

Stochastic background of Gravitational Waves from Primordial Black Holes

Davide Racco

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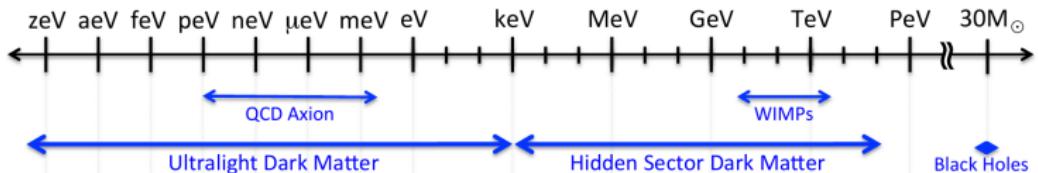
Galileo Galilei Institute, *Next Frontiers in the Search for Dark Matter*

25th September 2019

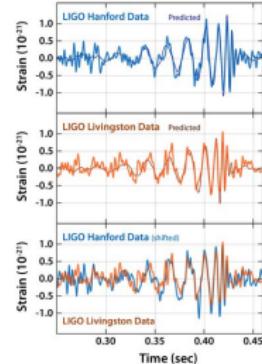
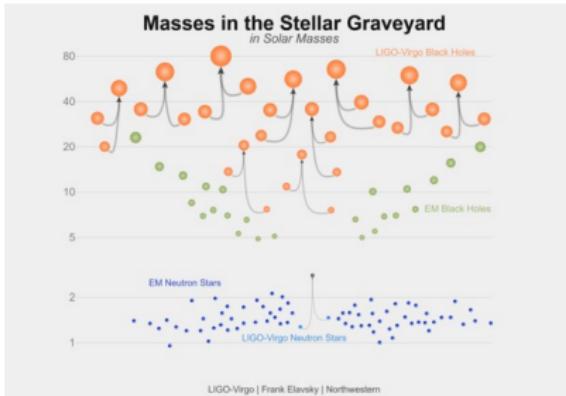


The quest for Dark Matter

- Our ignorance on the mass of Dark Matter spans 90 orders of magnitude.



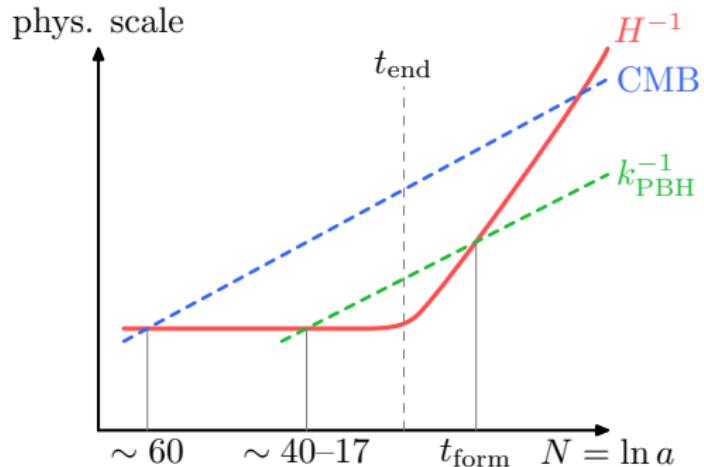
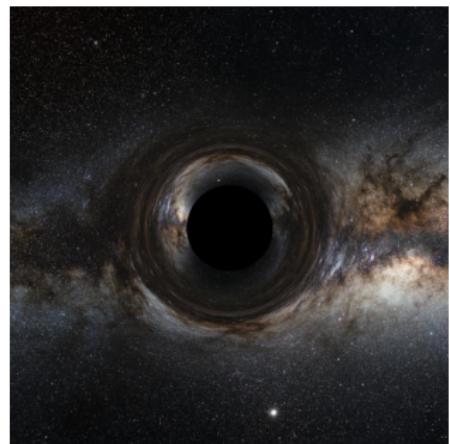
- Primordial Black Holes are still a viable candidate for the totality of DM.
- Exciting time for Black Holes in the Gravitational Wave era!



[LIGO-Virgo]

['67 Zeldovich, Novikov; '71 Hawking; '75 Chapline; '96 Garcia-Bellido, Linde, Wands; '16 Carr, Kühnel, Sandstad; '18 Carr, Silk; '18 Sasaki, Suyama, Tanaka, Yokoyama; ...]

- Black holes originating not from stellar evolution, but from collapse of large overdensities.
- Collapse happens for large curvature perturbations $\sim 10^{-1}$, much larger than primordial fluctuations $\sim 10^{-5}$.



$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

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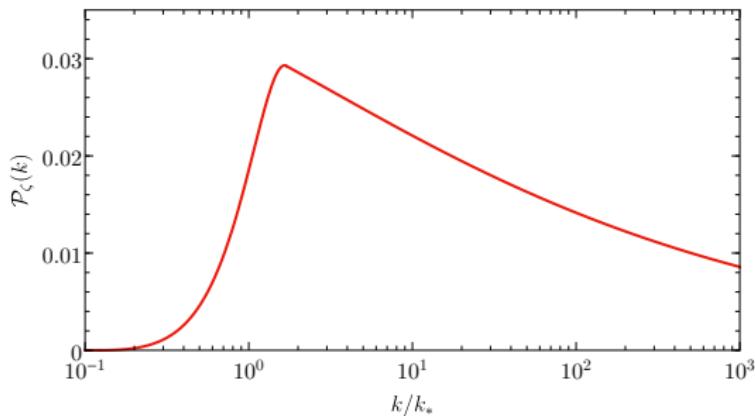
- M_{PBH} : mass contained in a sphere of volume $\sim H^{-3}$ at the time when k re-enters the Hubble radius.

$$M_{\text{PBH}} = \underbrace{\gamma}_{\sim 0.2} \frac{4\pi}{3} \rho H^{-3} \approx 10^{-15} M_\odot \left[\frac{k}{(10^{-14} \text{ Mpc})^{-1}} \right]^{-2}$$

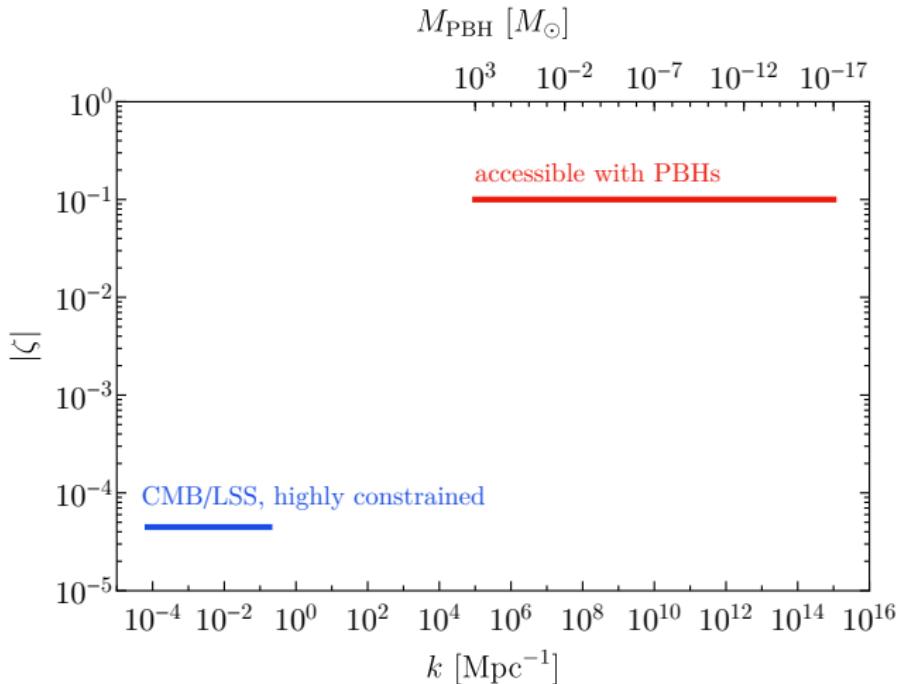
$$\stackrel{H \sim 10^{12} \text{ GeV}}{\approx} M_\odot e^{2(N-36)}.$$

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

- Power spectrum $\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} |\zeta_k|^2$ on small scales $k \gtrsim k_*$: much larger than the usual $\mathcal{O}(10^{-9})$ of CMB scales.

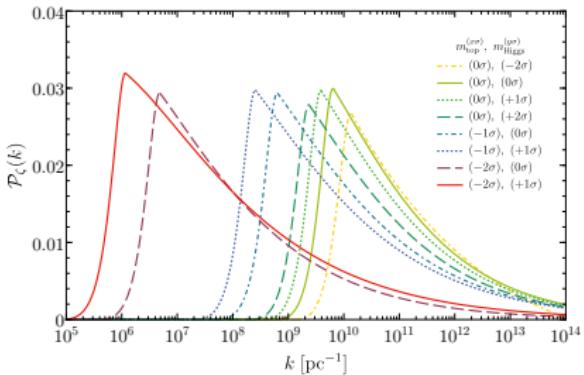
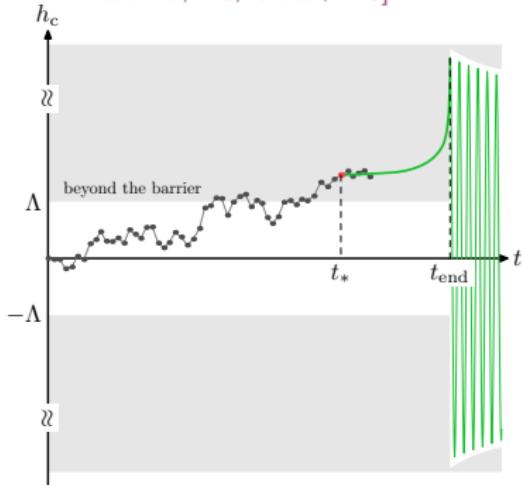


- PBH Abundance $\Omega_{\text{PBH}} \sim \Omega_{\text{DM}}$ for $\mathcal{P}_\zeta(k) \sim 10^{-2}$ at some k .



