Reevaluating Uncertainties in $\bar{B} \to X_s \gamma$ Decay

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Based on
A.G and Gil Paz "Reducing Uncertainties in $\bar{B} \to X_s \gamma$"
(To appear)

Introduction

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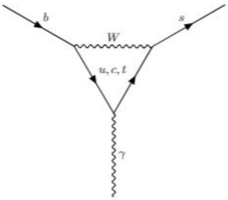
  - Can receive contributions from SM extensions.



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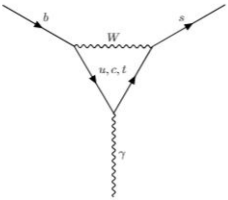

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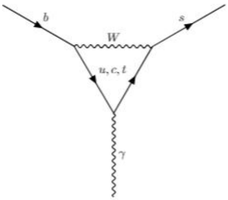

Figure: $b \to s\gamma$ flavor changing neural current (FCNC) in SM

  - SM extensions modify the $C_{7\gamma}$ Wilson coefficient
  - CP violation in $\bar{B} \to X_s\gamma$ can be enhanced by new physics

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- Also, gluon or quark pair can convert to photon

$$Q_{8g} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} G^{\mu\nu} (1 + \gamma_5) b$$

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- At leading power: Only $Q_{7\gamma} - Q_{7\gamma}$ contributes to decay rate
- At $1/m_b$: Γ get $Q_1 - Q_{7\gamma}$, $Q_{8g} - Q_{8g}$ and $Q_{7\gamma} - Q_{8g}$ contributions

Decay rate

- World average for experimental value:

$$\mathcal{B}\,(B \to X_s\gamma)\,(E_\gamma > 1.6\text{ GeV}) = (3.32 \pm 0.15) \times 10^{-4}$$

[Y. Amhis et. al. EPJC 77, 895 (2017)]

- NNLO prediction

$$\Gamma\left(\overline{B} \to X_q\gamma\right) = \underbrace{\Gamma\left(b \to X_q^p\gamma\right)}_{\text{Perturbatively calculable}} + \underbrace{\delta\Gamma_{\rm nonp}}_{\mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})}$$

- SM prediction (2015) [Misiak et. al. PRL 114, 221801 (2015)]

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- $\delta\Gamma_{\rm nonp} \equiv$ Non-perturbative contribution
  - The largest contribution to the error 5% from $\mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})$

Order $1/m_b$ power corrections to $\Gamma(\bar{B} \to X_s\gamma)$

- Non-perturbative effects arise from **Resolved Photon Contributions**

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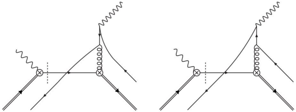

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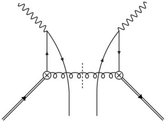
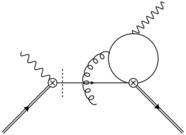


  - $Q_1 - Q_{7\gamma}$



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- 2010 estimates for non-perturbative contribution to error
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  - New Belle result for $Q_{7\gamma} - Q_{8g}$ contribution $\sim 2\%$

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- Now $Q_1^c - Q_{7\gamma}$ is the largest contribution to the error!

  Can we reduce it?

$Q_1^c - Q_{7\gamma}$ contribution

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$$\Lambda_{17} = e_c \,\mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - \underbrace{F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right)}_{\text{perturbative}} + \frac{m_b \omega_1}{12 m_c^2} \right] \underbrace{h_{17}(\omega_1)}_{\text{non-perturbative}}$$

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- Need a new model for h_{17} to **reduce the error**
  - New information on moments of h_{17}: constrain **new model**
  - What can we learn from moments?

Moments of h_{17}

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$$h_{17}(\omega_1) =$$
$$= \int \frac{dr}{2\pi} e^{-i\omega_1 r} \frac{\langle \bar{B} | (\bar{h} S_{\bar{n}})(0) \not{n} (1+\gamma_5) i\gamma^{\perp} \bar{n}_\beta (S_{\bar{n}} g G^{\alpha\beta} S_{\bar{n}})(r\bar{n})(S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle}{2M_B}$$

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$$S_n(x) = \mathbf{P}\exp\left(ig\int_{-\infty}^{0} dun \cdot A_s(x+un)\right)$$

  -

$$n^\mu \equiv (1,0,0,1) \text{ and } \bar{n}^\mu \equiv (1,0,0,-1)$$

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- New result Moments over ω_1

$$\langle \omega_1^k h_{17} \rangle = (-1)^k \frac{1}{2M_B} \langle \bar{B} | \bar{h} \cdots \underbrace{[i\bar{n}\cdot D, [i\bar{n}\cdot D, \cdots [i\bar{n}\cdot D,}_{k \text{ times}} [D^\alpha, i\bar{n}\cdot D] \cdots]] s^\lambda h | \bar{B} \rangle$$

