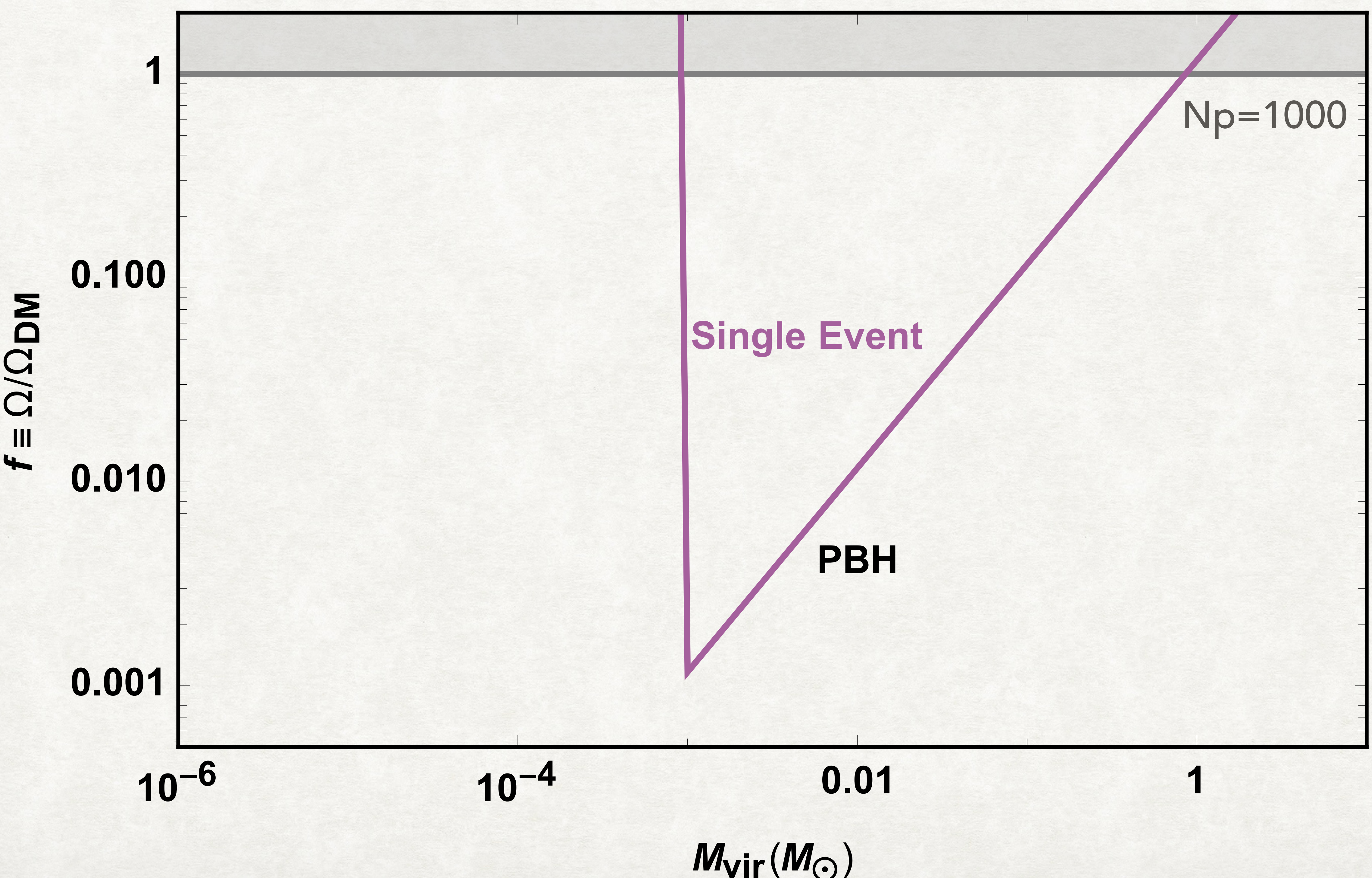
SINGLE EVENT DOPPLER



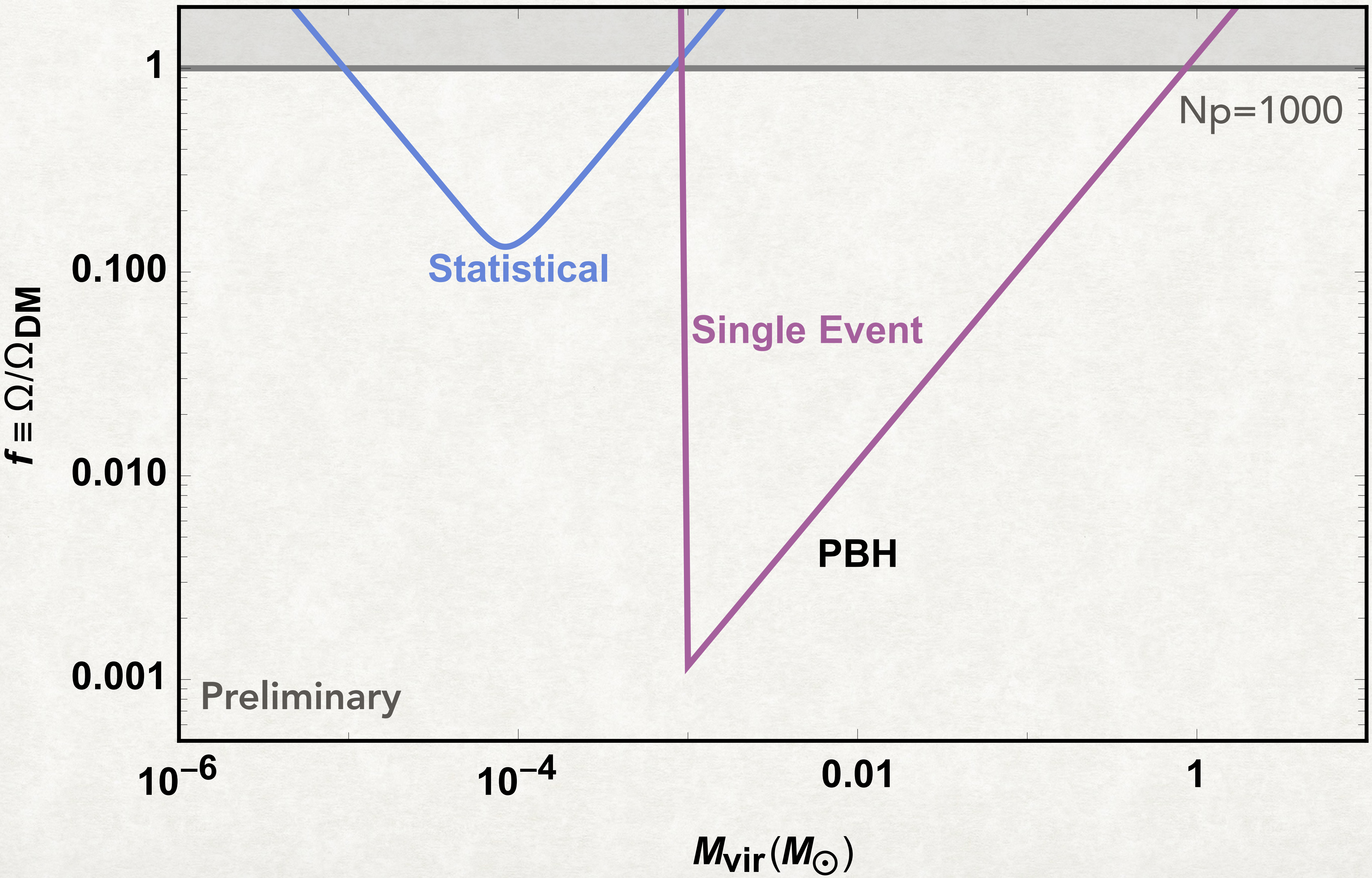
MULTI-EVENT DOPPLER



SINGLE-EVENT SHAPIRO



MULTI-EVENT SHAPIRO



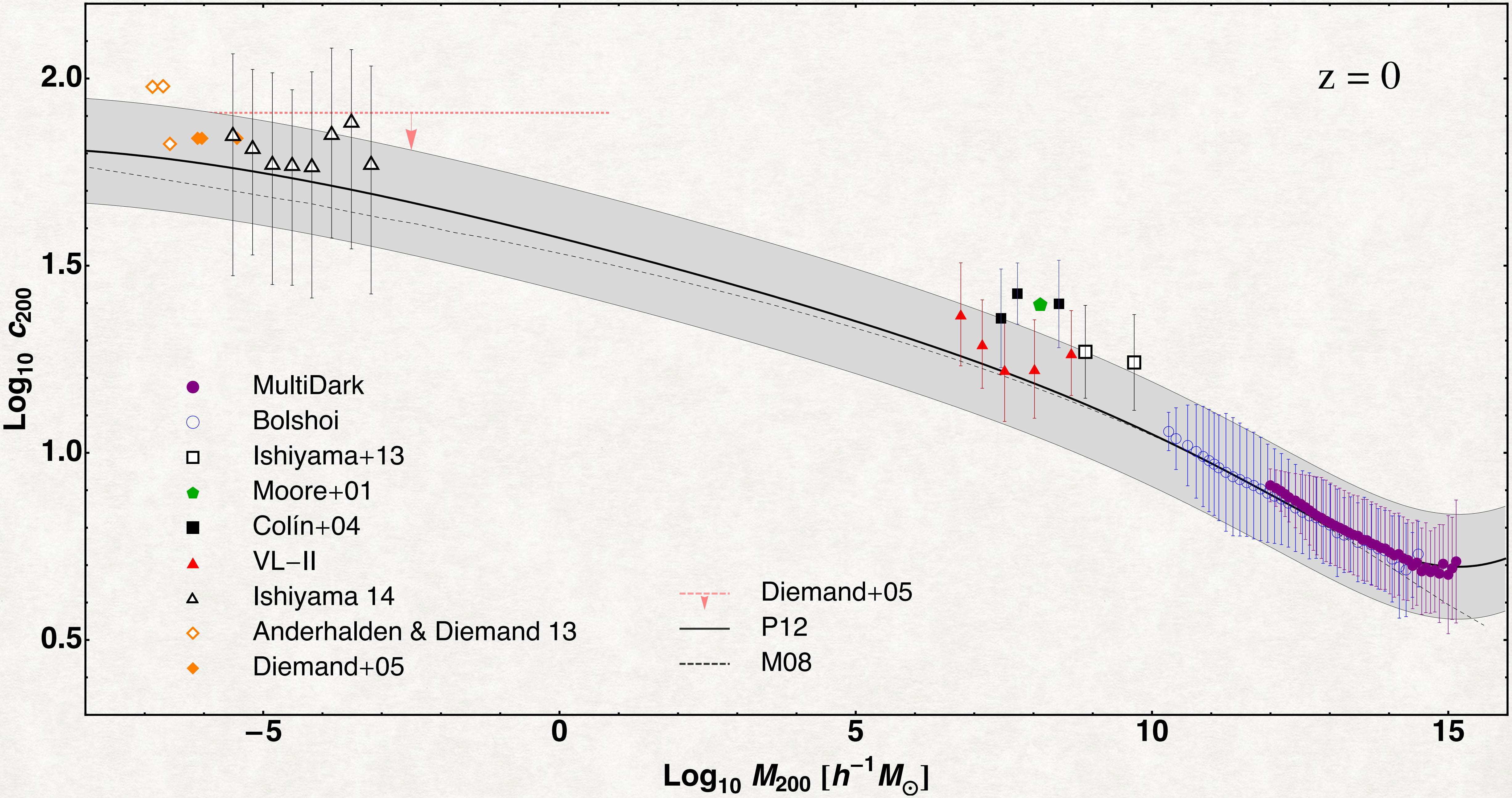
EXTENDED HALO MASS FUNCTION

- Assume typical scale-free Halo mass function from Press-Schechter.
- $dn/dM \sim M^{-2}$
- M_{min}, the IR cutoff
- M_{max}, the UV cutoff
- Equal amount of DM in every decade of masses,
- Even large M_{max}/M_{min} can be probed using sensitivity solely in a small subset window.

LIMIT SETTING PARAMETERS

- Set Limits for
- c, the concentration parameter
- f_{DM} the fraction of dark matter that has not disrupted
- Ignoring tidal disruption and sweeping it into c and f_{DM}

