Compact Perturbative Expressions for Oscillations with Sterile Neutrinos in Matter

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Introduction

We present a rotation method to calculate neutrino oscillations in matter when assuming existence of sterile neutrinos. The method has high precision and fast computing speed.

Neutrino Oscillations in Vacuum

Hamiltonian H

- Diagonal in Energy basis, Not in Flavor basis

PMNS matrix U

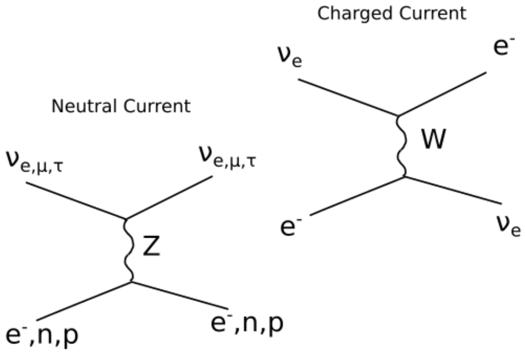
- Converts energy basis to flavor basis, i.e. Diagonalizes H_{flavor}
- Product of a series of (complex or real) rotations

Oscillation probabilities: 3 active + N sterile neutrinos

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_{j=1}^{3+N} U_{\alpha j}^* U_{\beta j} \, e^{-i \Delta m_{j1}^2 L/2E} \right|^2$$

- Δm_{j1}^2 =Eigenvalues, L =length of baseline, E =neutrino beam energy

Matter Effect



$$V_{NC} = \mp\sqrt{2}G_F N_n/2 \qquad V_{CC} = \pm\sqrt{2}G_F N_e$$

N_n and N_e are the number densities of the neutrons and electrons, respectively.

Oscillations in Matter

Hamiltonian H

$$H \to H_{\text{vacuum}} + \mathbf{V}$$

- **V** is Diagonal in Flavor basis, Not in vacuum Energy basis

PMNS matrix U

- Altered from the one in vacuum

Oscillation probabilities

- Eigenvalues of the new Hamiltonian
- Unitary matrix which Diagonalizes the new Hamiltonian in flavor basis

Possible Methods

Analytical solutions: arXiv:1808.03985

- Impossible for $N \geq 2$
- Complexity

Approximation methods: arXiv:1412.7524, arXiv:1712.02798, etc.

- Precision
- Computing Speed

Numerical methods: Eigen, HEigensystem, etc.

- Computing Speed

Rotate Hamiltonian

We will implement a series of (complex or real) rotations. Each rotation will only involve two flavors. Obviously basis and Hamiltonian must be rotated simultaneously.

For the Hamiltonian, the rotations do:

- Eliminate leading order (largest scale) off-diagonal elements
- Resolve crossings of diagonal elements
- Give 0th order results

After rotations

- Perturbative expansions

Example in details: 3+1

Preliminary assumptions

- Not very strong matter effect, for the earth crust: $E \lesssim 30\text{GeV}$
- Weak mixing with sterile neutrino: $U_{\alpha 4} \sim 0.1$
- The sterile neutrino mass: $\Delta m_{41}^2 \geq 0.1\text{eV}^2$

Step 0: Convention of the vacuum PMNS matrix

In a usual convention to define the PMNS matrix in vacuum, rotations mixing with the sterile neutrinos come after (from energy basis to flavor basis) the ones in the active neutrino space

$$U = U_{\text{sterile}}\, U_{23}\, U_{13}\, U_{12}$$

A different convention to define the PMNS matrix

$$U = U_{23}\, U_{\text{sterile}}\, U_{13}\, U_{12}$$

The matter potential term in the Hamiltonian (in flavor basis) is invariant under a transformation in the (2-3) sector. If U_{23} is the first (from flavor basis to energy basis) rotation, the following rotations process will be simplified.

Step 1: Vacuum U_{23} rotation

$$H_{\text{flavor}} \Rightarrow U_{23}^{\dagger}(\theta_{23}, \delta_{23})\, H_{\text{flavor}}\, U_{23}(\theta_{23}, \delta_{23})$$

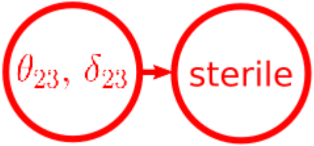
θ_{23} and δ_{23} are as in vacuum.

