Rare Decays and Lepton Flavor Universality Ratios

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• We will focus on rare decays of mesons containing a $b$-quark: “$B$-mesons”

• **Dominant decays of $b$-quarks are tree level transitions** $b \rightarrow cW^-$. There are also rare tree transition $b \rightarrow uW^-$ (suppressed by $|V_{cb}/V_{ub}|^2$).

• $b \rightarrow s$ (or, $b \rightarrow d$) quark level transitions are FCNCs and are forbidden in the SM at the tree level and can only occur at greatly suppressed rates through higher-order processes (penguin loops/box).
• The decays such as $B \rightarrow K(\ast) \ell^+ \ell^-$ are manifestations of quark level transitions $b \rightarrow s\ell^+ \ell^-$. 

The $\gamma/Z$ Penguin loops

$W^+W^-$ Box Diagram

a virtual $t$ quark contribution dominates, with secondary contributions from virtual $c$ and $u$ quarks.

• Sensitive to NP: Interference from the ‘possible’ contribution from the BSM.

• These decays are rich laboratories of NP studies on its own and offer hope of new physics.
• The decays such as $B \to K^{(*)} \ell^+ \ell^-$ are manifestations of quark level transitions $b \to s \ell^+ \ell^-$.  

\[
\begin{align*}
\text{\gamma/Z Penguin loops} & \quad \text{Box Diagram} \\
\end{align*}
\]

a virtual $t$ quark contribution dominates, with secondary contributions from virtual $c$ and $u$ quarks.

The relevant effective Hamiltonian:

\[
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C'_i O'_i) + \text{h.c.}
\]

The Operators which are most sensitive to the NP:

- $O_7$ dipole  
- $O_9$ and $O_{10}$ four fermion
First observation of a $b \rightarrow s\ell^+\ell^-$ decay (LP-2001)

$B^+ \rightarrow K^+\mu^+\mu^-$ Event

$B^+ \rightarrow K^+\mu^+\mu^-$
$B^0 \rightarrow K_s^0\mu^+\mu^-$
combined

29.5 fb$^{-1}$

9.5$^{+3.7}_{-3.1}$ events
4.8$\sigma$

Lepton Photon 01, 2001 July 23, Roma
Forward Backward asymmetry in $B \rightarrow K^* \ell^+ \ell^-$

Based on 657 M BB pairs, $\mu^+ \mu^-$ and $e^+e^-$ hadrons are produced:

- $B \rightarrow K^+\pi^-, K_S^0\pi^+, K^+\pi^0, K^+$ and $K_S^0$

$F_L$ and $A_{FB}$ are obtained from fit to $\cos\theta_{K^*}$ and $\cos\theta_{B\ell}$

\[
\begin{align*}
\frac{3}{2}F_L \cos^2\theta_{K^*} + \frac{3}{4}(1 - F_L)(1 - \cos^2\theta_{K^*}) & \varepsilon(\cos\theta_{K^*}) \\
\frac{3}{4}F_L (1 - \cos^2\theta_{B\ell}) + \frac{3}{8}(1 - F_L)(1 + \cos^2\theta_{B\ell}) & + A_{FB} \cos\theta_{B\ell} \varepsilon(\cos\theta_{B\ell})
\end{align*}
\]

For full $q^2$ range:

- $R(K^*) = 0.83 \pm 0.17 \pm 0.08$
- $R(K) = 1.03 \pm 0.19 \pm 0.06$

Till last year
Angular Analysis of $B \to K^* \ell^+ \ell^-$

- A number of angular observables in $B \to K^* \ell^+ \ell^-$ decays can be theoretically predicted with good control of the relevant form factor uncertainties.

- The observables $P_i'$ are considered to be largely free from form-factor related uncertainties (Introduced by LHCb in PRL 111, 191801 (2013)).

- Measurements are mostly compatible with the SM.
- LHCb observed deviation wrt SM predictions in $P_5'$ with $3.7 \sigma$.
- Belle observed similar central values for the $P_5'$ anomaly $q^2 \in (1.1, 6.0)$ GeV$^2$/c$^4$ with $2.5\sigma$ tension.

