Charged Lepton Flavour Violation Physics

Emilie Passemar*

Indiana University/Jefferson Laboratory

Division of Particles and Fields Meeting of the American Physical Society

Northeastern University, Boston, July 29, 2019

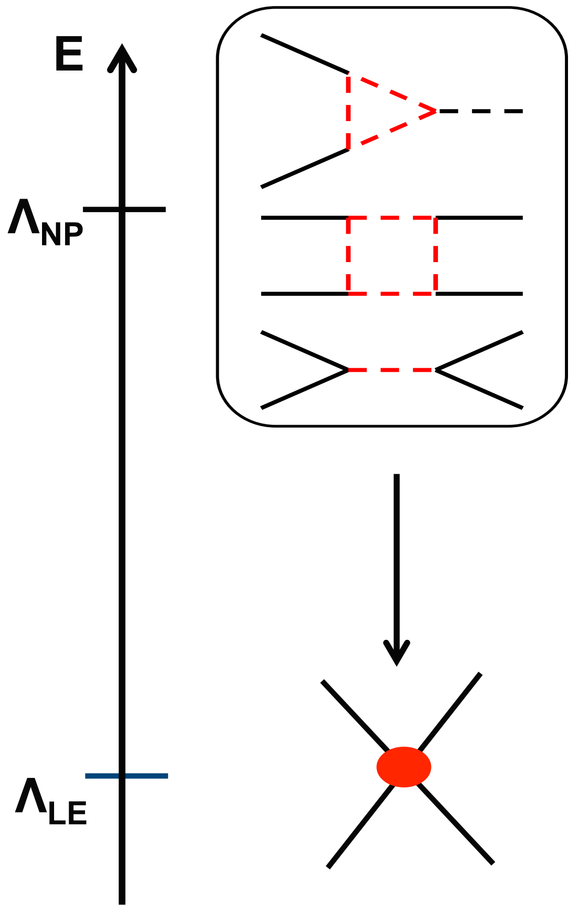
*Supported by NSF

Outline

1. Introduction and Motivation
2. Charged Lepton-Flavour Violation: Model discriminating power of muons and tau channels
3. Ex: Non-Standard LFV couplings of the Higgs boson
4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why study charged leptons?



- In the quest of New Physics, can be sensitive to very high scale:
  - Kaon physics: $\boxed{\frac{s\bar{d}s\bar{d}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^5 \text{ TeV}}$
    [ε_K]
  - Charged Leptons: $\boxed{\frac{\mu\bar{e}f\bar{f}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^4 \text{ TeV}}$
    [$\mu \to e\gamma$]

- At low energy: lots of experiments e.g., *MEG*, *COMET*, *Mu2e*, *E-969, BaBar*, *BelleI-II*, *BESIII, LHCb* ⇨ huge improvements on measurements and bounds obtained and more expected

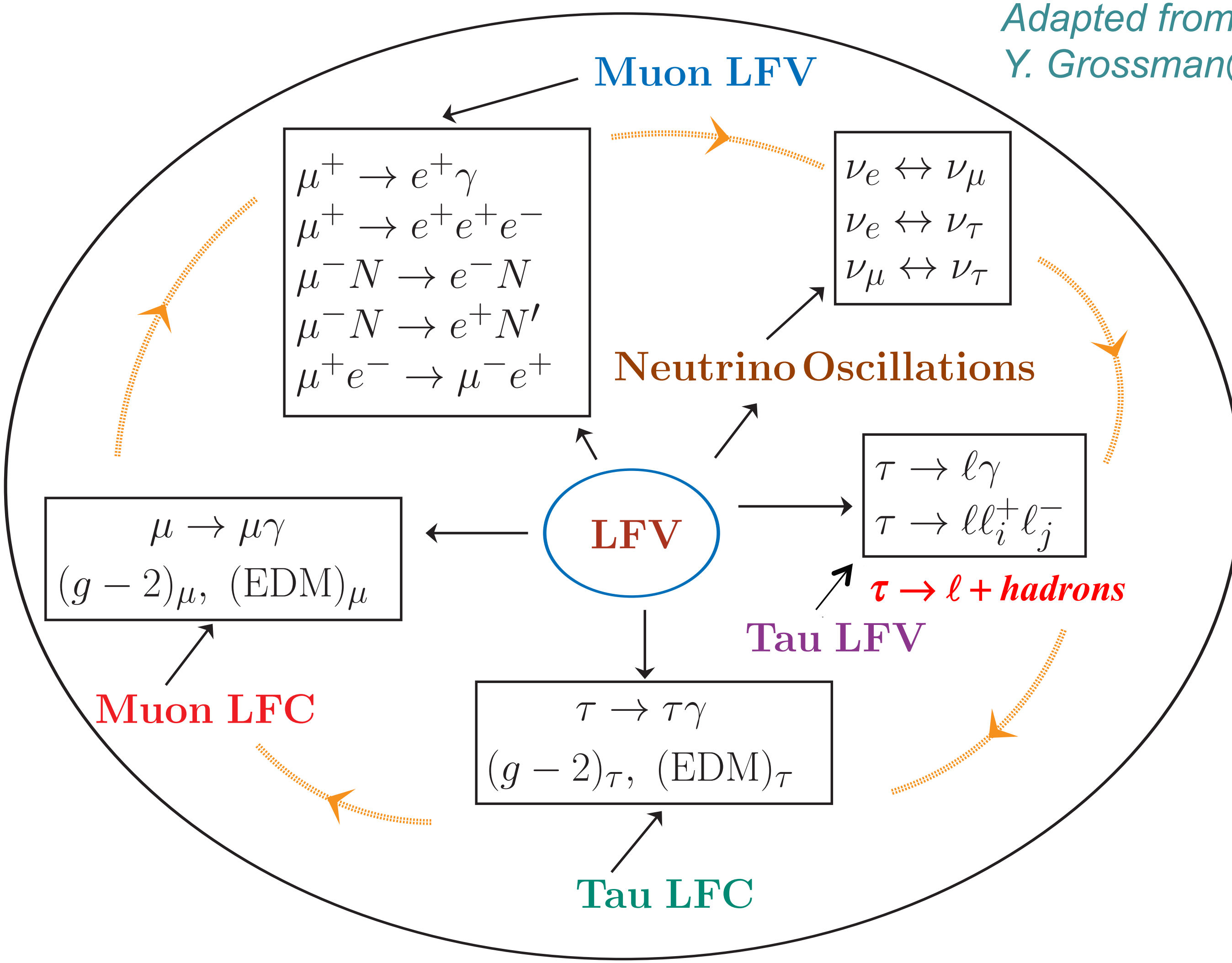
- In many cases no SM background: e.g., LFV, EDMs

- For some modes accurate calculations of hadronic uncertainties essential

⇨ Charged leptons very important to look for *New Physics*!

1.2 The Program

Adapted from Talk by
Y. Grossman@CLFV2013



2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

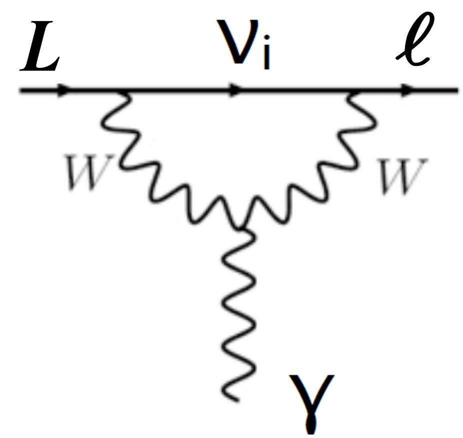
- Neutrino oscillations are the first evidence for lepton flavour violation

- How about in the charged lepton sector?

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br\left(\mu \to e\gamma\right) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Sl

$$\left[Br\left(\tau \to \mu\gamma\right) < 10^{-40}\right]$$

2.1 Introduction and Motivation

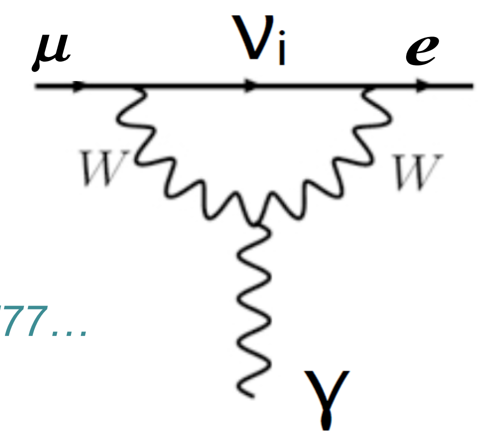
- Neutrino oscillations are the first evidence for lepton flavour violation

- How about in the charged lepton sector?

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇒ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

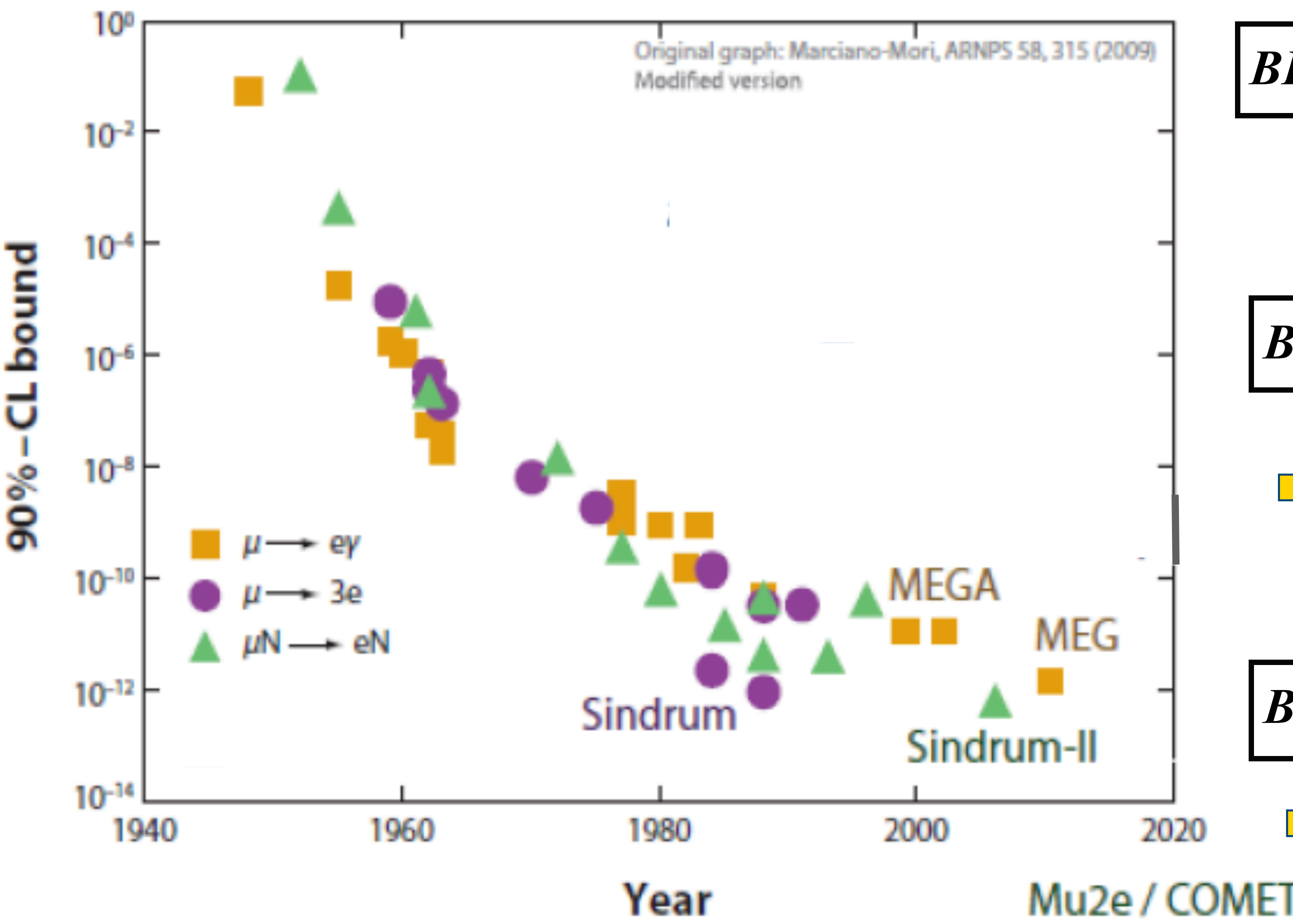
Talk by D. Hitlin @ CLFV2013

		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: muon decays

- Several processes: $\mu \to e\gamma,\ \mu \to e\bar{e}e,\ \mu(A,Z) \to e(A,Z)$



MEG'16

$$\boxed{BR(\mu \to e\gamma) < 4.2\times 10^{-13}}$$

$\Rightarrow$ 6×10^{-14}

Sindrum

$$\boxed{BR(\mu \to eee) < 1.0\times 10^{-12}}$$

$\Rightarrow$ $10^{-15} - 10^{-16}$

Mu3e

Sindrum II

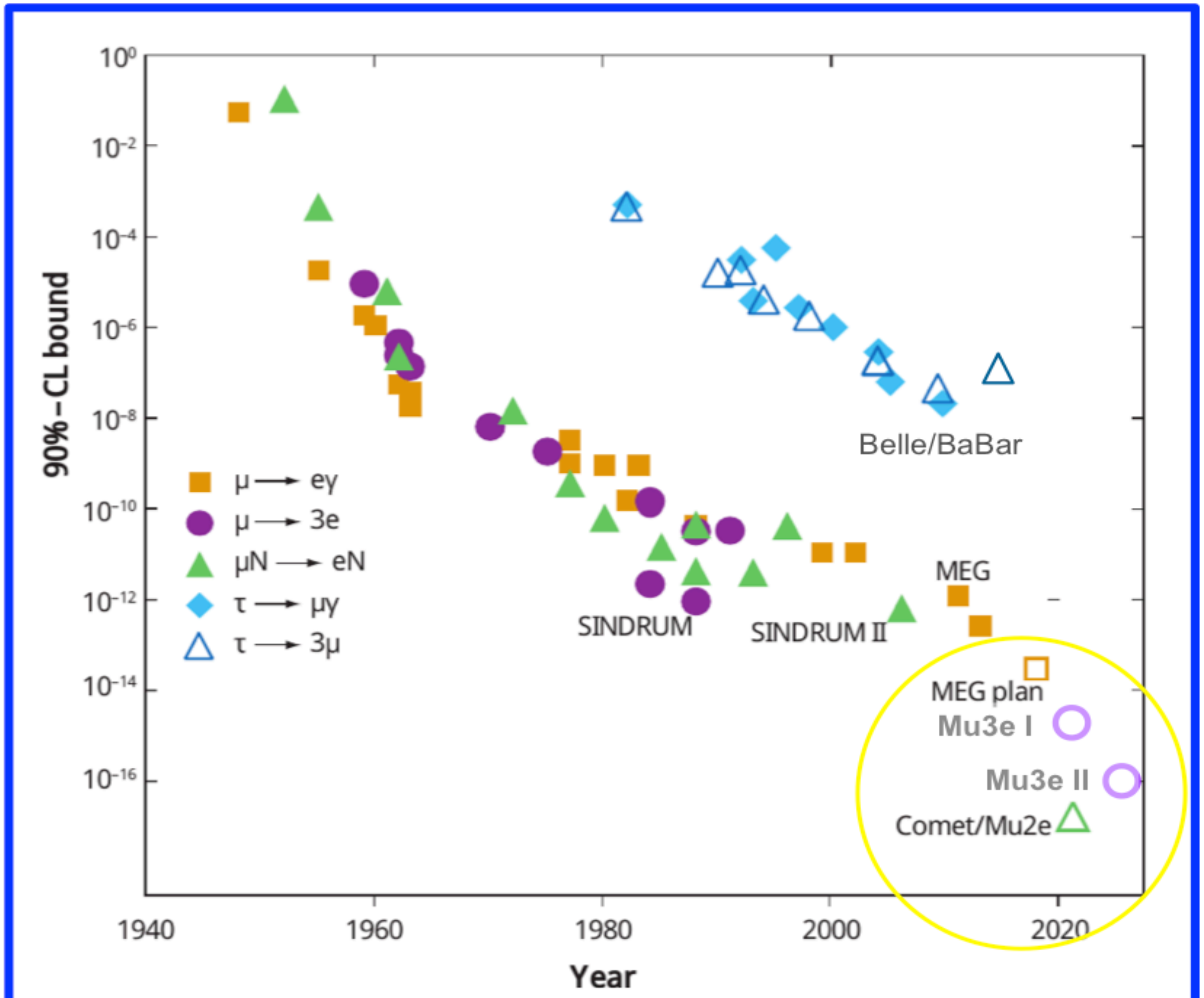
$$\boxed{BR^{Ti}_{\mu-e} < 4.3\times 10^{-12}}$$

$\Rightarrow$ $10^{-16} - 10^{-17}$

Mu2e/COMET

2.2 CLFV processes: muon decays

- Several processes: $\mu \to e\gamma,\ \mu \to e\bar{e}e,\ \mu(A,Z) \to e(A,Z)$



MEG'16

$$\boxed{BR(\mu \to e\gamma) < 4.2\times 10^{-13}}$$

$\Rightarrow$ 6×10^{-14}

Sindrum

$$\boxed{BR(\mu \to eee) < 1.0\times 10^{-12}}$$

$\Rightarrow$ $10^{-15} - 10^{-16}$

Mu3e

Sindrum II

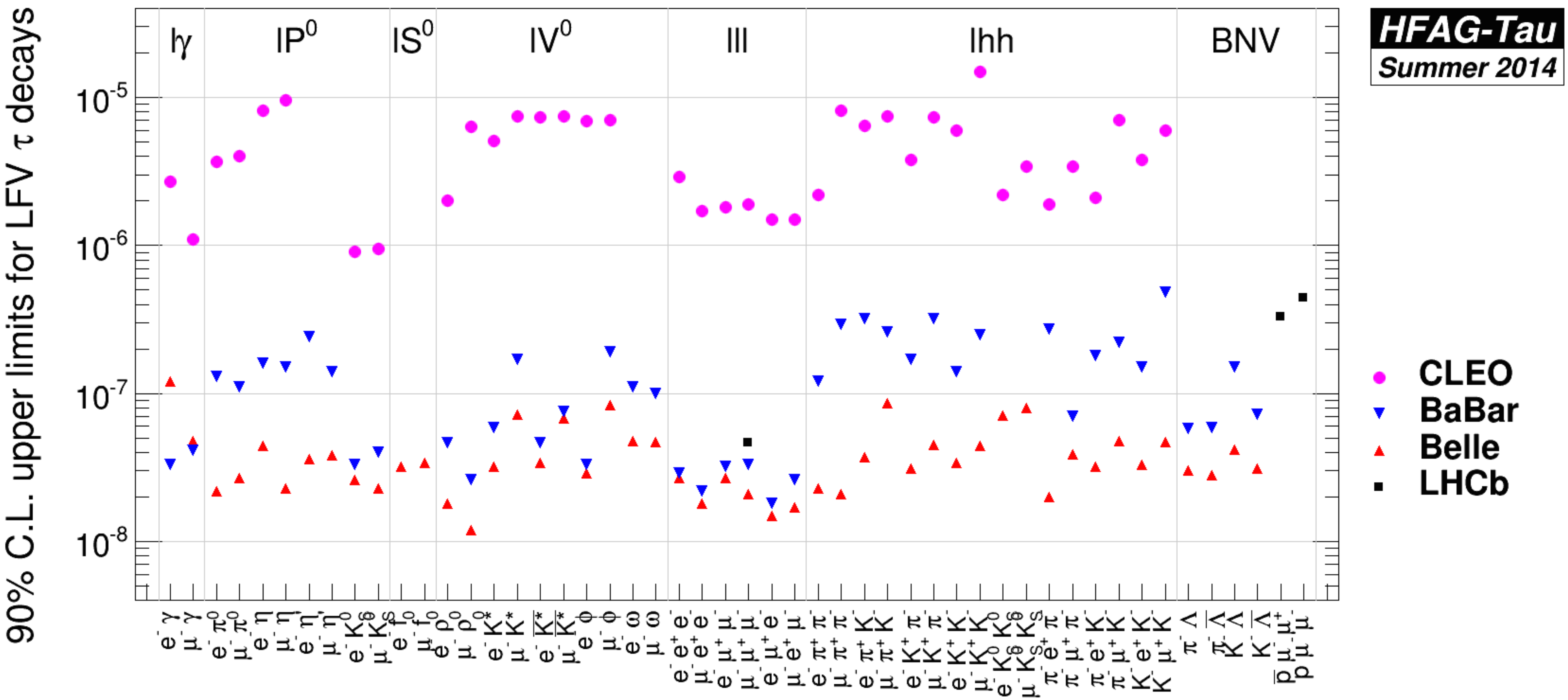
$$\boxed{BR^{Ti}_{\mu-e} < 4.3\times 10^{-12}}$$