Step 2: Vacuum U_{sterile} rotations.

$$U_{23}^{\dagger}(\theta_{23}, \delta_{23})\, H_{\text{flavor}}\, U_{23}(\theta_{23}, \delta_{23})$$

$$\Rightarrow \tilde{H} \equiv U_{\text{sterile}}^{\dagger} U_{23}^{\dagger}(\theta_{23}, \delta_{23})\, H_{\text{flavor}}\, U_{23}(\theta_{23}, \delta_{23}) U_{\text{sterile}}$$

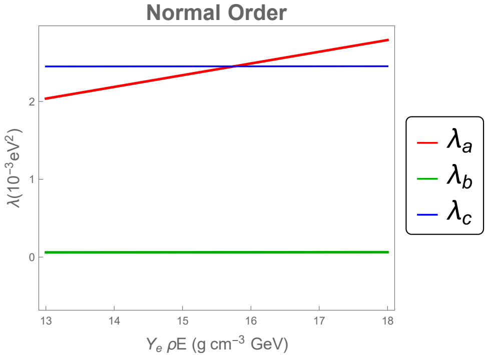
Rotations parameter (angles and phases) in U_{sterile} are still as in vacuum



Step 3: U_{13} rotation in matter

$$\tilde{H} = \frac{1}{2E}\begin{pmatrix} \lambda_a & \cdots & (\tilde{H})_{13} & \cdots \\ \vdots & \lambda_b & \vdots & \\ (\tilde{H})^*_{13} & \cdots & \lambda_c & \\ \vdots & & & \ddots \end{pmatrix}$$

- Kill $(\tilde{H})_{13}$
- Resolve the crossing of λ_a and λ_c



Step 3: Continued

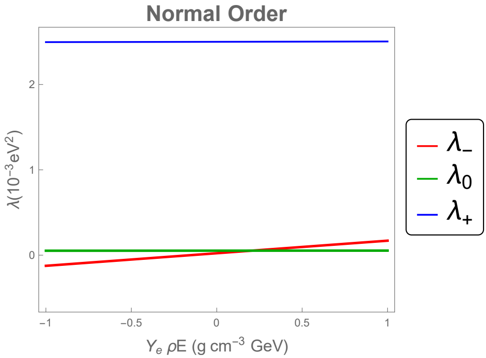
Diagonalize the (1-3) sector of $\tilde{H}$ by implementing a complex rotation $U_{13}(\tilde{\theta}_{13}, \alpha_{13})$

$$\tilde{H} \Rightarrow \hat{H} \equiv U_{13}^{\dagger}(\tilde{\theta}_{13}, \alpha_{13})\, \tilde{H}\, U_{13}(\tilde{\theta}_{13}, \alpha_{13})$$

Step 4: U_{12} rotation in matter

$$\hat{H} = \frac{1}{2E}\begin{pmatrix} \lambda_- & (\hat{H})_{12} & 0 & \cdots \\ (\hat{H})^*_{12} & \lambda_0 & (\hat{H})_{23} & \\ 0 & (\hat{H})^*_{23} & \lambda_+ & \\ \vdots & & & \ddots \end{pmatrix}$$

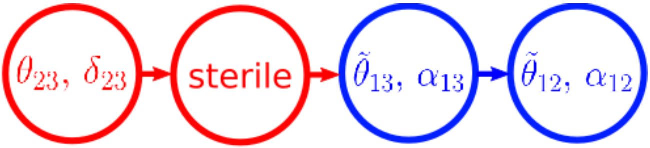
- Kill $(\hat{H})_{12}$
- Resolve the crossing of λ_- and λ_0 at the solar resonance.



Step 4: Continued

Diagonalize the (1-2) sector of $\hat{H}$ by implementing a complex rotation $U_{12}(\tilde{\theta}_{12}, \alpha_{12})$

$$\hat{H} \Rightarrow \check{H} \equiv U_{12}^{\dagger}(\tilde{\theta}_{12}, \alpha_{12})\, \hat{H}\, U_{12}(\tilde{\theta}_{12}, \alpha_{12})$$