P_i' and Q_i' in separate lepton flavors


- The Largest deviation in the muon mode with \(2.6\sigma\).
- Electron mode is deviating with \(1.1\sigma\).
- Belle (II) is equally efficient in e & \(\mu\) modes.

- With 2.8 ab\(^{-1}\) the uncertainty on \(P_5'(e \& \mu)\) at Belle II will be comparable to LHCb 3fb\(^{-1}\) (\(\mu\) only). B2TIP report | arXiv:1808.10567

- Test lepton flavor universality.
- Observables \(Q_i = P_i'\mu - P_i'e\).
  [JHEP 10, 075 (2016)]
- Deviation from zero very sensitive to NP.

- No significant deviation from zero is seen.
- \(Q_4\) and \(Q_5\) observables in agreement with SM and central values favoring NP scenario.
Lepton Flavor Universality Ratios

- The lepton flavor universality can be tested very precisely with the ratios:

\[
R_H[q_0^2, q_1^2] = \frac{\int_{q_0^2}^{q_1^2} dq^2 \frac{d\Gamma(B \rightarrow H\mu^+\mu^-)}{dq^2}}{\int_{q_0^2}^{q_1^2} dq^2 \frac{d\Gamma(B \rightarrow He^+e^-)}{dq^2}} \ ; H = K, K^*
\]

- In these ratios, hadronic uncertainties in theoretical predictions cancel and SM prediction is (very) close to unity.

- Experimentally, many sources of systematic uncertainties are also substantially reduced.

Status till last year 2018

Updates from Belle & LHCb this year

Updates from Belle this year

LHCb, Phys. Rev. Lett. 113 (2014) 151601

LHCb, JHEP08(2017)055


BaBar, Phys. Rev. D 86 (2012) 032012

[BIP, EPJC 76 440] [CDHMV, JHEP04(2017)016]

[EOS, PRD 95 035029] [flav.io, EPJC 77 377] [JC, PRD93 014028]
Recent R(K*) measurement at Belle

- Reconstructed B⁰ and B⁺: \( B \rightarrow K^*(892)\ell^+\ell^- \rightarrow \mu^+\mu^- \) and \( e^+e^- \)
- Full Belle data set 711 fb⁻¹
- Bremsstrahlung losses are recovered in electron candidates.

**Hierarchical NN Reconstruction**: A dedicated NN classifier is trained with MC samples to identify each particle type used in the decay chain.

- To further suppress \( (e^+e^- \rightarrow q\bar{q}) \) background events, variables related to event shape variables, vertex information are used in the NN.

- Large irreducible background events arise from the decay \( B \rightarrow K^*/\psi[\psi(2S)] \), which are vetoed by applying criteria on di-lepton invariant mass.

- However, the decays \( B \rightarrow K^*/\psi[\psi(2S)] \) serve as a very good Control Sample.

\[
\frac{BF[B \rightarrow K^*/\psi(\rightarrow \mu^+\mu^-)]}{BF[B \rightarrow K^*/\psi(\rightarrow e^+e^-)]} = 1.015 \pm 0.025 \pm 0.038
\]
Recent R(K*) measurement at Belle

- Signal is extracted in Beam Constrained Mass: \[ M_{bc} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_B|^2} \]
- Signal pdf: **Crystal Ball**, Combinatorial background pdf: **Argus shape**.
- For example, the fit presented below are for the \( q^2 > 0.045 \text{ GeV}^2/c^4 \)

**Analysis is performed in several \( q^2 \) bins \([0.045, 1.1], [1.1, 6.0], [0.1, 8.0], [15, 19], \) and \( > 0.045 \text{ GeV}^2/c^4 \)**
• Belle also provided first measurement of $R(K^{*+})$.
• Latest $R(K^*)$ measurement from Belle are consistent with the SM as well as with the previous measurements from LHCb (and BaBar).

