Compact Perturbative Expressions for Oscillations with Sterile Neutrinos in Matter

Xining Zhang, University of Chicago

DPF2019, Boston

July 30, 2019

Work done with Stephen Parke
arXiv:1905.01356
We present a rotation method to calculate neutrino oscillations in matter when assuming existence of sterile neutrinos. The method has high precision and fast computing speed.
Neutrino Oscillations in Vacuum

Hamiltonian $\mathcal{H}$

- Diagonal in Energy basis, Not in Flavor basis

PMNS matrix $U$

- Converts energy basis to flavor basis, i.e. Diagonalizes $H_{\text{flavor}}$
- Product of a series of (complex or real) rotations

Oscillation probabilities: 3 active + $N$ sterile neutrinos

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_{j=1}^{3+N} U_{\alpha j}^* U_{\beta j} e^{-i\Delta m^2_{j1} L/2E} \right|^2$$

- $\Delta m^2_{j1} =$Eigenvalues, $L =$length of baseline, $E =$neutrino beam energy
Matter Effect

\[ V_{NC} = \mp \sqrt{2} G_F N_n/2 \quad V_{CC} = \pm \sqrt{2} G_F N_e \]

\( N_n \) and \( N_e \) are the number densities of the neutrons and electrons, respectively.

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Oscillations in Matter

Hamiltonian $H$

$$H \rightarrow H_{\text{vacuum}} + V$$

- $V$ is **Diagonal** in **Flavor** basis, **Not** in vacuum **Energy** basis

PMNS matrix $U$

- Altered from the one in vacuum

Oscillation probabilities

- **Eigenvalues** of the new Hamiltonian
- Unitary matrix which **Diagonalizes** the new Hamiltonian in flavor basis
Possible Methods

  ▶ Impossible for $N \geq 2$
  ▶ Complexity

  ▶ Precision
  ▶ Computing Speed

Numerical methods: Eigen, HEigensystem, etc.
  ▶ Computing Speed
We will implement a series of (complex or real) rotations. Each rotation will only involve two flavors. Obviously basis and Hamiltonian must be rotated simultaneously.

For the Hamiltonian, the rotations do:

- Eliminate leading order (largest scale) off-diagonal elements
- Resolve crossings of diagonal elements
- Give 0th order results

After rotations

- Perturbative expansions
Example in details: 3+1

Preliminary assumptions

▶ Not very strong matter effect, for the earth crust: \( E \lesssim 30\text{GeV} \)
▶ Weak mixing with sterile neutrino: \( U_{\alpha 4} \sim 0.1 \)
▶ The sterile neutrino mass: \( \Delta m^2_{41} \geq 0.1\text{eV}^2 \)
In a usual convention to define the PMNS matrix in vacuum, rotations mixing with the sterile neutrinos come after (from energy basis to flavor basis) the ones in the active neutrino space

$$U = U_{\text{sterile}} U_{23} U_{13} U_{12}$$

A different convention to define the PMNS matrix

$$U = U_{23} U_{\text{sterile}} U_{13} U_{12}$$

The matter potential term in the Hamiltonian (in flavor basis) is invariant under a transformation in the (2-3) sector. If $U_{23}$ is the first (from flavor basis to energy basis) rotation, the following rotations process will be simplified.
Step 1: Vacuum $U_{23}$ rotation

$$H_{\text{flavor}} \Rightarrow U_{23}^\dagger(\theta_{23}, \delta_{23}) \ H_{\text{flavor}} \ U_{23}(\theta_{23}, \delta_{23})$$

$\theta_{23}$ and $\delta_{23}$ are as in vacuum.
Step 2: Vacuum $U_{\text{sterile}}$ rotations.

$$U_{23}^{\dagger}(\theta_{23}, \delta_{23}) H_{\text{flavor}} U_{23}(\theta_{23}, \delta_{23})$$

$$\Rightarrow \tilde{H} \equiv U_{\text{sterile}}^{\dagger} U_{23}^{\dagger}(\theta_{23}, \delta_{23}) H_{\text{flavor}} U_{23}(\theta_{23}, \delta_{23}) U_{\text{sterile}}$$

Rotations parameter (angles and phases) in $U_{\text{sterile}}$ are still as in vacuum
Step 3: $U_{13}$ rotation in matter

$$\tilde{H} = \frac{1}{2E} \left( \begin{array}{cccc} \lambda_a & \cdots & (\tilde{H})_{13} & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ (\tilde{H})^*_{13} & \cdots & \lambda_c & \cdots \\ \vdots & \ddots & \ddots & \ddots \end{array} \right)$$

- Kill $(\tilde{H})_{13}$
- Resolve the crossing of $\lambda_a$ and $\lambda_c$
Step 3: Continued

Diagonalize the (1-3) sector of $\tilde{H}$ by implementing a complex rotation $U_{13}(\tilde{\theta}_{13}, \alpha_{13})$

$$\tilde{H} \Rightarrow \hat{H} \equiv U_{13}^\dagger(\tilde{\theta}_{13}, \alpha_{13}) \tilde{H} U_{13}(\tilde{\theta}_{13}, \alpha_{13})$$
Step 4: $U_{12}$ rotation in matter

$$\hat{H} = \frac{1}{2E} \begin{pmatrix}
\lambda_- & (\hat{H})_{12} & 0 & \cdots \\
(\hat{H})_{12}^* & \lambda_0 & (\hat{H})_{23} & \\
0 & (\hat{H})_{23}^* & \lambda_+ & \\
\vdots & \vdots & \ddots & \ddots
\end{pmatrix}$$

- Kill $(\hat{H})_{12}$
- Resolve the crossing of $\lambda_-$ and $\lambda_0$ at the solar resonance.

![Normal Order](graph.png)
Step 4: Continued

Diagonalize the (1-2) sector of $\hat{H}$ by implementing a complex rotation $U_{12}(\tilde{\theta}_{12}, \alpha_{12})$

$$\hat{H} \Rightarrow \tilde{\hat{H}} \equiv U_{12}^\dagger(\tilde{\theta}_{12}, \alpha_{12}) \hat{H} U_{12}(\tilde{\theta}_{12}, \alpha_{12})$$
0th order PMNS matrix and Hamiltonian

In the 3+1 scheme

\[ U^{(0)}_{m} = U_{23} U_{34} U_{24} U_{14} U_{13} U_{12} \]

\[ \tilde{H} = \frac{1}{2E} \begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_2 & \lambda_3 \\ \lambda_3 & \lambda_4 \end{pmatrix} + \tilde{H}_1 \]

All diagonal elements of \( \tilde{H} \) have been absorbed into the 0th order Hamiltonian \( \tilde{H}_0 \)
$0^{\text{th}}$ order eigenvalues in the active neutrino space

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\end{itemize}
Perturbative expansion: Go to higher orders

\[
\lambda_i^{(\text{ex})} = \lambda_i + \lambda_i^{(1)} + \lambda_i^{(2)} + \cdots
\]

\[
U_{m}^{(\text{ex})} = U_{m}^{(0)} (1 + W_1 + W_2 + \cdots)
\]

\[
\epsilon \simeq \Delta m_{21}^2 / \Delta m_{31}^2 \simeq 0.03
\]
Presion test: oscillation possibilities

\[ \nu_\mu \rightarrow \nu_e, \; L=1300\,(\text{km}), \; \text{NO} \]
Summary

- Rotate Hamiltonian to eliminate largest (absolute value) off-diagonal elements and resolve crossings of diagonal elements
- Go to higher order precision by perturbative expansions
- Valid for all current and proposed accelerator oscillation experiments
- High precision and fast speed verified by numerical tests