0^{th} order PMNS matrix and Hamiltonian

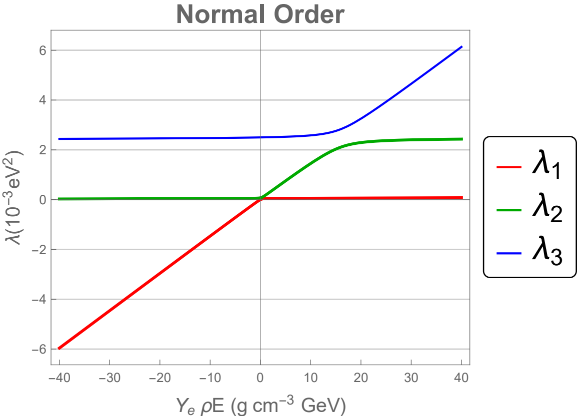
In the 3+1 scheme

$$U_{\text{m}}^{(0)} = U_{23} U_{34} U_{24} U_{14} U_{13} U_{12}$$

$$\check{H} = \underbrace{\frac{1}{2E} \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}}_{\check{H}_0} + \check{H}_1$$

All diagonal elements of $\check{H}$ have been absorbed into the 0^{th} order Hamiltonian $\check{H}_0$

0^{th} order eigenvalues in the active neutrino space

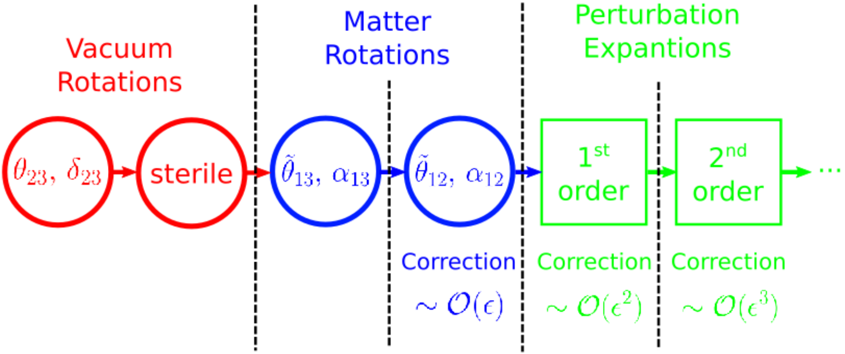


Perturbative expansion: Go to higher orders

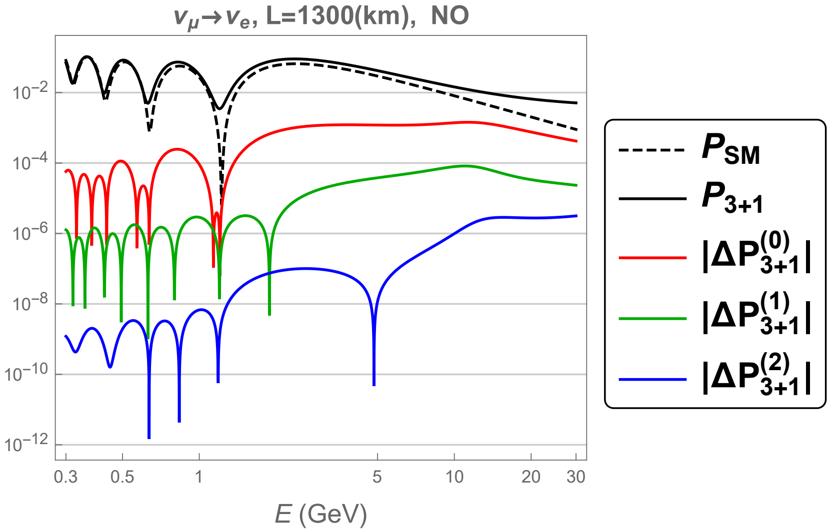
$$\lambda_i^{(\mathrm{ex})} = \lambda_i + \lambda_i^{(1)} + \lambda_i^{(2)} + \cdots$$

$$U_{\mathrm{m}}^{(ex)} = U_{\mathrm{m}}^{(0)}(\mathbb{1} + W_1 + W_2 + \cdots)$$

$$\epsilon \simeq \Delta m_{21}^2 / \Delta m_{31}^2 \simeq 0.03$$



Presion test: oscillation possibilities



Summary

- Rotate Hamiltonian to eliminate largest (absolute value) off-diagonal elements and resolve crossings of diagonal elements
- Go to higher order precision by perturbative expansions
- Valid for all current and proposed accelerator oscillation experiments
- High precision and fast speed verified by numerical tests