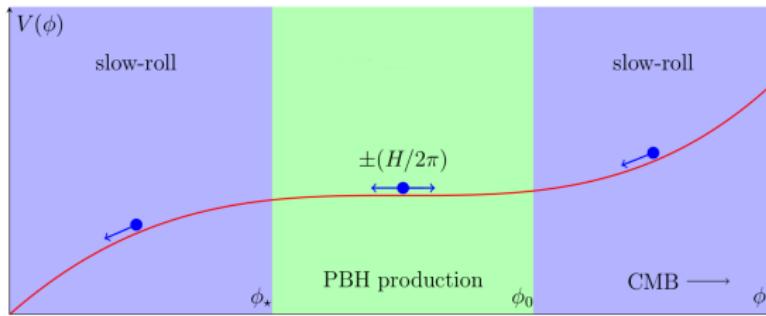
- Higgs field probing the unstable region at the end of inflation [Espinosa, DR, Riotto '17, '18; Gross+ '18]
- Violation of slow-roll condition: inflection point in inflaton potential, leading to ultra-slow roll phase. [García-Bellido, Ruiz Morales '17; Ballesteros, Taoso '17; Cicoli+ '18; Özsoy+ '18; Biagetti+ '18; Dalianis+ '18; ...]
- Multifield inflation: hybrid inflation, double inflation [García-Bellido+ '96; Yokoyama '98; Kawasaki+ '98; ...]
- Different mechanisms, not requiring large \mathcal{P}_ζ : oscillons from scalar fields [Cotner, Kusenko '16, '18]; domain walls [Ferrer+ '18]; ...

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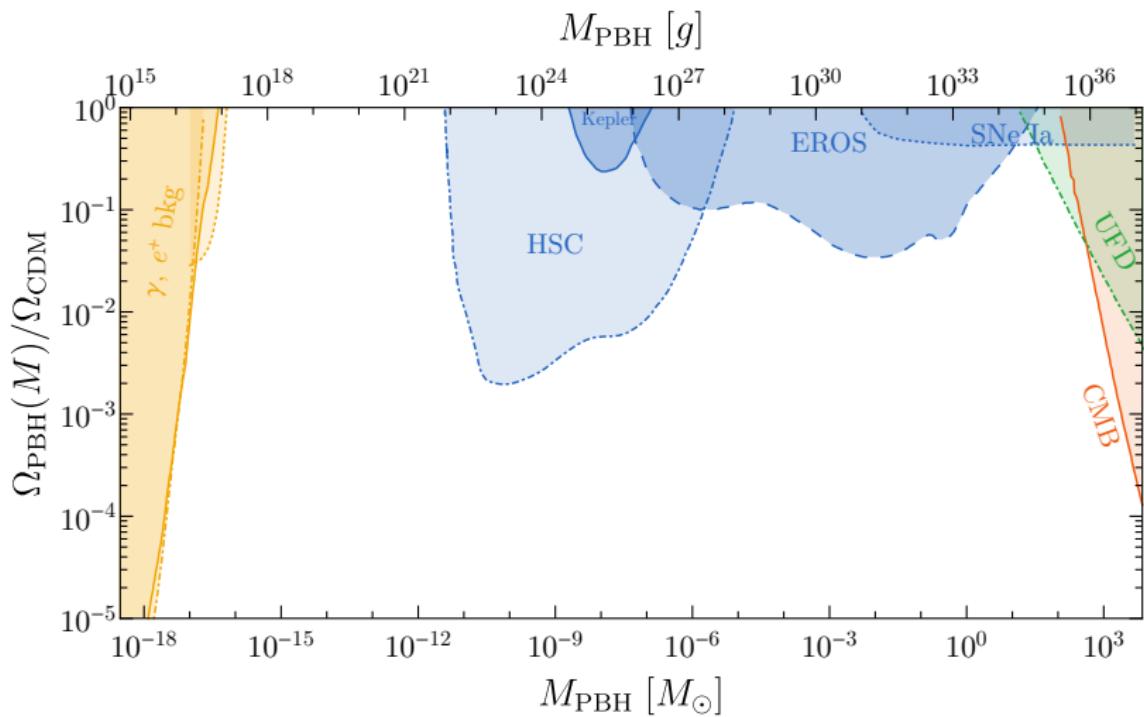


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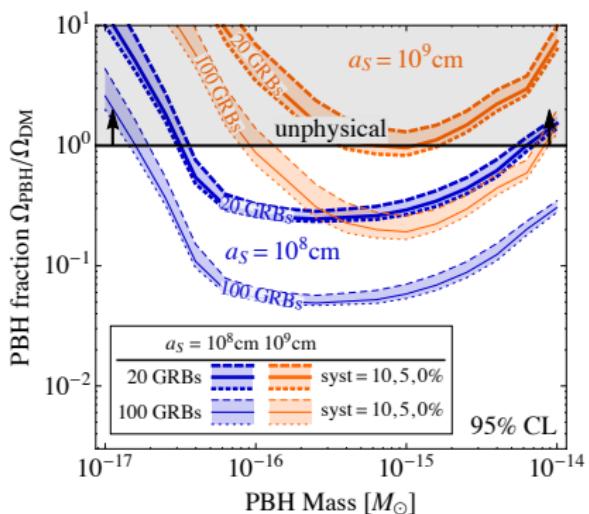
Constraints on Primordial Black Holes



- PBH capture in White Dwarves or Neutron Stars does not yield constraints [Montero-Camacho+ '19].
- Prospects with femtolensing:

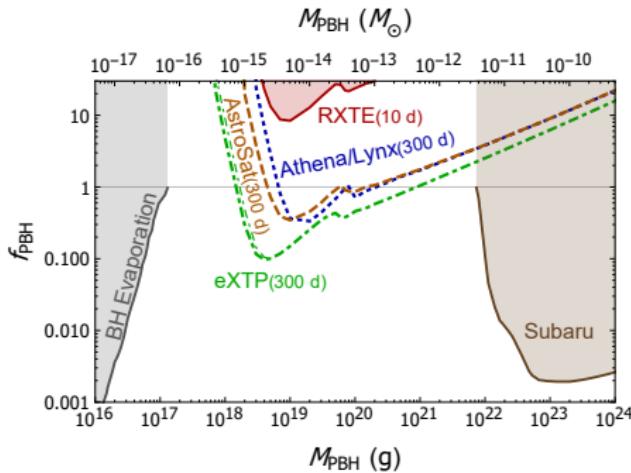
[Gould '92, Katz, Kopp, Sibiryakov, Xue '18]

Femtolensing of (compact) short Gamma Ray Bursts: fringes in their spectrum



[Bai, Orlofsky '18]

Femtolensing of X-ray binaries (NS/BH + star) in Milky Way satellites: transient magnification

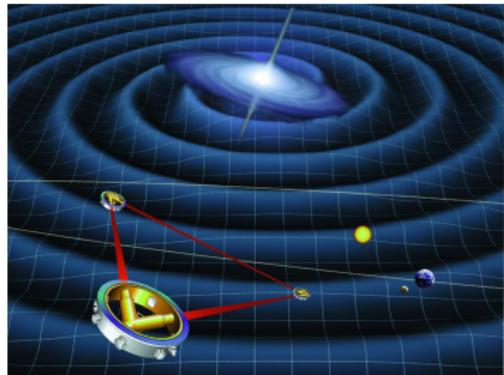


[Acquaviva+ '02; Mollerach, Harari, Matarrese '03; Ananda, Clarkson, Wands '06; Baumann+ '07]

- Tensor perturbations are excited in presence of large scalar perturbations when they cross the Hubble radius, then survive (redshifted) until today:

$$(\textcolor{red}{h_{ij}}'' + 2\mathcal{H}\textcolor{red}{h_{ij}}' - \nabla^2 \textcolor{red}{h_{ij}}) \sim \partial_i \textcolor{teal}{\zeta} \partial_j \textcolor{teal}{\zeta}$$

- This background of GWs can be detected by Advanced-LIGO, Einstein Telescope and especially LISA. \rightsquigarrow See talk by D. Croon

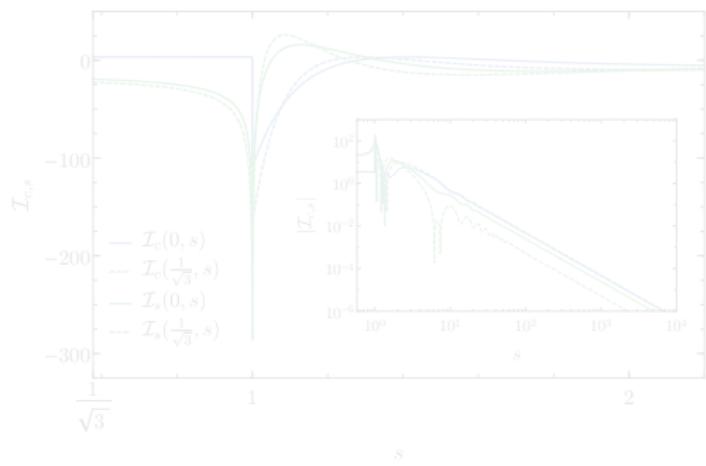
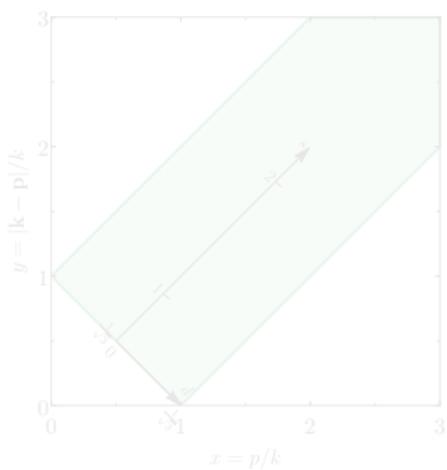


Solution for Gravitational Waves

Espinosa, DR, Riotto, JCAP 1809 (2018) no.09, 012, 1804.07731

$$h_{\mathbf{k}}^s(\eta) = \frac{4}{9} \frac{1}{k^3 \eta} \int \frac{d^3 p}{(2\pi)^3} \zeta(\mathbf{p}) \zeta(\mathbf{k} - \mathbf{p}) e^s(\mathbf{k}, \mathbf{p}) \cdot$$

