# Lattice QCD: New Tools, New Ideas, New Results

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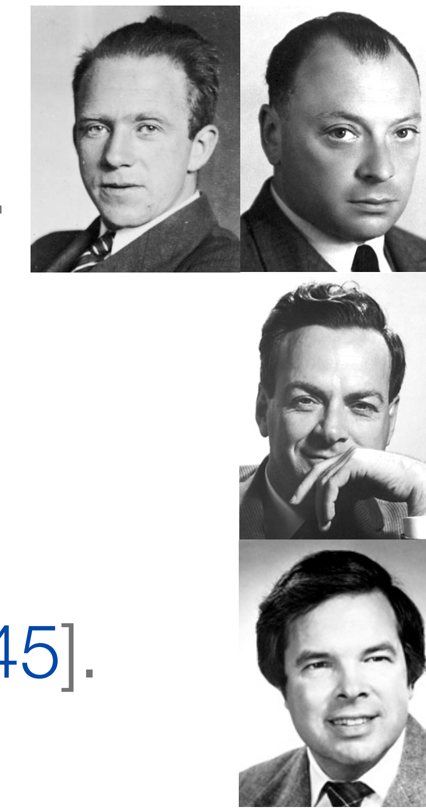
# Basic Idea



- Discretize space to have Kronecker $\delta_{xy}$ instead of Dirac $\delta(x-y)$ so the degrees of freedom are countable [Z. Phys. 56 (1929) 1].
- Set up path integral for these degrees of freedom, which introduces discrete time steps [RMP 20 (1948) 367].
- Figure out how to enforce gauge invariance [PRD 10 (1974) 2445].
- End up with mathematically well-posed definition of QFT:

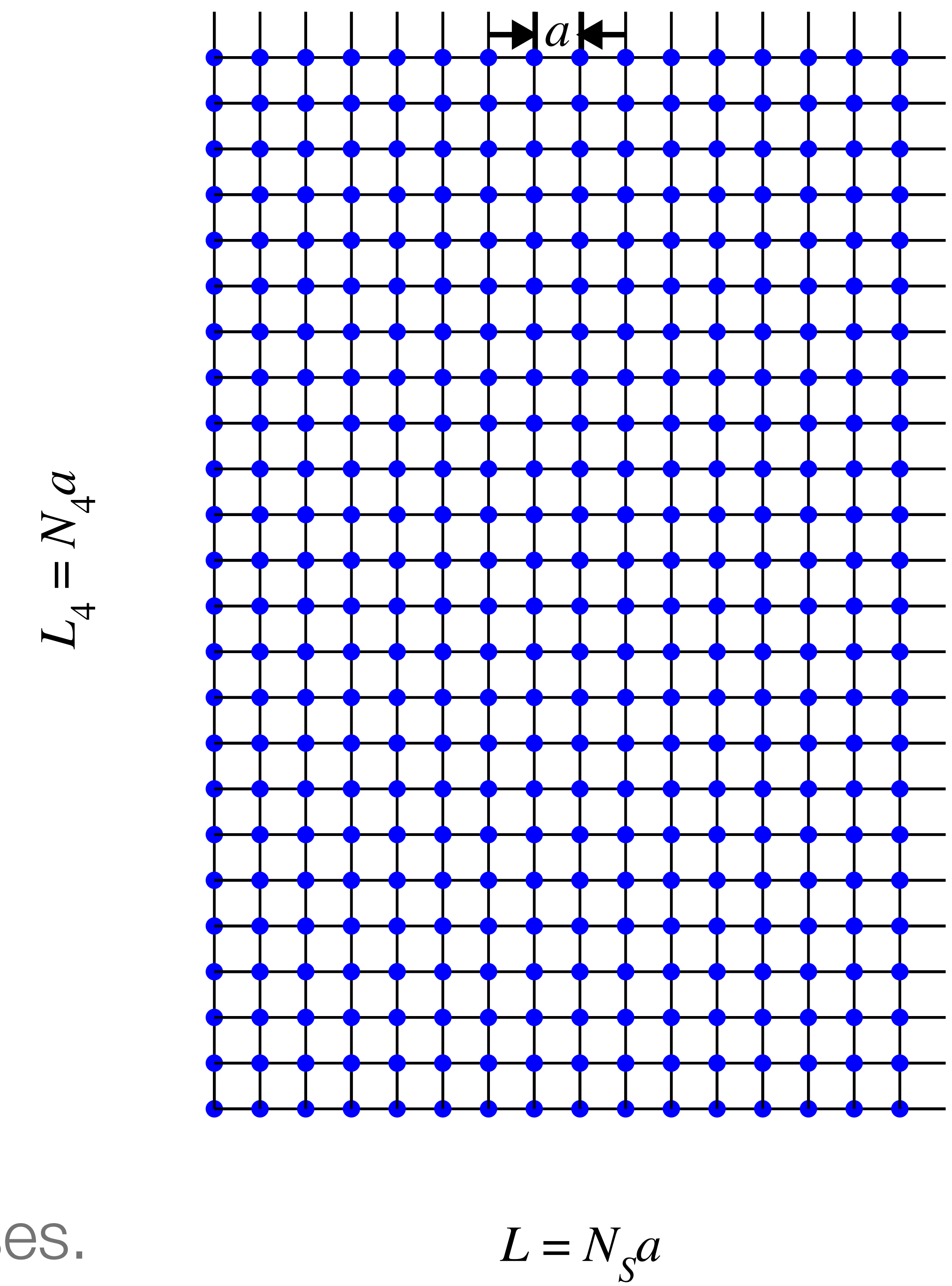
$$\langle\bullet\rangle = \frac{1}{Z}\int \underbrace{\mathcal{D}U}_{\text{MC}}\,\underbrace{\mathcal{D}\psi\mathcal{D}\bar{\psi}}_{\text{hand}}\exp\left(-S\right)[\bullet]$$

- In finite volume, the number of degrees of freedom is finite (albeit large).

- With a finite number of degrees of freedom, a computer can integrate.
- Random number generator (Markov chain Monte Carlo) produces lattice data varying

$$(a, L, L_4, \{m_{\text{sea}}\}, m_q, m_Q)$$

- Extract masses, matrix elements, etc., from various correlation functions.
- Use effective field theories to fit the lattice data to obtain results with
  - $a \to 0$,
  - $L, L_4 \to \infty$,
  - quark masses tuned to $n_f$ hadron masses.



# Resources

- USQCD Whitepapers:
  - Quark and lepton flavor, Lehner, Meinel *et al.* arXiv:1904.09479;
  - Beyond the standard model, Neil *et al.* arXiv:1904.09964;
  - Neutrino-nucleus scattering, ASK, Richards *et al.* arXiv:1904.09931;
  - Fundamental symmetries, Cirigliano, Davoudi *et al.* arXiv:1904.09704;
  - Hadrons and nuclei, Detmold, Edwards *et al.* arXiv:1904.09512;
  - Hot, dense QCD, Karsch, Mukherjee *et al.*, arXiv:1904.09951;
  - Calculations at the exascale, Joó, Jung *et al.*, arXiv:1904.09725;
- Flavor Lattice Averaging Group, arXiv:1902.08191, updates at FLAG2019.

# Outline

focus on material not covered in || sessions

- Introduction
- Something Cool
- Quark Flavor Physics: Form Factors and Decay Constants
- Higgs Physics: Quark Masses; $\alpha_s$; PDFs
- Lepton-Flavor Physics: Muon $g-2$; Nucleonic Ingredients for $\nu A$ Scattering
- Outlook

# Something Cool

# Work and Fields

arXiv:1803.05656

- The work done against an electric force is

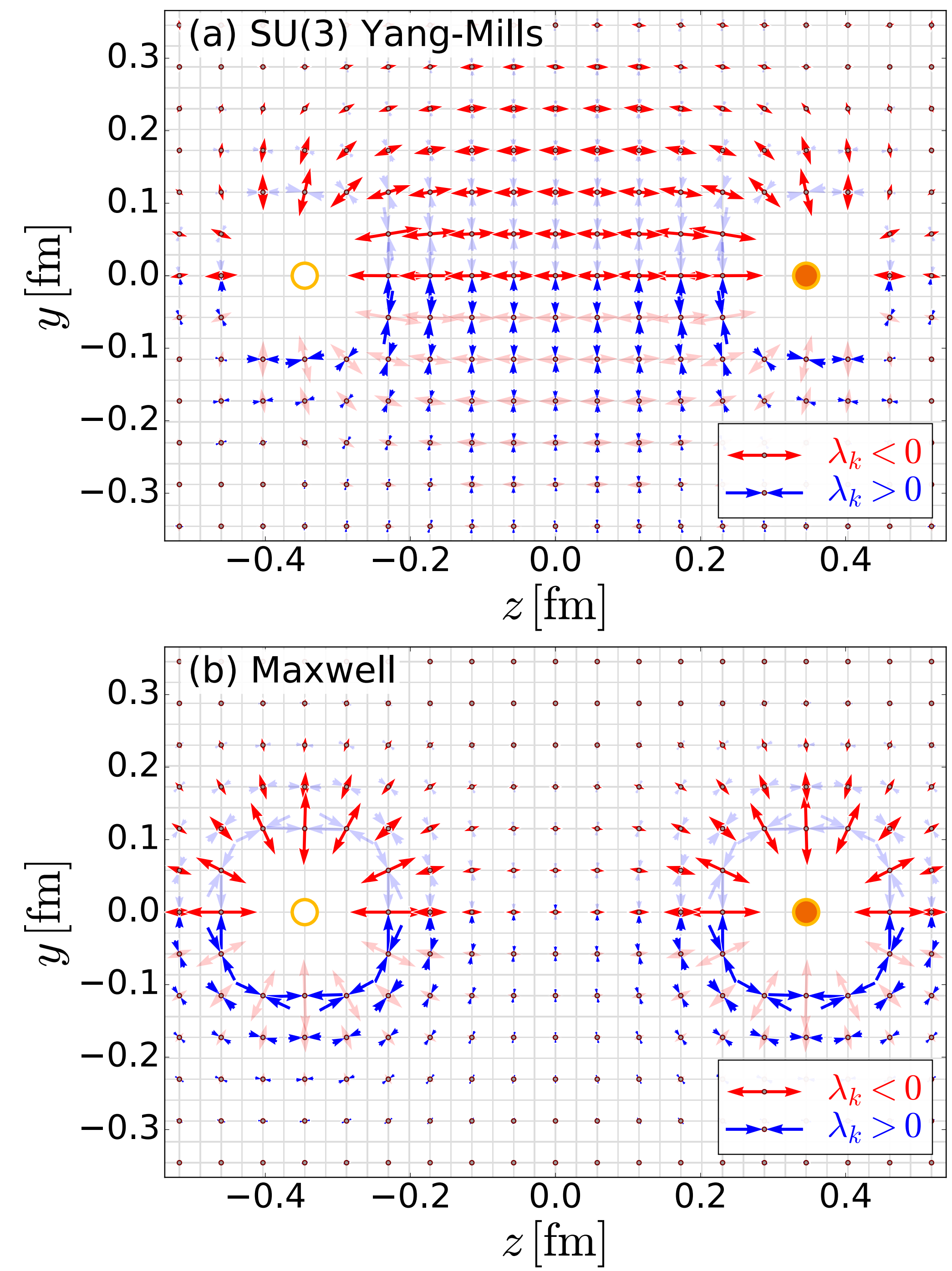
$$W = \mathbb{P} \int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2} \boldsymbol{F}(\boldsymbol{x}) \cdot d\boldsymbol{x}$$

where $\boldsymbol{F} = q\boldsymbol{E}$ and $\boldsymbol{E}$ is the field of the charge distribution apart from $q$.