$\Rightarrow$ $10^{-16} - 10^{-17}$

Mu2e/COMET

2.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P}, \ldots$



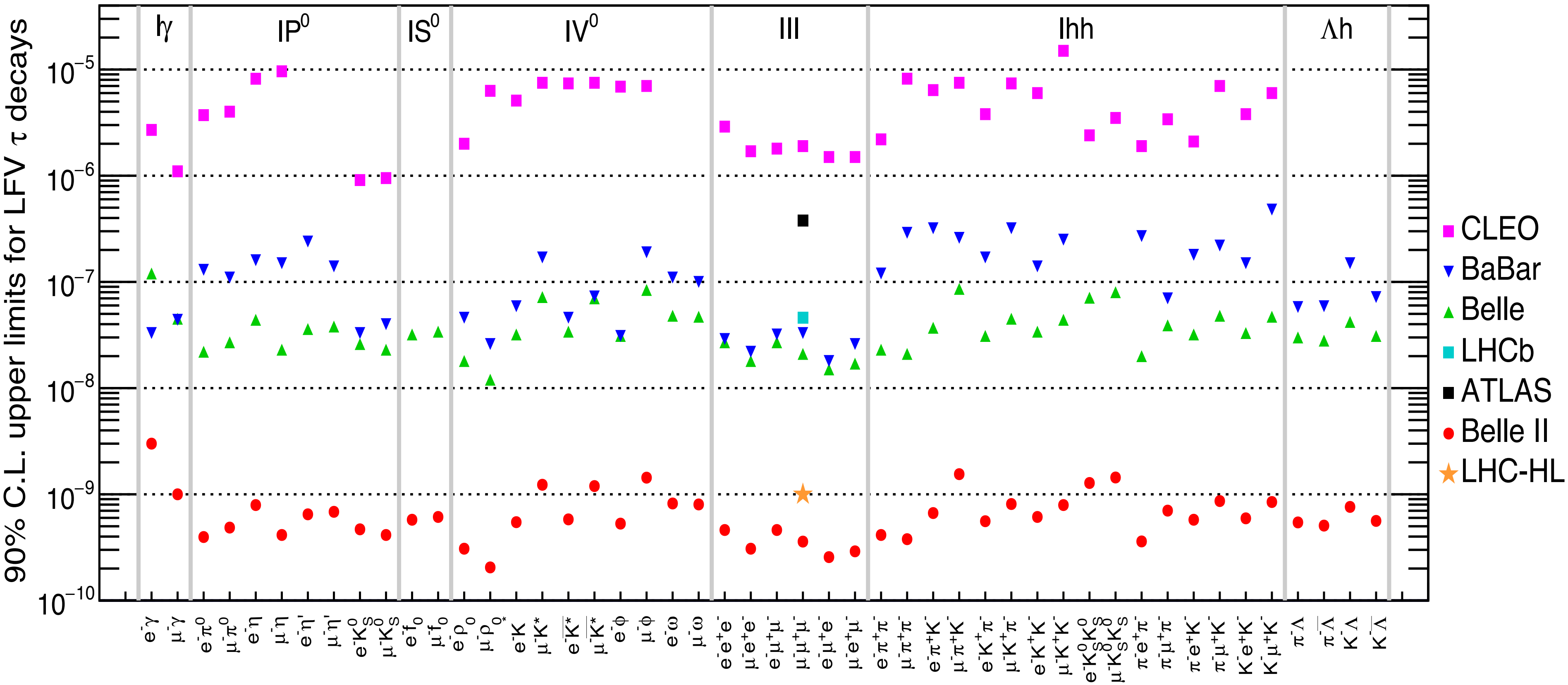
- 48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

Belle II Physics Book'18

HL-LHC&HE-LHC'18

- Several processes: $\tau \to \ell\gamma$, $\tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta$, $\tau \to \ell Y$ ← $P, S, V, P\bar{P}, \ldots$

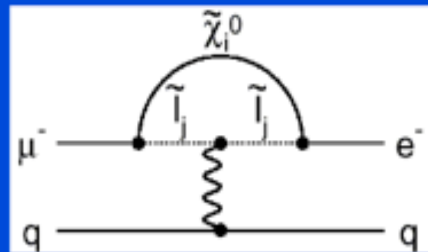


- Expected sensitivity 10^{-9} or better at *LHCb, Belle II, HL-LHC?*

A multitude of models…

Supersymmetry

Predictions at 10^{-15}

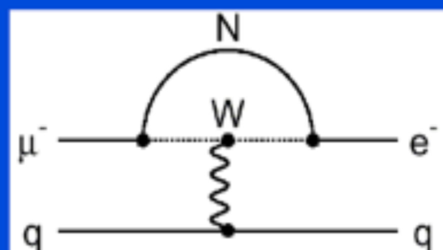


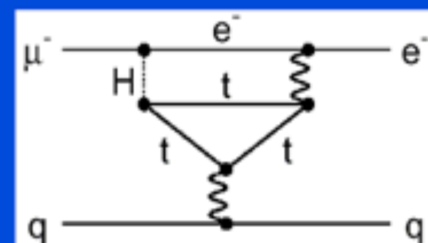
Compositeness

$\Lambda_c = 3000$ TeV

Heavy Neutrinos

$\left|U^*_{\mu N} U_{eN}\right|^2 = 8 \times 10^{-13}$





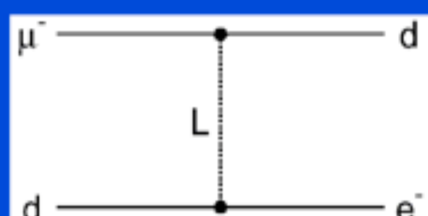
Second Higgs doublet

$g_{H_{\mu e}} = 10^{-4} \times g_{H_{\mu\mu}}$

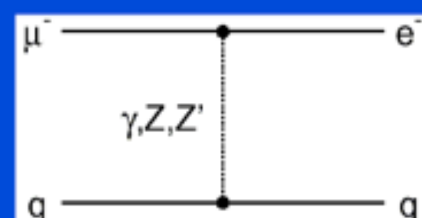
Leptoquarks

$M_L =$

$3000\sqrt{\lambda_{\mu d}\lambda_{ed}}$ TeV/c^2



After W. Marciano



Heavy Z', Anomalous Z coupling

$M_{Z'} = 3000$ TeV/c^2

$B(Z \to \mu e) < 10^{-17}$

James Miller, 2006

2.3 Effective Field Theory approach

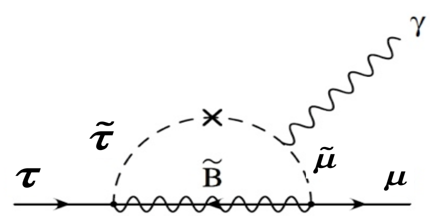
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \dots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole:

$$\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$$

e.g.



2.3 Effective Field Theory approach

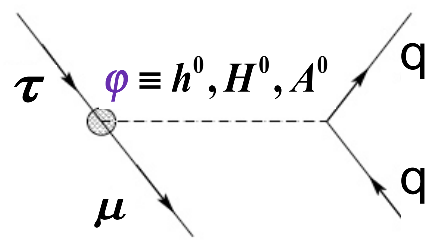
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

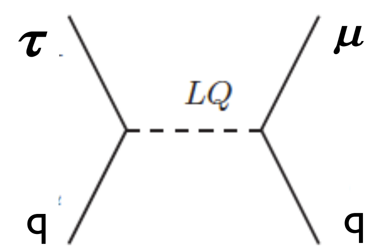
- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\boxed{\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\; \bar{q} \Gamma q}$$

e.g.



$\boxed{\Gamma \equiv 1}$



$\boxed{\Gamma \equiv \gamma^\mu}$

2.3 Effective Field Theory approach

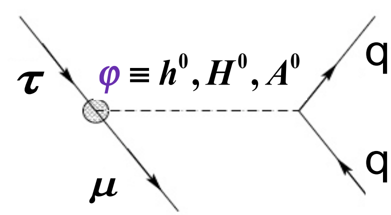
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \dots$$

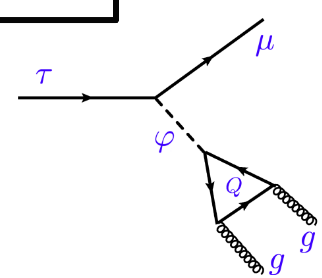
See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{q}\Gamma q}$
  - Integrating out heavy quarks generates *gluonic operator*

$$\boxed{\frac{1}{\Lambda^2}\bar{\mu}P_{L,R}\tau Q\bar{Q}} \rightarrow \boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu}P_{L,R}\tau\; G^a_{\mu\nu}G_a^{\mu\nu}}$$

Importance of this operator emphasized in *Petrov & Zhuridov'14*





2.3 Effective Field Theory approach

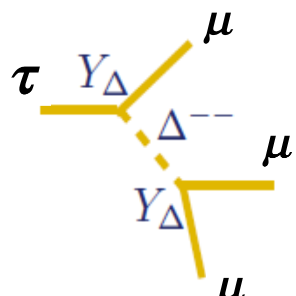
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

e.g.



2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R}\tau\, \bar{q}\Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\, G_{\mu\nu}^{a} G_{a}^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R}\tau\, \bar{\mu}\, \Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

2.4 Model discriminating power of muon processes

- Summary table:

Cirigliano@Beauty2014

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O_{S,V}^{4\ell}$	✓	–	–
O_D	✓	✓	✓
O_V^q	–	–	✓
O_S^q	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes
  ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.4 Model discriminating power of muon processes

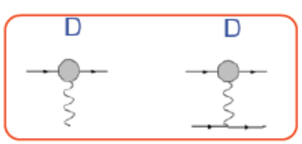
Cirigliano@Beauty2014

- Summary table:

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^{q}_{V}	–	–	✓
O^{q}_{S}	–	–	✓

- $\mu \to e\gamma$ vs. $\mu \to 3e$ ⇨ relative strength between *dipole* and *4L* operators

$$\frac{\Gamma_{\mu\to 3e}}{\Gamma_{\mu\to e\gamma}} = \frac{\alpha}{4\pi}\, I_{\mathrm{PS}} \left(1 + \sum_i \frac{c_i^{(\mathrm{contact})}}{c^{(\mathrm{dipole})}} \right)$$



6×10^{-3}

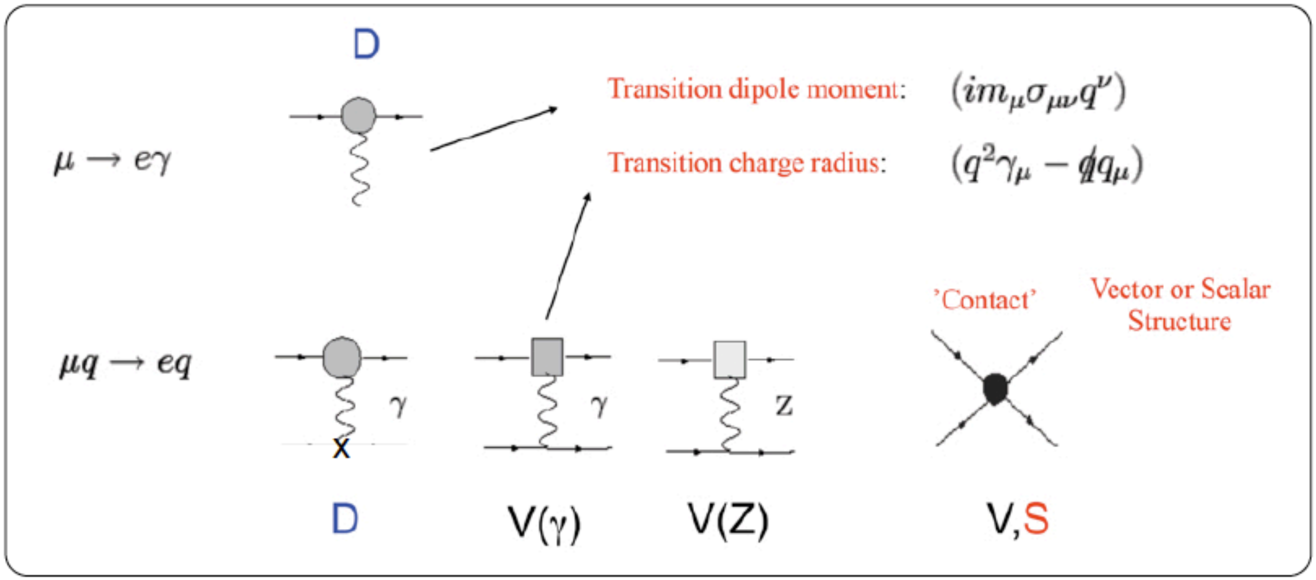
2.4 Model discriminating power of muon processes

Cirigliano@Beauty2014

- Summary table:

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O_{S,V}^{4\ell}$	✓	–	–
O_D	✓	✓	✓
O_V^q	–	–	✓
O_S^q	–	–	✓

- *$\mu \to e\gamma$* vs. *$\mu \to e$ conversion* ⇒ relative strength between *dipole* and *quark* operators

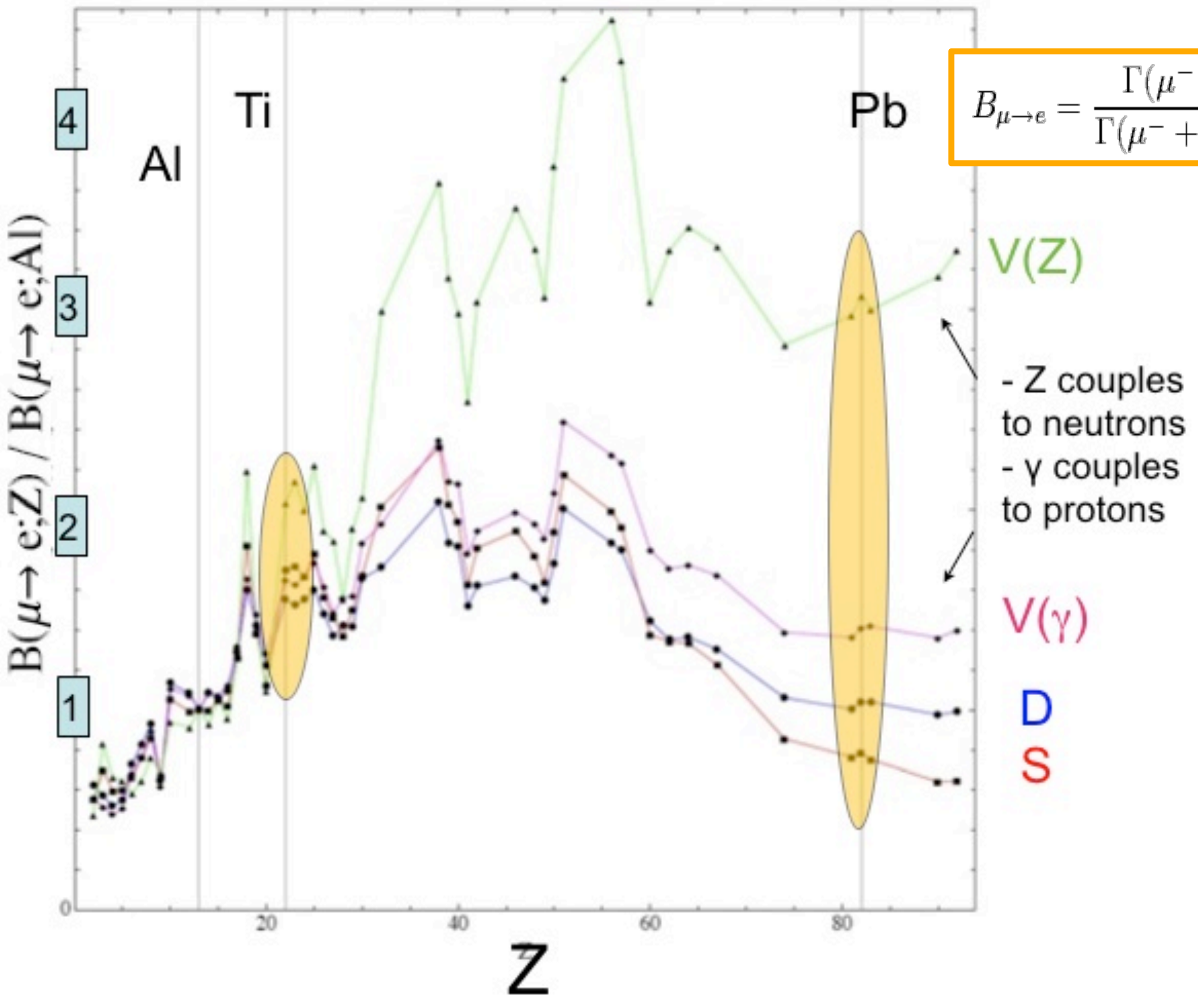


BR for $\mu \rightarrow$ e conversion

- For *$\mu \rightarrow$e conversion,* target dependence of the amplitude is different for V,D or S models

Cirigliano, Kitano, Okada, Tuzon'09

$$B_{\mu \to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$



2.5 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O_V^q	–	–	✓ (I=1)	✓(I=0,1)	–	–
O_S^q	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O_A^q	–	–	–	–	✓ (I=1)	✓ (I=0)
O_P^q	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important ⇒ sensitive to large number of operators!

- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. f_η, $f_{\eta'}$)

2.5 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- Form factors for $\tau \to \mu(e)\pi\pi$ determined using *dispersive techniques*
- Hadronic part:

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13
Celis, Cirigliano, E.P.'14

$$\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(Lorentz\ \text{struct.}\right)^i_\mu F_i(s)}$$

with

$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

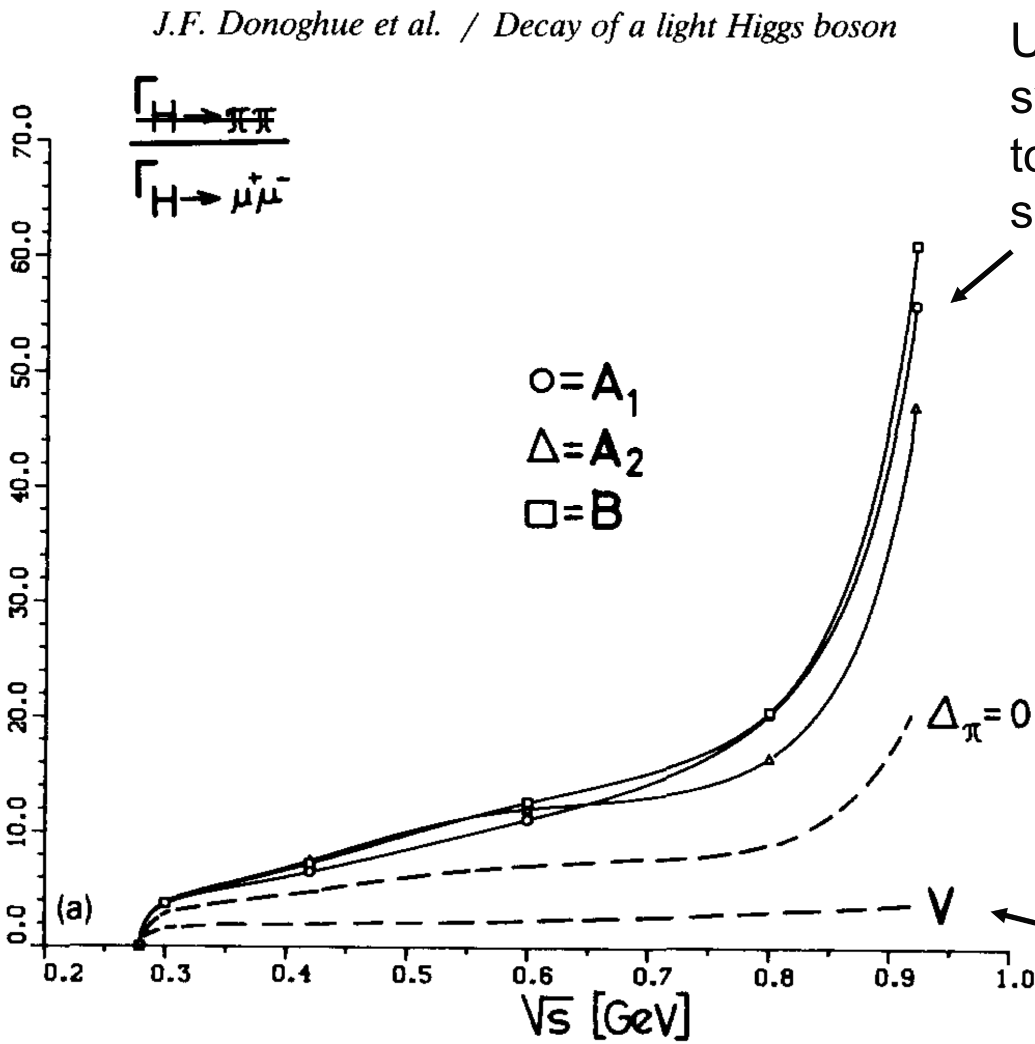
- 2-channel unitarity condition is solved with I=0 S-wave ππ and KK scattering data as input

$$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$$n = \pi\pi, K\bar{K}$$

How to describe the form factors?