**LHCb measurements for $R(K^*)$:**

\[
0.66^{+0.11}_{-0.07}\text{ (stat)} \pm 0.03 \text{ (sys.)} \\
\text{for } q^2 \in (0.045,1.1) \text{ GeV}^2/c^4 \\
2.1 - 2.3 \sigma \text{ from SM}
\]

\[
0.69^{+0.11}_{-0.07}\text{ (stat)} \pm 0.05 \text{ (sys.)} \\
\text{for } q^2 \in (1.1,6.0) \text{ GeV}^2/c^4 \\
2.4 - 2.5 \sigma \text{ from SM}
\]

**Belle measurements for $R(K^*)$:**

\[
0.52^{+0.36}_{-0.26}\text{ (stat)} \pm 0.05 \text{ (sys.)} \\
\text{for } q^2 \in (0.045,1.1) \text{ GeV}^2/c^4
\]

\[
0.96^{+0.29}_{-0.27}\text{ (stat)} \pm 0.11 \text{ (sys.)} \\
\text{for } q^2 \in (1.1,6.0) \text{ GeV}^2/c^4
\]

SM example: JHEP 1801 (2018) 093
for $q^2 \in (0.045,1.1) \text{ GeV}^2/c^4$ : 0.92 ± 0.02
for $q^2 \in (1.1,6.0) \text{ GeV}^2/c^4$ : 1.00 ± 0.01
Recent R(K) measurement at LHCb

- Reconstructed $B^+ : B^+ \rightarrow K^+ \ell^+\ell^-$ using 5 fb$^{-1}$ of pp collision at CM energy 7, 8 and 12 TeV
- Significantly different reconstruction of decays with muons in the final state as compared to decays with electrons (brem losses and different trigger selection).

**Measure $R_K$ as a double ratio:**

$$R_K = \frac{\mathcal{B}(B^+ \rightarrow K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K^+)} \bigg/ \frac{\mathcal{B}(B^+ \rightarrow K^+e^+e^-)}{\mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow e^+e^-)K^+)}$$

- Several cross-checks are used to verify the analysis procedure.
  - Single ratio $r(J/\psi) = [1.014 \pm 0.035]$ is found to be consistent with unity (also as a function momentum of leptons and dilepton opening angle.)
  - Double ratio $R_{K}^{\psi(2S)} = [0.986 \pm 0.013]$ is determined close to 1.
Recent R(K) measurement at LHCb

- Reconstructed $B^+ : B^+ \to K^+ \ell^+ \ell^-$ using 5 fb$^{-1}$ of pp collision at CM energy 7, 8 and 12 TeV

- Significantly different reconstruction of decays with muons in the final state as compared to decays with electrons (brem losses and different trigger selection).

- A total of $1943 \pm 49 \ B^+ \to K^+ \mu^+ \mu^-$ decays are observed.

- The value of $R_K \ [q^2 \in (1.1, 6.0) \text{ GeV}^2/c^4] = 0.846^{+0.016}_{-0.014} \text{ (sys.)}$
Recent R(K) measurement at Belle

- Reconstructed $B^0$ and $B^+$: $B \rightarrow K \ell^+ \ell^-$
- Full Belle data set $711 \text{ fb}^{-1}$
- Charged tracks are required to originate near the interaction region (except $K_S^0$) and further selected based on particle identification.
- Bremsstrahlung losses are recovered in electron candidates.
- A NN is trained with input variables related event shape, vertex quality and decay kinematics to suppress the background from continuum and generic $B$ decays.
- Large irreducible background events arise from the decay $B \rightarrow K J/\psi[\psi(2S)]$, which are vetoed by applying criteria on di-lepton invariant mass.
- Also a veto $[M_{K\pi} \notin (1.85, 1.88) \text{ GeV}/c^2]$ is applied to suppress events arising from the decay $B^- \rightarrow D^0[K^- \pi^+] \pi^-$ due to particle mis-identification.
- The decays $B \rightarrow K J/\psi[\psi(2S)]$ served as a good control sample.