- On the other hand, it should also be the change in energy of the total field, $\boldsymbol{E}_{\text{tot}}$, including that of $q$:

$$\mathscr{E} = \frac{1}{2} \int d^3x \left[ |\boldsymbol{E}_{\text{tot}}|^2_{q \text{ at } \boldsymbol{x}_2} - |\boldsymbol{E}_{\text{tot}}|^2_{q \text{ at } \boldsymbol{x}_1} \right]$$

- Yanagihara *et al*. show this works in YM.



- Yanagihara *et al*. [arXiv:1803.05656] did more, showing how the stress part of the stress-energy tensor produces the force even in a confining quantum system.

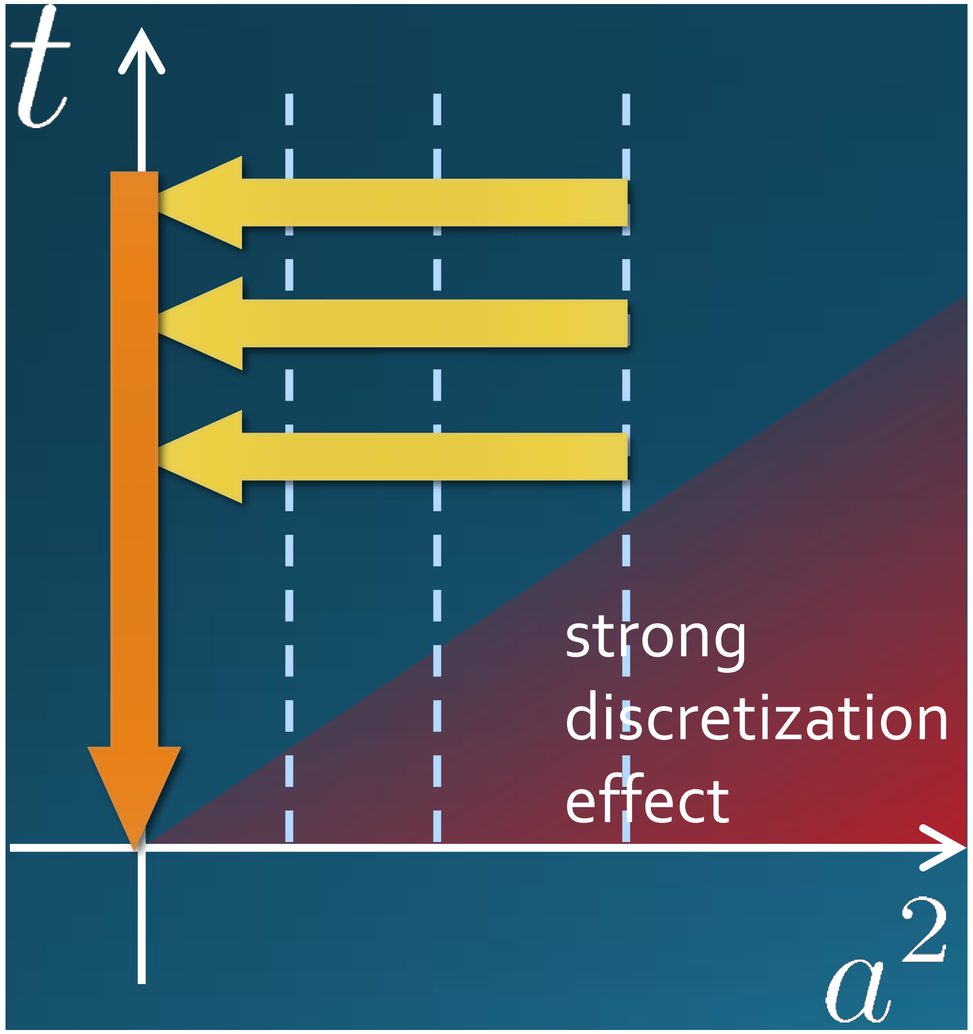
- To do so, they had to overcome a problem with the lattice-UV-regulated stress-energy tensor: power-law mixing with the unit operator.

- Old idea, more recently developed into a powerful tool: "gradient flow".

- Smears gauge field over distance $\sqrt{8t}$; flow time $t$ has dimension distance$^2$.

- No power law divergences, if you take $a \to 0$ first, then $t \to 0$.

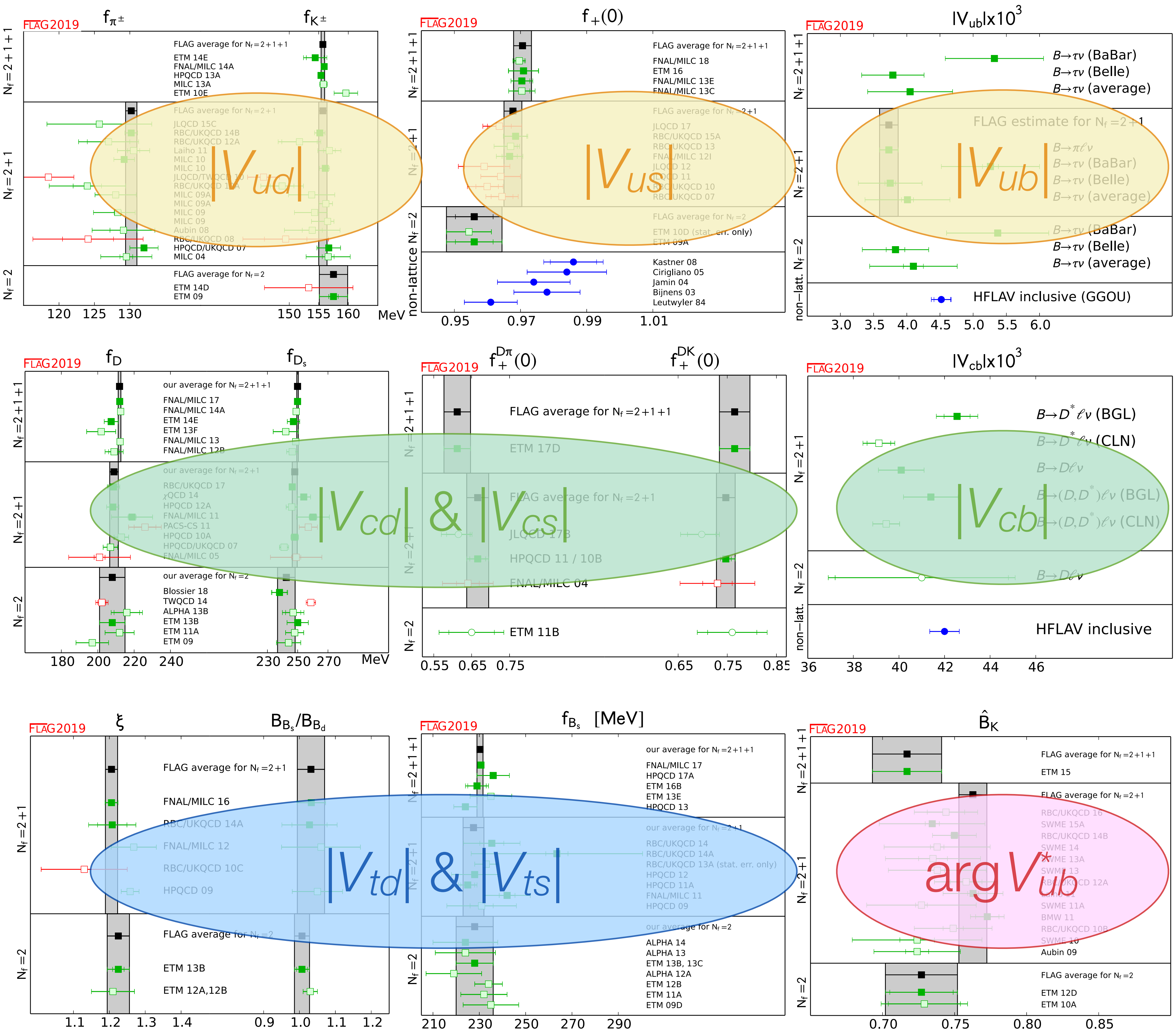
- Use to renormalize any operator.



