Reevaluating Uncertainties in $\bar{B} \to X_s\gamma$ Decay

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Based on
A.G and Gil Paz “Reducing Uncertainties in $\bar{B} \to X_s\gamma$”
(To appear)
Introduction
Why $\bar{B} \rightarrow X_s \gamma$
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- $\bar{B} \rightarrow X_s \gamma$ decay is an important **New Physics** probe
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  - It is suppressed at tree level in SM
  - Can receive contributions from SM extensions.

**Figure:** $b \rightarrow s \gamma$ flavor changing neural current (FCNC) in SM
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**Figure:** $b \rightarrow s \gamma$ flavor changing neutral current (FCNC) in SM

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**Figure**: $b \to s \gamma$ flavor changing neural current (FCNC) in SM

- SM extensions modify the $C_{7\gamma}$ Wilson coefficient
- CP violation in $\bar{B} \to X_s \gamma$ can be enhanced by new physics
Photon production

Photon can be produced directly:

\[ Q_7^\gamma = -\frac{e}{\pi} \frac{\sigma_{\mu\nu} F_{\mu\nu} (1 + \gamma_5)}{2 m_b \bar{s}} \]

Also, gluon or quark pair can convert to photon

\[ Q_8^g = -\frac{e}{\pi} \frac{\sigma_{\mu\nu} G_{\mu\nu} (1 + \gamma_5)}{2 m_b \bar{s}} \]

\[ Q_8^{q_1} = (\bar{q}_b V^\text{A} - A^\text{V} \bar{s} q_b)^V - A^\text{A} \]

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Effective Lagrangian

• The effective Lagrangian to describe $\bar{B} \to X_s \gamma$
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$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}^* V_{qs}$$

$$+ \left( C_1 Q_1^q + \sum_{i=2}^{6} C_i Q_i + C_7 \gamma Q_7 \gamma + C_{8g} Q_{8g} \right) + \text{h.c.}$$
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- $Q_1^q = (\bar{q}b)_{\nu-A}(\bar{s}q)_{\nu-A}$, $Q_{8g} = -\frac{e}{8\pi^2} m_b \bar{s}s \sigma_{\mu\nu} G^{\mu\nu} (1 + \gamma_5) b$

- At leading power: Only $Q_7\gamma - Q_7\gamma$ contributes to decay rate
- At $1/m_b$: $\Gamma$ get $Q_1 - Q_7\gamma$, $Q_{8g} - Q_{8g}$ and $Q_7\gamma - Q_{8g}$ contributions
Decay rate

- World average for experimental value:

\[ \mathcal{B}(B \to X_s \gamma) (E_\gamma > 1.6 \text{ GeV}) = (3.32 \pm 0.15) \times 10^{-4} \]

[ Y. Amhis et. al. EPJC 77, 895 (2017)]

- NNLO prediction

\[ \Gamma(B \to X_q \gamma) = \Gamma(b \to X^p_q \gamma) + \delta \Gamma_{\text{nonp}} \]

Perturbatively calculable

\[ \mathcal{O}(\frac{\Lambda_{\text{QCD}}}{m_b}) \]

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for \( E_{\gamma} > 1.6 \text{ GeV} \)

• \( \delta\Gamma_{\text{nonp}} \equiv \text{Non-perturbative contribution} \)

- The largest contribution to the error 5% from \( \text{O}(\frac{\Lambda_{QCD}}{m_b}) \)
Order $1/m_b$ power corrections to $\Gamma(\bar{B} \to X_s\gamma)$

- Non-perturbative effects arise from **Resolved Photon Contributions**

$$\Delta \Gamma \sim \begin{array}{l}
\underbrace{J} \\
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\end{array} \otimes \begin{array}{l}
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- $Q_{7\gamma} - Q_{8g}$

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- $Q_1 - Q_{7\gamma}$
Contribution to the non-perturbative error

• 2010 estimates for non-perturbative contribution to error
  - From $Q_1^c - Q_7^\gamma \in [-1.7, +4.0]\%$
  - From $Q_8^g - Q_8^g \in [-0.3, +1.9]\%$
  - From $Q_7^\gamma - Q_8^g \in [-4.4, +5.6]\%$

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where

$$\Lambda_{17} = e_c \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[ 1 - F \left( \frac{m_c^2 - i\varepsilon}{m_b\omega_1} \right) + \frac{m_b\omega_1}{12m_c^2} \right]$$

perturbative

$$h_{17}(\omega_1)$$

non-perturbative

Need a new model for $h_{17}$ to reduce the error.