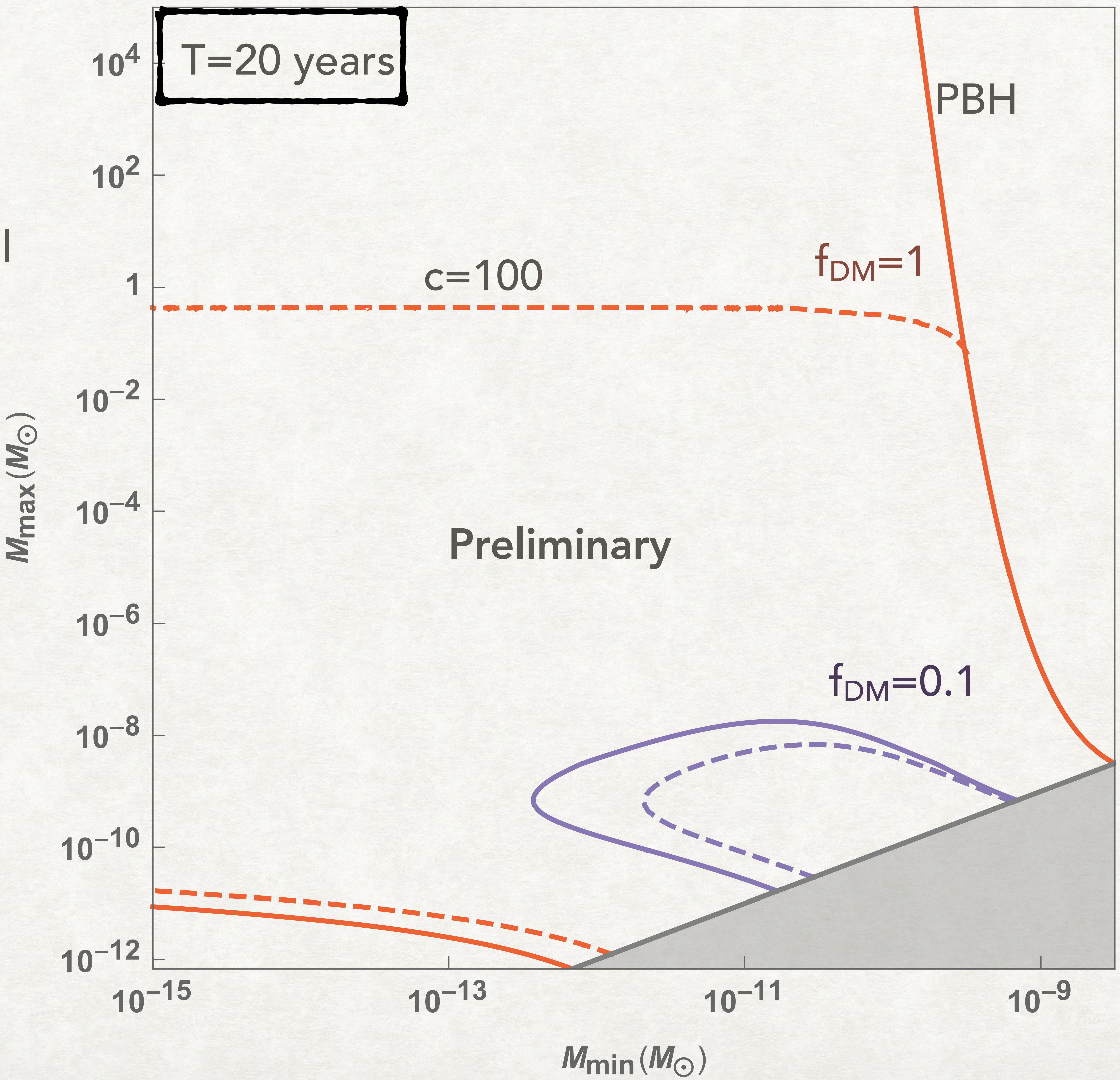
GOAL: C=100



M usually cut off at 10^{-6} because WIMPs wash out small-scale structure…

LIMIT PROJECTIONS ON EXTENDED MASS

Doppler Statistical Only