# Quark-Flavor Physics

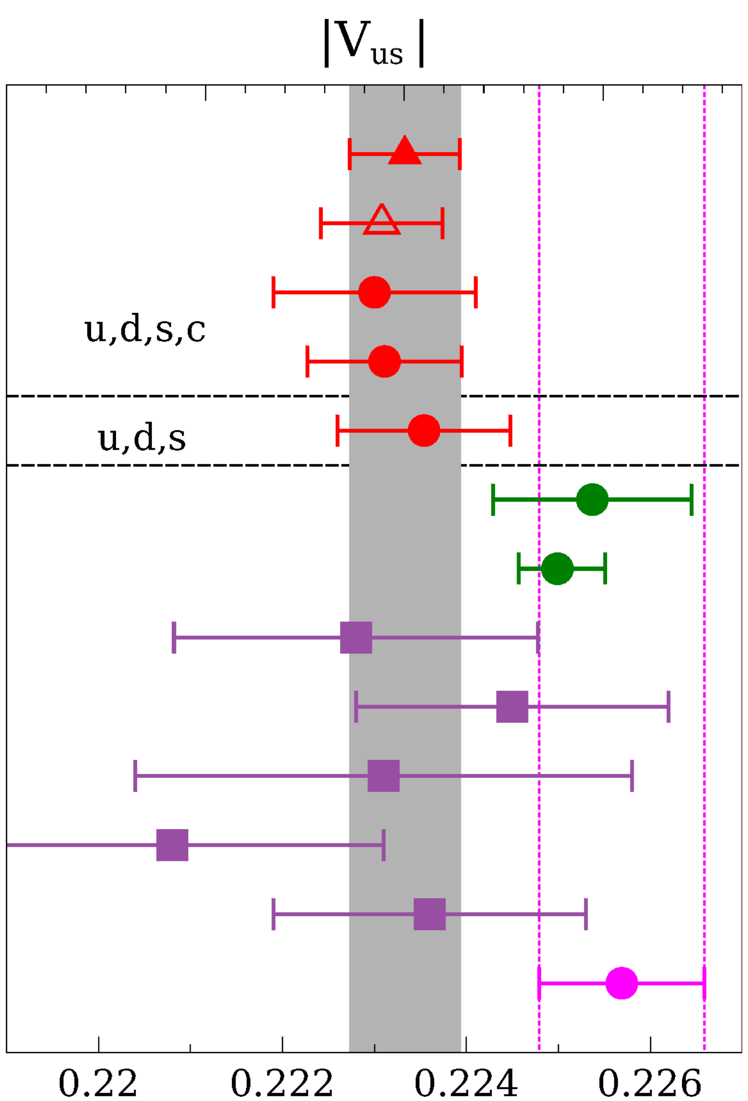
# FLAG 2019

arXiv:1902.08191



# $|V_{us}|$: Semileptonic

- QCD uncertainty in $|V_{us}|$ now the same as experiment [arXiv:1809.02827].




This work $|V_{us}| = 0.22333(42)_{\rm expt}(43)_{f_+(0)}$

This work (only neutral kaon exp. data)

$K_{l3}$ ETMC 2016

$K_{l3}$ Fermilab Lattice/MILC 2014

$K_{l3}$ RBC/UKQCD 2014

$K_{l2}$ FLAG 2016 + $f_K$ FLAG $N_f$=2+1

$K_{l2}$ + $f_K/f_\pi$ Fermilab Lattice/MILC 2017

$\tau \to$ s inclusive, Boyle et al. 2018

$\tau \to$ s inclusive + $K_{l2}$ input, Boyle et al. 2018

$\tau \to$ s inclusive, Hudspith et al. 2017

$\tau \to$ s inclusive, Hudspith et al. 2017 + HFLAV 2016 exp. input

$\tau \to K \ell \nu / \tau \to \pi \ell \nu$ HFLAV2017+ $f_K/f_\pi$ Fermilab Lattice/MILC 2017

Unitarity with $|V_{ud}|$=0.97420(21), RC from Marciano & Sirlin 2005

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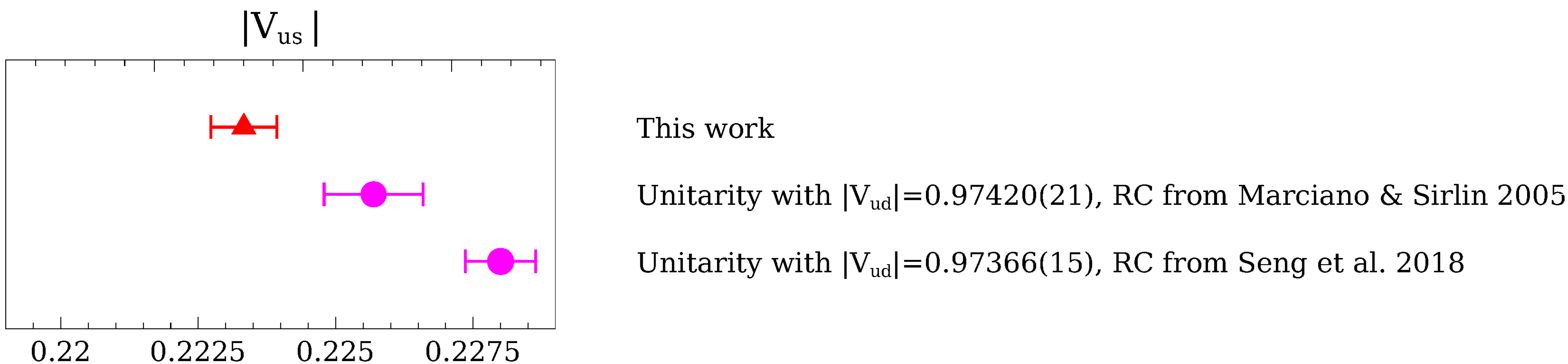
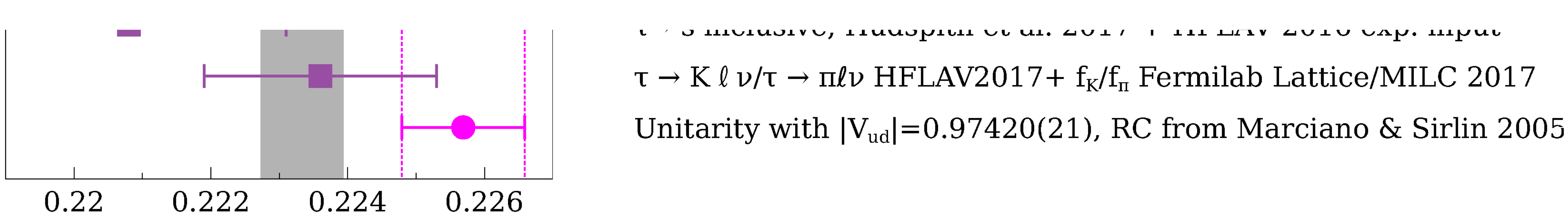
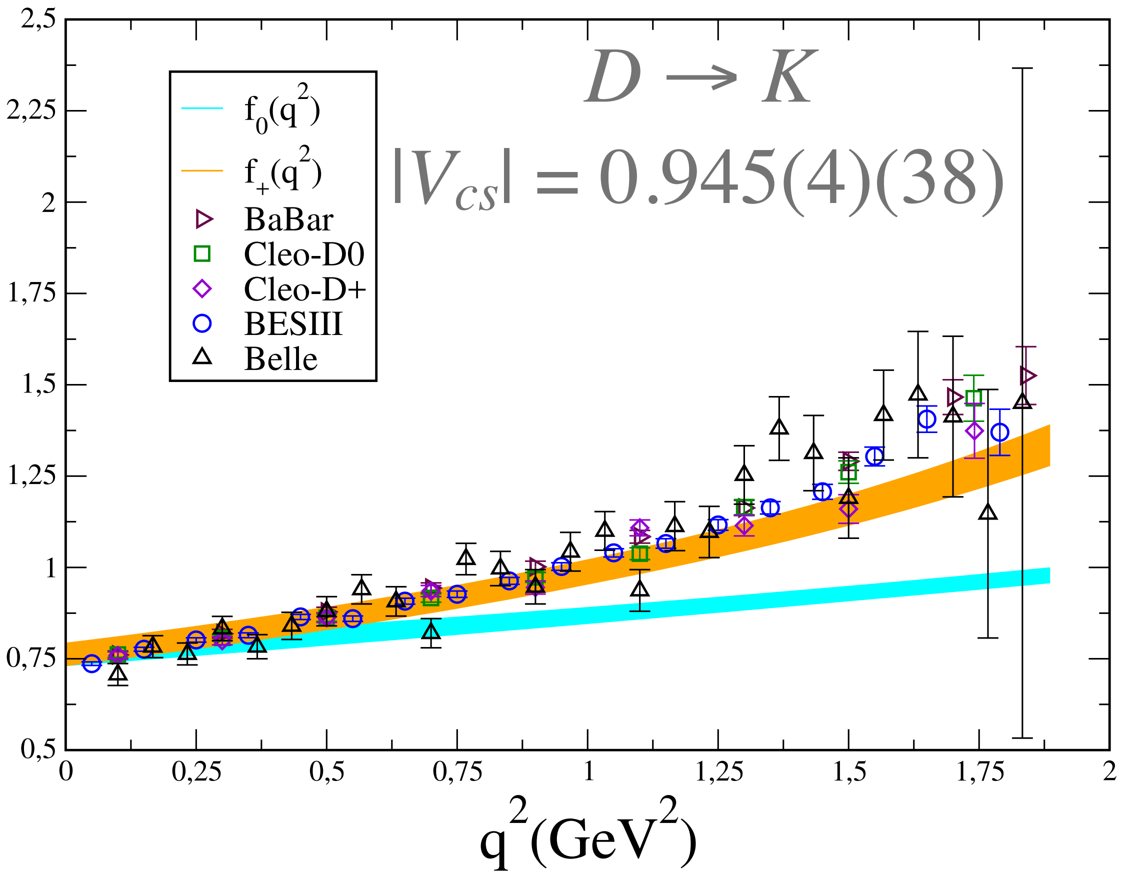