Donoghue, Gasser, Leutwyler'90



Using the triple constraints of chiral symmetry, analyticity, and unitarity, together with exp. input from pion scattering

very far from the naive expectation

$$\frac{\Gamma(h \to \mu^+\mu^-)}{\Gamma(h \to \pi^+\pi^-)} \sim \frac{m_\mu^2}{m_\pi^2}$$

Voloshin'85

Unitarity

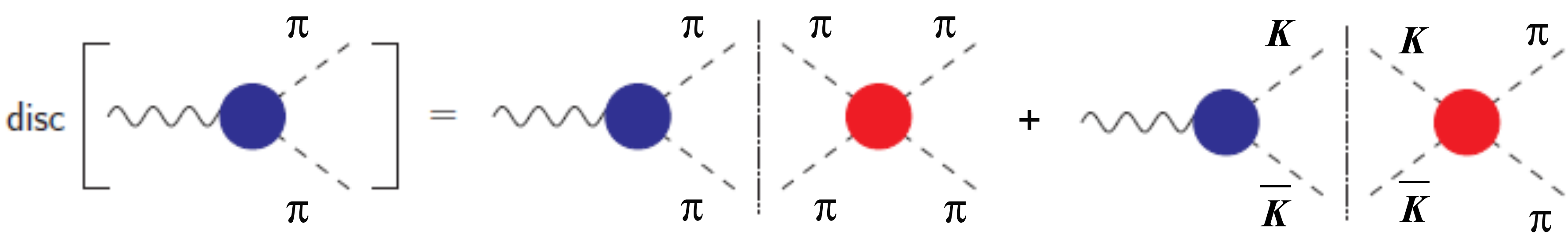
Celis, Cirigliano, E.P.'14

- Elastic approximation breaks down for the ππ S-wave at $K\bar{K}$ threshold due to the strong inelastic coupling involved in the region of *f_0(980)*

⇨ Need to *solve a Coupled Channel* *Mushkhelishvili-Omnès* problem

Donoghue, Gasser, Leutwyler'90
Osset & Oller'98
Moussallam'99

- Unitarity ⇨ the discontinuity of the form factor is known



⇨ $$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

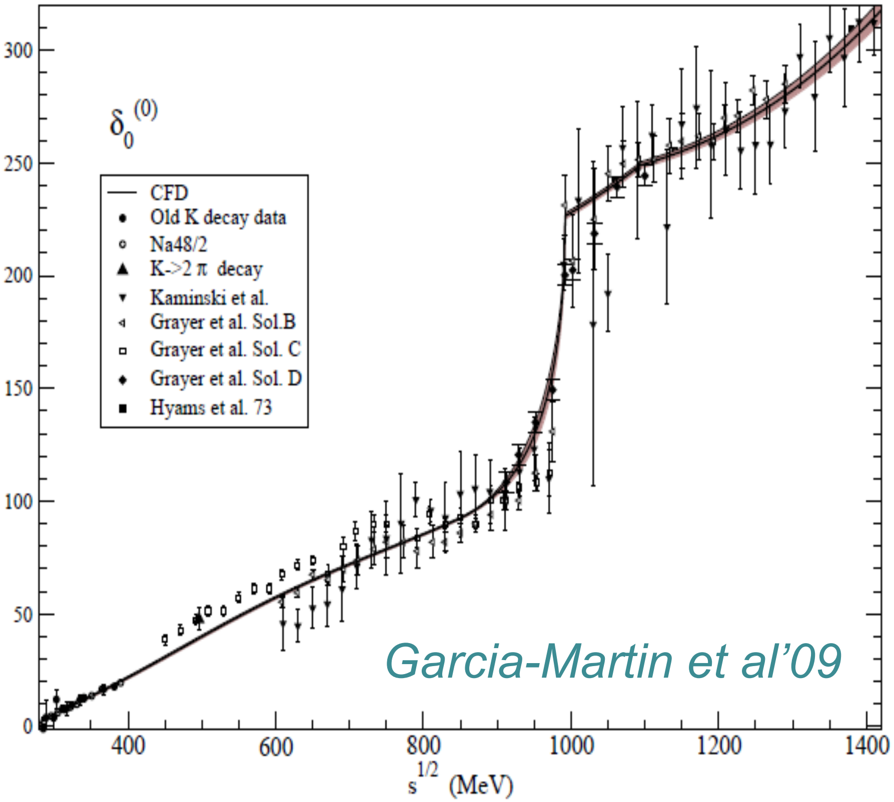
$n = \pi\pi, K\bar{K}$

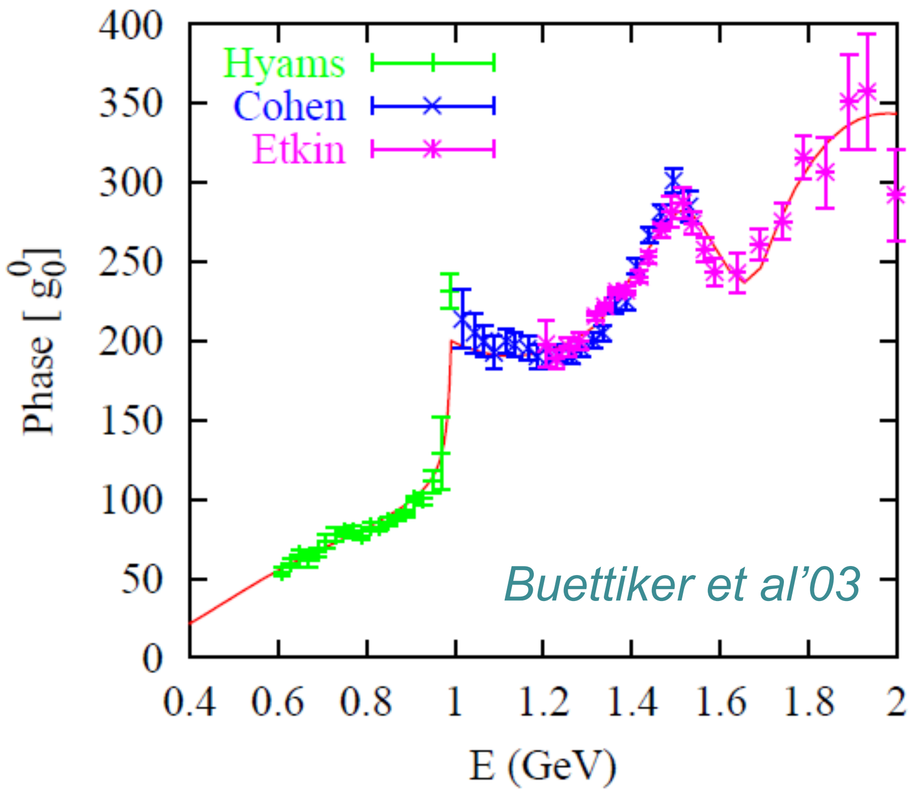
Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

Inputs for the coupled channel analysis

- Inputs : $\pi\pi \rightarrow \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

Dispersion relations

Celis, Cirigliano, E.P.'14

- General solution to *Mushkhelishvili-Omnès* problem:

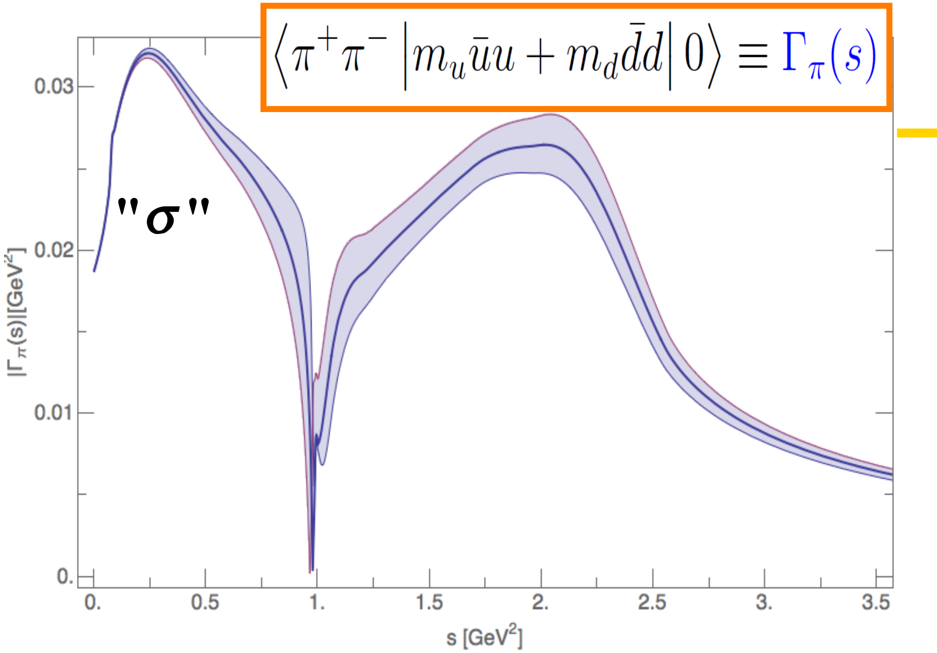
$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

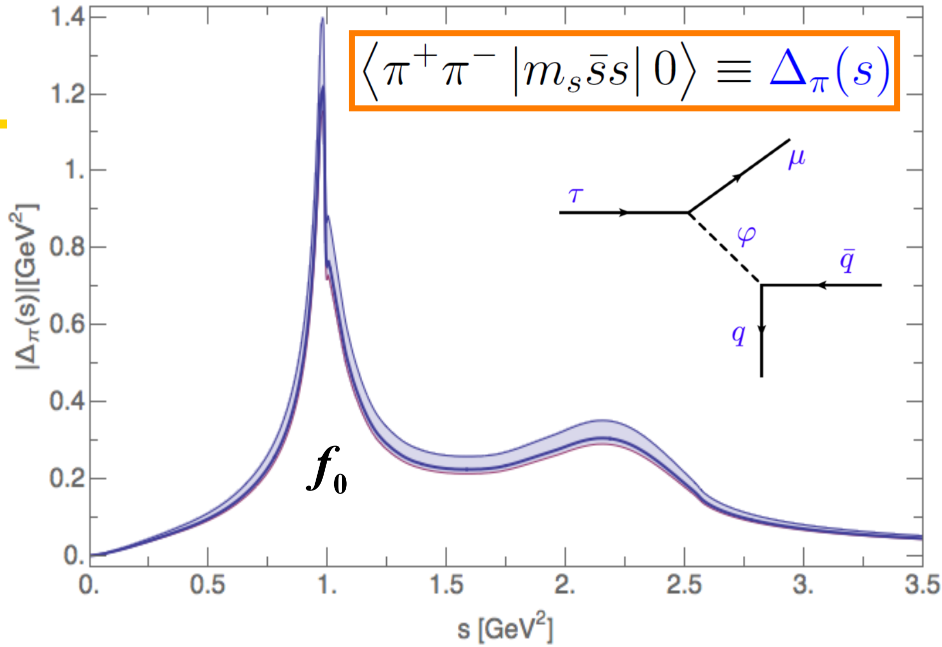
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

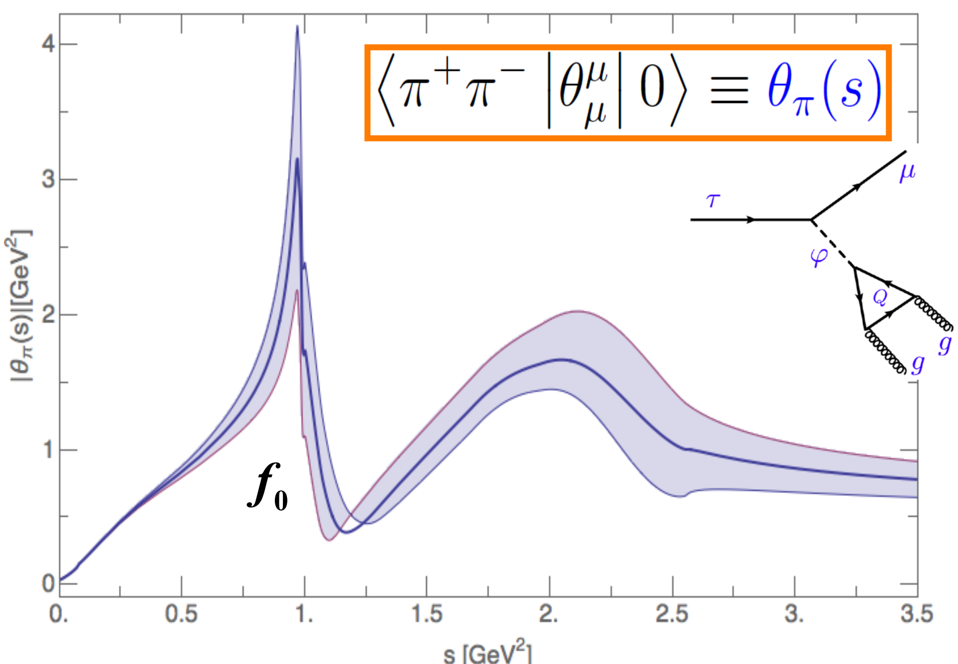
Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving dispersive integral equations iteratively starting with *Omnès functions* that are solutions of the one-channel unitary condition

$$\boxed{\Omega_{\pi,K}(s) \equiv \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{dt}{t}\frac{\delta_{\pi,K}(t)}{(t-s)}\right]}$$







Celis, Cirigliano, E.P.'14

- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs

See also *Daub et al.'13*

2.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

2.6 Model discriminating of BRs

- Studies in specific models

Buras et al.'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 0.1$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.02\ldots 0.04$	$0.03\ldots 1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots 2$	~ 5	$0.3\ldots 0.5$	$1.5\ldots 2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots 1.6$	~ 0.2	$5\ldots 10$	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots 0.15$	$10^{-12}\ldots 26$

⇨ Disentangle the *underlying dynamics* of NP

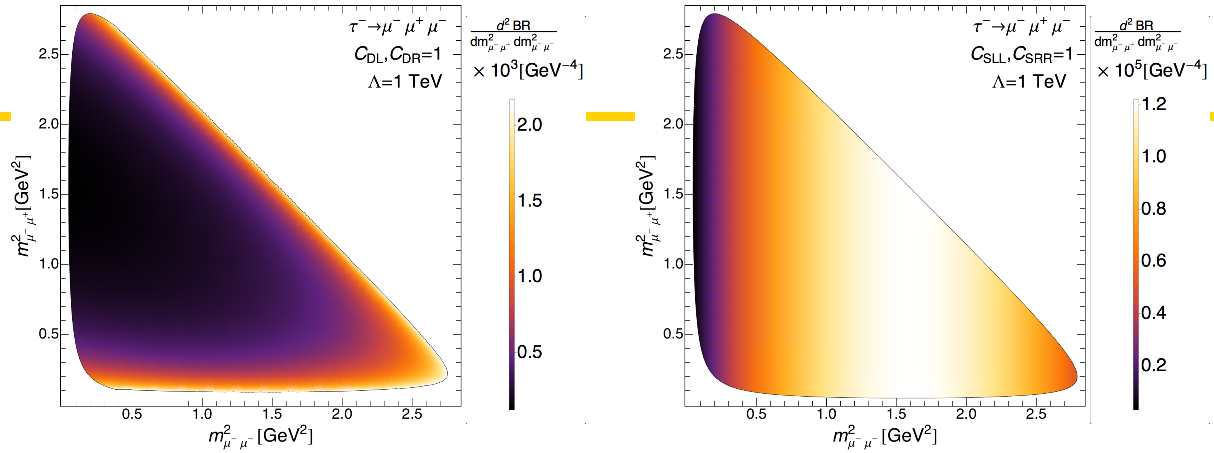


Figure 3: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of* $\mathrm{C_{DL,DR}} = 1$ *(left) and* $\mathrm{C_{SLL,SRR}} = 1$ *(right), taking $\Lambda = 1$ TeV in both cases. Colors denote the density for* $\mathrm{d^2BR}/(dm^2_{\mu^-\mu^+} dm^2_{\mu^-\mu^-})$*, small values being represented by darker colors and large values in lighter ones. Here $m^2_{\mu^-\mu^+}$ represents m^2_{12} or m^2_{23}, defined in Sec. 3.1.*

Dassinger, Feldman, Mannel, Turczyk' 07
Celis, Cirigliano, E.P.'14

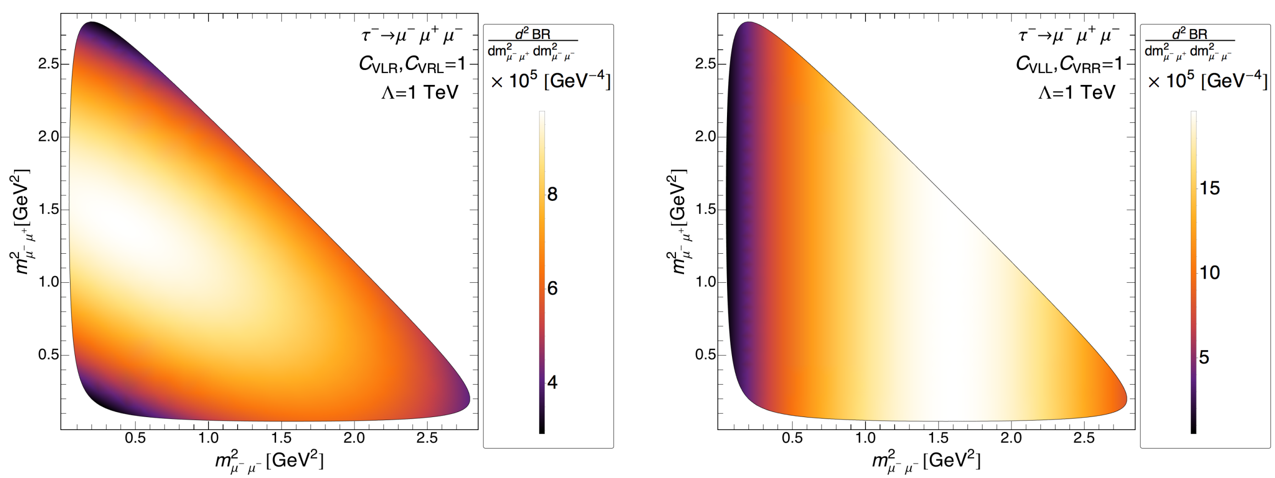


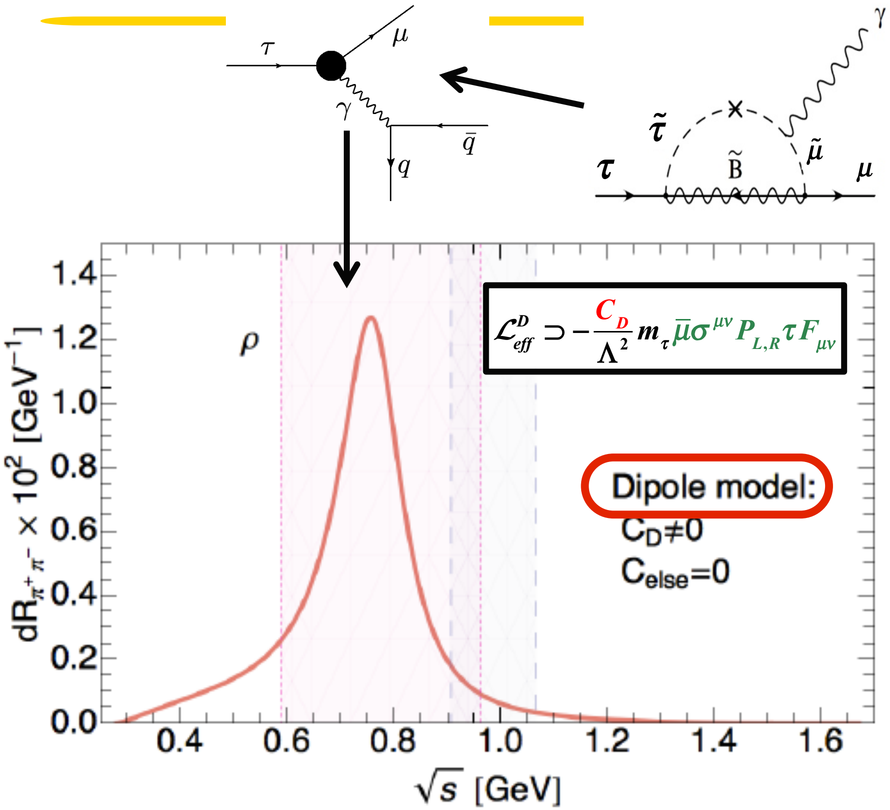
Figure 4: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of* $\mathrm{C_{VRL,VLR}} = 1$ *(left) and* $\mathrm{C_{VLL,VRR}} = 1$ *(right), taking $\Lambda = 1$ TeV in both cases. Colors are defined as in Fig. 3.*