Moments of the g_{17}

- Procedure to obtain these HQET matrix elements derived in
  [A. Gunawardana and G. Paz, JHEP 07(2017)137 [arXiv:1702.08904]]

$$\langle h_{17} \rangle = 2\lambda_2 = 2\mu_G^2/3$$
$$\left\langle \omega_1^2 h_{17} \right\rangle = \frac{2}{15}\left(5m_5 + 3m_6 - 2m_9\right) \text{ New result}$$

- m_i were extracted from data for the first time in 2016
  [P. Gambino, K. J Healey, S. Turczyk PLB 763, 60 (2016)]

$$\mu_G^2 = 0.355 \pm 0.060 \text{ GeV}^2 \qquad m_5 = 0.072 \pm 0.045 \text{ GeV}^4$$
$$m_6 = 0.060 \pm 0.164 \text{ GeV}^4 \qquad m_9 = -0.280 \pm 0.352 \text{ GeV}^4$$

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- Expect in future
  - Further improvements on HQET matrix elements
  - Belle II or LQCD data $\Rightarrow$ Better constrains on moments

Applications

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  - Real and even function over ω_1
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- Our model: $$h_{17}(\omega_1) = \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sqrt{2}\sigma}\right) e^{\frac{-\omega_1^2}{2\sigma}}$$
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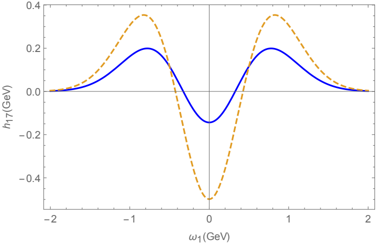
- $|h_{17}| < 1$ GeV and no peaks beyond $\omega_1 = 1$ GeV

New model vs 2010 model



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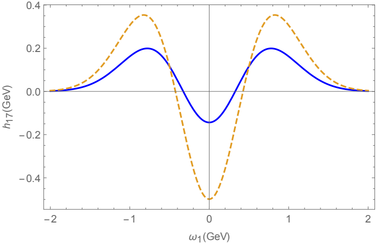

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- Orange dashed line: 2010 model $h_{17}(\omega_1, \mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$

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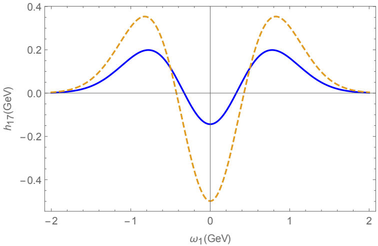

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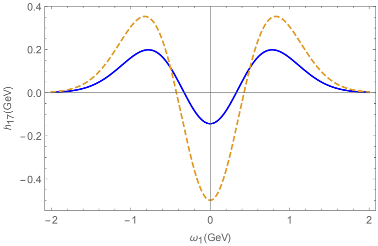

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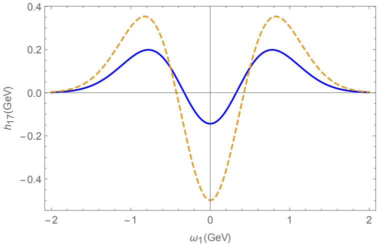

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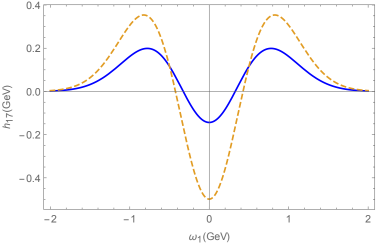

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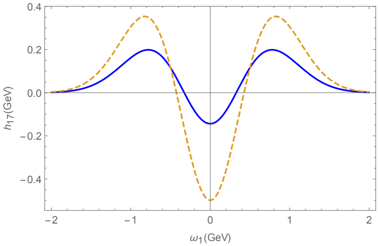

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- Consider also unknown higher moments, up to 6 Hermite polynomials

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- Direct CP Asymmetry experimental bound:

$$A_{CP} = (1.5 \pm 2.0)\,\%$$

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- We plan to improve these estimates

Conclusion

- $\bar{B} \to X_s\gamma$ is a important New Physics probe
- Non perturbative error of the decay rate is 5%
- $Q_1^c - Q_{7\gamma}$ is the largest contribution to the error
- Better estimates for $Q_1^c - Q_{7\gamma}$ obtained from moments of h_{17}
- New estimates for CP asymmetry
- Reduce non-perturbative error on rate and CP asymmetry