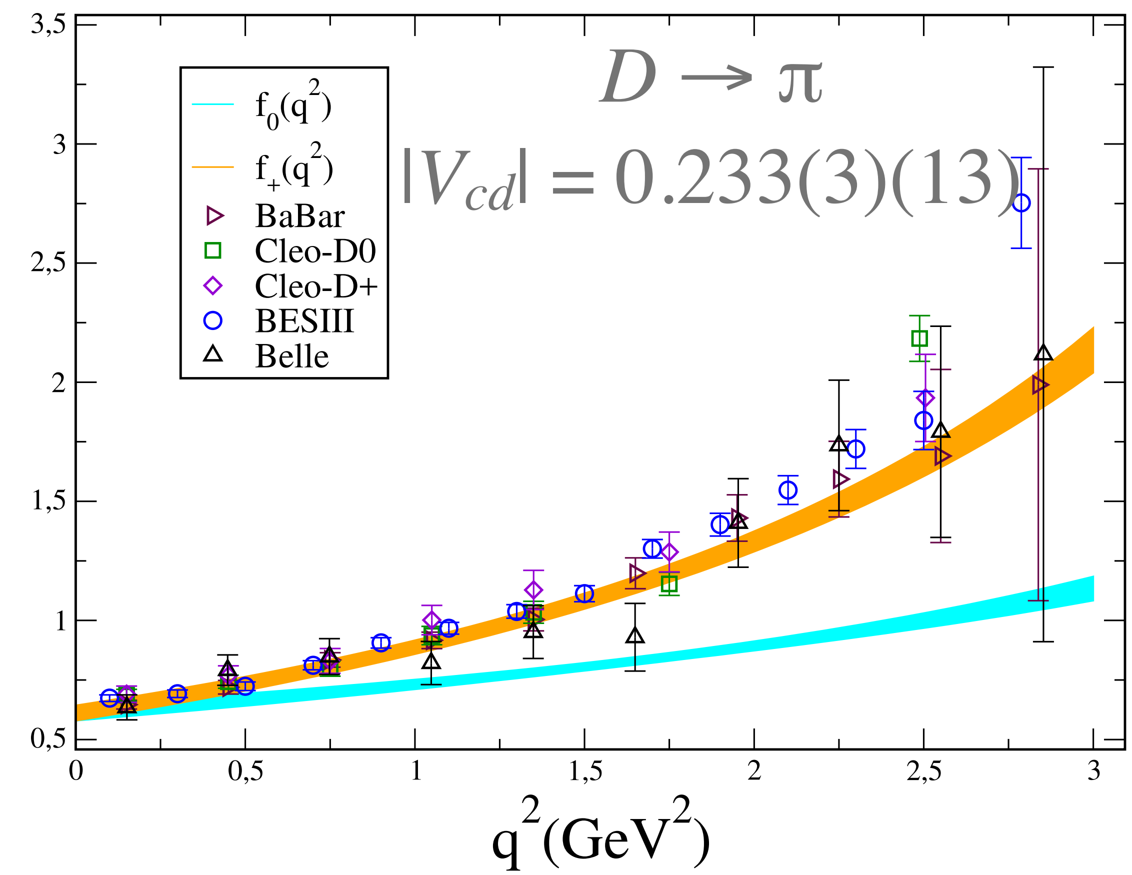

~2σ violation of unitarity becomes >5σ with new RC



# $|V_{cd}|$ and $|V_{cs}|$: Semileptonic

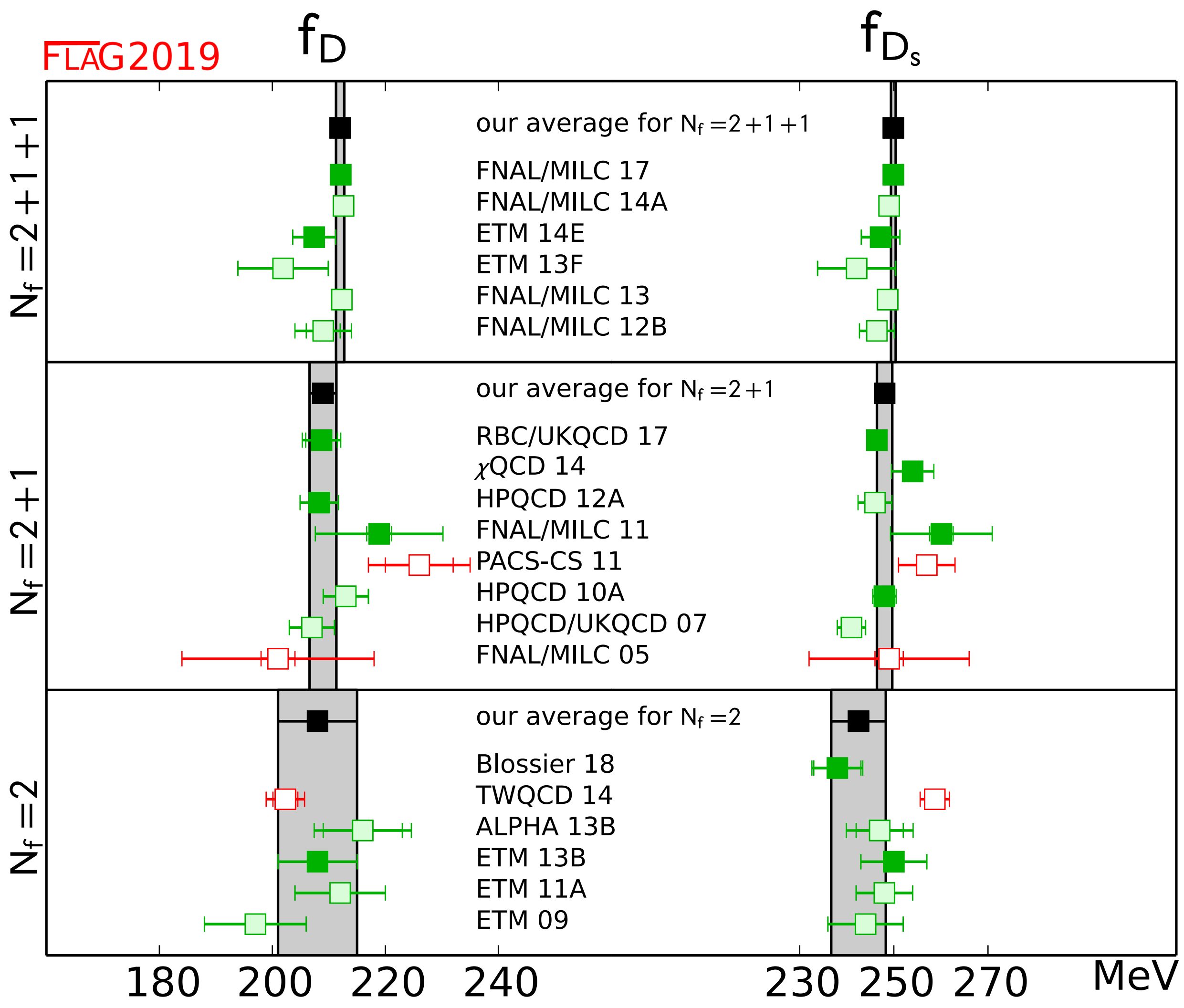
- Most recent entry in FLAG 2016 was HPQCD 2010 ($D \to K$), 2011 ($D \to \pi$); see also arXiv:1305.1462 (unpublished).
- Newer results from ETM removes hypercubic artifacts [arXiv:1706.03017] and determines CKM [arXiv:1706.03657].