- ① $h \sim \mathcal{P}_\zeta$
- ② Dimensionless integral over time that we compute analytically, as a function of $p/k, |\mathbf{k} - \mathbf{p}|/k$:



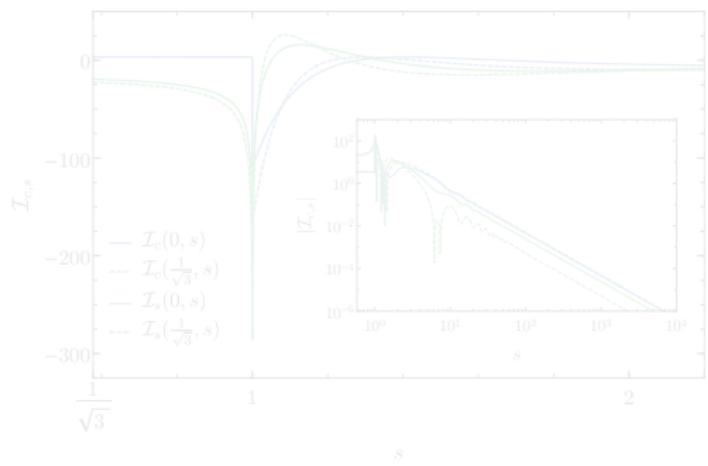
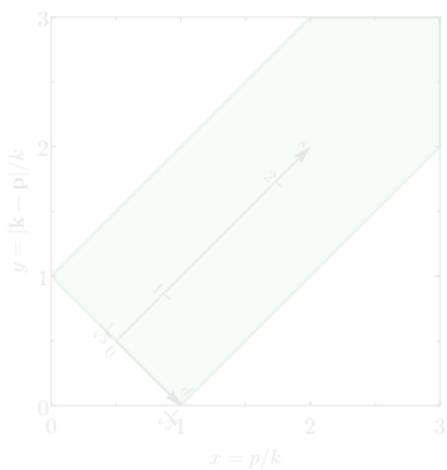
- ③ Integration over momenta

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- ➊ $h \sim \mathcal{P}_\zeta \cdot \left[\int_{k^{-1}}^\eta d\tilde{\eta} (\text{Green \& Transfer functions})(\mathbf{p}, |\mathbf{k} - \mathbf{p}|) \right]$
- ➋ Dimensionless integral over time that we compute analytically, as a function of $p/k, |\mathbf{k} - \mathbf{p}|/k$:



- ➌ Integration over momenta

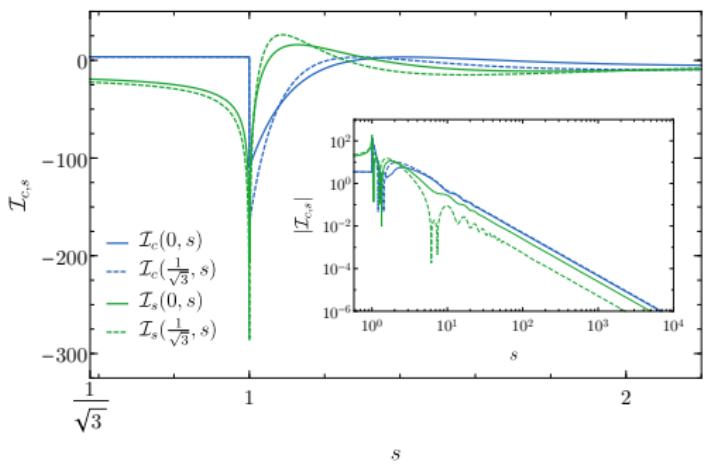
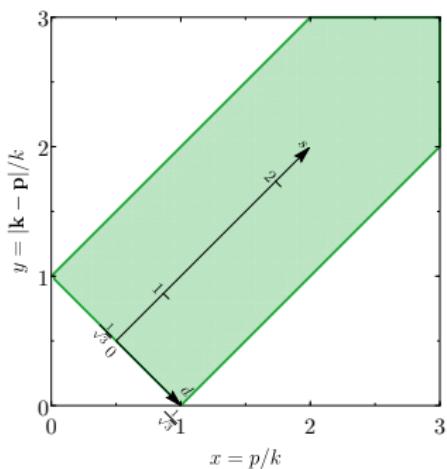
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- \textcircled{3} Integration over momenta

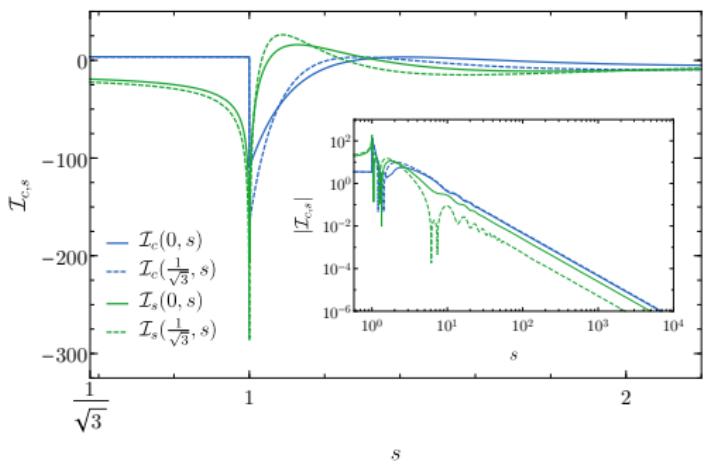
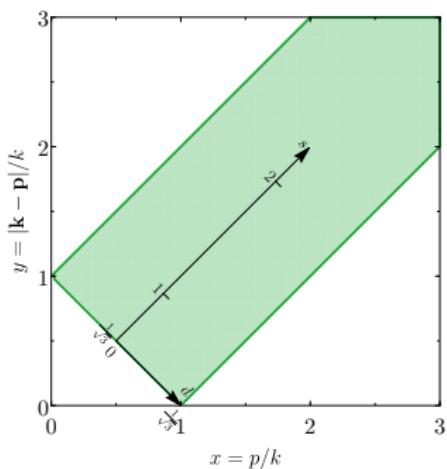
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- From the solution for $h_{\mathbf{k}}$ we can compute

$$\langle h_{\mathbf{k}_1} h_{\mathbf{k}_2} \rangle \sim \langle \zeta \zeta \zeta \zeta \rangle$$

$$\langle h_{\mathbf{k}_1} h_{\mathbf{k}_2} h_{\mathbf{k}_3} \rangle \sim \langle \zeta \zeta \zeta \zeta \zeta \zeta \rangle$$

which involve just 2D and 3D numerical integrals over momenta.

- 2-pt function $\leftrightarrow \Omega_{\text{GW}}$, energy density of GW.
- Final result [Ando, Inomata, Kawasaki, Mukaida, Yanagida, '17; Espinosa, DR, Riotto '18; Kohri, Terada '18]

$$\Omega_{\text{GW}}(\eta_0, k) = \frac{\Omega_{r,0}}{972} \iint_{\mathcal{S}} dx dy \frac{x^2}{y^2} \left[1 - \frac{(1+x^2-y^2)^2}{4x^2} \right]^2 \cdot \mathcal{P}_{\zeta}(kx) \mathcal{P}_{\zeta}(ky) [\mathcal{I}_c(x,y)^2 + \mathcal{I}_s(x,y)^2]$$

- ~ See talk by C. Ünal about effect of non-Gaussianities in ζ .

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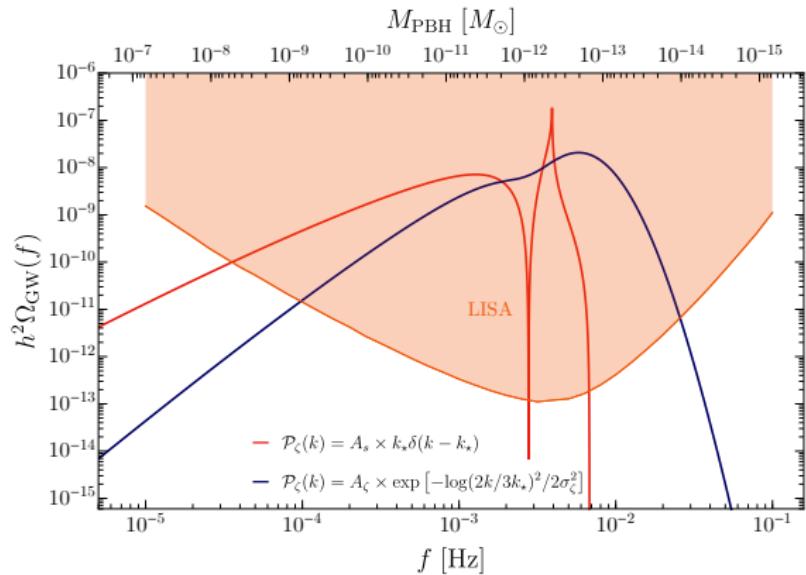
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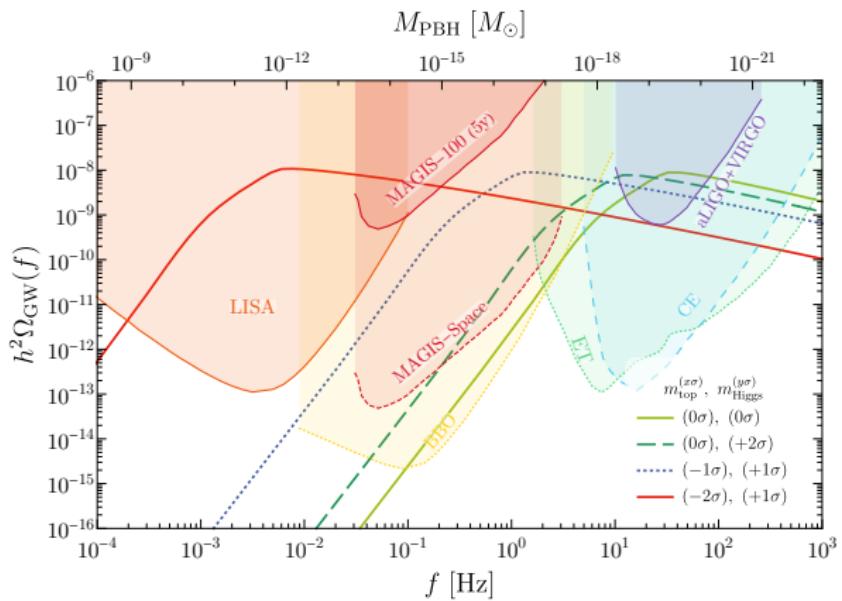
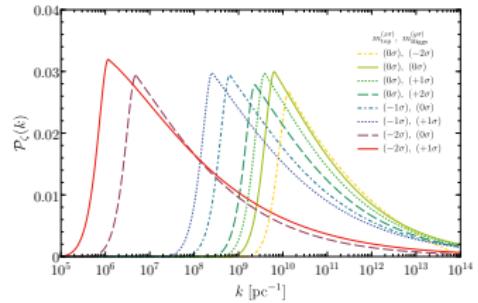
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Red: Curvature $\mathcal{P}_\zeta \sim \text{Dirac delta}$