LIMIT PROJECTIONS ON EXTENDED MASS

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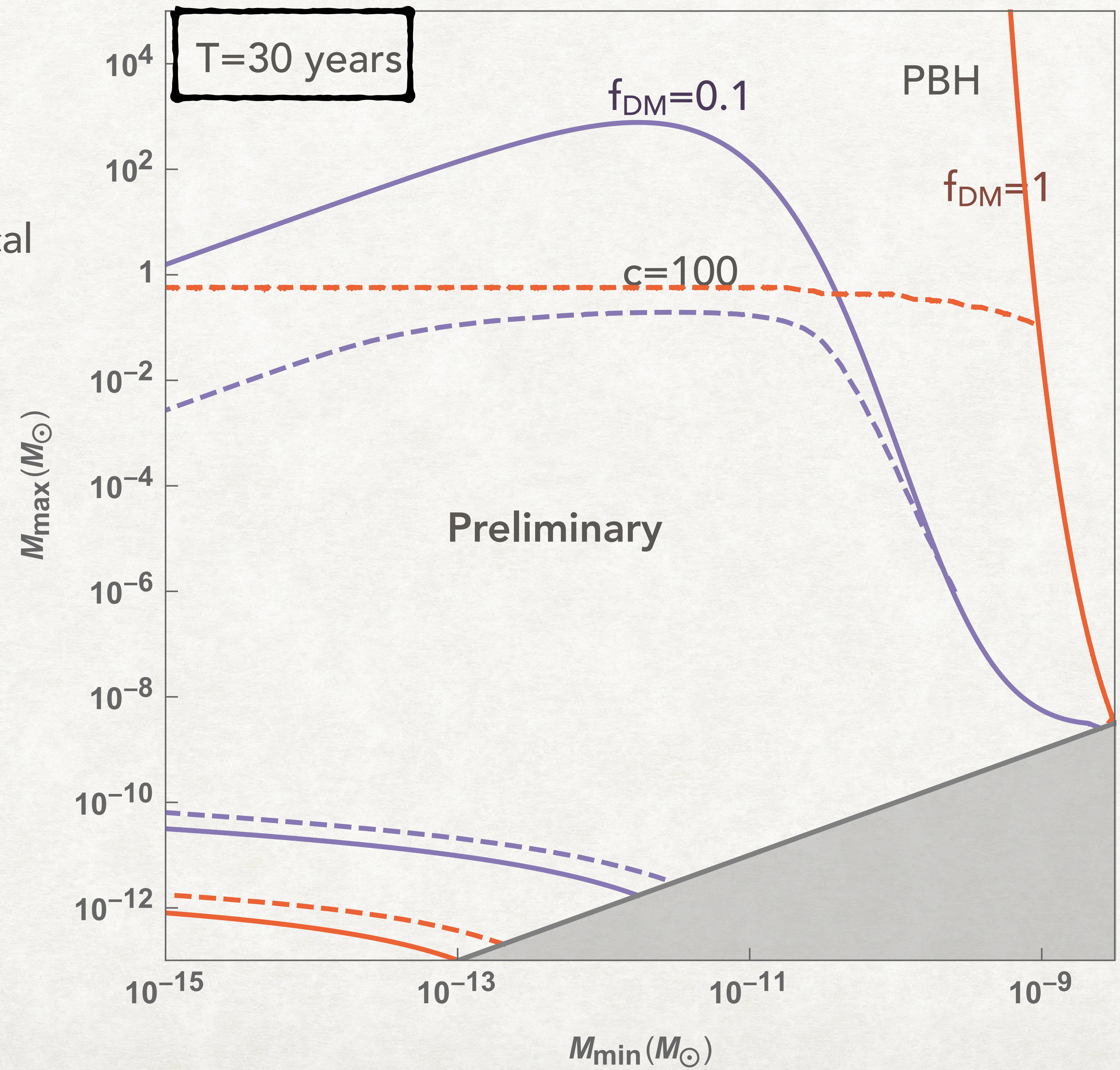


TO DO LIST

- MSPs across the GC?
- in DM rich environments?
- Extra galactic MSPs?