- New information on moments of $h_{17}$: constrain new model
- What can we learn from moments?
$Q_1^c - Q_7\gamma$ contribution

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Moments of $h_{17}$
Definition of $h_{17}$

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$$h_{17}(\omega_1) =$$

$$= \int \frac{dr}{2\pi} e^{-i\omega_1 r} \langle \bar{B} | (\bar{h} S_{\bar{n}})(0) \gamma(1 + \gamma_5) i \gamma^\perp \bar{n}_\beta (S_{\bar{n}} g G^{\alpha\beta} S_{\bar{n}})(r \bar{n})(S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle$$

$$= \frac{2M_B}{2M_B}$$

- $S_n(x)$ is the Wilson line
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- $S_n(x)$ is the Wilson line

$$S_n(x) = P \exp \left( ig \int_{-\infty}^{0} du \cdot A_s(x + un) \right)$$

- $n^\mu \equiv (1, 0, 0, 1) \text{ and } \bar{n}^\mu \equiv (1, 0, 0, -1)$
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- $k$ th moment of $h_{17}$; Obtained using $\frac{\partial^k}{\partial r^k} e^{-i\omega_1 r}$
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$$\langle \omega_1^k h_{17} \rangle = (-1)^k \frac{1}{2M_B} \left \langle \bar{B} \left | (\bar{h} S_{\bar{n}}) (0) \cdot [i \bar{n} \cdot \partial]^k \left ( S_{\bar{n}}^\dagger g G^{\alpha \beta}_{\bar{s}} S_{\bar{n}} \right ) (r \bar{n}) \left ( S_{\bar{n}}^\dagger h \right ) (0) \right | \bar{B} \right \rangle \bigg |_{r=0}$$
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- Using the (new) identity

$$i\bar{n} \cdot \partial \left( S_\bar{n}^\dagger (x) O(x) S_\bar{n}(x) \right) = S_\bar{n}^\dagger (x)[i\bar{n} \cdot D, O(x)] S_\bar{n}(x)$$
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- Apply this for $k$ derivatives $\Rightarrow k$ commutators of $i\overline{n} \cdot D$
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- **New result** Moments over $\omega_1$

$$\langle \omega_{1}^{k} h_{17} \rangle = (-1)^{k} \frac{1}{2M_B} \left[ \bar{B} \right| \bar{h} \cdot \ldots \cdot \left[ i \bar{n} \cdot D, \ldots [i \bar{n} \cdot D, [D^\alpha, i \bar{n} \cdot D] \ldots ] \right] s^\lambda h \left| B \right]$$

**k times**
Moments of the $g_{17}$

- Procedure to obtain these HQET matrix elements derived in [A. Gunawardana and G. Paz, JHEP 07(2017)137 [arXiv:1702.08904]]

$$\langle h_{17} \rangle = 2\lambda_2 = 2\mu_G^2/3$$

$$\langle \omega_1^2 h_{17} \rangle = \frac{2}{15} (5m_5 + 3m_6 - 2m_9) \text{ New result}$$

- $m_i$ were extracted from data for the first time in 2016 [P. Gambino, K. J Healey, S. Turczyk PLB 763, 60 (2016)]

$$\mu_G^2 = 0.355 \pm 0.060 \text{ GeV}^2$$

$$m_5 = 0.072 \pm 0.045 \text{ GeV}^4$$

$$m_6 = 0.060 \pm 0.164 \text{ GeV}^4$$

$$m_9 = -0.280 \pm 0.352 \text{ GeV}^4$$
What we learn from moments

- Relative errors are large:

\[ \langle \omega_0 \rangle \text{ Numerical error is 17\% for } \]
\[ \langle \omega_1 h \rangle \text{ Numerical error is 80\% for } \]

- These moments still give useful information

- 2019 estimate
  \[ \langle \omega_2 h \rangle \in (0.03, 0.27) \text{ GeV} \]
- 2010 models provide
  \[ \langle \omega_2 h \rangle \in (-0.31, 0.49) \text{ GeV} \]

- These older models were constructed before \( m_i \) were extracted
- New estimate is significantly smaller than old estimate.