Angular analysis with polarized taus

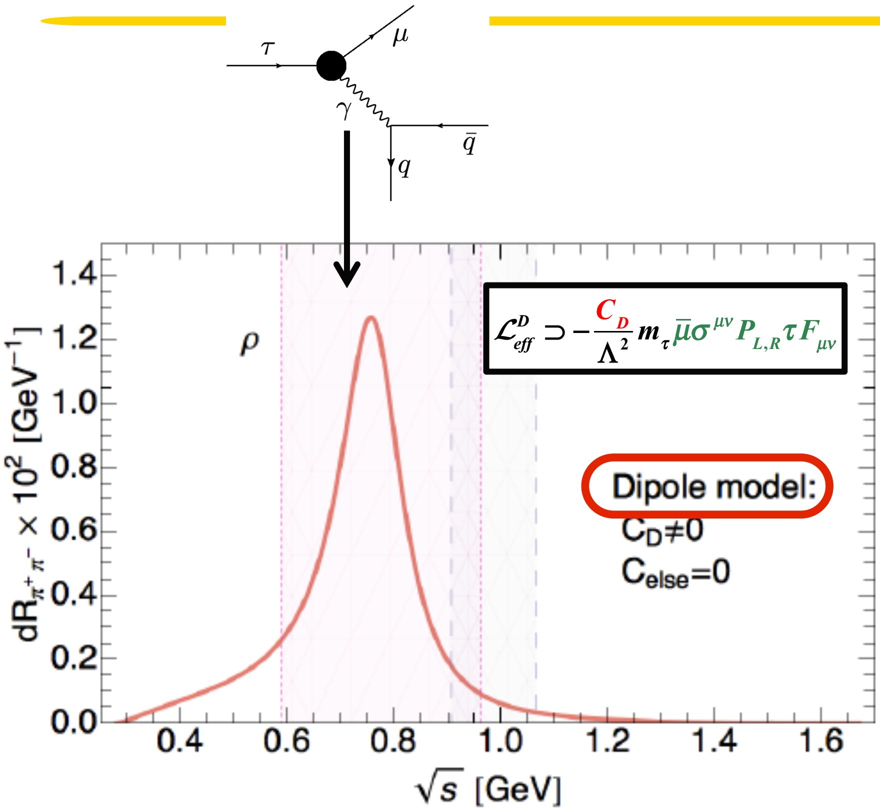
Dassinger, Feldman, Mannel, Turczyk' 07

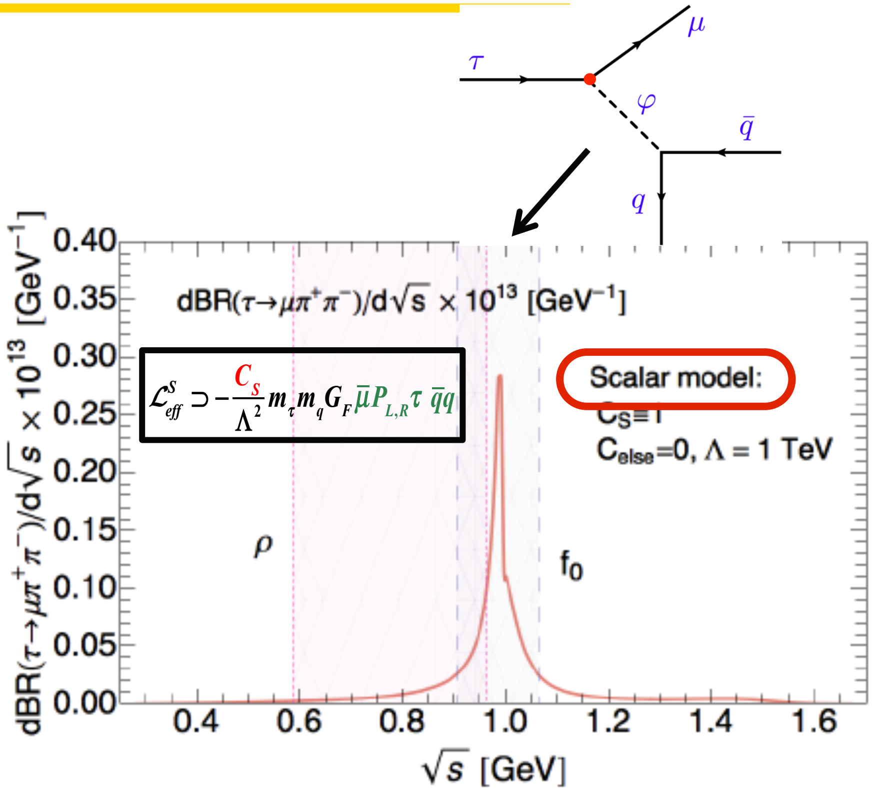
4.7 Discriminating power of τ→μ(e)ππ decays

Celis, Cirigliano, E.P.'14

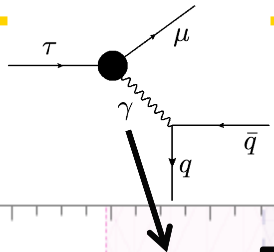


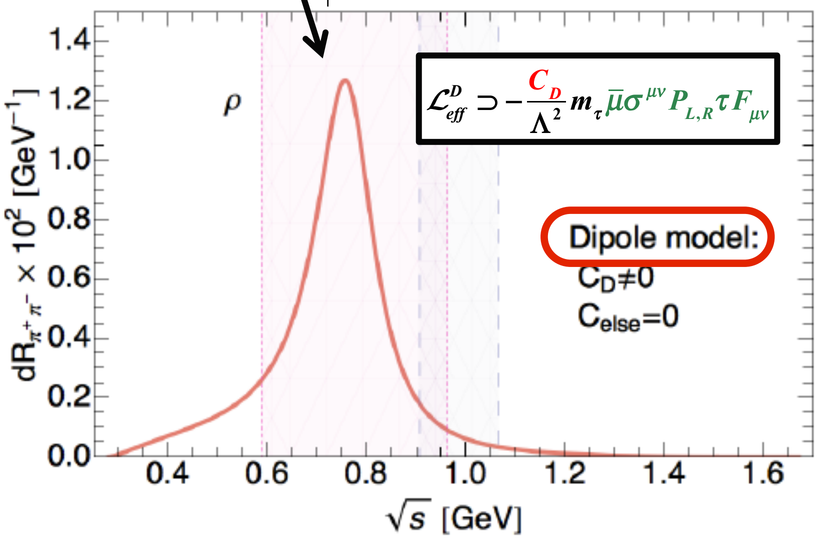
4.7 Discriminating power of τ→μ(e)ππ decays

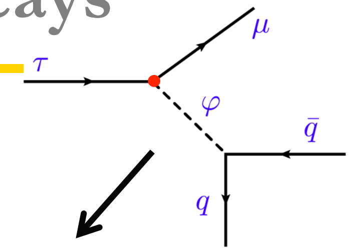


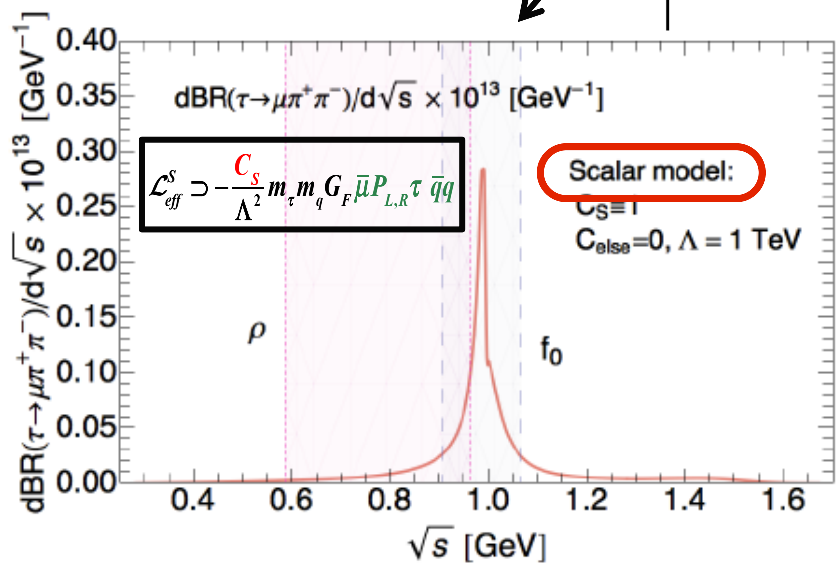


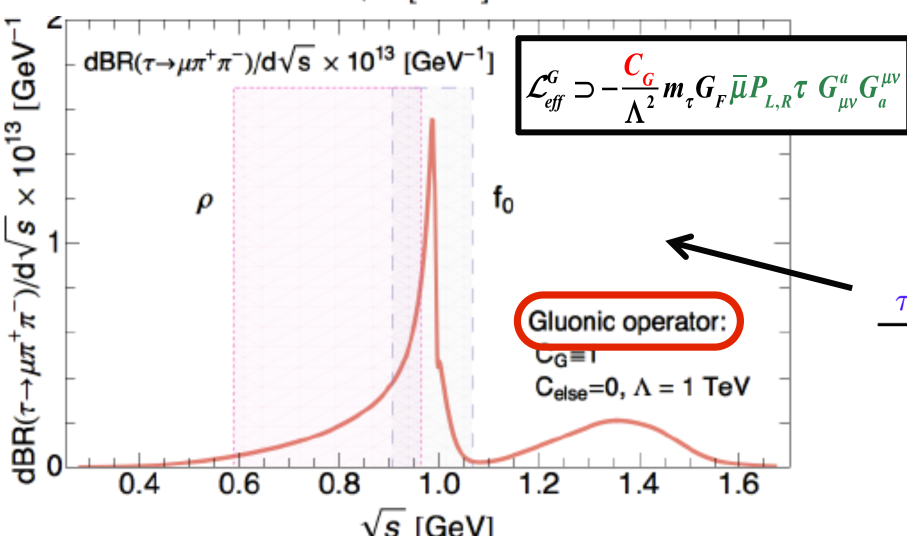
4.7 Discriminating power of τ→μ(e)ππ decays



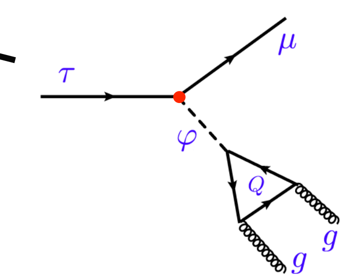








Different distributions according to the *operator!*



3. Ex: Charged Lepton-Flavour Violation and Higgs Physics

3.1 Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

$$\boxed{\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - h\left(Y_{e\mu}\bar{e}_L\mu_R + Y_{e\tau}\bar{e}_L\tau_R + Y_{\mu\tau}\bar{\mu}_L\tau_R\right) + \ldots}$$

- Arise in several models Cheng, Sher'97, Goudelis, Lebedev,Park'11
  Davidson, Grenier'10

Cheng, Sher'97

- Order of magnitude expected $\Rightarrow$ No tuning: $\boxed{|Y_{\tau\mu}Y_{\mu\tau}| \lesssim \frac{m_\mu m_\tau}{v^2}}$

- In concrete models, in general further parametrically suppressed

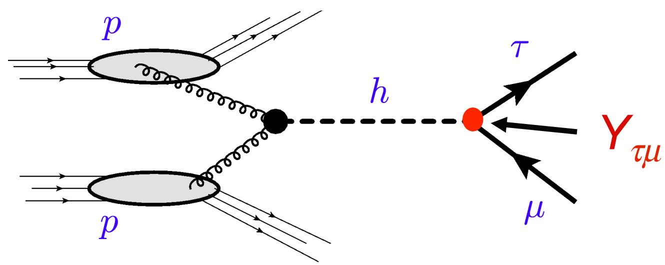
3.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

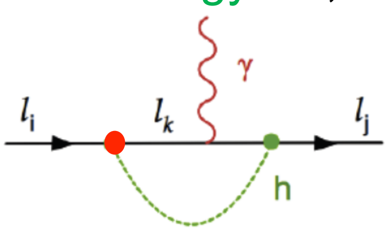
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

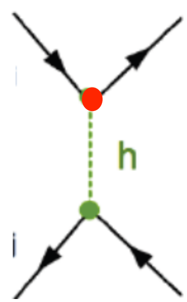
- High energy : LHC

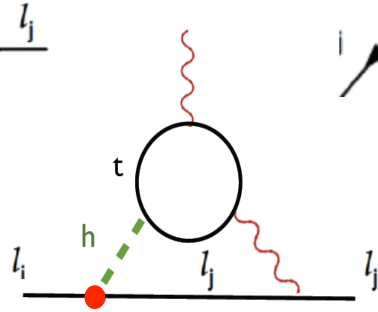


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







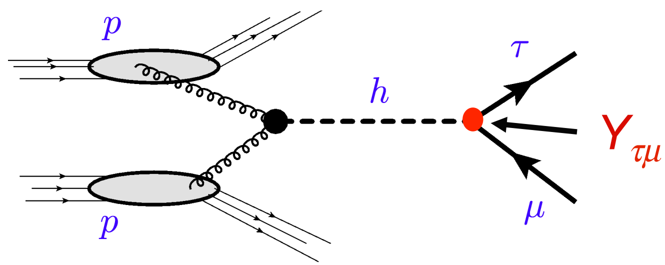
3.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

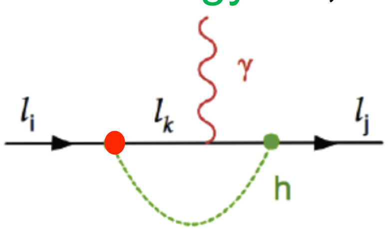
- High energy : LHC

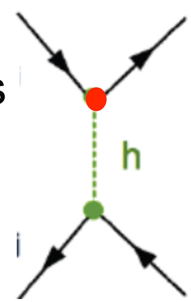


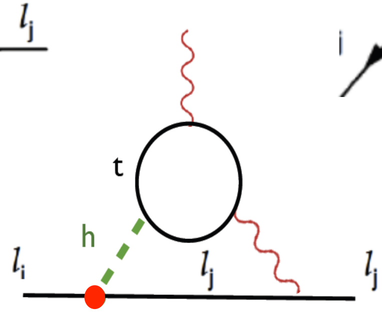
Hadronic part treated with perturbative QCD

Reverse the process

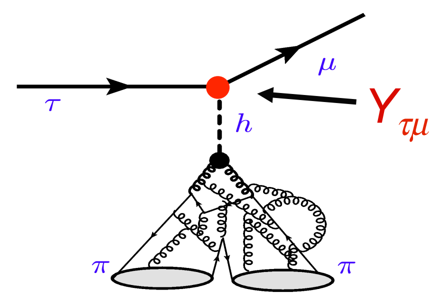
- Low energy : D, S, G operators







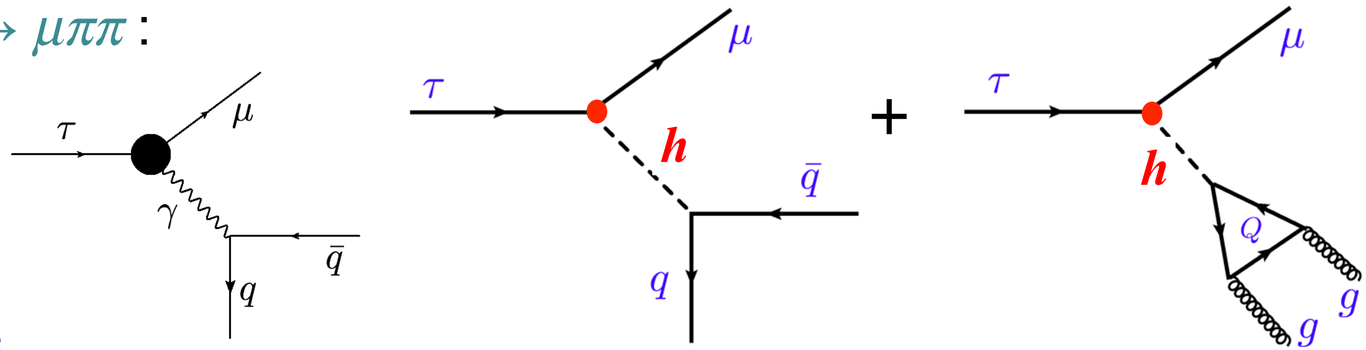
+

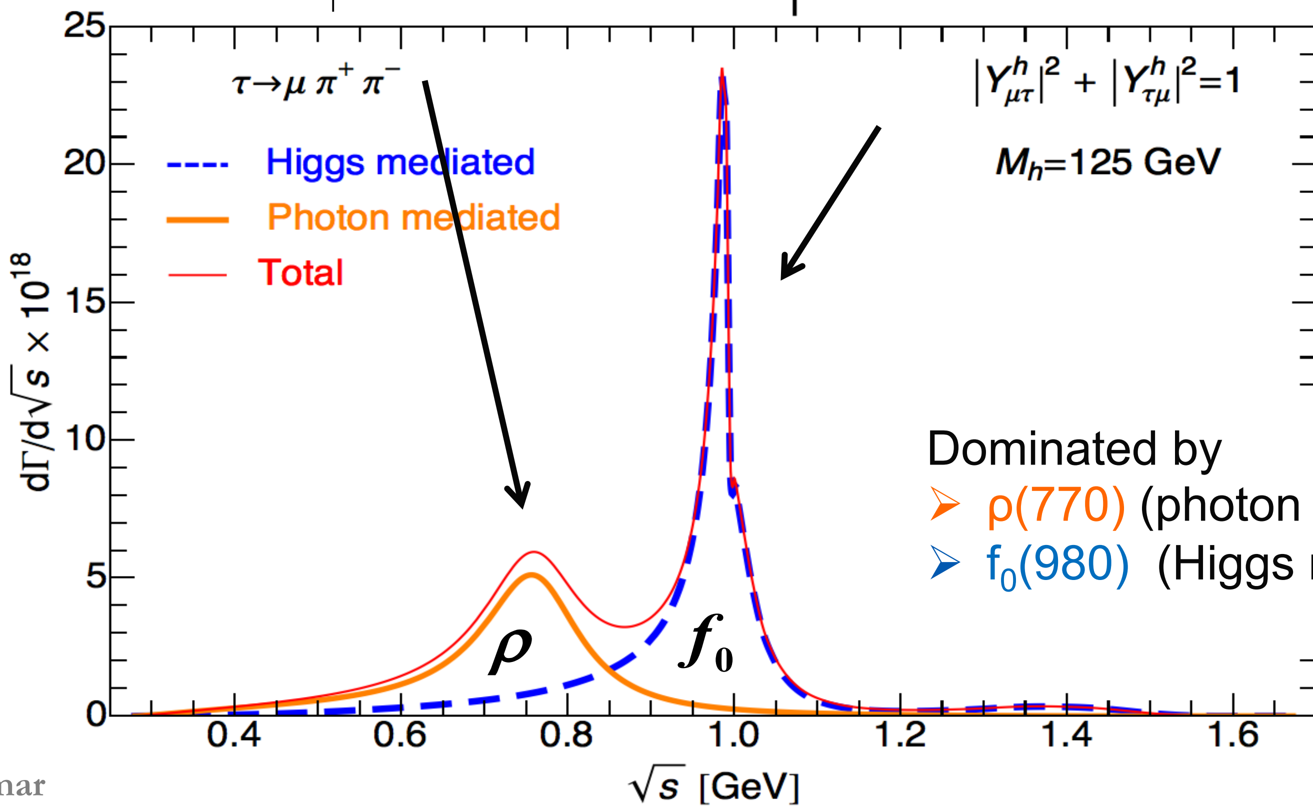


Hadronic part treated with non-perturbative QCD

3.2 Constraints in the τμ sector

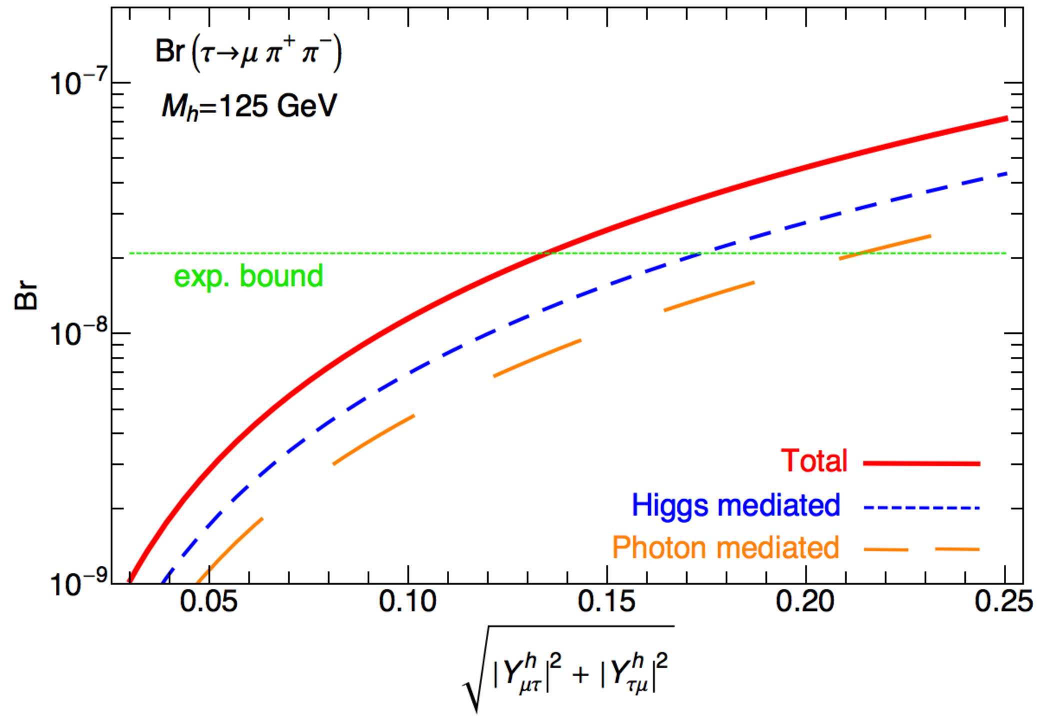
- At low energy
  - $\tau \to \mu\pi\pi$:





Dominated by
- ρ(770) (photon mediated)
- $f_0(980)$ (Higgs mediated)

3.2 Constraints in the τμ sector



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

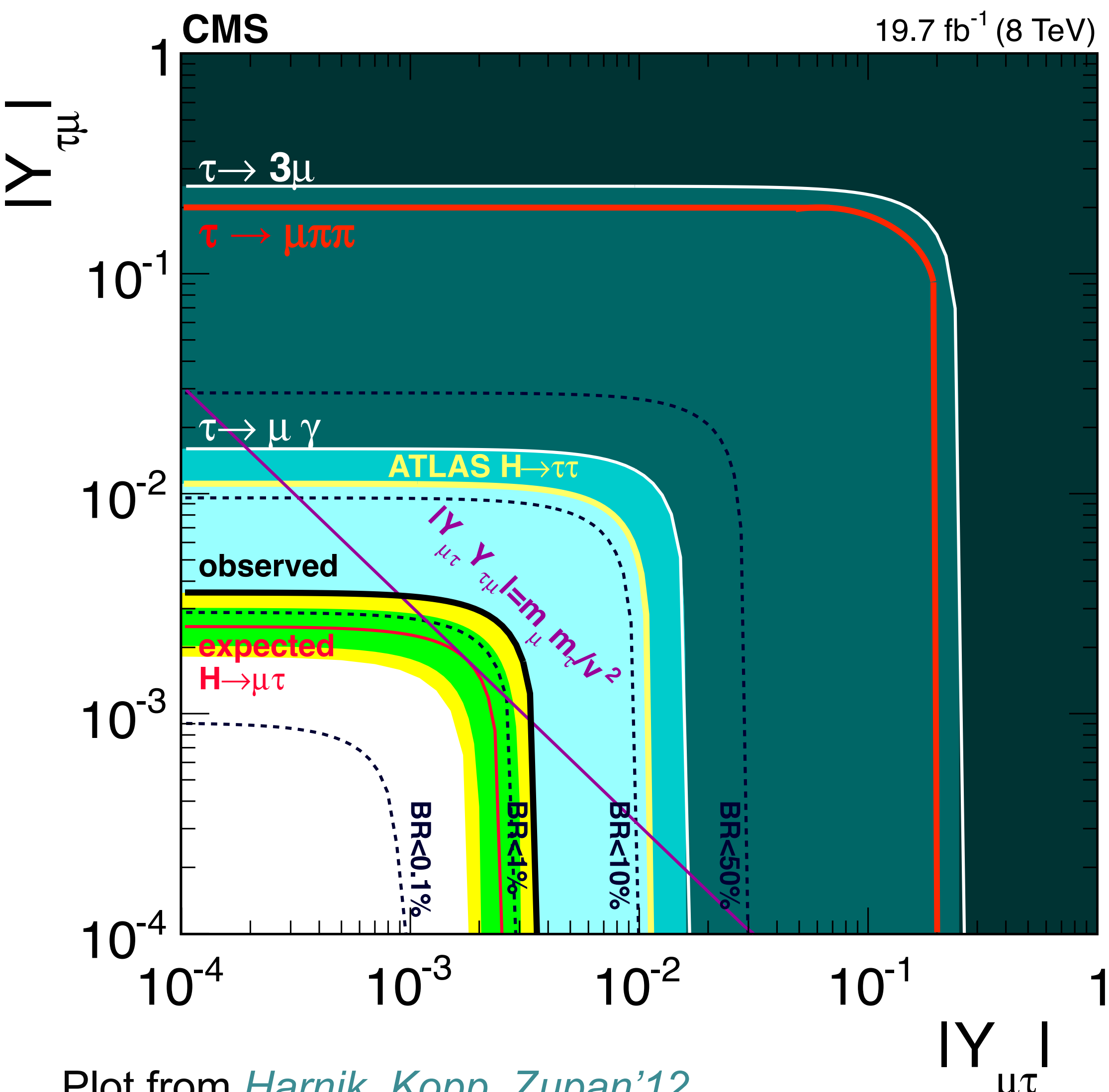
Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

?

Less stringent but more robust handle on LFV Higgs couplings

Belle'08'11'12 except last from *CLEO'97*

3.2 Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

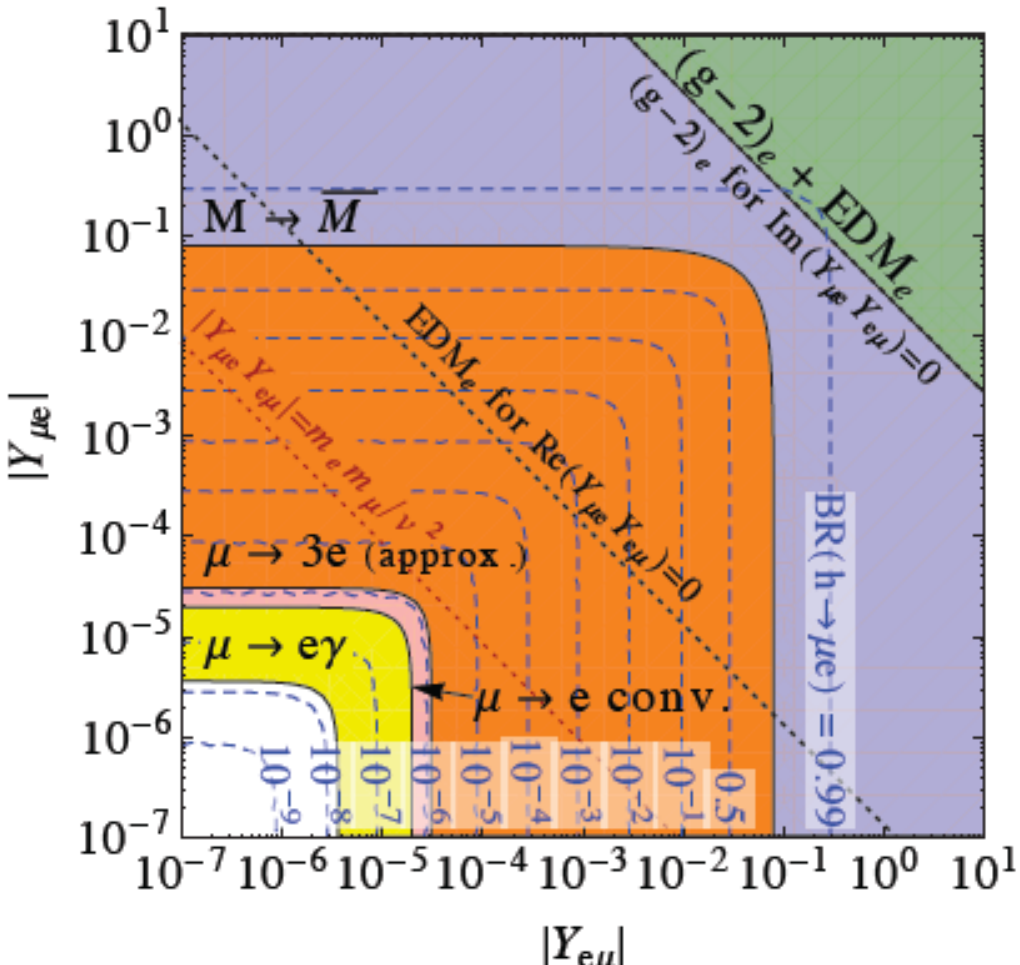
- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ⇒ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ⇒ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