\[
\frac{BF[B^+ \rightarrow K^+ J/\psi(\rightarrow \mu^+ \mu^-)]}{BF[B^+ \rightarrow K^+ J/\psi(\rightarrow e^+ e^-)]} = 0.992 \pm 0.011 \\
\frac{BF[B^0 \rightarrow K_S^0 J/\psi(\rightarrow \mu^+ \mu^-)]}{BF[B^0 \rightarrow K_S^0 J/\psi(\rightarrow e^+ e^-)]} = 1.048 \pm 0.020
\]
• The NN output (O) is translated to (O’) using the formula:

\[ O' = \log \frac{O - O_{\text{min}}}{O_{\text{max}} - O} \]

• Requirement \( O_{\text{min}} > -0.6 \) reduces 75\% bkg with 4-5\% signal loss.

• Extended maximum likelihood fit is performed in 3-dimensions: \( M_{bc}, \Delta E (E_B - E_{\text{beam}}), \) and \( O' \). (parameterized with MC, control samples and off-resonance data).

• For example, the fit presented below are for the \( q^2 > 0.1 \text{ GeV}^2/c^4 \)

\[ B^+ \rightarrow K^+ \mu^+ \mu^- \quad 137 \pm 14 \]
\[ B^0 \rightarrow K_S^0 \mu^+ \mu^- \quad 27.3^{+6.6}_{-5.8} \]

\[ B^+ \rightarrow K^+ e^+ e^- \quad 138 \pm 15 \]
\[ B^0 \rightarrow K_S^0 \mu^+ \mu^- \quad 21.8^{+7.0}_{-6.1} \]

• The value of \( R_K [q^2 \in (1.0, 6.0) \text{ GeV}^2/c^4] = 0.98^{+0.27}_{-0.23} \text{ (stat.) } \pm 0.06 \text{ (sys. )} \)
• **Belle measured $R(K)$ in several $q^2$ bins** and also reported first measurement of $R(K_{s0})$.

• In all the bins Belle’s $R(K)$ is **consistent with SM** value.

- **Belle measurement** $R(K)$
  \[0.98^{+0.27}_{-0.23} \pm 0.06\] [in $q^2 \in (1.0, 6.0) \text{ GeV}^2/c^4$] **is consistent with LHCb measurement** of $R(K) = 0.846^{+0.060}_{-0.054}^{+0.016}_{-0.014}$ [in $q^2 \in (1.1, 6.0) \text{ GeV}^2/c^4$].
  
- **LHCb measurement** is compatible with SM at 2.5$\sigma$.

• In fact, Belle $R(K^+) = 1.31^{+0.34}_{-0.31} \pm 0.07$ [in $q^2 \in (1.0, 6.0) \text{ GeV}^2/c^4$]
Recent A(I) measurement at Belle

• Another theoretically clean observable is CP averaged isospin asymmetry:

\[ A_I = \frac{\left( \frac{\tau_{B^+}}{\tau_{B^0}} \right) \times \mathcal{B}(B^0 \rightarrow K^0 \ell \ell) - \mathcal{B}(B^+ \rightarrow K^+ \ell \ell)}{\left( \frac{\tau_{B^+}}{\tau_{B^0}} \right) \times \mathcal{B}(B^0 \rightarrow K^0 \ell \ell) + \mathcal{B}(B^+ \rightarrow K^+ \ell \ell)} \]

The value of A(I) is expected to be close to zero in the SM.

J. Lyon and R. Zwicky,

• Earlier, BaBar [PRD 86, 032012 (2012)], Belle [PRL103, 171801 (2009)] and LHCb [JHEP 06 (2014)133] had reported A(I) to be significantly below zero, especially in the \( q^2 \) region below the \( J/\psi \) resonance.

• Belle’s A(I) measurement is consistent with the previous measurements.