Blue: Curvature $\mathcal{P}_\zeta \sim \text{Gaussian distribution}$



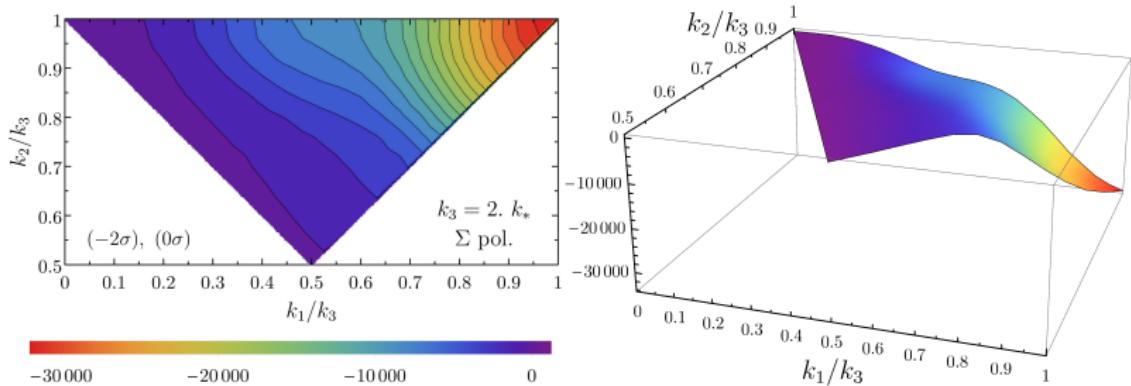
Power spectrum of Gravitational Waves generated at second order



$$\Omega_{\text{GW}}(f) \simeq 10^{-8} (f/f_*)^{n_T}$$

Spectral tilt of GWs: $n_T \begin{cases} \sim 3 & \text{before the peak} \\ \sim 2 \cdot n_s & \text{after the peak} \end{cases}$

Primordial Bispectrum of Gravitational Waves



- The shape of the bispectrum typically peaks in equilateral configurations. In case of detection of the 3-pt function, its shape could help to identify the origin of the signal.



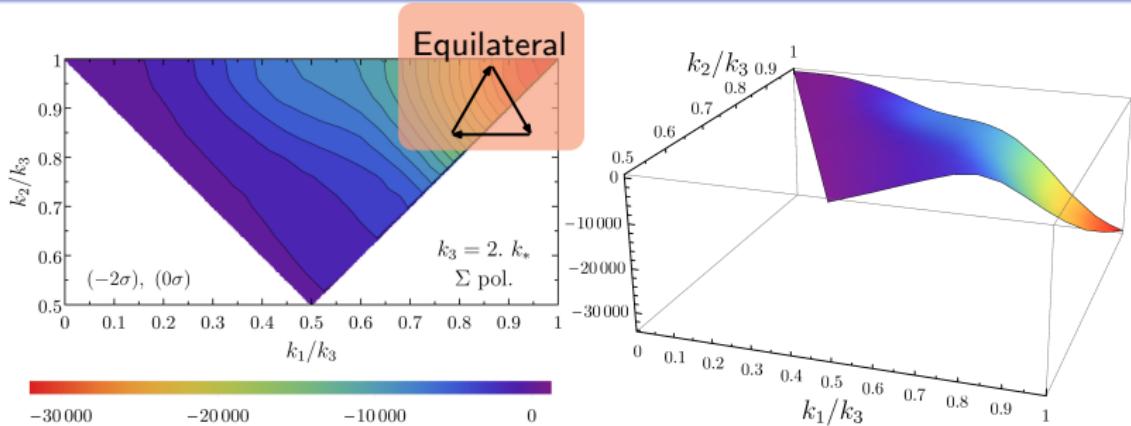
But tensor non-Gaussianity is not locally observable.¹

[Bartolo, De Luca, Franciolini, Lewis, Peloso, DR, Riotto, 1810.12218, 1810.12224]

A detector of the GW background receives signals in all frequencies and directions from many uncorrelated patches, so the resulting sum is Gaussian (Central Limit Theorem).

¹Except for ultra-squeezed limit, see [Dimastrogiovanni, Fasiello, Tasinato '19].

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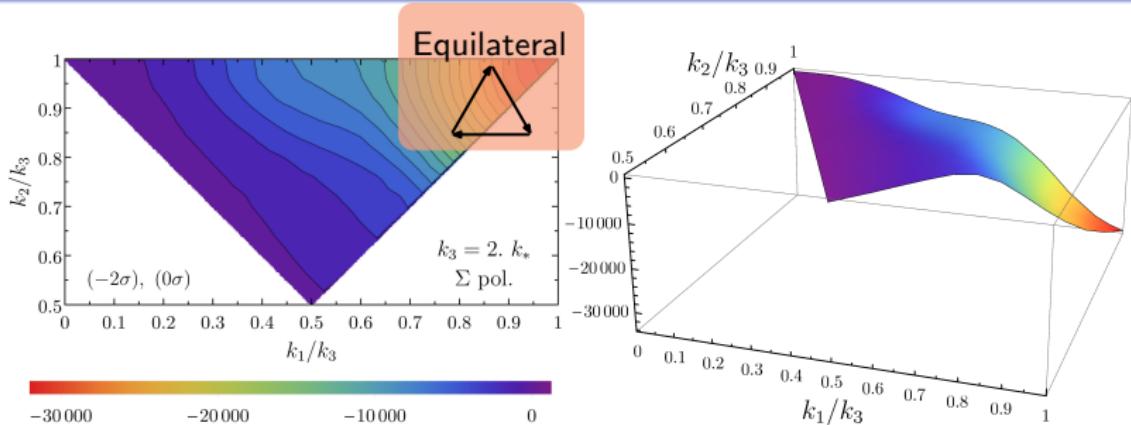
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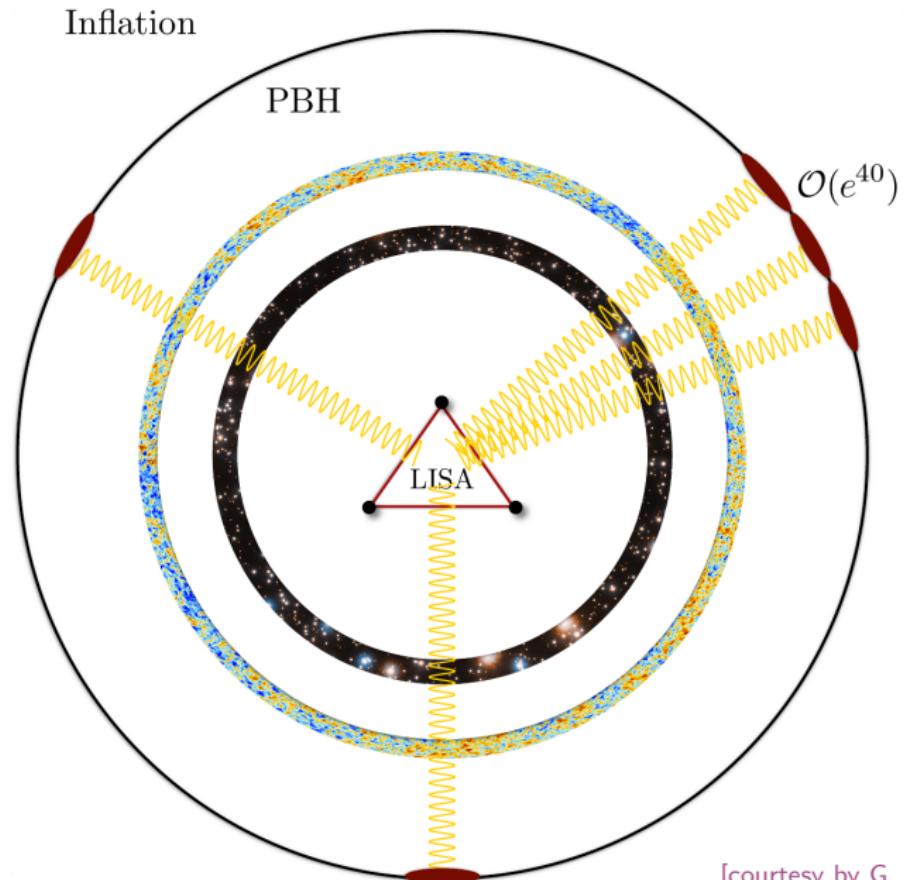


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Undetectability of the GW bispectrum

[Bartolo, De Luca, Franciolini, Lewis, Peloso, DR, Riotto, '18]