⇒ $\boxed{BR(\tau \to \mu\gamma) < 2.2\times 10^{-9}}$

$\boxed{BR(\tau \to \mu\pi\pi) < 1.5\times 10^{-11}}$

- Constraints from Higgs decay (LHC) vs. low energy LFV and LFC observables

Harnik, Kopp, Zupan'12



- Best constraints coming from low energy: $\mu \to e\gamma$

MEG'13

$$\boxed{BR(\mu \to e\gamma) < 5.7\ 10^{-13}}$$

$$\Rightarrow \boxed{BR(h \to \mu e) < 10^{-7}}$$

3.4 Hint of New Physics in h → τμ?

$$BR\left(h \to \tau\mu\right) = \left(0.84^{+0.39}_{-0.37}\right)\%$$

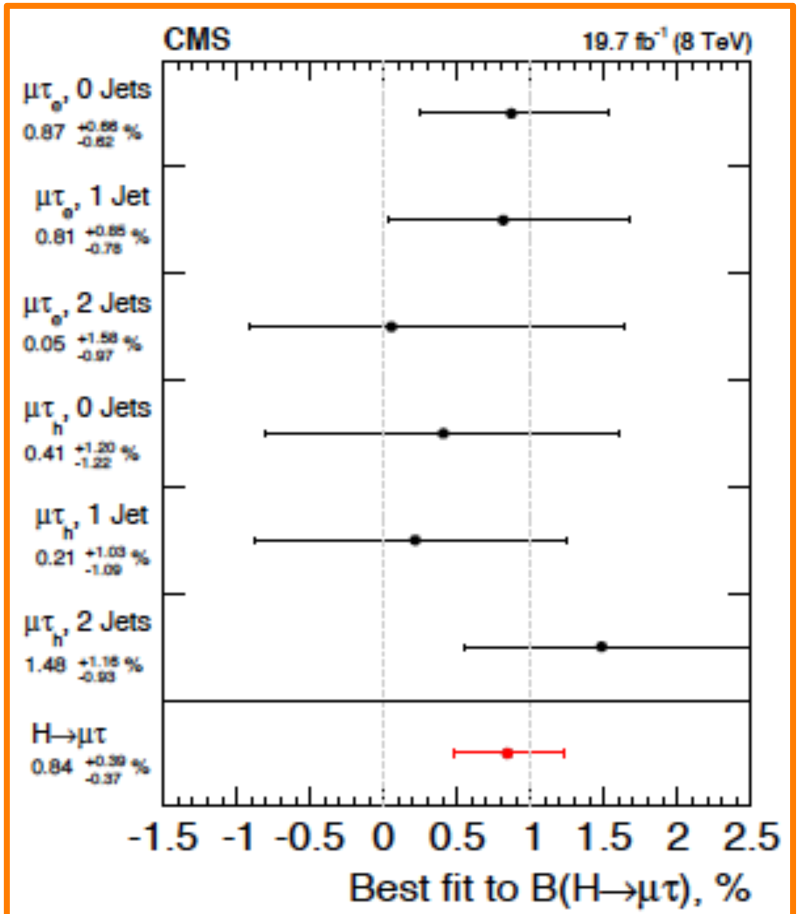
@2.4σ

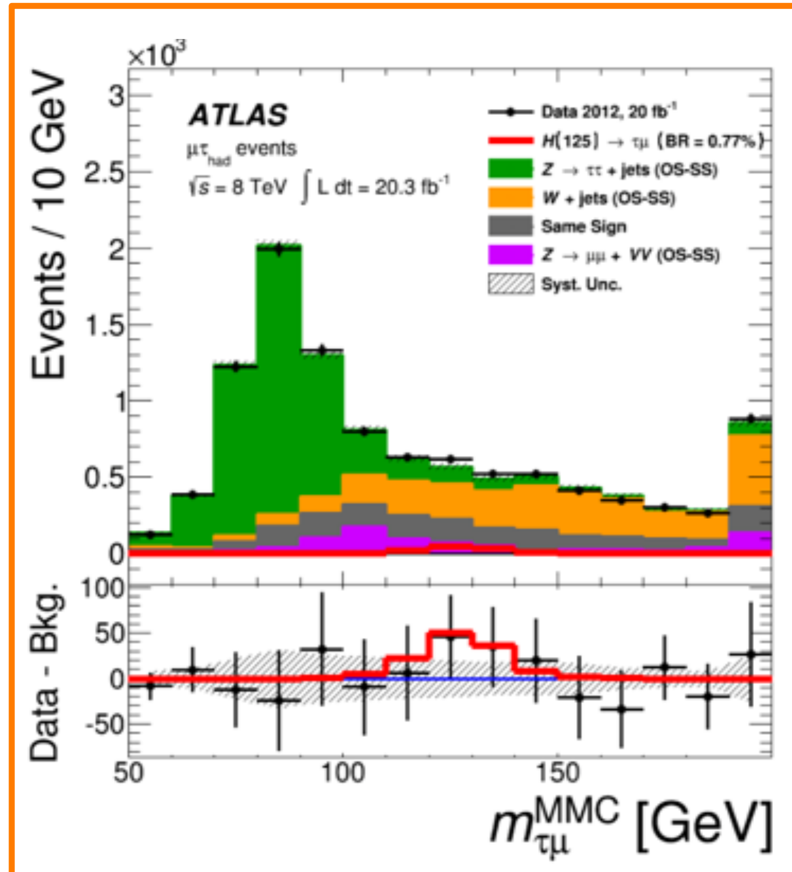
CMS’15

$$BR\left(h \to \tau\mu\right) = \left(0.53 \pm 0.51\right)\%$$

@1σ

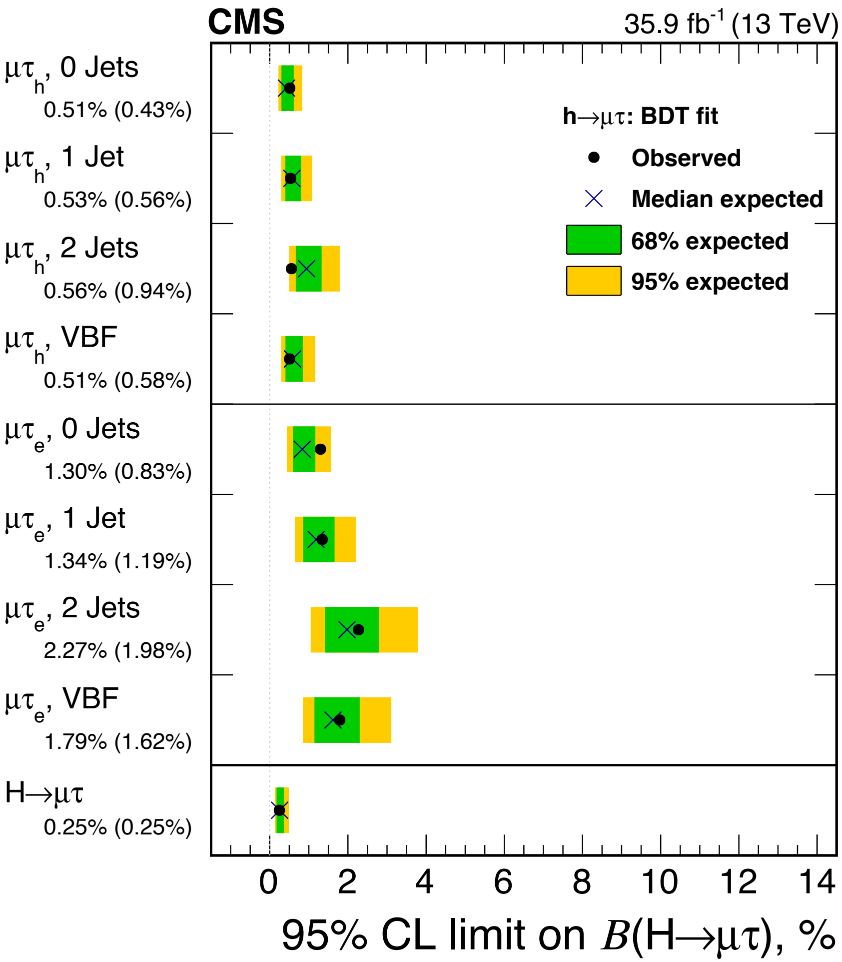
ATLAS’15

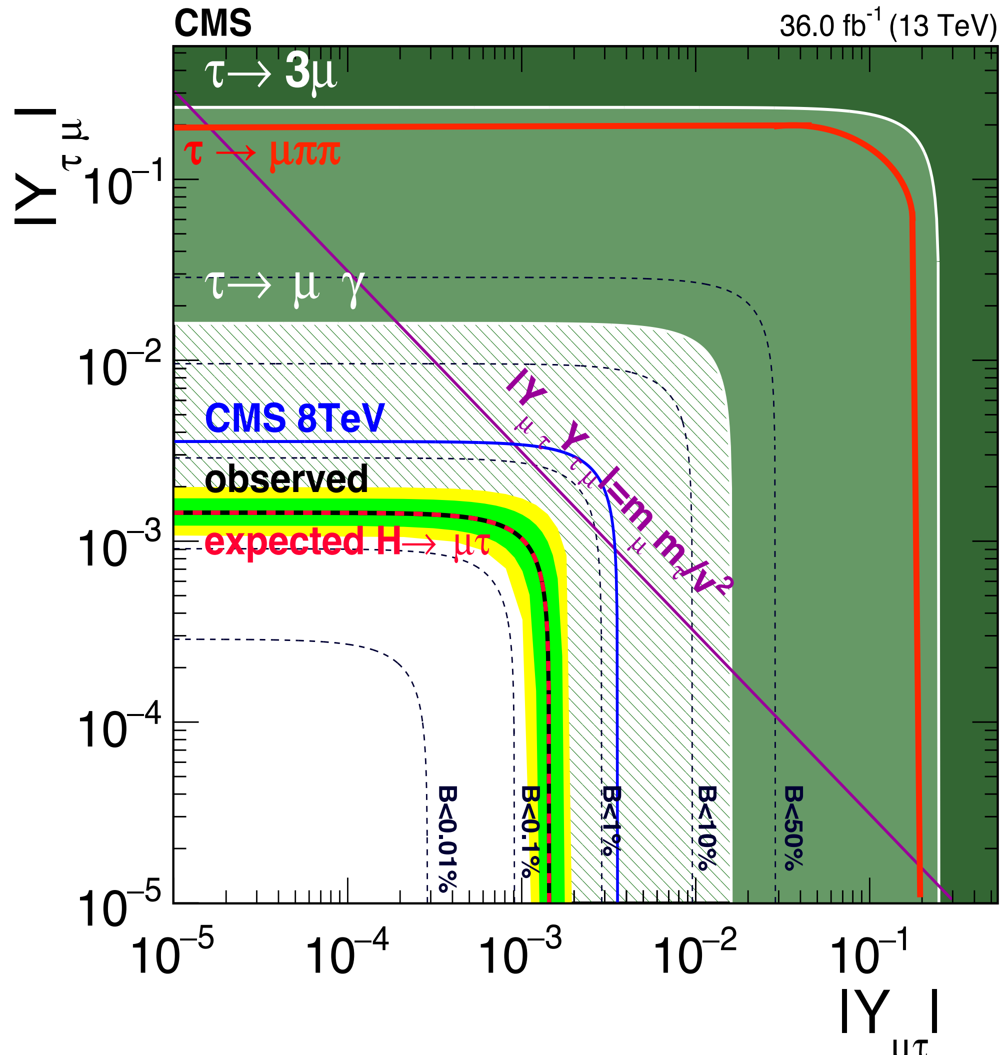




3.4 Hint of New Physics in $h \to \tau\mu$?

CMS'17

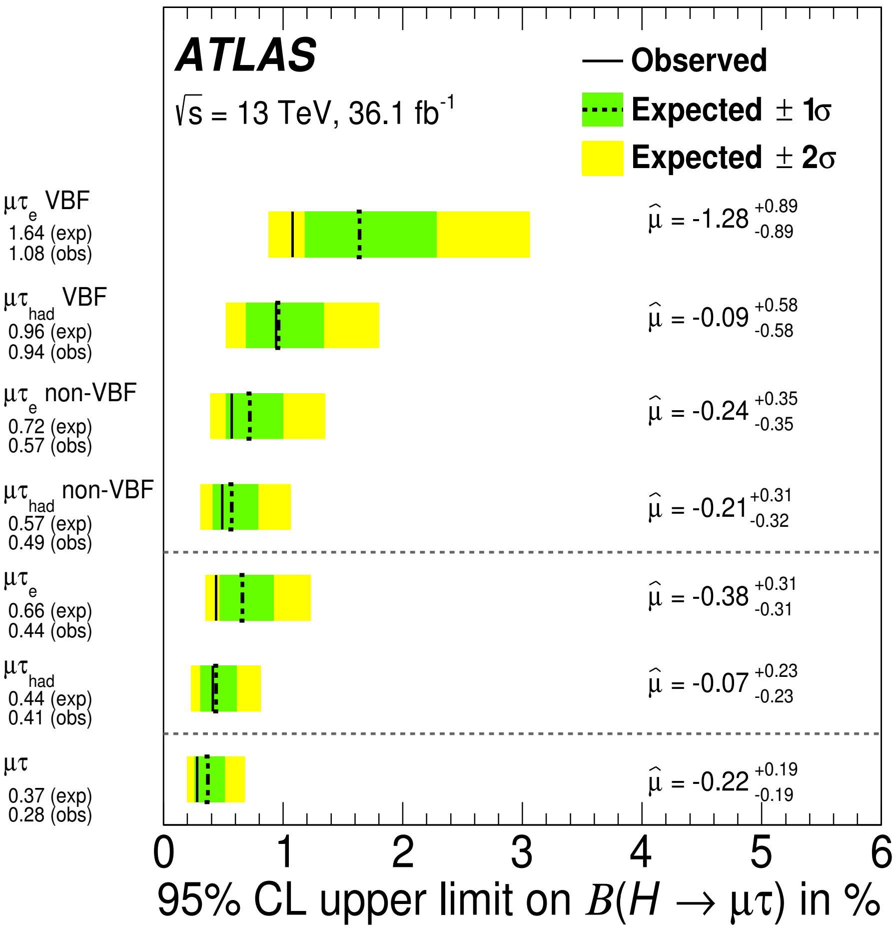


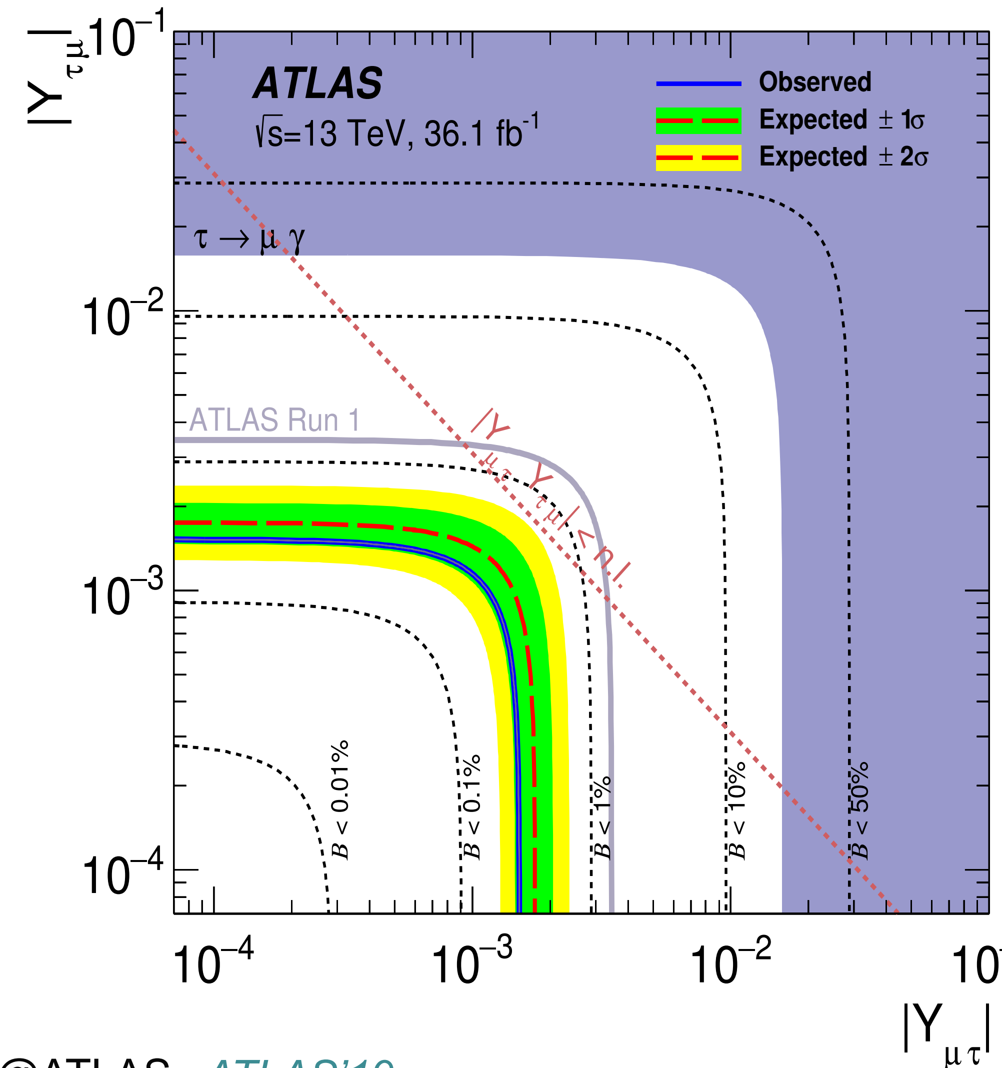


$$BR(h \to \tau\mu) = (0.25 \pm 0.25)\%$$ 13 TeV@CMS CMS'17

3.4 Hint of New Physics in $h \to \tau\mu$?

ATLAS'19





$BR(h \to \tau\mu) \leq 0.28\%$ 13 TeV@ATLAS ATLAS'19

4. Conclusion and Outlook

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

- *Charged leptons* offer an important spectrum of possibilities:
  - ➢ LFV measurements have SM-free signal
  - ➢ Current experiments and mature proposals promise orders of magnitude sensitivity improvements
  - ➢ In addition to leptonic and radiative decays ⇨ hadronic decays important, e.g. $\tau \to \mu(e)\pi\pi$, $\mu N \to eN$
  - ➢ New physics models usually strongly correlate these sectors

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

- *Charged leptons* offer an important spectrum of possibilities:
  - ➢ We show how CLFV decays offer an excellent model discriminating tools giving indications on
    - the *mediator* (operator structure)
    - the *source of flavour breaking* (comparison $\tau\mu$ vs. τe vs. μe)

- Interplay low energy and collider physics: LFV of the Higgs boson

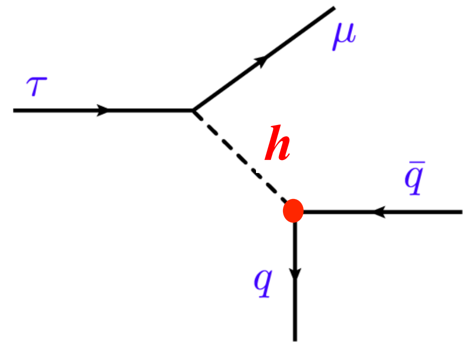
- Several experimental programs:
  MEG, Mu3e, COMET, *Mu2e*, *Belle II*, *BESIII, LHCb, LHC-HL*

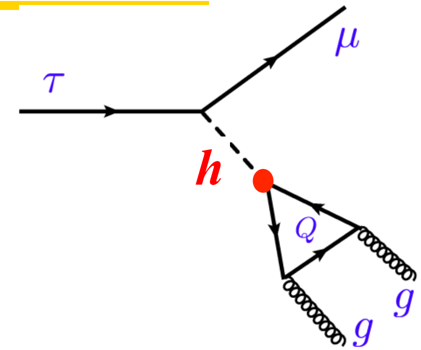
5. Back-up

3.4 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL





- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!

- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

below present experimental limits!

- If discovered ⇒ among other things *upper limit* on $Y_{u,d,s}$!

  ⇒ Interplay between high-energy and low-energy constraints!

2.4 Constraints at Low Energy

Harnik, Kopp, Zupan'12

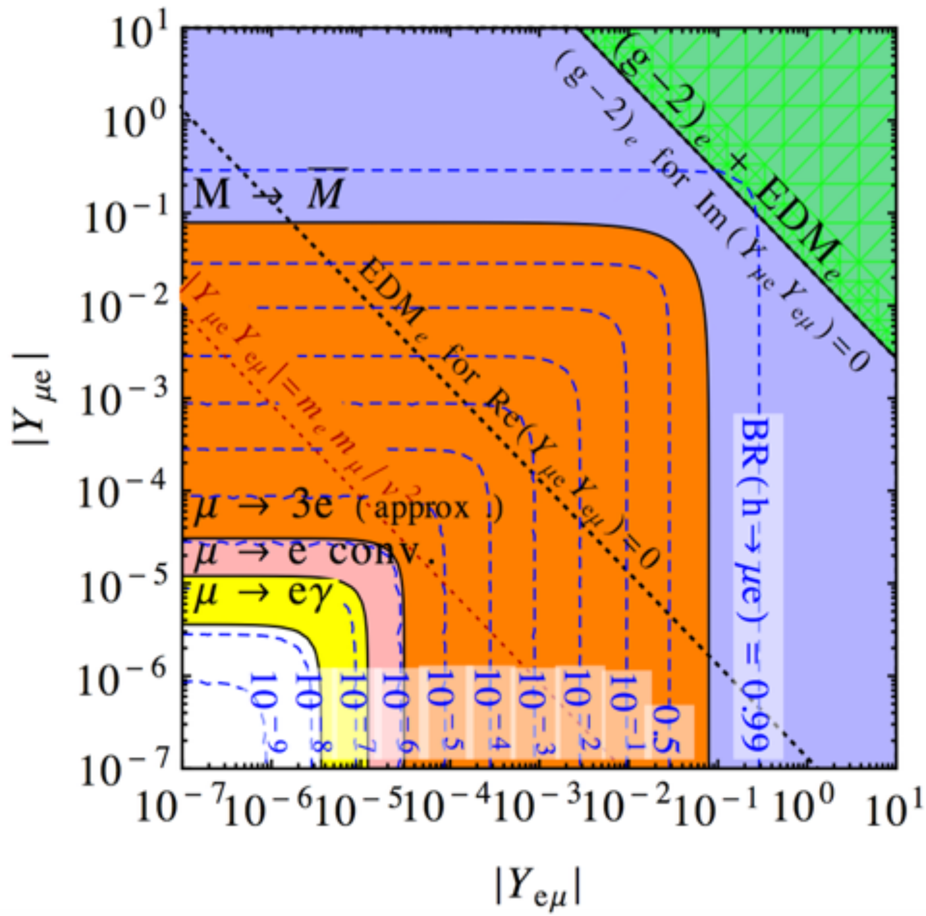
- Results :

Channel	BR 90% CL	$\sqrt{\left\|Y^h_{ij}\right\|^2 + \left\|Y^h_{ji}\right\|^2}$
$\mu \to e\gamma$	$< 4.2 \times 10^{-12}$	$< 3.6 \times 10^{-6}$
$\mu \to 3e$	$< 1 \times 10^{-12}$	$\lesssim 3.1 \times 10^{-5}$
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	< 0.014
$\tau \to 3e$	$< 2.7 \times 10^{-8}$	$\lesssim 0.12$
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	< 0.016
$\tau \to 3\mu$	$< 2.1 \times 10^{-8}$	$\lesssim 0.25$

- Bounds from flavour factories : *MEG*, *Belle*, *Babar* and *LHCb* for $\tau \to 3\mu$

- Strong constraint from $\mu(\tau) \to e(\mu)\gamma$ loop induced process, very sensitive to UV completion ⇨ Model dependent

- For μe: best constraints from LE

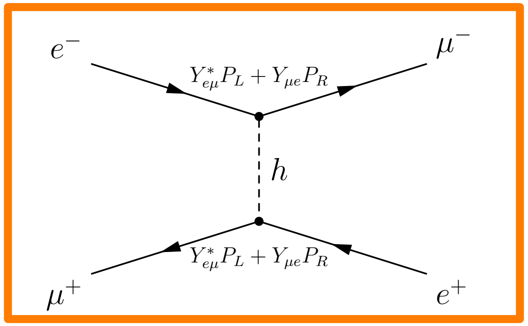


N.B.: Diagonal couplings set to the SM values

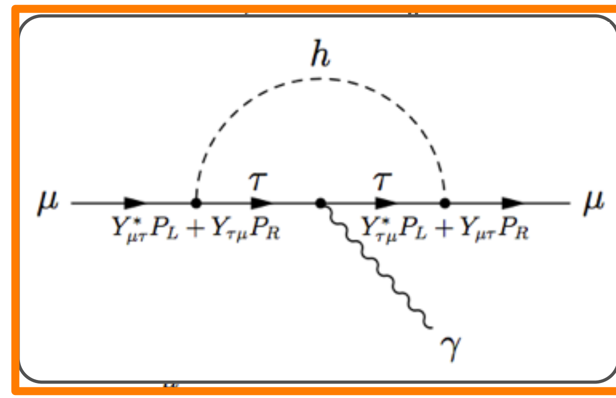
Constraints at Low Energy on LFV

- Muonion-antimuonic oscillations

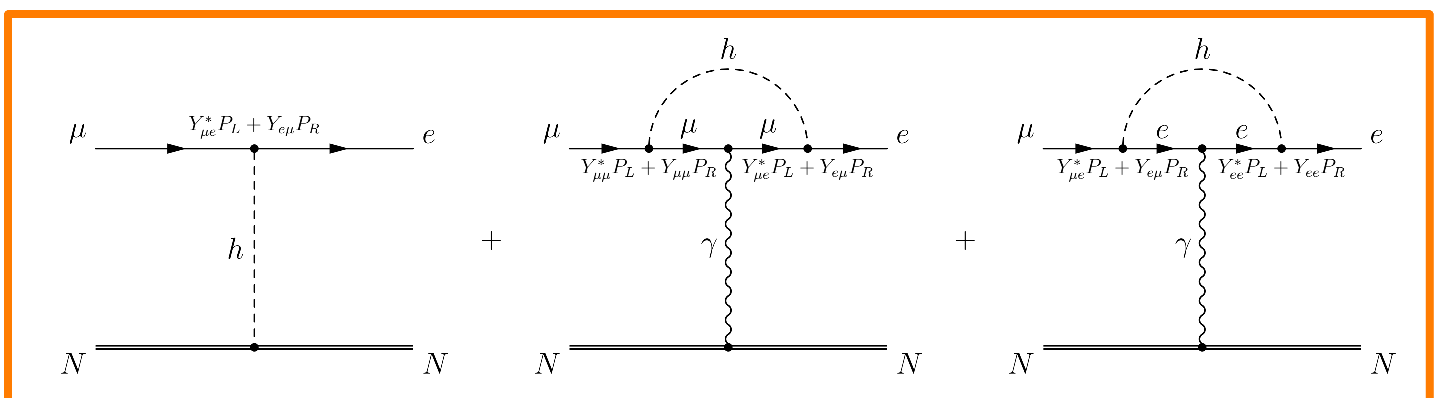
Harnik, Kopp, Zupan'12



- Anomalous magnetic moment of the muon:

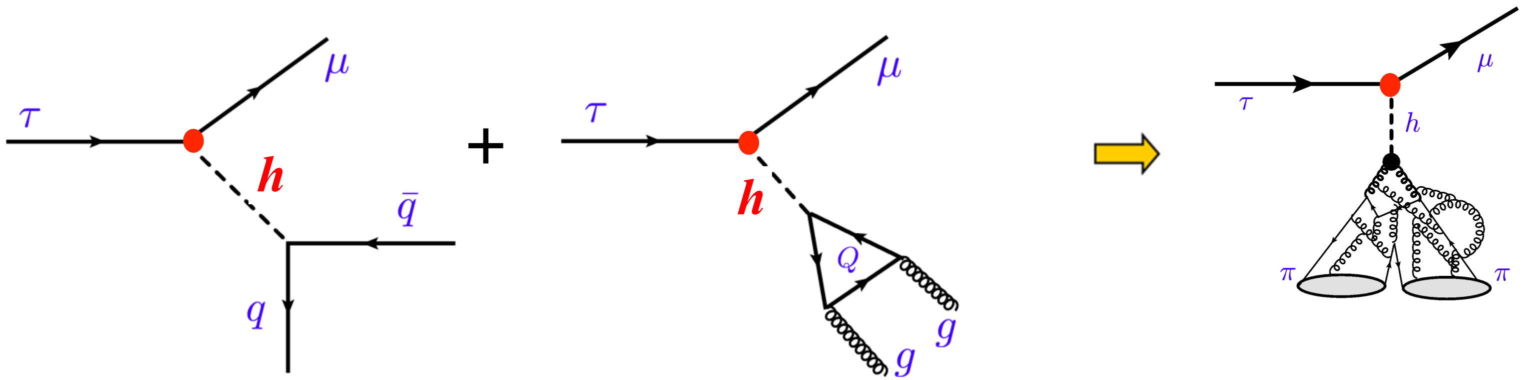


- Mu to e conversion:



2.6 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

  ⇨ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

  Daub, Dreiner, Hanart, Kubis, Meissner'13

  Celis, Cirigliano, E.P.'14

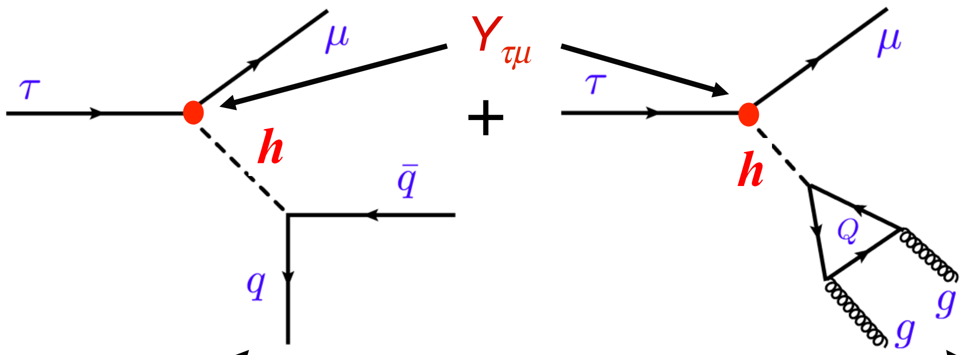
- Dispersion relations: based on unitarity, analyticity and crossing symmetry