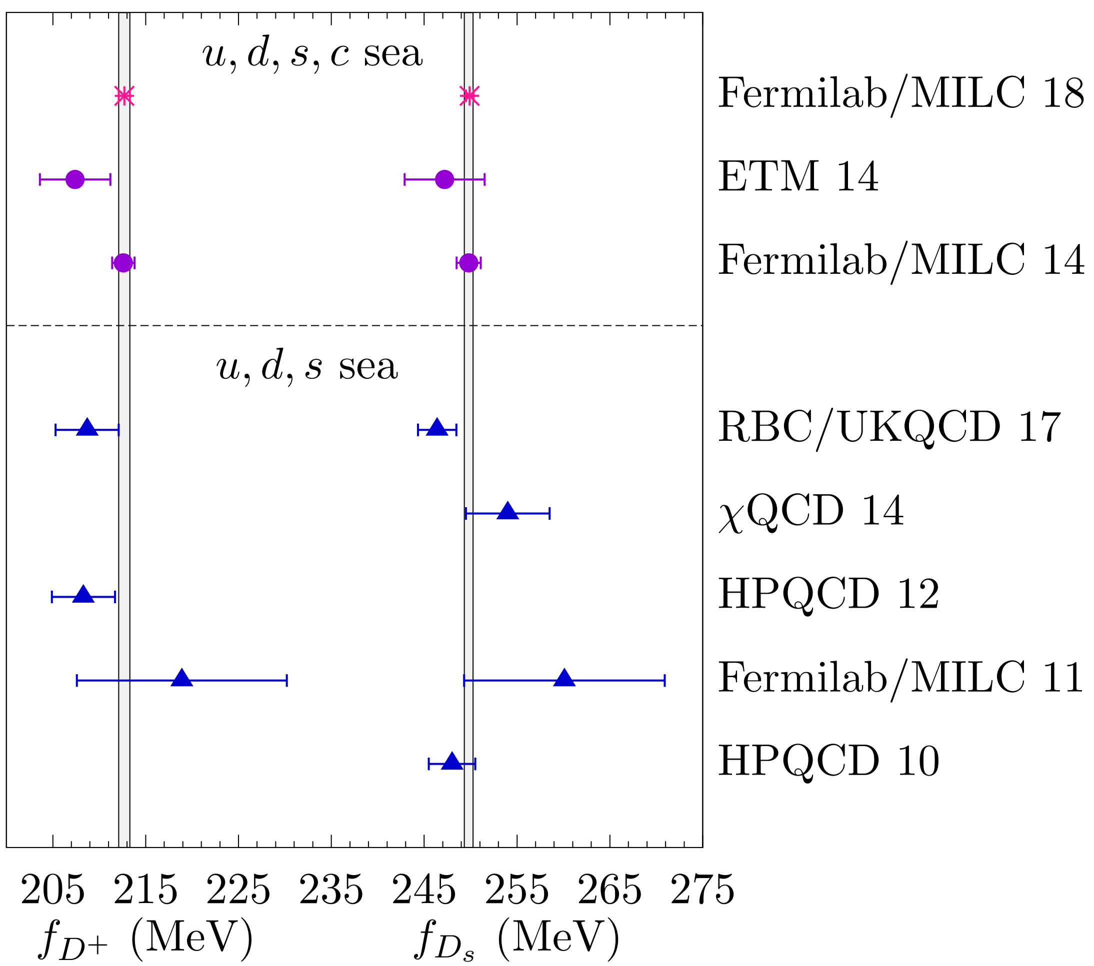


# $|V_{cd}|$ and $|V_{cs}|$: Leptonic

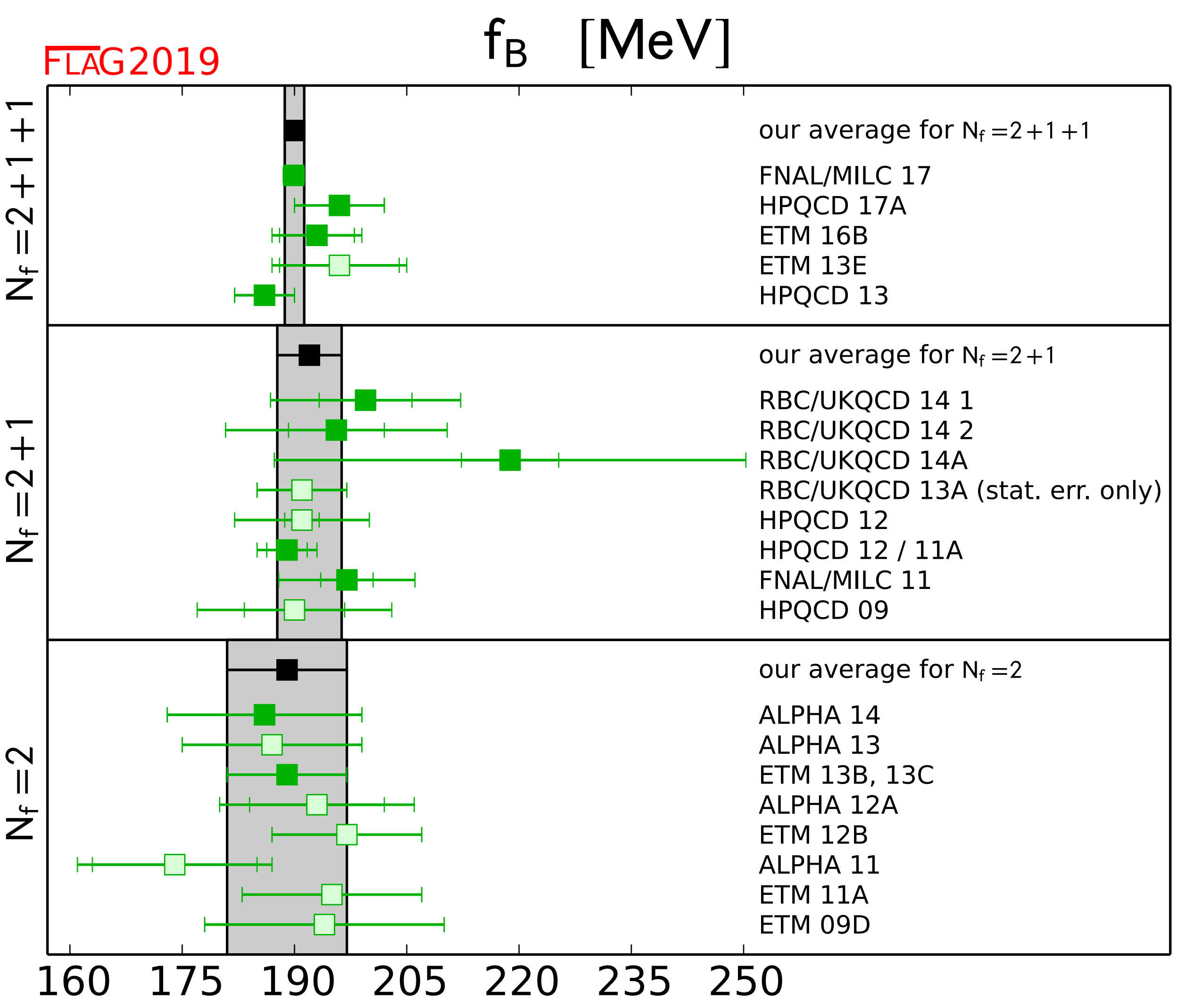
- Major update from Fermilab Lattice/MILC [arXiv:1712.09262]: ~0.2%
- Also RBC/UKQCD [arXiv:1701.02644].

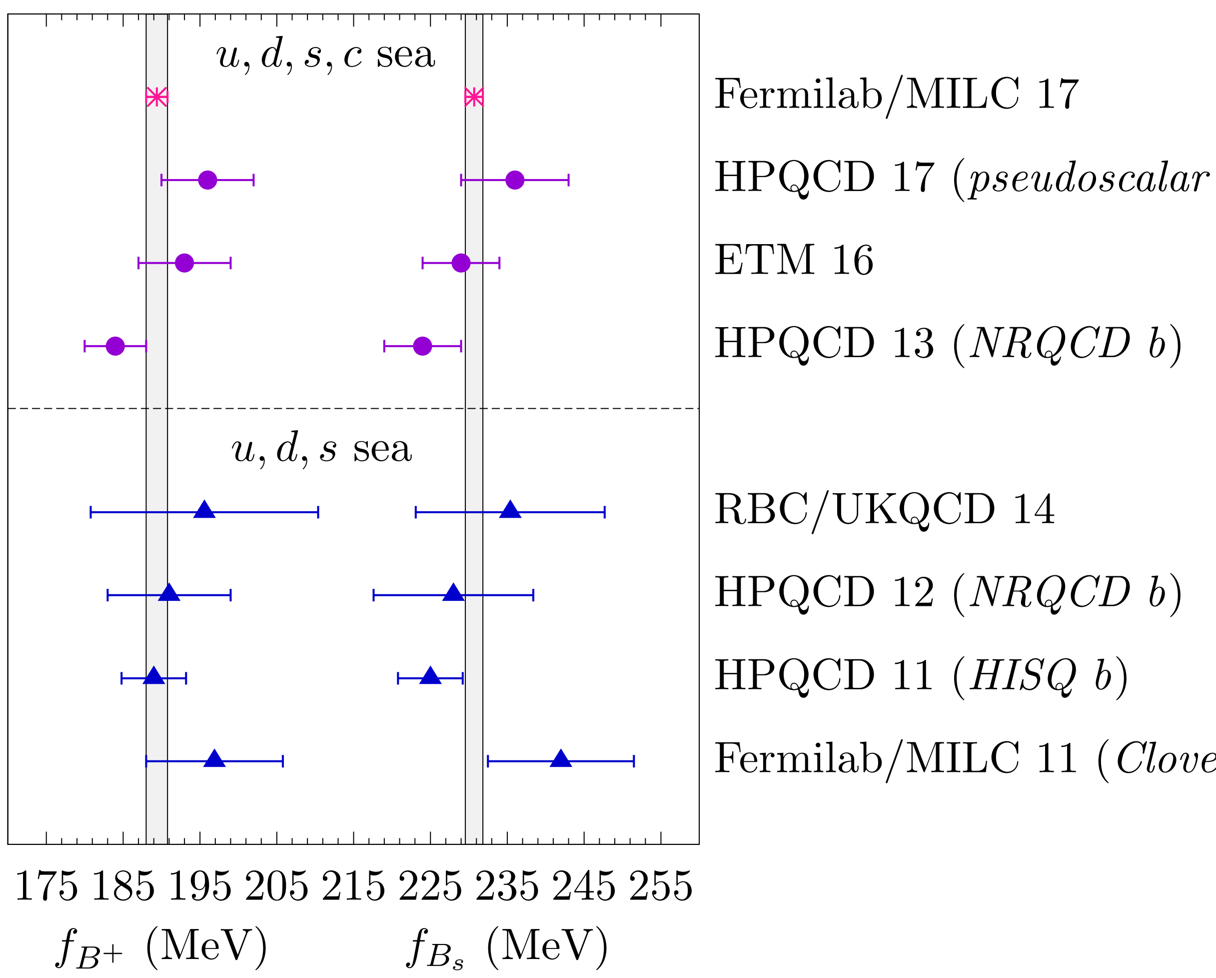


# $|V_{ub}|$: Leptonic

- Updates from Fermilab Lattice/MILC [arXiv:1712.09262]: ~0.7%
- Other updates from HPQCD [arXiv:1711.09981], ETM [arXiv:1603.04306].



# $|V_{ub}|$ and $|V_{cb}|$

- Older $B \to \pi l \nu$ from RBC/UKQCD [arXiv:1501.05373] and Fermilab Lattice/MILC [arXiv:1503.07839].
- Baryon decay $\Lambda_b \to p l \nu$: Detmold, Lehner, Meinel [arXiv:1503.01421 and S. Meinel, Weds. 14:00].
- $B_s \to K l \nu$ done [arXiv:1808.09285, arXiv:1901.02561] or underway.
- Updates to $B \to \pi l \nu$ coming [arXiv:1710.09442 and W. Jay, Tues. 17:20].
- Aim to keep QCD uncertainties similar to Belle 2 and LHCb uncertainties.
- Explore prospects for, e.g., $B \to K^* ll$ [S. Meinel, Weds. 14:00].
  ↳ $K\pi$

# Lepton-Flavor Physics

# Muon Anomalous Magnetic Moment

- Fermilab experiment aims to improve precision by a factor of 4.

- By-far dominant theoretical uncertainties stem from hadronic contributions, which can be computed with lattice QCD.

- Hadronic vacuum polarization (HVP); *cf.*, S. Gottlieb, Mon. 17:40.

- Hadronic light-by-light (HLbL):
    - first calculation at physics light-quark masses [arXiv:1610.04603];
    - suggests "Glasgow consensus" is not way off.

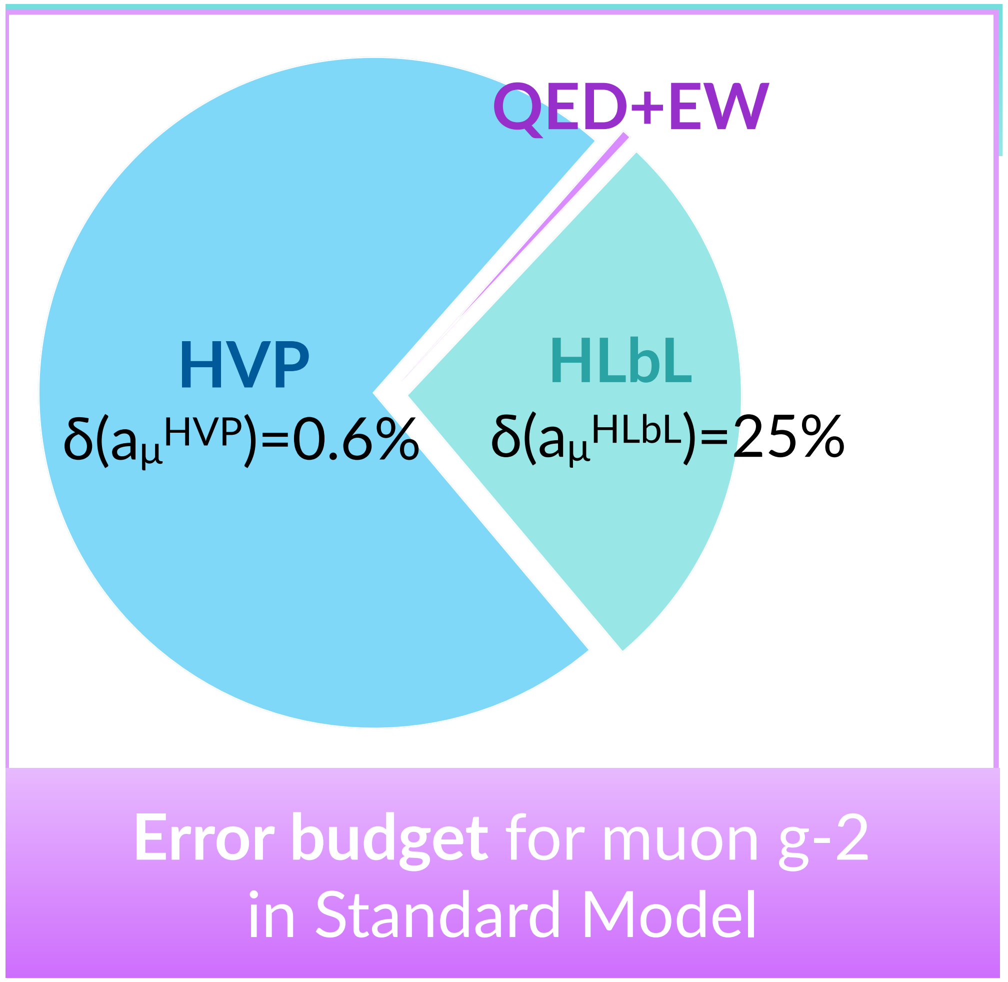



image from Ruth Van de Water

# Neutrino Physics—$\nu N$ Scattering

- Key issue in neutrino oscillations is to reconstruct the neutrino energy, but the nuclear remnant (and possibly other particles) are not detected.
- Therefore, nuclear models are needed.
- The models require nucleonic ingredients which should come from QCD:
- Therefore, aim to get scattering amplitudes from first principles:
  - $\bar{\nu}p$ or $\nu d$ scattering experiments in the Tokai, NuMI, or LBNF beam;
  - lattice QCD with error budgets as comprehensive as those for CKM.