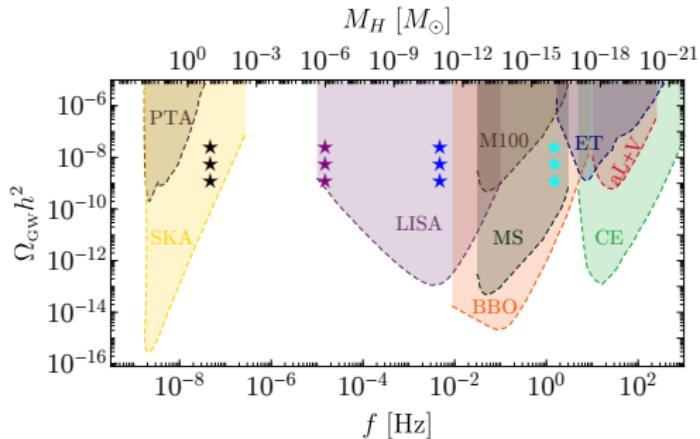


[courtesy by G. Franciolini]

- Large density peaks which do not collapse into PBHs can source GWs through their quadrupole [De Luca, Desjacques, Franciolini, Riotto '19]
⇒ Subleading effect wrt GWs generated at 2nd order.
- Merger events of PBH binaries also create a GW bkg. [Clesse, Garcia-Bellido '16; Raidal, Vaskonen, Veermäe '17; Wang, Terada, Kohri '19]
⇒ different frequencies wrt GWs at 2nd order; for $M_{\text{PBH}} \lesssim 10^{-5} M_{\odot}$, hardly observable.
- Background from mergers of astrophysical BHs [Cusin, Dvorkin, Pitrou, Uzan '19] ⇒ Expected to be smaller than GWs at 2nd order
- The study of angular anisotropies of Ω_{GW} can allow to discriminate the origin of the background [Cusin, Dvorkin, Pitrou, Uzan '19; Bartolo+ '19]
The study of the frequency dependence of inhomogeneities could unveil non-Gaussianity [Bartolo+ '19].

Other Gravitational Wave backgrounds

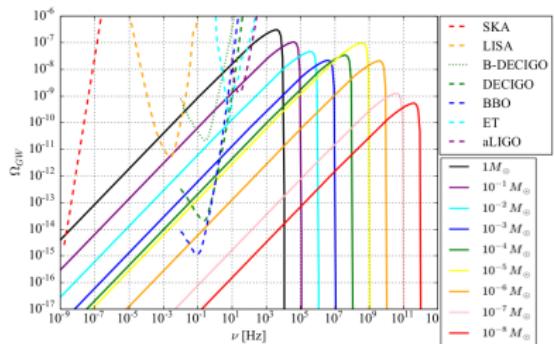
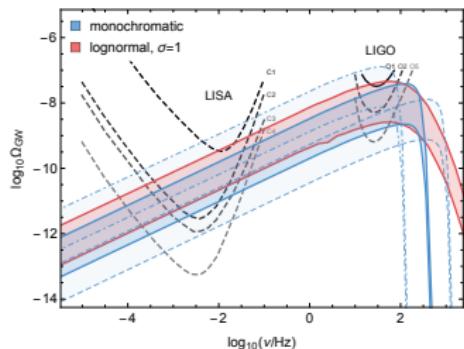
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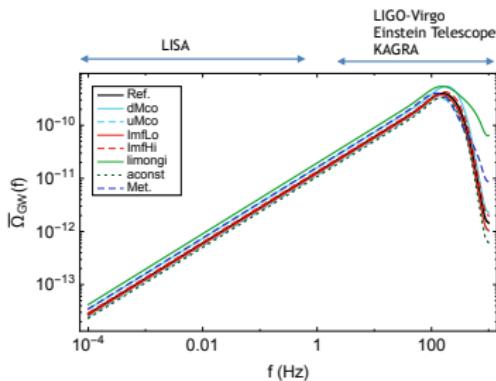
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- Large density peaks which do not collapse into PBHs can source GWs through their quadrupole [De Luca, Desjacques, Franciolini, Riotto '19]
⇒ Subleading effect wrt GWs generated at 2nd order.
- Merger events of PBH binaries also create a GW bkg. [Clesse, Garcia-Bellido '16; Raidal, Vaskonen, Veermäe '17; Wang, Terada, Kohri '19]
⇒ different frequencies wrt GWs at 2nd order; for $M_{\text{PBH}} \lesssim 10^{-5} M_{\odot}$, hardly observable.
- Background from mergers of astrophysical BHs [Cusin, Dvorkin, Pitrou, Uzan '19] ⇒ Expected to be smaller than GWs at 2nd order
- The study of angular anisotropies of Ω_{GW} can allow to discriminate the origin of the background [Cusin, Dvorkin, Pitrou, Uzan '19; Bartolo+ '19]
The study of the frequency dependence of inhomogeneities could unveil non-Gaussianity [Bartolo+ '19].

Primordial Black Holes represent a viable candidate for Dark Matter, which can be probed today in many ways.

When PBHs are sourced by large scalar perturbations, these generate also a background of Gravitational Waves.

The PBH mass range $10^{-15} - 10^{-12} M_{\odot}$ which is currently unconstrained can be probed by LISA through the GW background.

Thanks for your attention!



BACKUP SLIDES

- ① Primordial Black Holes
- ② Gravitational Wave Background associated to PBHs
- ③ PBHs and GWs from the SM Higgs instability

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

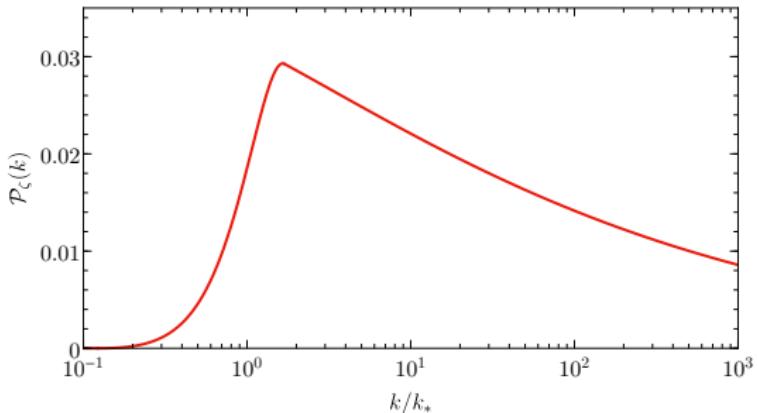
- M_{PBH} : mass contained in a sphere of volume $\sim H^{-3}$ at the time when k re-enters the Hubble radius.

$$M_{\text{PBH}} = \underbrace{\gamma}_{\sim 0.2} \frac{4\pi}{3} \rho H^{-3} \approx 10^{-15} M_\odot \left[\frac{k}{(10^{-14} \text{ Mpc})^{-1}} \right]^{-2}$$

$$\stackrel{H \sim 10^{12} \text{ GeV}}{\approx} M_\odot e^{2(N-36)}.$$

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

- Power spectrum $\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} |\zeta_k|^2$ on small scales $k \gtrsim k_*$: much larger than the usual $\mathcal{O}(10^{-9})$ of CMB scales.



$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

- The density contrast is the relevant quantity [14 Young, Byrnes, Sasaki]

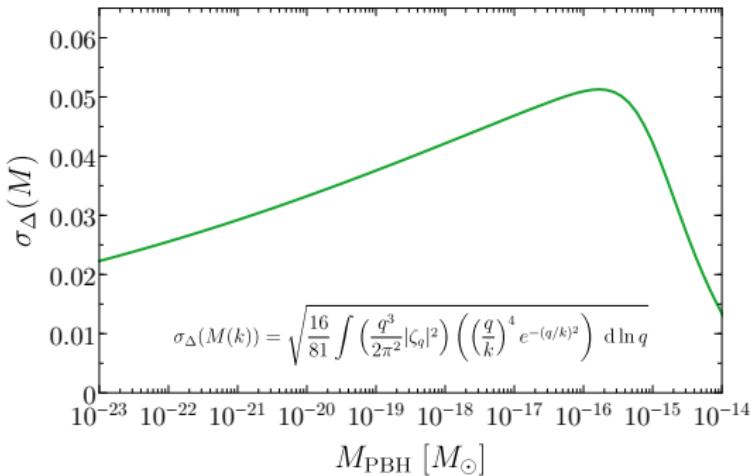
$$\Delta(t, \mathbf{x}) = \frac{4}{9} \left(\frac{1}{aH} \right)^2 \nabla^2 \zeta(\mathbf{x}),$$

physically corresponding to spatial curvature of the metric.

- Threshold for collapse depends on full spatial profile of $\Delta(t, \mathbf{x})$. [18 Yoo, Harada, Garriga, Kohri; 18 Germani, Musco; 19 Kalaja+]
- Approximation: collapse happens when Δ crosses a threshold $\Delta_c \approx 0.45$.

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

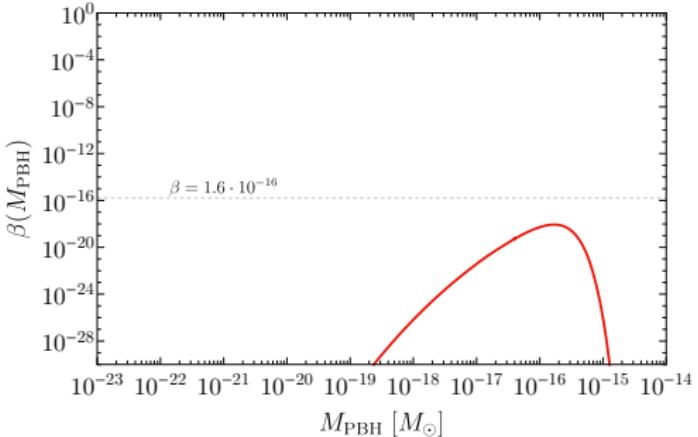
- How likely it is for Δ to cross the threshold Δ_c ?
- Approximation: Gaussian distribution, with variance σ_Δ .