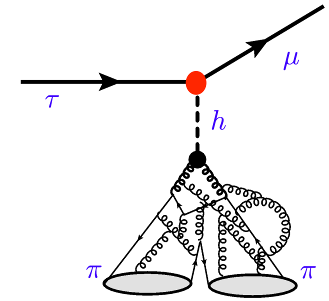
  ⇨ Take *all rescattering* effects into account

  $\pi\pi$ final state interactions important

2.6 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange





$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\left\langle \pi^+\pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s) \qquad \left\langle \pi^+\pi^- \left| \theta^\mu_\mu \right| 0 \right\rangle \equiv \theta_\pi(s)$$

$$\left\langle \pi^+\pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s)$$

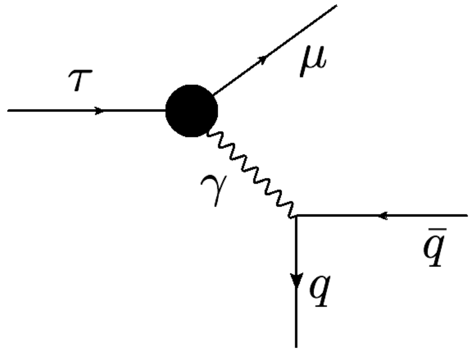
$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2\sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2}$$

$$f\left(y^h_q\right)$$

2.6 Constraints from $\tau \to \mu\pi\pi$

- Contribution from dipole diagrams



$$\boxed{L_{eff} = c_L Q_{L\gamma} + c_R Q_{R\gamma} + h.c.}$$

with the dim-5 EM penguin operators :

$$\boxed{Q_{L\gamma,R\gamma} = \frac{e}{8\pi^2} m_\tau \left(\mu\sigma^{\alpha\beta} P_{L,R}\tau\right) F_{\alpha\beta}}$$

- $$\boxed{\frac{d\Gamma(\tau \to \ell\pi^+\pi^-)}{d\sqrt{s}} = \frac{\alpha^2 |F_V(s)|^2 \left(|c_L|^2 + |c_R|^2\right)}{768\pi^5\, m_\tau} \frac{(s - 4m_\pi^2)^{3/2}(m_\tau^2 - s)^2(s + 2m_\tau^2)}{s^2}}$$

with the vector form factor :

$$\boxed{C_{L,R} = f\left(Y_{\tau\mu}\right)}$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \middle| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \middle| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \to \mu^-\pi^+\pi^-$ absent for $\tau^- \to \mu^-\pi^0\pi^0$

⇨ neutral mode more model independent

3.2 Dispersion relations: Method

- Solution: Use analyticity to reconstruct the form factor in the entire space

⇨ Omnès representation : $F_I(s) = P_I(s)\,\Omega_I(s)$

polynomial Omnès function

- Omnès function : $\boxed{\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right]}$

- Polynomial: $P_I(s)$ not known but determined from a matching to experiment or to ChPT at low energy

3.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

3.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{\left(s' - s - i\varepsilon\right)}\right]}$$

3.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

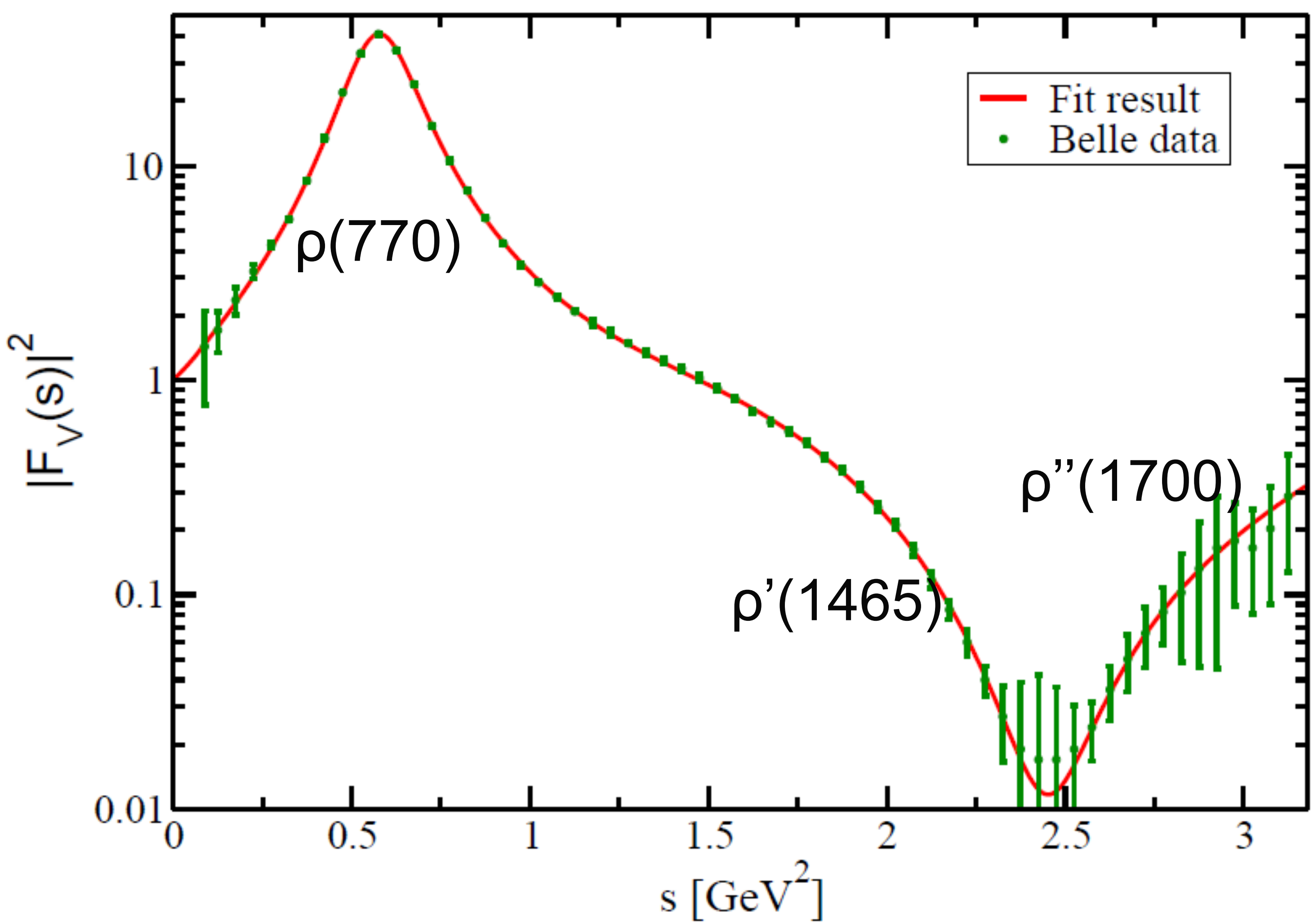
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the Belle data $\tau^- \to \pi^0\pi^-\nu_\tau$

Determination of $F_V(s)$

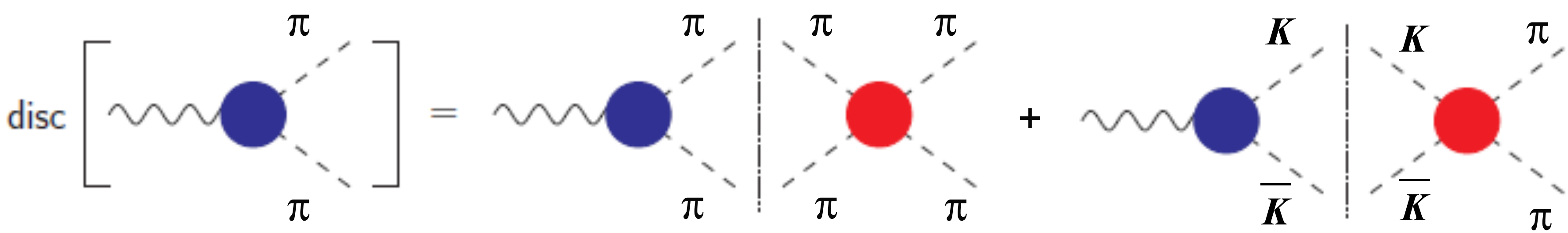


Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- Here no experimental data to determine the polynomial

- $\boxed{4m_\pi^2 < s < \left(m_\tau - m_\mu\right)^2 \sim \left(1.77\ \text{GeV}\right)^2}$ two channels contribute $\pi\pi$ and $K\bar{K}$



Unitarity

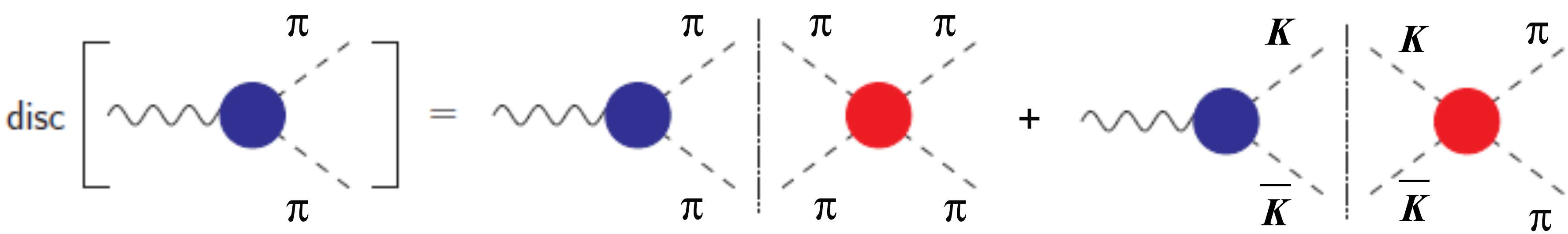
- *Coupled channel analysis* up to √s~1.4 GeV: *Mushkhelishvili-Omnès* approach

  Inputs: I=0, S-wave ππ and KK data

  Donoghue, Gasser, Leutwyler'90
  Moussallam'99
  Daub, Dreiner, Hanart, Kubis, Meissner'13
  Celis, Cirigliano, E.P.'14

  See also *Osset & Oller'98*
  Lahde & Meissner'06

- Unitarity ⇒ the discontinuity of the form factor is known



⇒ $$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

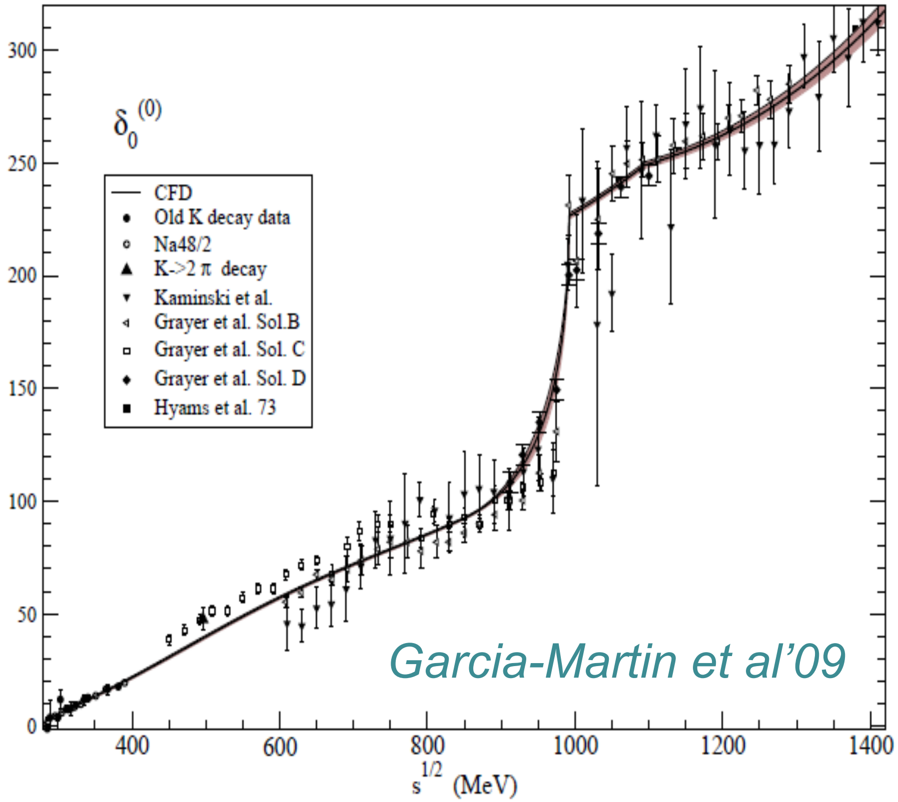
$n = \pi\pi, K\bar{K}$

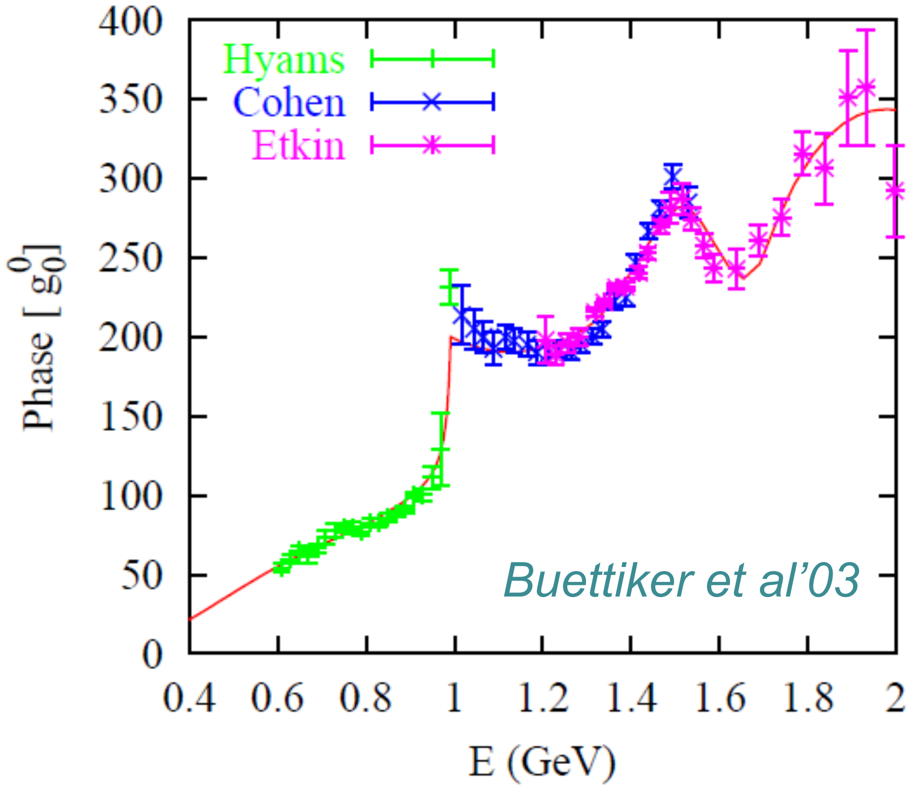
Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

Inputs for the coupled channel analysis

- Inputs : $\pi\pi \to \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

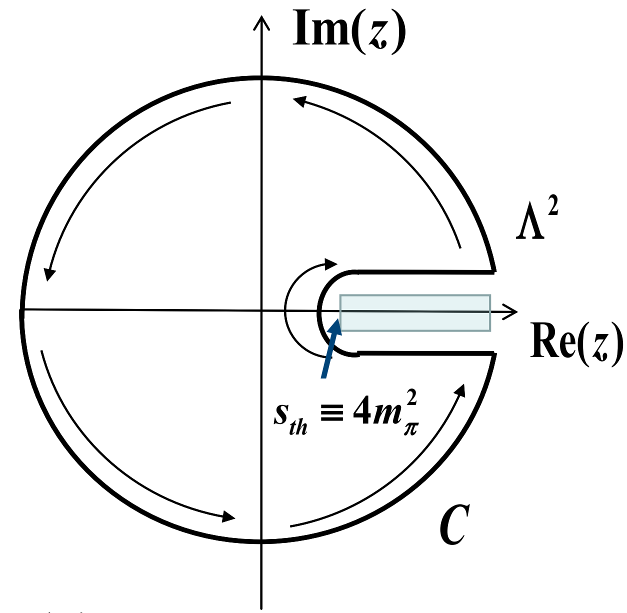
$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

Canonical solution $X(s) = C(s), D(s)$:

- Knowing the discontinuity of X(s) ⇨ write a dispersion relation for it
- Analyticity of the FFs: X(z) is
  - real for z < s_{th}
  - has a branch cut for z > s_{th}
  - analytic for complex z



- Cauchy Theorem and Schwarz reflection principle:

$$X(s) = \frac{1}{\pi} \oint_C dz \frac{X(z)}{z-s}$$

$$= \frac{1}{2i\pi} \int_{s_{th}=4M_\pi^2}^{\Lambda^2} dz \frac{disc\left[F(z)\right]}{z-s-i\varepsilon} + \frac{1}{2i\pi} \int_{|z|=\Lambda^2} dz \frac{F(z)}{z-s}$$

$\Lambda \to \infty$ ⇨

$$\boxed{X(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dz \frac{\mathrm{Im}\left[X(z)\right]}{z-s-i\varepsilon}}$$

X(s) can be reconstructed everywhere from the knowledge of Im X(s)

Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\boxed{\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} T_{mn}^* \sigma_m(s) X_m^{(N)}(s)} \longrightarrow \boxed{X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} X_n^{(N+1)}(s')}{s'-s}}$$

Determination of the polynomial

- Fix the polynomial with requiring $\boxed{F_P(s) \rightarrow 1/s}$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\boxed{\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2}$$

$$\boxed{\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2}$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2)$$

Determination of the polynomial

- Fix the polynomial with requiring $\boxed{F_P(s) \to 1/s}$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\boxed{\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d}\right) M_P^2}$$

$$\boxed{\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s}\right) M_P^2}$$

- At LO in ChPT:

$$\begin{aligned} M_{\pi^+}^2 &= (m_u + m_d)\, B_0 + O(m^2) \\ M_{K^+}^2 &= (m_u + m_s)\, B_0 + O(m^2) \\ M_{K^0}^2 &= (m_d + m_s)\, B_0 + O(m^2) \end{aligned} \quad \Rightarrow \quad \boxed{\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2\right) + \cdots \end{aligned}}$$

Determination of the polynomial

- At LO in ChPT:

$$M^2_{\pi^+} = (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2)$$

- For the scalar FFs:

$$\boxed{\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}}M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2}M_\pi^2\right) + \cdots \end{aligned}}$$

- Problem: large corrections in the case of the kaons!