- Expect in future
  - Further improvements on HQET matrix elements
  - Belle II or LQCD data
  \[ \Rightarrow \text{ Better constrains on moments} \]
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Applications
New model for $h_{17}$

- Properties of $h_{17}$
New model for \( h_{17} \)

- Properties of \( h_{17} \)
  - Real and even function over \( \omega_1 \)
  - \( \langle \omega_1^k h_{17}(\omega_1) \rangle = 0 \) for \( k = 1, 3, 5, \ldots \)
  - \( h_{17} \) has a dimension of mass
  - Range of \( \omega_1 \Rightarrow -\infty < \omega_1 < \infty \)

- We use Hermite polynomials \( H_n(x) \)

- Our model:
  \[
  h_{17}(\omega_1) = \sum_n a_{2n} H_{2n}(\omega_1 \sqrt{2\sigma}) e^{-\omega_1^2 / 2\sigma}
  \]
  - Where \( a_0 = \langle \omega_0^1 h_{17} \rangle \sqrt{2\pi |\sigma|}, a_{2n} = \langle \omega_{2n}^1 h_{17} \rangle - \sigma^2 \langle \omega_0^1 h_{17} \rangle / 4 \sqrt{2\pi |\sigma|}, a_{4n} = \cdots \)

- \( |h_{17}| < 1 \) GeV and no peaks beyond \( \omega_1 = 1 \) GeV
New model for $h_{17}$

- Properties of $h_{17}$
  - Real and even function over $\omega_1$
    - $\langle \omega_1^k h_{17}(\omega_1) \rangle = 0$ for $k = 1, 3, 5, \cdots$
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    $a_4 = \cdots$
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  - where
    $$a_0 = \frac{\langle \omega_1^0 h_{17} \rangle}{\sqrt{2\pi}|\sigma|}, \quad a_2 = \frac{\langle \omega_1^2 h_{17} \rangle - \sigma^2 \langle \omega_1^0 h_{17} \rangle}{4\sqrt{2\pi}|\sigma|^3}, \quad a_4 = \cdots$$

- $|h_{17}| < 1 \text{ GeV}$ and no peaks beyond $\omega_1 = 1 \text{ GeV}$
New model vs 2010 model

Figure: 2019 model vs 2010 model for $h_{17}$
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- Orange dashed line: 2010 model
  \[
  h_{17}(\omega_1, \mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}
  \]

- Blue line: 2019 model
  \[
  \sigma = 0.5 \text{ GeV}, \quad \Lambda = 0.425 \text{ GeV}
  \]

- New function is 50% smaller than the 2010 model

- New model gives better constraints on $Q_1 - Q_7$ contribution

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  [ Y. Amhis et. al. EPJC 77, 895 (2017)]
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  \(-330\text{ MeV} < \tilde{\Lambda}_{u17} < +525\text{ MeV}\)
  \(-9\text{ MeV} < \tilde{\Lambda}_{c17} < +11\text{ MeV}\)
  

\[ A_{Xs\gamma}^{SM} = \left( 1.15 \times \frac{\tilde{\Lambda}_{u17} - \tilde{\Lambda}_{c17}}{300\text{MeV}} + 0.71 \right) \% \text{ CP asymmetry} \]

\[ \tilde{\Lambda}_{u17} = \frac{2}{3} h_{17}(0) \]

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Non-perturbative
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Conclusion

- $\bar{B} \to X_s \gamma$ is a important New Physics probe
- Non perturbative error of the decay rate is 5%
- $Q_1^c - Q_7^\gamma$ is the largest contribution to the error
- Better estimates for $Q_1^c - Q_7^\gamma$ obtained from moments of $h_{17}$
- New estimates for CP asymmetry
- Reduce non-perturbative error on rate and CP asymmetry