# Axial Form Factor from Lattice QCD

- PNDME ⊂ USQCD, [arXiv:1901.00060]() and [Lattice '19 talk]():

- Lattice slopes smaller than that extracted from $\nu d$.

- Even continuum limit:

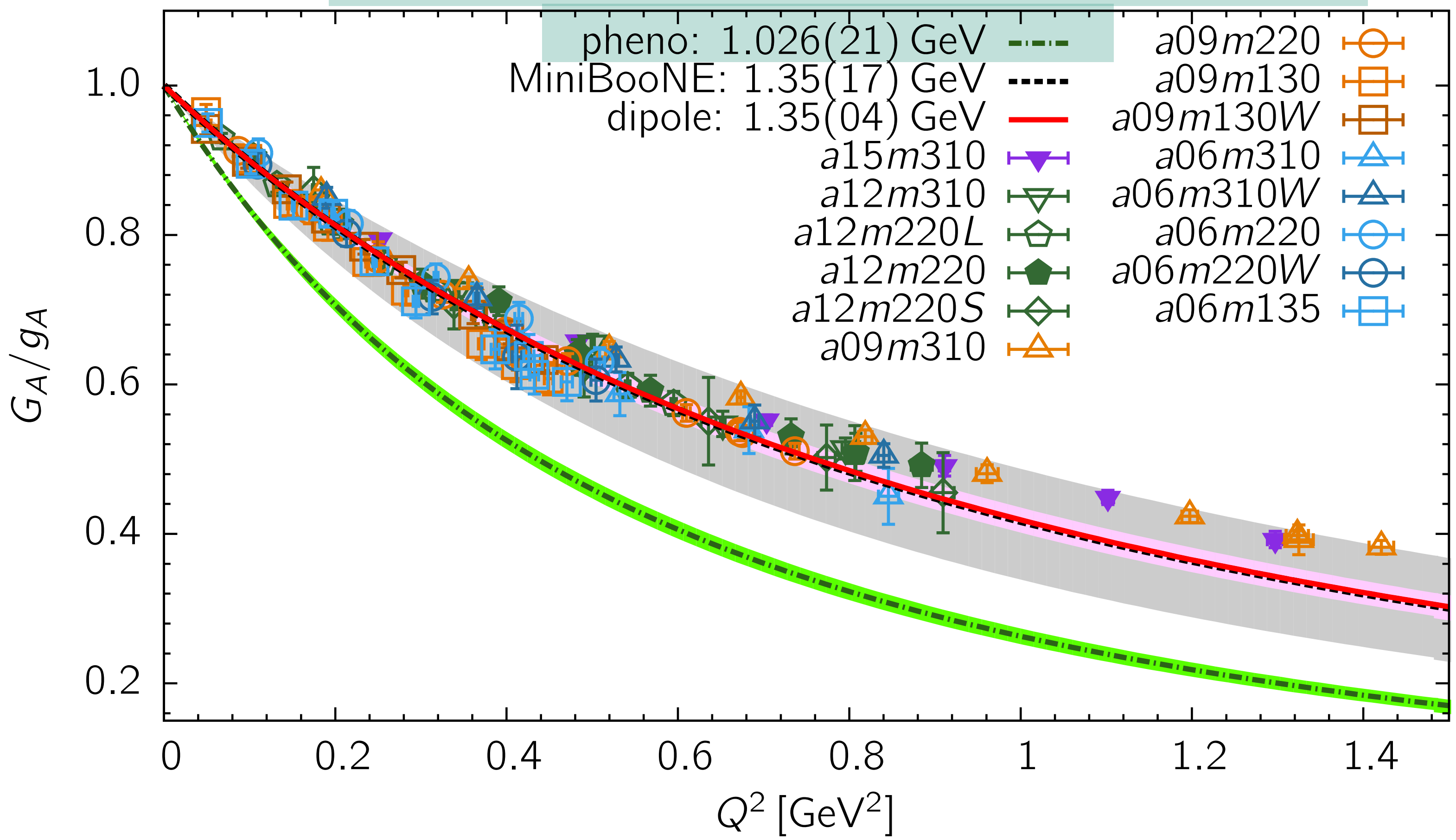
  $$r_A = 0.481(85) \text{ fm},$$

  vs Meyer *et al*.:

  $$r_A = 0.68(16) \text{ fm}.$$

- *Caveat emptor* extraction from $\nu d$ data.

complicated extraction from $\nu d$ data



# Quark Masses

# Four Methods

- Mass renormalization [$\bar{m} = m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$]:
  - $\bar{m} = Z_m(m_0 - m_{\text{cr}})$, with $Z_m$ either ≤ 2 loops or nonperturbative. new!
- Ward identities:
  - $m_{\text{AW}} Z_P \langle P \rangle = Z_A \langle \partial \cdot A \rangle$, run $m_{\text{AW}}$ to high scale μ & convert to $m_{\overline{\text{MS}}}(\mu)$.
- Continuum limit ⊗ continuum pQCD:
  - $\lim_{a \to 0} G_n(Qa) = 1 + \sum_{k=1}^{K \geq 3} G_n^{(k)} \alpha_s^k(Q)$, *e.g.*, quarkonium moments. new!
- Continuum limit ⊗ HQET:
  - $M = m_{\text{MRS}} + \bar{\Lambda}_{\text{MRS}} + \cdots$, *e.g.*, heavy-light meson masses. new!

# Charmonium Correlators

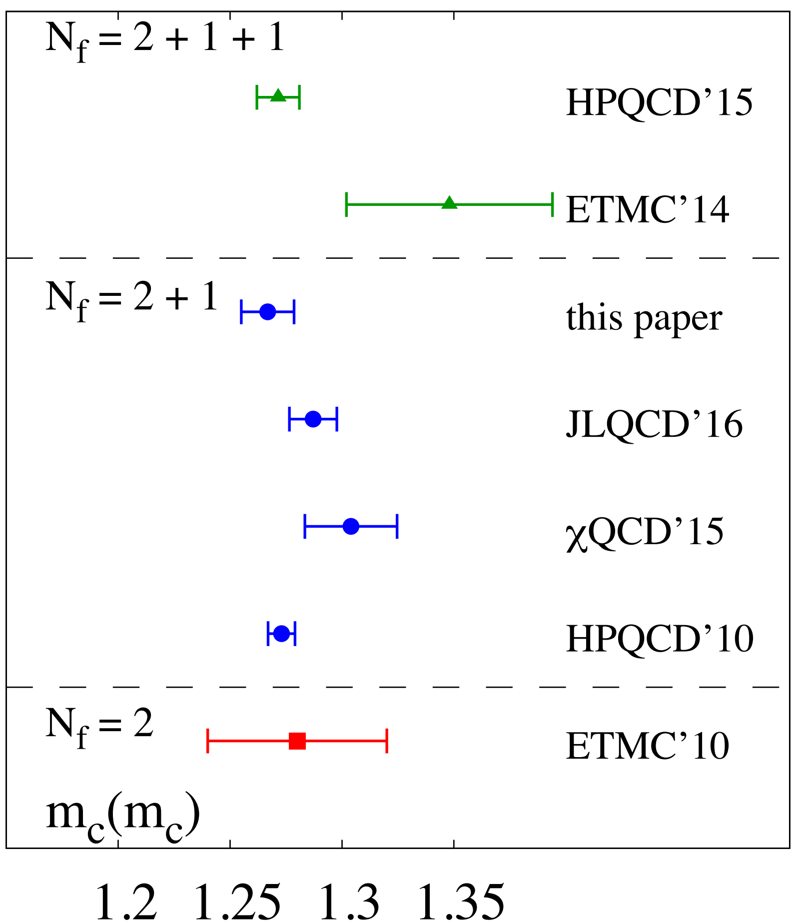
- Lately, the most precise determinations of heavy-quarks come from:

$$\lim_{a\to 0} \frac{G_n}{G_n^{\text{tree}}} = g_n(\alpha_s(\mu), m_{c,\overline{\text{MS}}}/\mu)$$

$$G_n = \sum_t t^n m_0^2 \left\langle \bar{c}\gamma^5 c(t)\, \bar{c}\gamma^5 c(0) \right\rangle, \quad \overset{\text{even}}{n} \geq 4$$

Bochkarev & de Forcrand [hep-lat/9505025], Allison et al. [HPQCD, arXiv:0805.2999].

- New analysis on 11 ensembles from hotQCD ($n_f = 2+1$ HISQ) by Maezawa & Petreczky [arXiv:1606.08798] compatible with earlier HPQCD results [arXiv:1408.4169].