- Formation rate: probability of exceeding threshold Δ_c .^a

$$\beta(M) = \int_{\Delta_c}^{\infty} \frac{d\Delta}{\sqrt{2\pi} \sigma_\Delta(M)} \exp\left(-\frac{\Delta^2}{2\sigma_\Delta^2(M)}\right)$$



^aNon-Gaussianities play an important role. [Franciolini, Kehagias, Matarrese, Riotto '18; De Luca, Franciolini, Kehagias, Peloso, Riotto, Ünal '19]

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}}$$

- After formation, $\rho_{\text{PBH}} \sim a^{-3}$ and behave as collisionless CDM. After equality, they scale as the rest of matter.

$$f(M) \equiv \frac{\Omega_{\text{PBH}}(M)}{\Omega_{\text{CDM}}} = \frac{\beta(M)}{1.6 \cdot 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}} \left(\frac{g_*(T_f)}{106.75}\right)^{-\frac{1}{4}} \left(\frac{M}{10^{-15} M_\odot}\right)^{-\frac{1}{2}}.$$

- For PBHs of mass $\sim 10^{-15} M_\odot$, the right abundance is achieved for $\sigma_\Delta \sim 0.05$.

Equation of motion for gravity waves:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{lm}\mathcal{S}_{lm}, \quad (1)$$

where the transverse projector and the source are

$$\widehat{\mathcal{T}}_{ij}^{lm}(\mathbf{k}) = e_{ij}^{(+)}(\mathbf{k})e^{(+lm)}(\mathbf{k}) + e_{ij}^{(\times)}(\mathbf{k})e^{(\times lm)}(\mathbf{k}), \quad (2)$$

$$\mathcal{S}_{ij} = 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{4}{3(1+w)}\partial_i\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)\partial_j\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right). \quad (3)$$

Solution with the Green function method in RD:

$$h_{\mathbf{k}}^s(\eta) = \frac{1}{a(\eta)} \int^\eta d\tilde{\eta} g_{\mathbf{k}}(\eta, \tilde{\eta}) a(\tilde{\eta}) \widehat{\mathcal{S}}^s(\tilde{\eta}, \mathbf{k}), \quad (4)$$

$$g_{\mathbf{k}}(\eta, \tilde{\eta}) = \frac{\sin[k(\eta - \tilde{\eta})]}{k} \theta(\eta - \tilde{\eta}). \quad (5)$$

Transfer function $T(\eta, k)$ in RD:

$$\widehat{\Psi}(\eta, \mathbf{k}) = \frac{2}{3}T(\eta, k)\zeta(\mathbf{k}), \quad (6)$$

$$T(\eta, k) = \mathcal{T}(k\eta), \quad \mathcal{T}(z) = \frac{9}{z^2} \left[\frac{\sin(z/\sqrt{3})}{z/\sqrt{3}} - \cos(z/\sqrt{3}) \right]. \quad (7)$$

The solution reads

$$h_{\mathbf{k}}^s(\eta) = \frac{4}{9} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{k^3 \eta} e^s(\mathbf{k}, \mathbf{p}) \zeta(\mathbf{p}) \zeta(\mathbf{k} - \mathbf{p}) [\mathcal{I}_c(x, y) \cos(k\eta) + \mathcal{I}_s(x, y) \sin(k\eta)] \quad (8)$$

where the polarisation tensor is

$$e^s(\mathbf{k}, \mathbf{p}) \equiv e^{s,ij}(\mathbf{k}) p_i p_j = \begin{cases} \frac{1}{\sqrt{2}} p^2 \sin^2 \theta \cos 2\phi & \text{for } s = (+), \\ \frac{1}{\sqrt{2}} p^2 \sin^2 \theta \sin 2\phi & \text{for } s = (\times), \end{cases} \quad (9)$$

and we have defined the dimensionless integrals

$$\begin{aligned} \mathcal{I}_c(x, y) &= \int_1^\infty d\tau \tau (-\sin \tau) \cdot 4 \left\{ 2\mathcal{T}(x\tau)\mathcal{T}(y\tau) + [\mathcal{T}(x\tau) + x\tau \mathcal{T}'(x\tau)][\mathcal{T}(y\tau) + y\tau \mathcal{T}'(y\tau)] \right\}, \\ \mathcal{I}_s(x, y) &= \int_1^\infty d\tau \tau (\cos \tau) \cdot 4 \left\{ 2\mathcal{T}(x\tau)\mathcal{T}(y\tau) + [\mathcal{T}(x\tau) + x\tau \mathcal{T}'(x\tau)][\mathcal{T}(y\tau) + y\tau \mathcal{T}'(y\tau)] \right\}. \end{aligned} \quad (10)$$

The dimensionless power spectrum is defined as

$$\langle h^r(\eta, \mathbf{k}_1) h^s(\eta, \mathbf{k}_2) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \delta^{rs} \frac{2\pi^2}{k_1^3} \mathcal{P}_h(k_1) \quad (11)$$

which gives a time-averaged power spectrum $\overline{\mathcal{P}_h(\eta, k)}$

$$\begin{aligned} \overline{\mathcal{P}_h(\eta, k)} &= \frac{2}{81} \frac{1}{k^2 \eta^2} \iint_{\mathcal{S}} dx dy \frac{x^2}{y^2} \left[1 - \frac{(1+x^2-y^2)^2}{4x^2} \right]^2 \cdot \\ &\quad \cdot \mathcal{P}_\zeta(kx) \mathcal{P}_\zeta(ky) \sqrt{\mathcal{I}_c(x, y)^2 + \mathcal{I}_s(x, y)^2}. \end{aligned} \quad (12)$$

The corresponding energy density reads

$$\Omega_{\text{GW}}(\eta_0, k) = \Omega_{r,0} \Omega_{\text{GW}}(\eta_f, k) = \frac{\Omega_{r,0}}{24} \frac{k^2}{\mathcal{H}(\eta_f)^2} \overline{\mathcal{P}_h(\eta_f, k)}. \quad (13)$$

- From the 3-pt function of GW we introduce the bispectrum B_h

$$\langle h^r(\eta, \mathbf{k}_1) h^s(\eta, \mathbf{k}_2) h^t(\eta, \mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

- The dimensionless normalised shape $S_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

$$S_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = k_1^2 k_2^2 k_3^2 \frac{B_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{\sqrt{\mathcal{P}_h(k_1) \mathcal{P}_h(k_2) \mathcal{P}_h(k_3)}}$$

identifies the strength of the bispectrum signal.

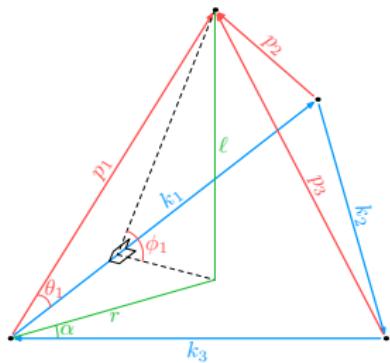
- $S_h(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is a “fingerprint” of the GW signal.

The bispectrum is defined through

$$\langle h^r(\eta, \mathbf{k}_1) h^s(\eta, \mathbf{k}_2) h^t(\eta, \mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \quad (14)$$

and its envelope over time reads

$$B_h^{rst}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 8 \left(\frac{4}{9} \right)^3 \pi^3 \int d^3 p_1 \frac{1}{k_1^3 k_2^3 k_3^3 \eta^3} e^r(\mathbf{k}_1, \mathbf{p}_1) e^s(\mathbf{k}_2, \mathbf{p}_2) e^t(\mathbf{k}_3, \mathbf{p}_3) \cdot \\ \cdot \frac{\mathcal{P}_\zeta(p_1)}{p_1^3} \frac{\mathcal{P}_\zeta(p_2)}{p_2^3} \frac{\mathcal{P}_\zeta(p_3)}{p_3^3} \sqrt{\mathcal{I}_c \left(\frac{p_1}{k_1}, \frac{p_2}{k_1} \right)^2 + \mathcal{I}_s \left(\frac{p_1}{k_1}, \frac{p_2}{k_1} \right)^2} \cdot \\ \cdot \sqrt{\mathcal{I}_c \left(\frac{p_2}{k_2}, \frac{p_3}{k_2} \right)^2 + \mathcal{I}_s \left(\frac{p_2}{k_2}, \frac{p_3}{k_2} \right)^2} \sqrt{\mathcal{I}_c \left(\frac{p_3}{k_3}, \frac{p_1}{k_3} \right)^2 + \mathcal{I}_s \left(\frac{p_3}{k_3}, \frac{p_1}{k_3} \right)^2} \quad (15)$$



$$\int d^3 p_1 \rightarrow \int_{-\infty}^{+\infty} d\ell \int_0^{+\infty} r dr \int_0^{2\pi} d\alpha,$$

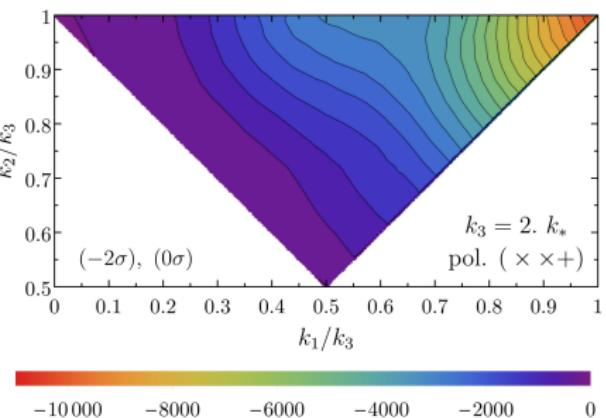
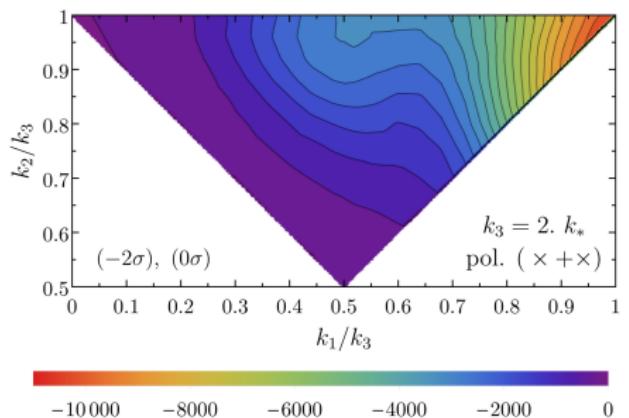
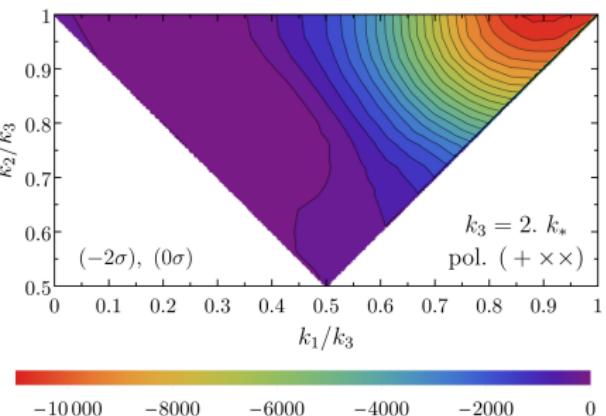
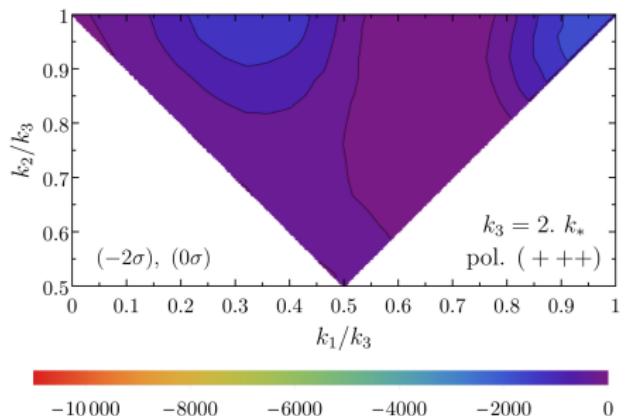
$$\mathbf{p}_1 = (r \cos \alpha, r \sin \alpha, \ell),$$