• In all bins A(I) is below zero.
Recent A(I) measurement at Belle

- Another theoretically clean observable is CP averaged isospin asymmetry:

\[
A_I = \frac{\left( \frac{\tau_{B^+}}{\tau_{B^0}} \right) \times \mathcal{B}(B^0 \to K^0 \ell \ell) - \mathcal{B}(B^+ \to K^+ \ell \ell)}{\left( \frac{\tau_{B^+}}{\tau_{B^0}} \right) \times \mathcal{B}(B^0 \to K^0 \ell \ell) + \mathcal{B}(B^+ \to K^+ \ell \ell)}
\]

The value of A(I) is expected to be close to zero in the SM.


- Earlier, BaBar [PRD 86, 032012 (2012)], Belle [PRL103, 171801 (2009)] and LHCb [JHEP 06 (2014)133] had reported A(I) to be significantly below zero, especially in the \( q^2 \) region below the J/ψ resonance.

\[B \rightarrow K\mu^+\mu^-\]

\[B \rightarrow K\mathbf{e}^+\mathbf{e}^-\]
Belle II Projections for R(K) and R(K*)

- Upcoming Belle II measurements will be helpful in reducing statistical uncertainties.
- Total uncertainties on R(K) and R(K*) measurements can reach down to below 5% with full data-set at Belle II.
- Uncertainties are still statistical dominant (total systematic is below 1% with dominating uncertainty from lepton identification ~ 0.4%)


Belle II Commissioning, First Results, and Future Prospects: Z. Liptak (Jul 31st)

Quark & Lepton Flavor Session
Summary

- The decays $B \rightarrow K^{(*)} \ell^+ \ell^-$ are FCNC processes and is a laboratory of New Physics studies on its own.

- The ratio of branching fractions $R(K)$ and $R(K^*)$ are theoretically as well as experimentally clean variables.

- Most precise results come from LHCb collaboration and reported (this year: **PRL122 (2019) 191801**) a deviation of $2.6\sigma$ in $q^2 \in (1.1,6.0)$ GeV$^2$/c$^4$ for $R(K)$ and similar deviations were reported earlier in $R(K^*)$ **JHEP08(2017)055**.

- Belle measurements of $R(K^*)$ (**arXiv:1904.02440**) and $R(K)$ are compatible with both SM as well as with past measurements.

- Belle A(I) measurement for $B \rightarrow K \ell^+ \ell^-$ is found significantly below zero (specially in $q^2 \in (1.0,6.0)$ GeV$^2$/c$^4$ for decay $B \rightarrow K \mu^+ \mu^-$ with deviation $\sim 2.7\sigma$).

- Belle II experiment has started physics runs and expected to accumulate larger data sample, which will be crucial for rare decays measurements.

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**No penguins were harmed in any of these measurements**
Extra Slides
The APS (DPF) Meeting 2019 | S. Sandilya

LHCb prospects

From slides of Julián García Pardiñas (Win 2019) & Vitalii Lisovskyi (EPSHEP 2019)
**arXiv:1903.0961**
[Alok, Amol, Shireen, Dinesh]

- analyze all the scenarios where the NP contributes to a pair of $(O_9, O_{10}, O'_9, O'_{10})$ operators at a time.

- Scenarios with new physics contributions to the $(C^\text{NP}_9, C'_9)$ or $(C^\text{NP}_9, C^\text{NP}_{10})$ pair remain the most favored ones.
Forward Backward asymmetry in $B \rightarrow K^* \ell^+ \ell^-$

$B \rightarrow K^{(*)} \ell^+ \ell^-$ based on 657 M BB pairs

$\mu^+ \mu^-$ and $e^+ e^-$

$K^+ \pi^-, K_S^0 \pi^+, K^+ \pi^0, K^+$ and $K_S^0$

$F_L$ and $A_{FB}$ are obtained from fit to $\cos \theta_{K^*}$ and $\cos \theta_{B\ell}$

For illustration in $q^2 \in (0, 2.0) \text{ GeV}^2/c^2$

$R(K^*) = 0.83 \pm 0.17 \pm 0.08$

$R(K) = 1.03 \pm 0.19 \pm 0.06$

Till this year
Belle II prospects for $B \rightarrow X_s \ell^+ \ell^-$

- Belle II can significantly improve upon this situation and with its expected larger statistics.