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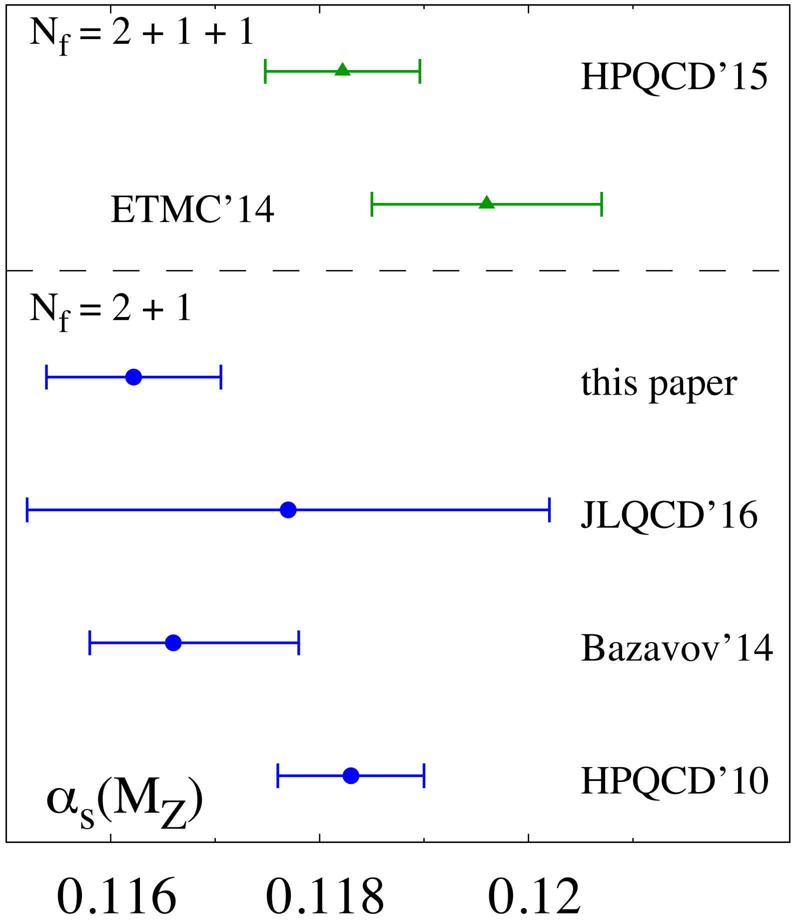
$\alpha_s(M_Z)$



# RI-SMOM Method

arXiv:1805.06225

- Fix gauge and use quark vertex function:

$$\Lambda_P = \langle S^{-1}(p_1)\rangle\langle\bar{\psi}\gamma^5\psi(p_1,p_2)\rangle\langle S^{-1}(p_2)\rangle$$

$$|p_1+p_2| = |p_1| = |p_2| \equiv \mu$$

$$Z_P^{\mathrm{SMOM}}(\mu) \equiv \frac{\Lambda_P(\mu)}{\Lambda_P^{(0)}}$$

$$Z_m^{\mathrm{SMOM}}(\mu) = 1/Z_P^{\mathrm{SMOM}}(\mu)$$

- Trickiest part of the analysis is that this gauge-fixed quantity has “gluon-mass condensate” contribution ~$\langle A^2\rangle/\mu^2$.

- Run $m^{\mathrm{SMOM}}(\mu) = Z_m^{\mathrm{SMOM}}(\mu)m_0$ to a high scale and convert to $m_{\overline{\mathrm{MS}}}(\mu)$.

# Heavy-light Meson Masses

- From HQET (or other approaches to the $1/m_h$ expansion):

$$M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- This formula used to determine $\bar{\Lambda}$, $\mu_\pi^2$, and $\mu_G^2(m_b)$ from lattice QCD in hep-ph/0006345; new, improved theory [arXiv:1712.04983].

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mass of spin-$J$ meson

mass of heavy quark

energy of gluons and light quarks

kinetic energy of heavy quark

$1$ for $B$, $-\frac{1}{3}$ for $B^*$

spin-orbit interaction

- This formula used to determine $\bar{\Lambda}$, $\mu_\pi^2$, and $\mu_G^2(m_b)$ from lattice QCD in hep-ph/0006345; new, improved theory [arXiv:1712.04983].

# Minimal Renormalon Subtraction

arXiv:1701.00347, arXiv:1712.04983

- New idea how to isolate the leading $\mathrm{O}(\Lambda)$ ambiguity in the pole mass, yielding an unambiguous definition of $\bar{\Lambda}$.

- For the bottom quark (tree, 1-loop, 2-loop, 3-loop, 4-loop):

$$m_{\mathrm{b,pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$$

$$m_{b,\mathrm{MRS}}/\bar{m}_b = (1.157, 1.133, 1.131, 1.132, 1.132)$$

- For the top quark (tree, 1-loop, 2-loop, 3-loop, 4-loop):

$$m_{t,\mathrm{MRS}}/\bar{m}_t = (1.0687, 1.0576, 1.0573, 1.0574, 1.0574)$$

could have implications for top physics at LHC.

# Results from MRS/HQET

$\alpha_s$ parametric not PT truncation

- Masses in numerical form:

$$m_{l,\overline{\rm MS}}(2\ {\rm GeV}) = 3.404(14)_{\rm stat}(08)_{\rm syst}(19)_{\alpha_s}(04)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

$$m_{u,\overline{\rm MS}}(2\ {\rm GeV}) = 2.118(17)_{\rm stat}(32)_{\rm syst}(12)_{\alpha_s}(03)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

$$m_{d,\overline{\rm MS}}(2\ {\rm GeV}) = 4.690(30)_{\rm stat}(36)_{\rm syst}(26)_{\alpha_s}(06)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

$$m_{s,\overline{\rm MS}}(2\ {\rm GeV}) = 92.52(40)_{\rm stat}(18)_{\rm syst}(52)_{\alpha_s}(12)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

$$m_{c,\overline{\rm MS}}(3\ {\rm GeV}) = 984.3(4.2)_{\rm stat}(1.6)_{\rm syst}(3.2)_{\alpha_s}(0.6)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

$$m_{b,\overline{\rm MS}}(m_{b,\overline{\rm MS}}) = 4203(12)_{\rm stat}(1)_{\rm syst}(8)_{\alpha_s}(1)_{f_{\pi,\rm PDG}}\ {\rm MeV}$$

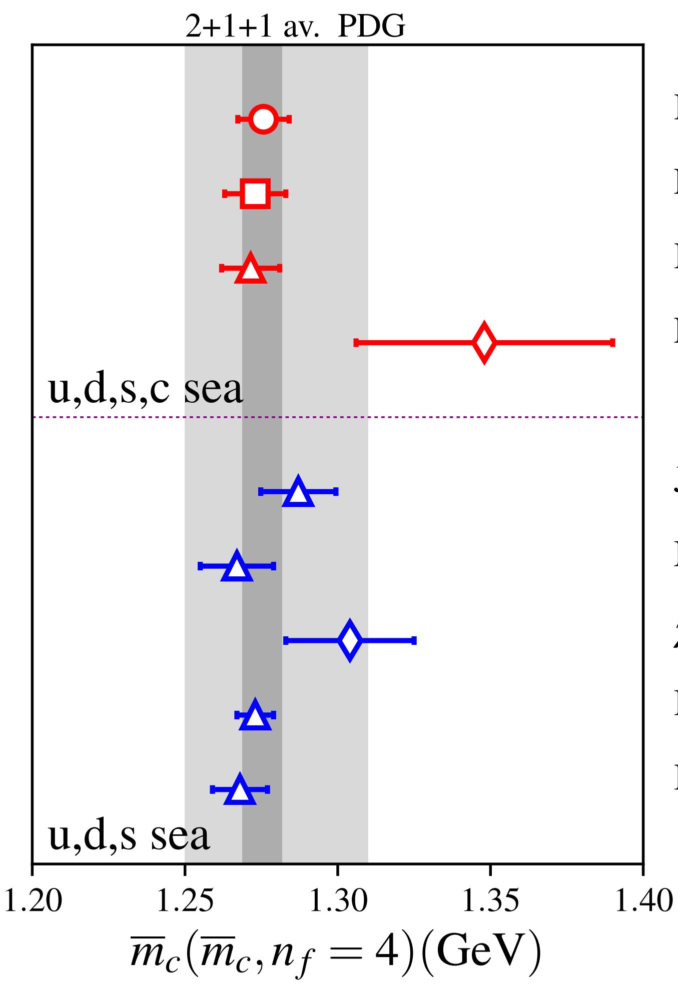
- Mass ratios:

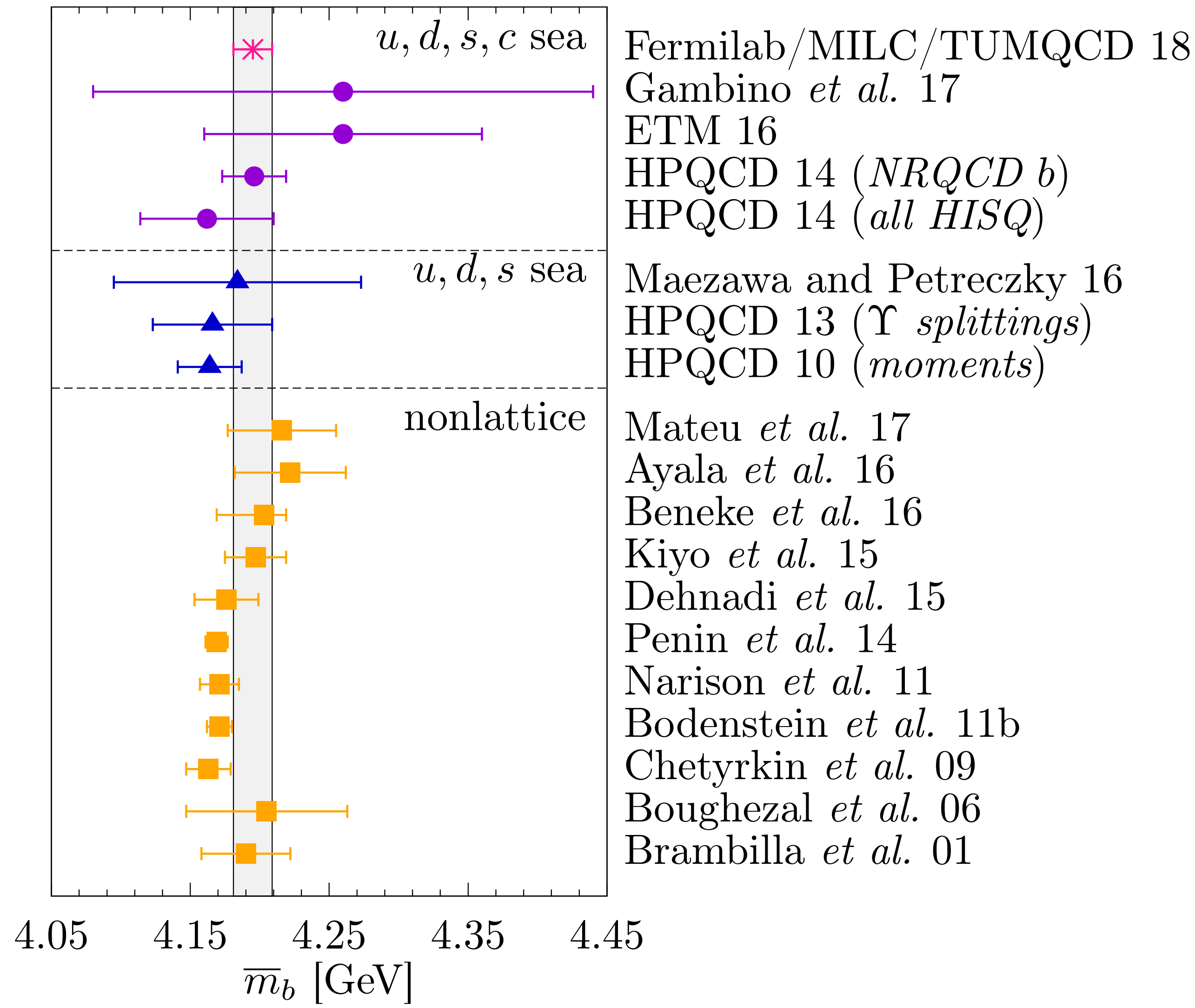
$$m_c/m_s = 11.784(11)_{\rm stat}(17)_{\rm syst}(00)_{\alpha_s}(08)_{f_{\pi,\rm PDG}}$$