BACKUP

PROBES VS OBJECT SIZE

$$r_{\rm PTA} \sim 10^{-3}\ {\rm pc} \times \begin{cases} \frac{M}{10^{-9}\, M_\odot} & \text{(Doppler Dynamic)} \\ \left(\frac{M}{10^{-3}\, M_\odot}\right)^2 & \text{(Shapiro Dynamic)} \end{cases}$$

$$r_{\rm PTA} \sim 10^{-3}\ {\rm pc} \times \begin{cases} \left(\frac{M}{10^{-8}\, M_\odot}\right)^{\frac{1}{3}} & \text{(Doppler Static)} \\ \left(\frac{M}{10^{-3}\, M_\odot}\right)^{\frac{1}{3}} & \text{(Shapiro Static)} \end{cases}$$

$$r_E \sim \begin{cases} 10^{-6}\ {\rm pc} \left(\frac{M}{10^{-4}\, M_\odot}\right)^{\frac{1}{2}} & \text{(Stellar Lensing)} \\ 10^{-2}\ {\rm pc} \left(\frac{M}{10\, M_\odot}\right)^{\frac{1}{2}} & \text{(Supernovae Lensing)} \end{cases}$$

TOWARDS A STATISTICAL DESCRIPTION OF SIGNALS

- Gravitational effects can be related to the local density
- Correlations in delay are measuring correlations in density and hence measuring the matter power spectrum
- Can handle multiple lower threshold events
- Need to be careful about how much is sampled - cuts in momentum space
- Thus not only Power, but halo mass function important
- Doppler - Random walk signal
- Shapiro - Statistics of Multiple Blips
- More careful with background

TO DO

- Limits on current power spectrum.
- Can then project that to limits on particle models, by assuming a rough clustering model (or N-body simulations?)
- Pulsars near Galactic center to increase local DM density.
- Extra-galactic pulsars to increase baseline.

CONCLUSIONS

- Pulsar timing can probe structure at a wide range of small scales.
- Doppler and Shapiro delays, especially in the dynamic regime, can provide a compelling discovery signal for DM subhalos
- Limits on PBH provide a second probe of the lensing region
- Limits on more diffuse objects could do much better than lensing

COMPACT OBJECTS

- Parametrize the profile as NFW or UCMH.

$$\rho(r, M_{\rm vir}) = \frac{\rho_s}{(r/r_s)^\alpha \, (1 + r/r_s)^\beta}$$

$$\alpha = 1,\ \beta = 2$$

NFW

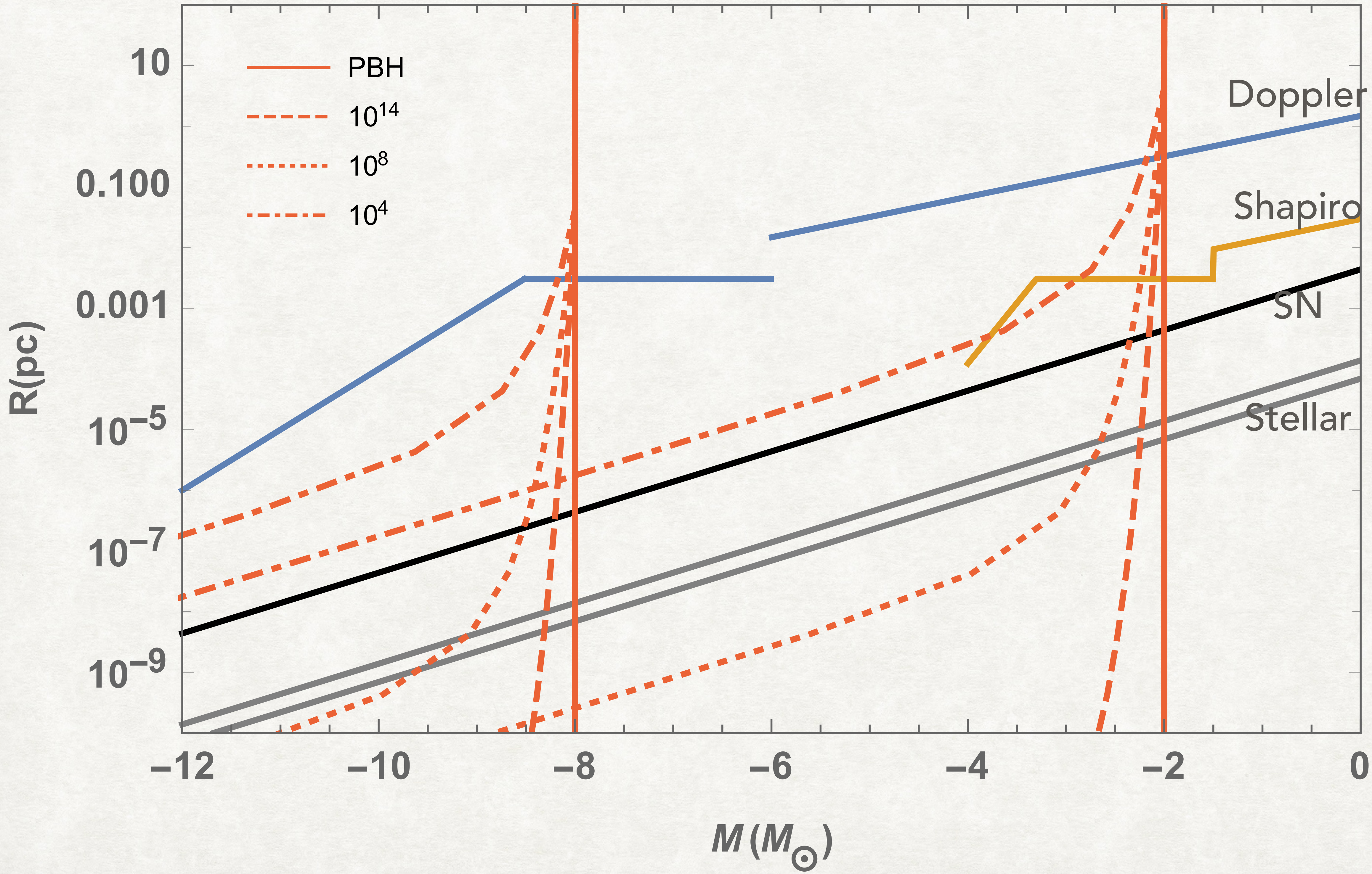
$$\alpha = 9/4,\ \beta = 0$$

UCMH

$$r_{\rm vir} \equiv (3M_{\rm vir}/800\pi\rho_c)^{1/3}$$

$$c \equiv r_{\rm vir}/r_s$$

SENSITIVITY TO CONCENTRATION



Limits iff red line intersects a probe radius

SIGNAL TO NOISE RATIO (STATIC SIGNALS)