- In the beginning, Belle II will still have to rely on the sum-of-exclusive method but later fully inclusive analysis can also be attempted.

<table>
<thead>
<tr>
<th>Observables</th>
<th>Belle 0.71 ab$^{-1}$</th>
<th>Belle II 5 ab$^{-1}$</th>
<th>Belle II 50 ab$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Br(B \rightarrow X_s \ell^+ \ell^-)$ ([1.0, 3.5] GeV$^2$)</td>
<td>29%</td>
<td>13%</td>
<td>6.6%</td>
</tr>
<tr>
<td>$Br(B \rightarrow X_s \ell^+ \ell^-)$ ([3.5, 6.0] GeV$^2$)</td>
<td>24%</td>
<td>11%</td>
<td>6.4%</td>
</tr>
<tr>
<td>$Br(B \rightarrow X_s \ell^+ \ell^-)$ (&gt; 14.4 GeV$^2$)</td>
<td>23%</td>
<td>10%</td>
<td>4.7%</td>
</tr>
<tr>
<td>$A_{FB}(B \rightarrow X_s \ell^+ \ell^-)$ ([1.0, 3.5] GeV$^2$)</td>
<td>26%</td>
<td>9.7%</td>
<td>3.1%</td>
</tr>
<tr>
<td>$A_{FB}(B \rightarrow X_s \ell^+ \ell^-)$ ([3.5, 6.0] GeV$^2$)</td>
<td>21%</td>
<td>7.9%</td>
<td>2.6%</td>
</tr>
<tr>
<td>$A_{FB}(B \rightarrow X_s \ell^+ \ell^-)$ (&gt; 14.4 GeV$^2$)</td>
<td>19%</td>
<td>7.3%</td>
<td>2.4%</td>
</tr>
</tbody>
</table>


5% CL on the $R_{9,10} = \frac{C_{9,10}}{C_{9,10}^{SM}}$

In SM: $R_{9,10} = 1$

Belle II reach measurement of the forward-backward asymmetry from global analysis

Exclusion contours in the $C_{9}^{NP} - C_{10}^{NP}$ plane resulting from future BF and $A_{FB}$ for $B \rightarrow X_s \ell^+ \ell^-$
Measurement of $A_{FB}(B \rightarrow X_s \ell^+ \ell^-)$

- Inclusive measurement is theoretically cleaner than the exclusive, but experimentally more challenging.

- Sum-of-exclusive technique (10 modes with $M[X_s] < 2.0 \text{ GeV}/c^2$) used to measured $A_{FB}$ (corresponds to ~50% of the inclusive rate).

$\Leftarrow$ For illustration in $q^2 \in (1, 6) \text{ GeV}^2/c^2$

- The result is consistent with a SM prediction within error (1.8$\sigma$ tension in low-$q^2$).

- Results are statistically dominated $\rightarrow$ Belle II (with its expected larger statistics).

- Also, fully inclusive analysis can also be attempted in Belle II.
The differential decay rate for $B \rightarrow K^* \ell^+ \ell^-$ can be written as