$$m_b/m_s = 53.93(7)_{\rm stat}(8)_{\rm syst}(1)_{\alpha_s}(5)_{f_{\pi,\rm PDG}}$$

$$m_b/m_c = 4.577(5)_{\rm stat}(7)_{\rm syst}(0)_{\alpha_s}(1)_{f_{\pi,\rm PDG}}$$

# Heavy Comparisons

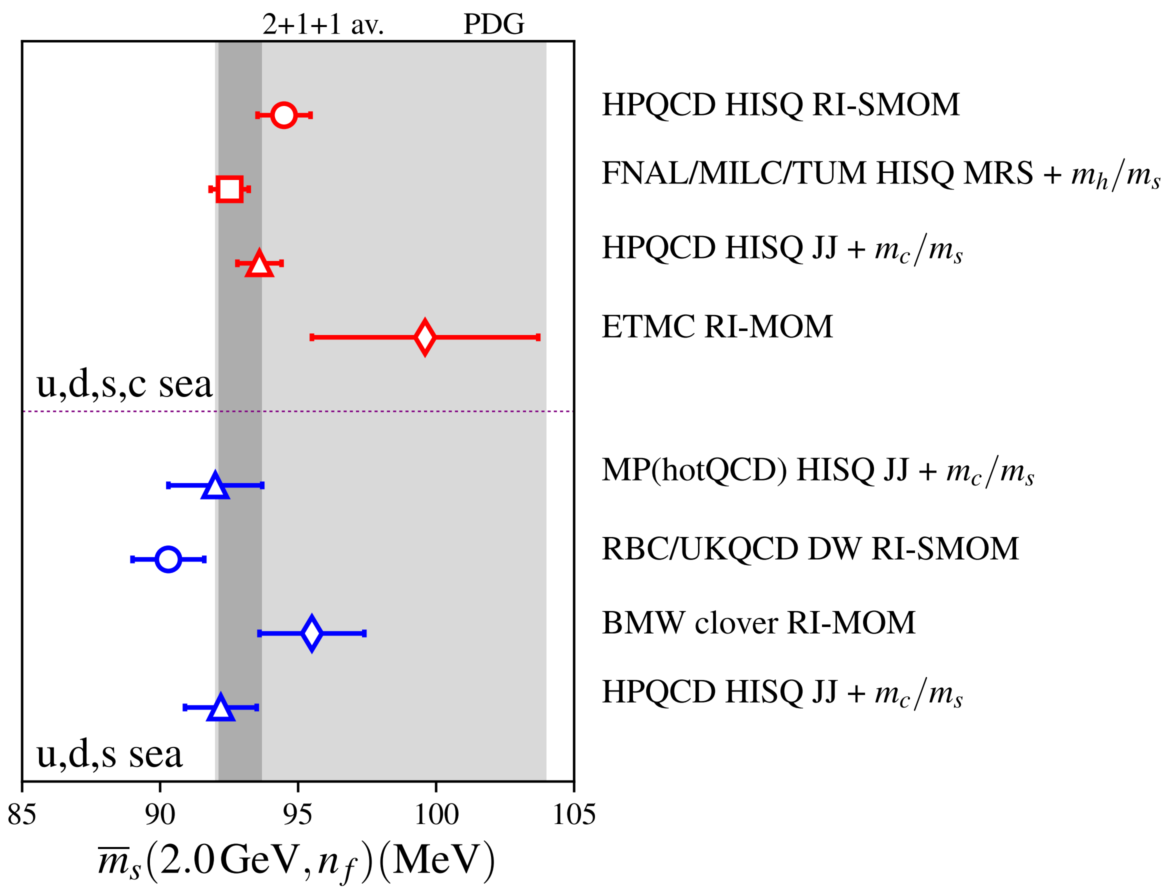


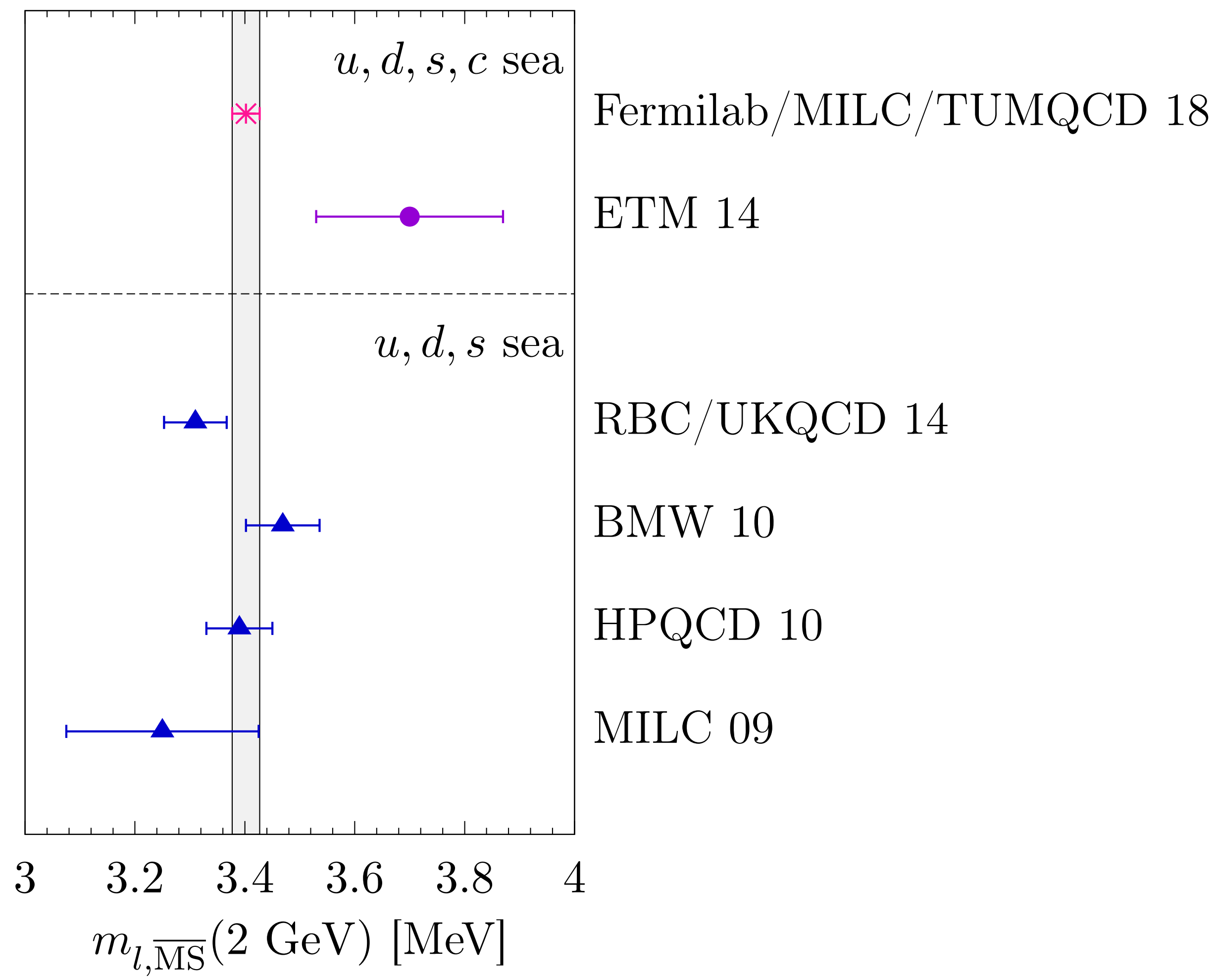




- Precision: 0.3% for bottom to 0.5% for charm.

plots from arXiv:1802.04248, arXiv:1805.06225

# Light Comparisons




plots from arXiv:1802.04248
arXiv:1805.06225

- Precision: 2% for up quark.

Consistent picture: all quarks but top

# Higgs Physics

- Quark masses and $\alpha_s$ relevant for Higgs physics.
- Anything else?
- Moments of parton distribution functions (PDFs):
  - tame power-law divergences with gradient flow [arXiv:1612.01584].
- Dependence on Bjorken $x$:
  - numerous techniques (quasi-pdf, pseudo-pdf, "lattice cross sections", Ioffe-time matrix elements, ...) stemming from Ji [arXiv:1305.1539].
- Community whitepaper [arXiv:1711.07916] $\Longleftarrow$ lattice-pheno synergy.

# Outlook

- Lattice QCD now has had huge impact in quark-flavor physics:
  - results are increasingly precise: sub-% threshold has been passed for $f_{B(s)}$, $f_{D(s)}$, $m_b$, $m_c$ (and good enough for Belle 2, BES III);
  - quark masses already more than good enough for ILC Higgs studies.
- Excellent consistency of different quark-mass methods suggests that PDG estimates for quark masses must be updated.
- Lattice QCD is on track to reduce hadronic uncertainties in muon $g$–2.
- Bright future for nucleon matrix elements (form factors for neutrino physics, PDFs for LHC, many others for the precision frontier).

Thank you!