- In the limit that you don't see the whole signal, Taylor expand.
- A constant first derivative i.e. spin-down or sometimes even spin-up is already observed (and hard to ascertain from first principles).
- Subtracted as part of the fitting procedure.
- Second derivative much less common.
- Non-observation of second derivative can be used to set constraints.

Doppler

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{2GMv}{r_{\min}^3} \sim 3 \times 10^{-32} \left(\frac{N_P f}{200}\right) \ \mathrm{Hz}^2$$

Shapiro

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{16GMv^3}{r_{\times,\min}^3}$$

$$\sim 8 \times 10^{-33} \left(\frac{N_P f}{200}\right)^{\frac{3}{2}} \left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \left(\frac{d}{\mathrm{kpc}}\right)^{\frac{3}{2}} \ \mathrm{Hz}^2$$

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Uncertainty in second derivative purely from rms fluctuations

$$\sigma_{\ddot{\nu}/\nu} = 6\sqrt{\frac{2800\Delta t}{T}\frac{t_{\rm RMS}}{T^3}}$$

$$f_{\rm D,\ stat} \lesssim 0.4 \left(\frac{200}{N_P}\right) \left(\frac{20\ \rm yr}{T}\right)^{\frac{7}{2}}$$

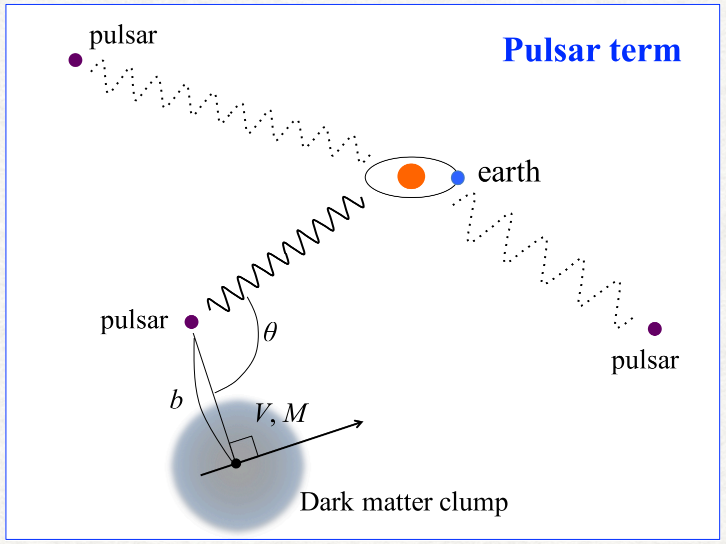
$$f_{\rm S,\ stat} \lesssim \left(\frac{200}{N_P}\right) \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} \left(\frac{20\ \rm yr}{T}\right)^{\frac{7}{3}} \left(\frac{\rm kpc}{d}\right)$$

Notice no M dependence here

DO WE SEE THE WHOLE SIGNAL?

Timescales

- To determine typical timescale, let us determine object of closest approach
- Cross-section for Doppler, is a circle.
- Remembering $|\mathbf{b}| = \tau v$



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$$\sim \frac{20\ \rm yr}{\sqrt{N_P f}} \left(\frac{M}{10^{-9}\, M_\odot}\right)^{\frac{1}{2}} \left(\frac{20\ \rm yr}{T}\right)^{\frac{1}{2}}$$

N_P pulsars

N_P x cross-section

Number of pulsars

Fraction of DM in M mass PBH

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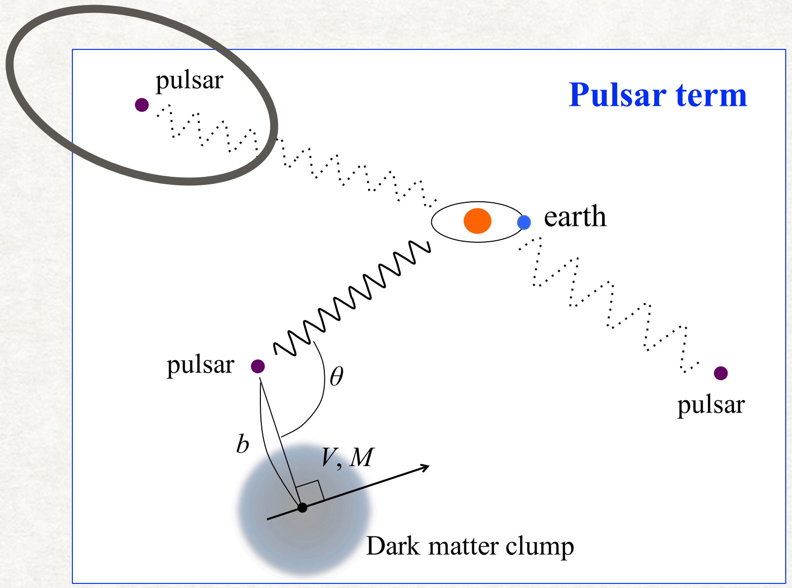