$$\mathbf{p}_2 = (-k_{1x} + r \cos \alpha, -k_{1y} + r \sin \alpha, \ell),$$

$$\mathbf{p}_3 = (k_3 + r \cos \alpha, r \sin \alpha, \ell),$$

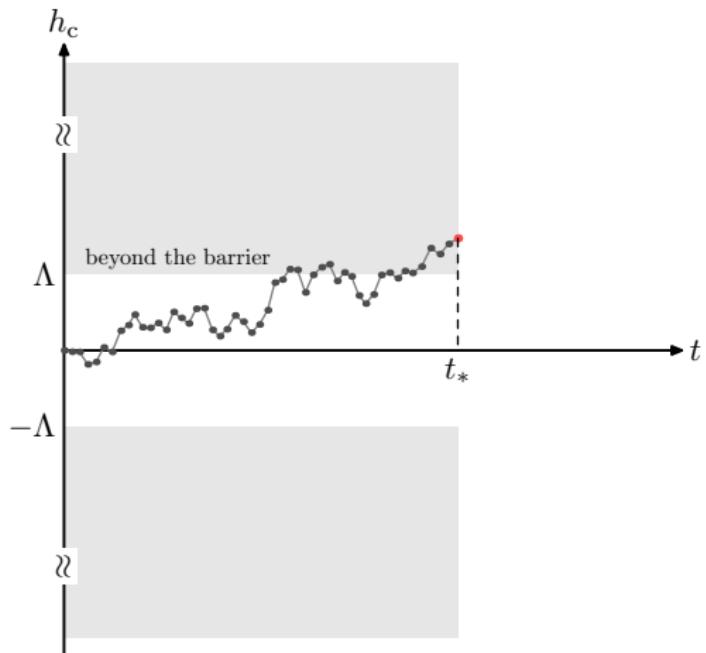
$$p_i^2 \sin^2 \theta_i = p_i^2 - \frac{|\mathbf{p}_i \cdot \mathbf{k}_i|^2}{k_i^2}, \quad \sin \phi_i = \frac{\ell k_i}{|\mathbf{p}_i \times \mathbf{k}_i|}.$$

Bispectrum of Gravitational Waves



Evolution of the background of the Higgs

1710.11196 Espinosa, DR, Riotto, Phys. Rev. Lett. **120** (2018) no.12, 121301

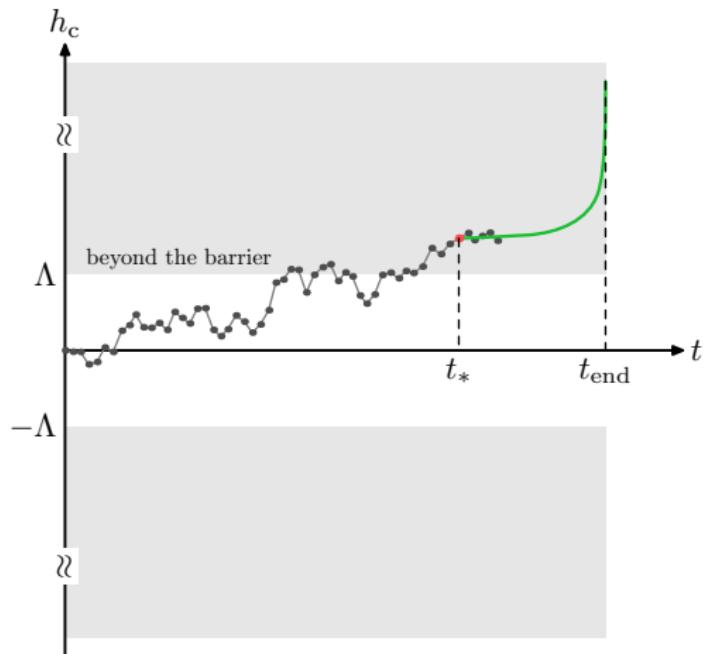


- We assume that $V(h) < 0$ at some scale.
- During inflation, the background field $h_c(t)$ has random fluctuations

$$\Delta_q h_c \sim \pm \frac{H}{2\pi} .$$

Evolution of the background of the Higgs

1710.11196 Espinosa, DR, Riotto, Phys. Rev. Lett. **120** (2018) no.12, 121301

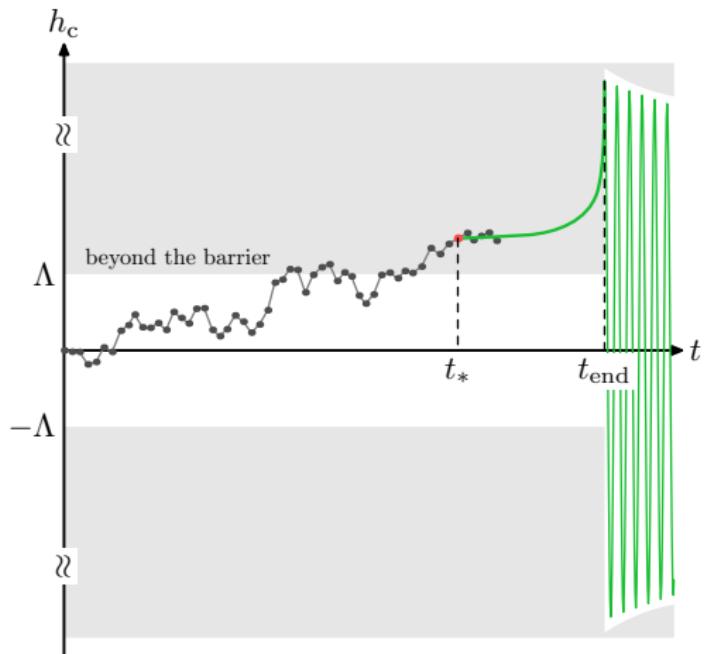


- If classical evolution prevails over quantum fluctuations, h_c begins to slow roll down the negative potential.

$$\text{classical} \quad \overbrace{h_c \Delta t} \gtrsim \text{quantum} \quad \overbrace{\frac{H}{2\pi}}$$

Evolution of the background of the Higgs

1710.11196 Espinosa, DR, Riotto, Phys. Rev. Lett. **120** (2018) no.12, 121301



- At the end of inflation, if the reheating temperature is high enough and Higgs has not gone too far down the potential, thermal corrections can rescue h and bring it back around 0.
- Decay to radiation: in a very short time the Higgs field decays to radiation.

- Define the fluctuations of the Higgs, $h(t, \mathbf{x}) = h_c(t) + \delta h(t, \mathbf{x})$.

$$\ddot{\delta h}_k + 3H\dot{\delta h}_k + \frac{k^2}{a^2} \delta h_k + V''(h_c) \delta h_k = 0$$

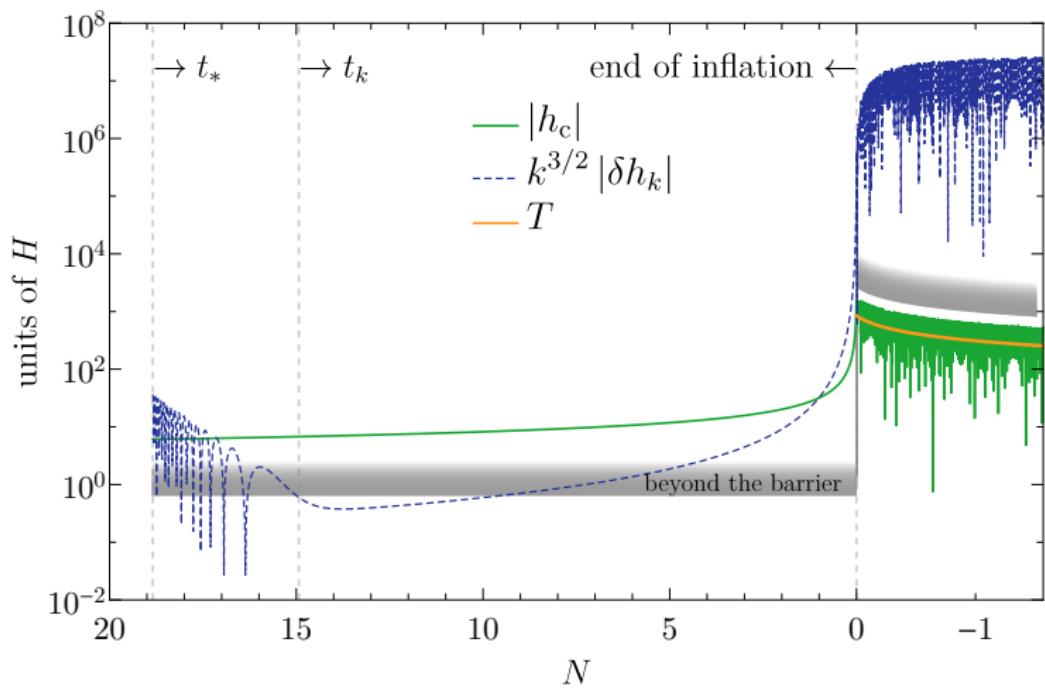
Sub-Hubble

Oscillation term dominates.