$$\frac{1}{d\Gamma/dq^2 d\cos \theta_L d\cos \theta_K d\phi dq^2} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right. \\
+ \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_L \\
- F_L \cos^2 \theta_K \cos 2\theta_L + S_3 \sin^2 \theta_K \sin^2 \theta_L \cos 2\phi \\
+ S_4 \sin 2\theta_K \sin 2\theta_L \cos \phi + S_5 \sin 2\theta_K \sin \theta_L \cos \phi \\
+ S_6 \sin^2 \theta_K \cos \theta_L + S_7 \sin 2\theta_K \sin \theta_L \sin \phi \\
+ S_8 \sin 2\theta_K \sin 2\theta_L \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_L \sin 2\phi \left. \right] ,$$
In the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, followed by $K^{*0} \rightarrow K^+ \pi^-$, the direction of the four outgoing particles can be described by three angles, shown in Fig. 11. The forward-backward asymmetry $A_{FB}$ is defined as the relative difference between the number of positive and negative leptons going along the direction of the $B^0$ meson in the rest frame of the two-lepton system. This corresponds to an asymmetry in the distribution of the $\theta_\ell$ angle. Similarly, the $K^{*0}$ polarisation fraction $F_L$ depends on the angle $\theta_K$, defined analogously to $\theta_\ell$. Other asymmetries can be constructed from the other angles or combinations of them. The $P_5'$ asymmetry suggested by Ref. 101 is based on the angles $\theta_K$ and $\phi$. It is defined as the relative difference between the number of decays in the regions in red and blue in Fig. 12, divided by $\sqrt{F_L(1-F_L)}$. Quantities based on several angles are more difficult to measure than single-angle ones as they require a better understanding of the reconstruction efficiencies depending on the kinematics of the outgoing particles.
Angular Analysis of $B \rightarrow K^* \ell^+ \ell^-$ at Belle

The differential decay rate for $B \rightarrow K^* \ell^+ \ell^-$ can be written as

$$
\frac{1}{d\Gamma / dq^2} \frac{d^4 \Gamma}{d \cos \theta_L \, d \cos \theta_K \, d\phi \, dq^2} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \\
+ \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_L \\
- F_L \cos^2 \theta_K \cos 2\theta_L + S_3 \sin^2 \theta_K \sin^2 \theta_L \cos 2\phi \\
+ S_4 \sin 2\theta_K \sin 2\theta_L \cos \phi + S_5 \sin 2\theta_K \sin \theta_L \cos \phi \\
+ S_6 \sin^2 \theta_K \cos \theta_L + S_7 \sin 2\theta_K \sin \theta_L \sin \phi \\
+ S_8 \sin 2\theta_K \sin 2\theta_L \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_L \sin 2\phi \right]
$$

JHEP 01 (2009) 019

The observables are considered to be largely free from form-factor related uncertainties.

$P'_{4, 5, 6, 8} = \frac{S_{i=4,5,7,8}}{\sqrt{F_L(1 - F_L)}}$

$P'_{4, 5, 6, 8}$, $S_4, S_5$

- $P'_{4, 5, 6, 8}$, $S_4, S_5$ defined as
- $\phi \rightarrow -\phi$ for $\phi < 0$
- $\phi \rightarrow \pi - \phi$ for $\theta_L > \pi/2$
- $\theta_L \rightarrow \pi - \theta_L$ for $\theta_L > \pi/2$
Folding Procedure

\[ P'_4, S_4 : \begin{cases} 
\phi \rightarrow -\phi & \text{for } \phi < 0 \\
\phi \rightarrow \pi - \phi & \text{for } \theta_L > \pi/2 \\
\theta_L \rightarrow \pi - \theta_L & \text{for } \theta_L > \pi/2,
\end{cases} \]

\[ P'_5, S_5 : \begin{cases} 
\phi \rightarrow -\phi & \text{for } \phi < 0 \\
\theta_L \rightarrow \pi - \theta_L & \text{for } \theta_L > \pi/2,
\end{cases} \]

- With a transformation of the angles, the dimension is reduced to three free parameters.
- Each transformation remains three observables \( S_j, F_L \) and \( S_3 \).
- The observables
  \[ P'_{i=4,5,6,8} = \frac{S_{i=4,5,7,8}}{\sqrt{F_L(1 - F_L)}}, \]
  are considered to be largely free from form-factor uncertainties (J. High Energy Phys. 05 (2013) 137).
- Transverse polarization asymmetry
  \[ A_T^{(2)} = \frac{2S_3}{(1 - F_L)}. \]
Angular Analysis of $B \rightarrow K^* \ell^+ \ell^-$ at Belle

- Data is divided in the $q^2$ bins.
- Signal and background fraction is obtained by fitting $M_{bc}$ distribution.
- The data is split into a sideband and signal region.
- Shape of the background can be determined in the sideband region.
- Final fit in signal region for each transformation.