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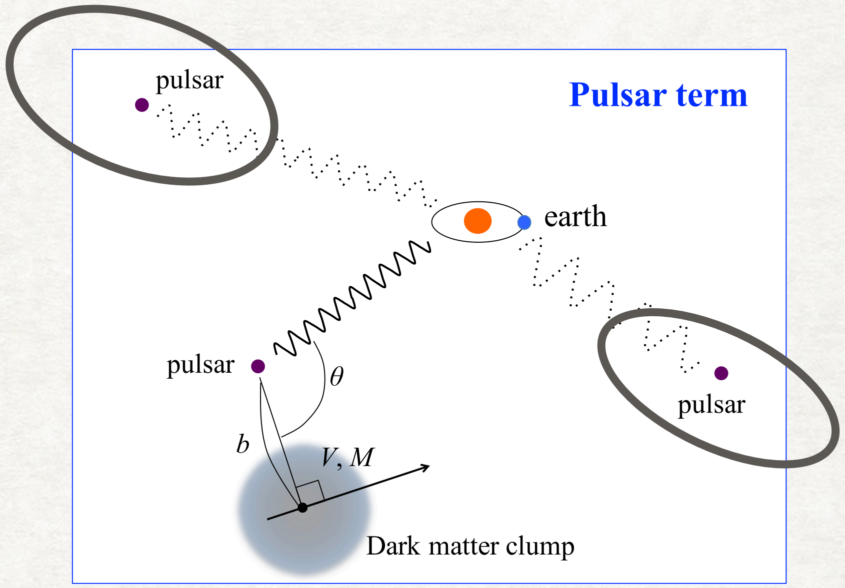

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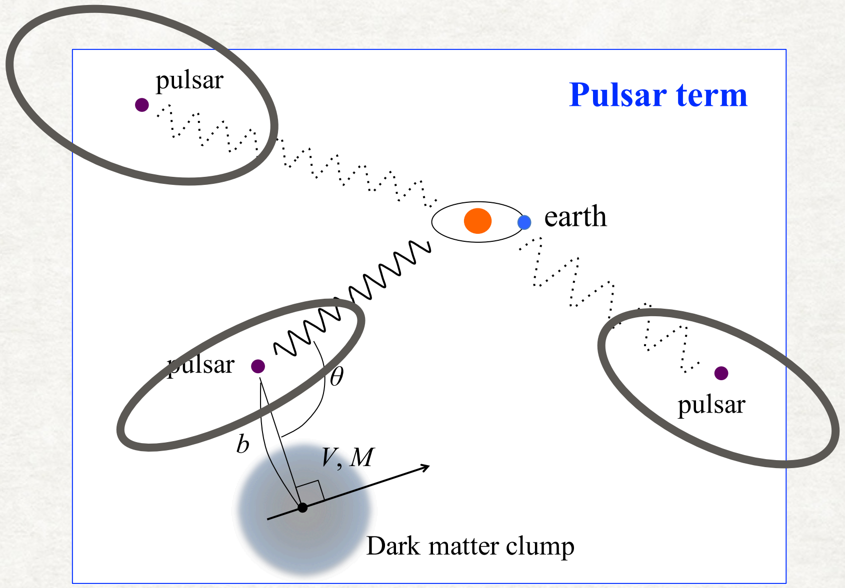

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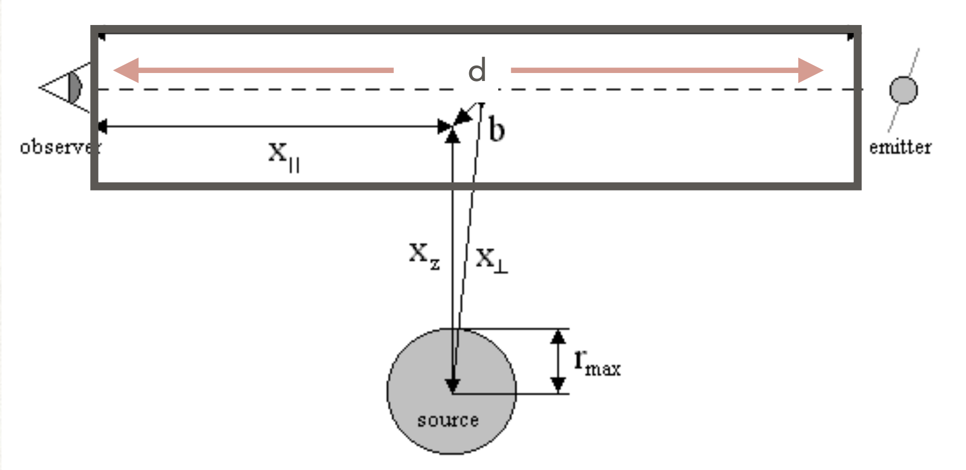
Number of pulsars

Fraction of DM in M mass PBH

DO WE SEE THE WHOLE SIGNAL?