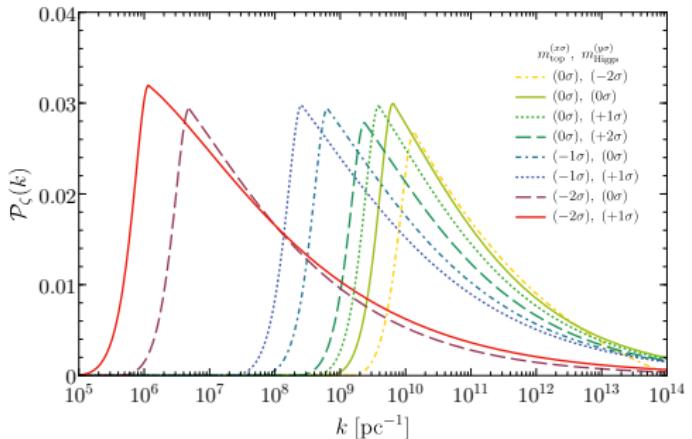
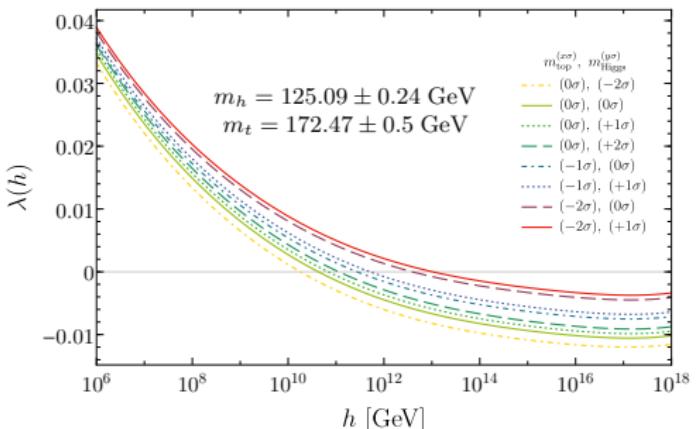
Super-Hubble

After the Hubble crossing $k \sim aH$, which happens at t_k , dominates.
 $V''(h_c) < 0 \Rightarrow$ source term which drives δh_k .

Evolution of Higgs background and fluctuations



Power spectrum of curvature perturbations

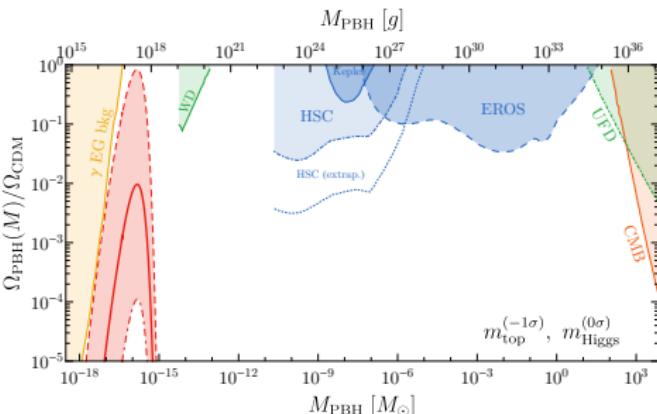


1st signature: Primordial Black Holes

- Position of the peak: depends on t_* , thus on the slope of $V(h)$.

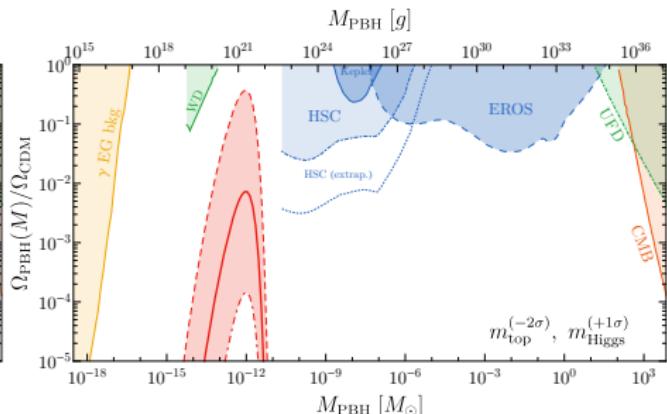
$$m_{\text{Higgs}} = 125.09 \text{ GeV}$$

$$m_{\text{top}} = 172 \text{ GeV}$$



$$m_{\text{Higgs}} = 125.33 \text{ GeV}$$

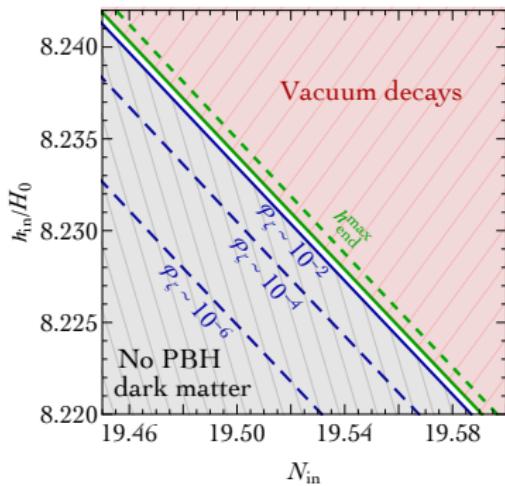
$$m_{\text{top}} = 171.47 \text{ GeV}$$



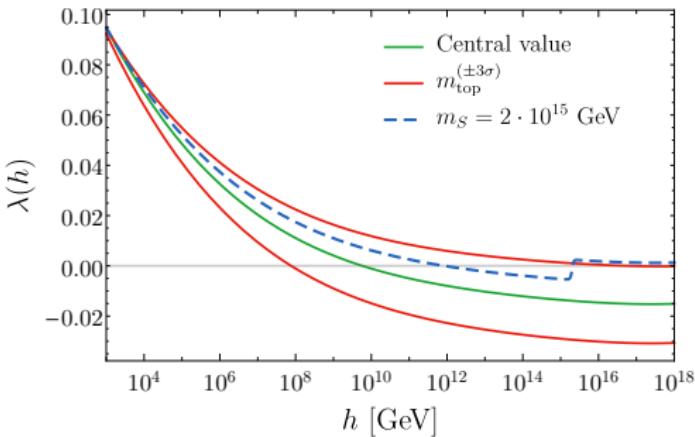
- Height of the peak: depends on how much the fluctuations grew.
Finely tuned, as in any model for PBH formation.
In SM + inflation, DM is provided by this mechanism \Rightarrow anthropic explanations: without DM there would be no Large Scale Structures.
- Important discussion about spatial inhomogeneities:

[1803.10242 Gross, Polosa, Strumia, Urbano, Xue; 1804.07731 Espinosa, Racco, Riotto]

- Small fluctuations of order $H/(2\pi) \sim 0.16H$ occur on a scale H^{-1} .
- These are larger than the precision required on the initial value h_* .
- A small overshooting on h_* prevents the reheating from rescuing h , leaving an expanding AdS region.



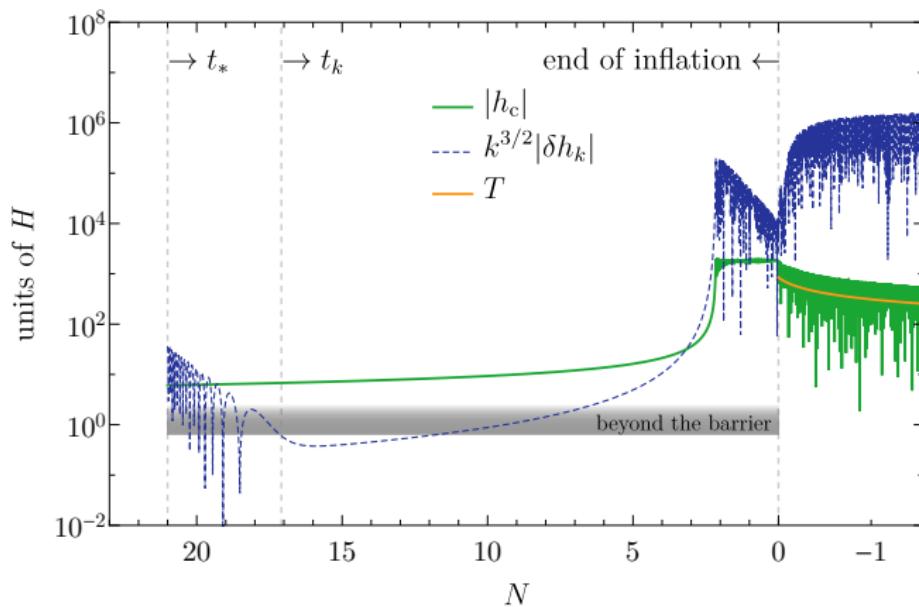
[1803.10242 Gross, Polosa, Strumia, Urbano, Xue]



- Example of BSM contributions which can lift $\lambda(h)$ to positive values: extra scalar S interacting with the Higgs, yielding a positive contribution to λ above m_S [1203.0237, Elias-Miró, Espinosa, Giudice, Lee, Strumia].
- Higgs potential has an absolute minimum at $\sim m_S$.
- Assume $m_S \lesssim T_{\text{RH}}$, so that the Higgs is always rescued at reheating. No fine-tuning here: we can allow variations $\sim (10 - 20)\%$ for m_S , not to spoil the PBH generation.

Evolution in the presence of an absolute minimum

- If h_c undershoots (starts too late, or too low), nothing changes.
- If h_c overshoots (starts too early, or too high), it reaches the minimum, where it oscillates. During this phase h and δh_k slowly decrease as matter.



Evolution in the presence of an absolute minimum