Timescales

- Cross-section for Shapiro is a rectangle



$$\tau_{\rm min} \simeq \frac{2}{v}\frac{M}{N_P f \rho_{\rm DM} v T d},$$

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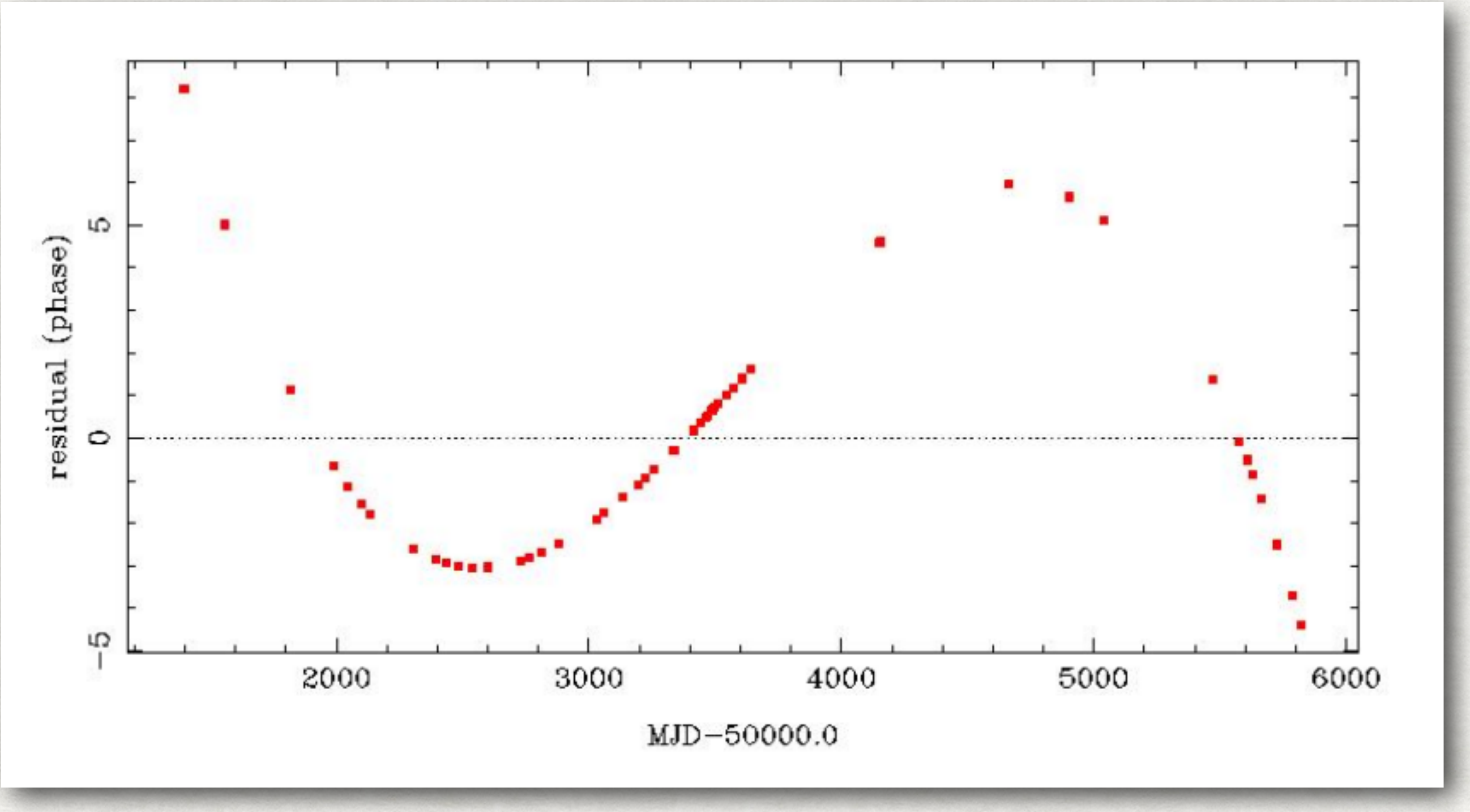
Notice no M dependence here

STATIC BACKGROUNDS

- A few pulsars already display non-zero second derivative.
- Will need to supplement with E&M observations to subtract known nearby objects.



MONTECARLO SIMULATION

- Assume PBHs randomly distributed
- Isotropic Maxwell distribution in velocity truncated at v_{esc}.
- Simulate N_p randomly distributed pulsars at appropriate distances.
- Simulate order $O(10^4)$ universes and require more than 95% universes pass SNR>4.

PTA FOR DARK MATTER

- Ultralight DM causing GW like delays
- [Khmelnitsky, Rubakov - 1309.5888], [Graham, Kaplan, Mardon, Rajendran, Terrano - 1512.06165]
- PTAs are sensitive accelerometers: Doppler Delay
- [Seto,Corray - astro-ph/0702586] , [Baghram,Afshordi,Zurek - 1101.5487] [Kashiyama, Seto - 1208.4101],[Kazumi, Oguri, Masamune - 1801.07847]
- Gravitational potential wells along the light path: Shapiro Delay
- [Siegel, 0801.3458], [Siegel,Hertzberk,Fry - astro-ph/0702546], [Baghram,Afshordi,Zurek - 1101.5487], [Clark,Lewis,Scott - 1509.02938] , [Schutz, Liu - 1610.04234]
- Our Work
  Explicit calculations for SNR calculations for Doppler and Shapiro Delays
  Correcting several mistakes in literature
  Extending analysis to diffuse halos