  ⇨ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\ M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05}\,(M_K^2 - 1/2 M_\pi^2)$$

Daub, Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
The unsubtracted DR is not saturated by the 2 states

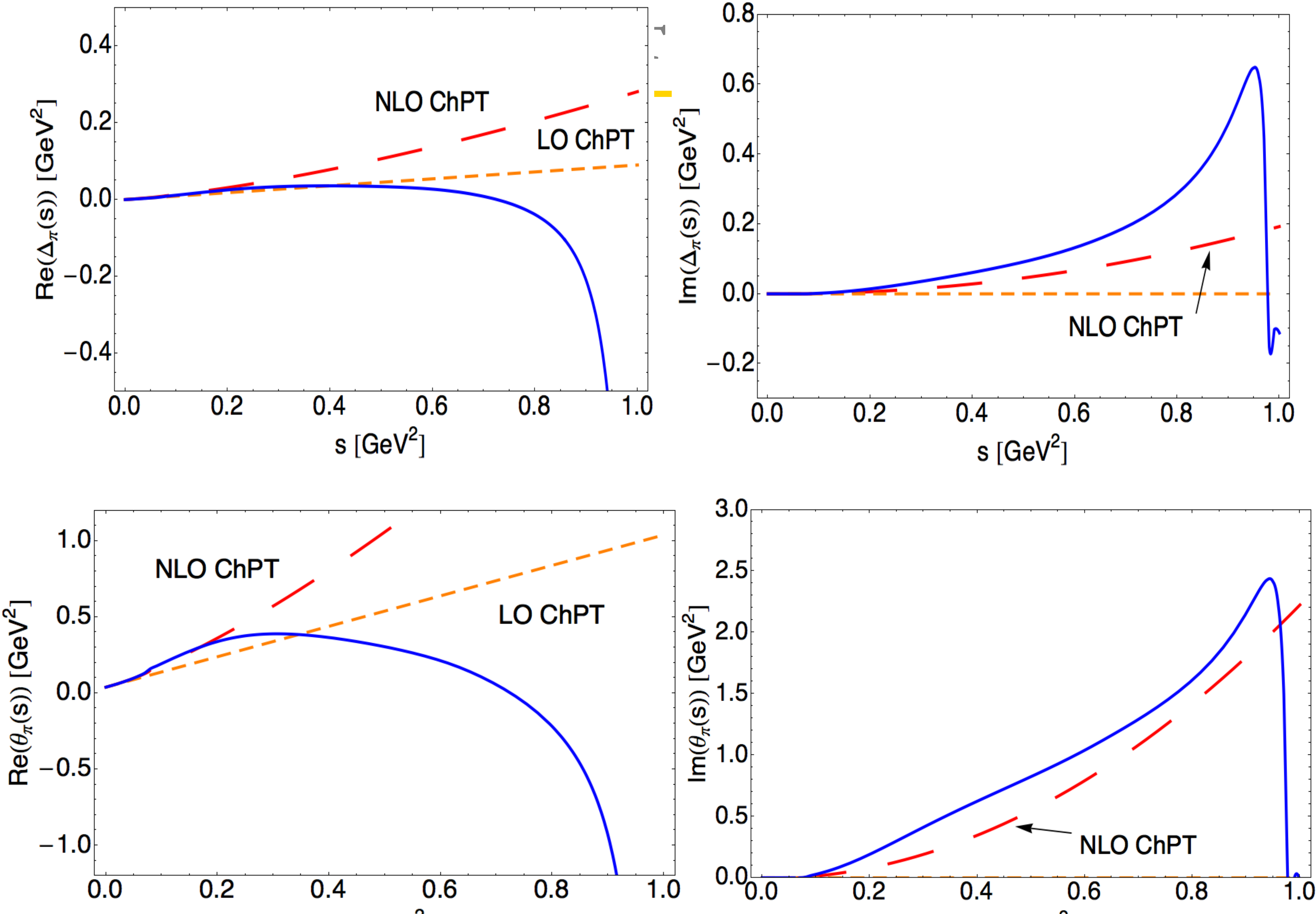
⇨ Relax the constraints and match to ChPT

$$\boxed{\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2\dot{C}_1 - \frac{4M_K^2}{\sqrt{3}}\dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}}M_K^2 + \frac{2}{\sqrt{3}}\left(\dot{\theta}_K - \sqrt{3}M_\pi^2\dot{C}_2 - 2M_K^2\dot{D}_2\right) s \end{aligned}}$$

with $\dot{f} = \left(\frac{df}{ds}\right)_{s=0}$

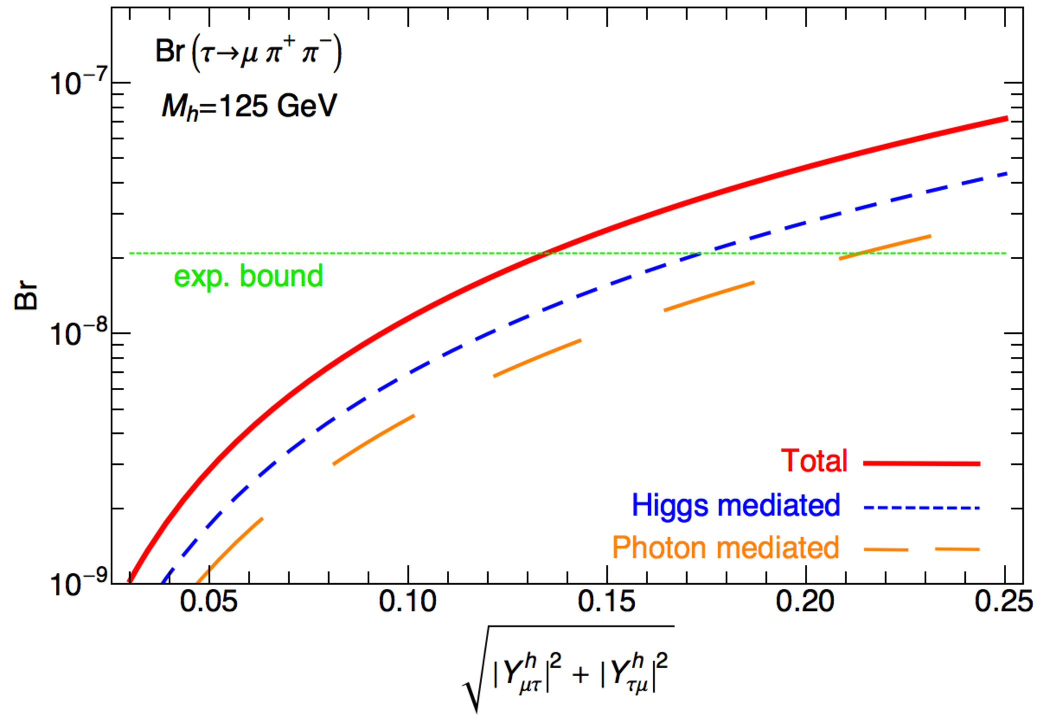
- At LO ChPT: $\dot{\theta}_{\pi,K} = 1$

- Higher orders ⇨ $\boxed{\dot{\theta}_K = 1.15 \pm 0.1}$



NLO ChPT
LO ChPT
Re(Δπ(s)) [GeV²]
Im(Δπ(s)) [GeV²]
Re(θπ(s)) [GeV²]
Im(θπ(s)) [GeV²]
s [GeV²]

3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

Process	(BR $\times 10^8$) 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	< 1.4×10^3 [87]	< 6.3	Scalar, Gluon

Less stringent but more robust handle on LFV Higgs couplings

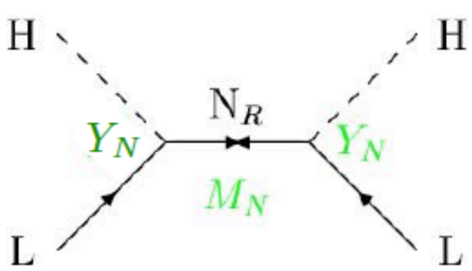
?

Belle'08'11'12 except last from *CLEO'97*

CLFV in see-saw models

Type I:
Fermion singlet

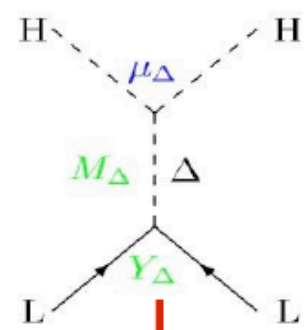
N_{R_i}



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Type II:
Scalar triplet

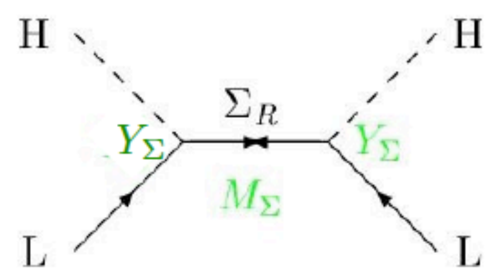
$\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Type III:
Fermion triplet

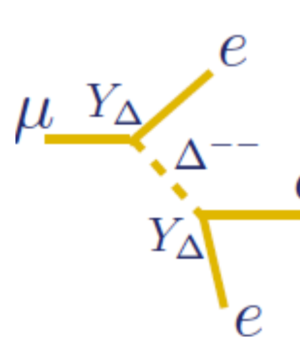
$\Sigma_i \equiv (\Sigma_i^{+}, \Sigma_i^{0}, \Sigma_i^{-})$

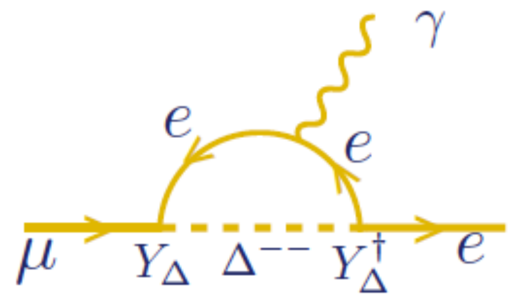


$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

- Observable CLFV if see-saw scale low (with protection of LN)
- Each model leads to specific CLFV pattern

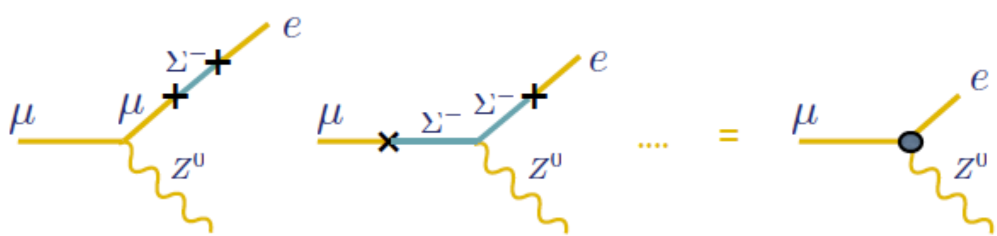
- CLFV in Type II seesaw: tree-level 4L operator (D,V at loop) → 4-lepton processes most sensitive





- CLFV in Type III seesaw: tree-level LFV couplings of Z ⇒

  μ →3e and μ →e conversion at tree level, μ →eγ at loop



Abada-Biggio-Bonnet-Gavela-Hambye '07, '08

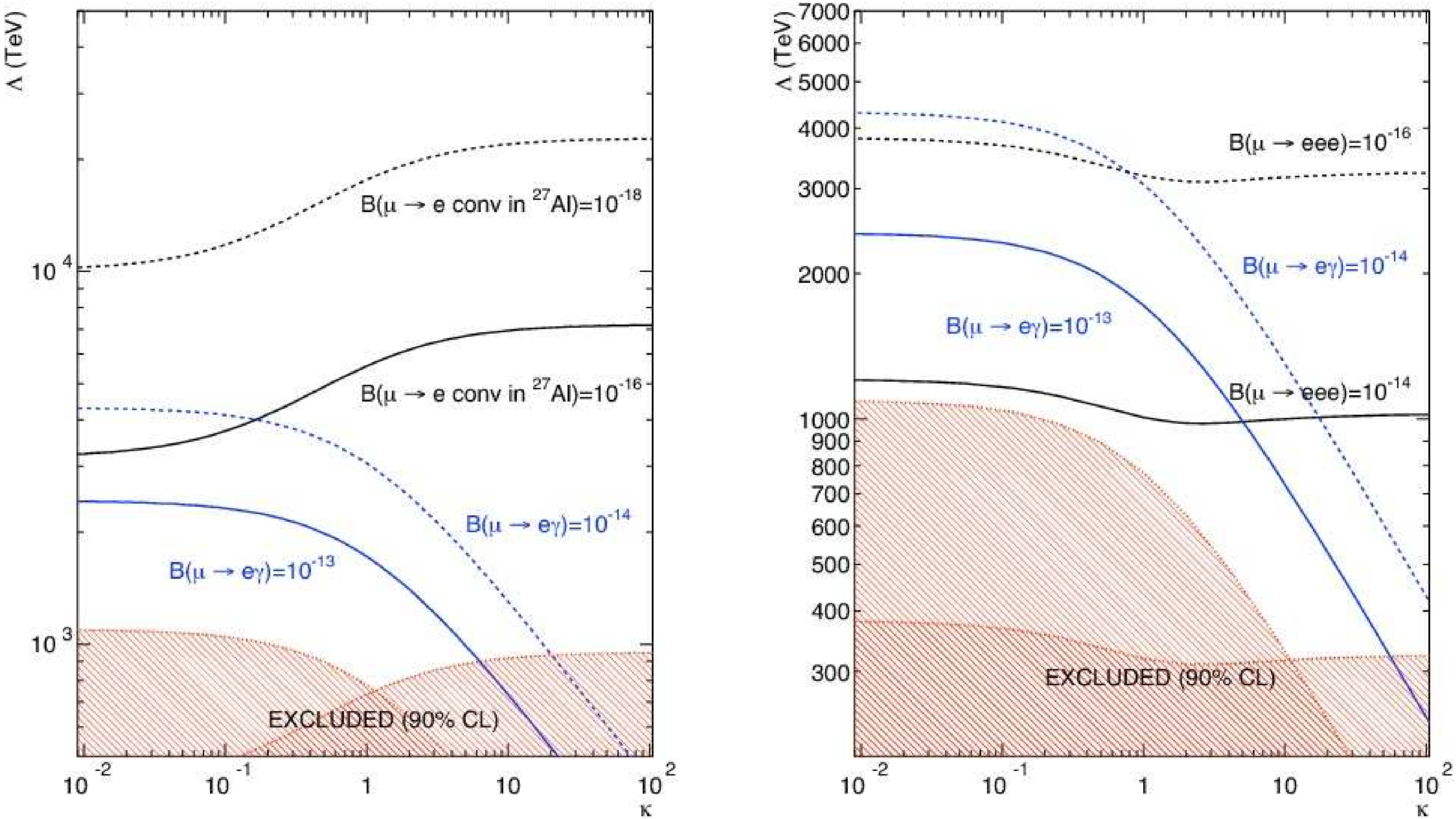
- Ratios of 2 processes with same flavor transition are fixed

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 3.1 \cdot 10^{-4} \cdot R^{\mu \to e}_{Ti} \\
Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) \\
Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee)
\end{aligned}
$$

2.4 Model discriminating power of muon processes

- Dependence: NP scale Λ versus ratio of two operators $\kappa = \frac{C_1}{C_2}$

DeGouvea & Vogel'13



2.5 Model discriminating power of Tau processes

- Two handles:

Celis, Cirigliano, E.P.'14

  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

- Benchmarks:
  - Dipole model: $C_D \neq 0$, $C_{else} = 0$
  - Scalar model: $C_S \neq 0$, $C_{else} = 0$
  - Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
  - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

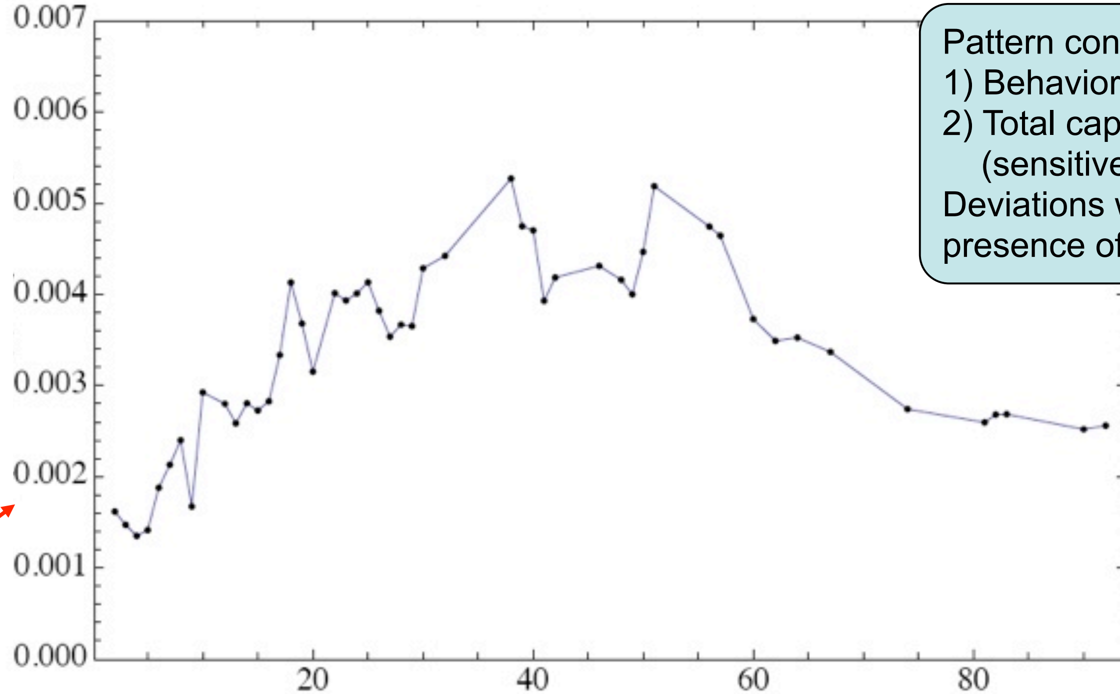
$\mu \to e$ vs $\mu \to e\gamma$

- Assume dipole dominance:

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

Kitano-Koike-Okada '02
VC-Kitano-Okada-Tuzon '09

$$\boxed{\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}}$$



Z

O(α/π)

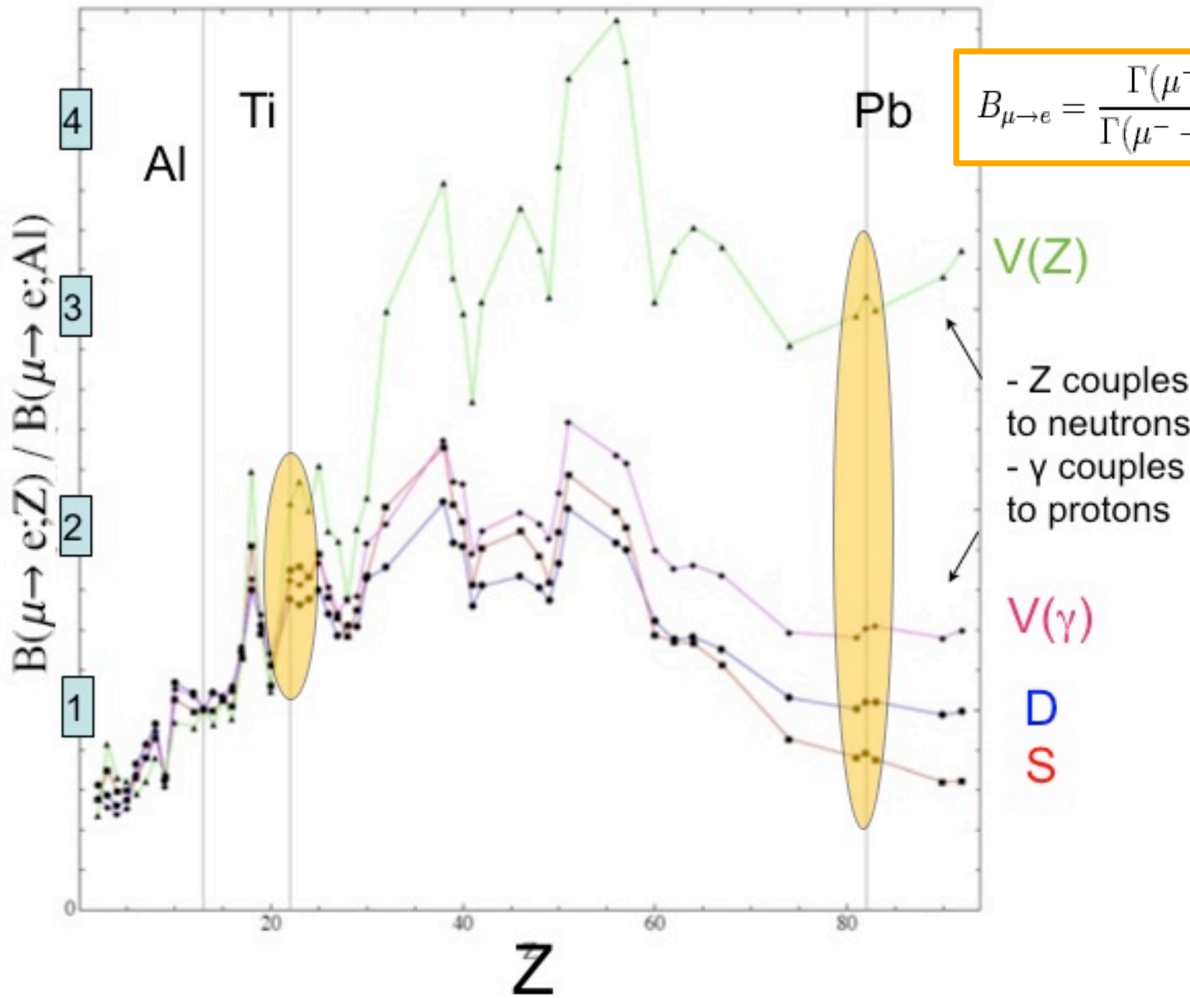
Pattern controlled by:
1) Behavior of overlap integrals
2) Total capture rate
(sensitive to nuclear structure)
Deviations would indicate presence of scalar / vector terms

2.6.1 BR for $\mu \rightarrow$ e conversion

- For *$\mu \rightarrow$e conversion,* target dependence of the amplitude is different for V,D or S models

Cirigliano, Kitano, Okada, Tuzon'09

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$



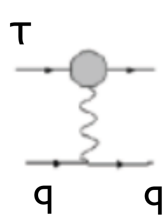
2.5 Model discriminating power of Tau processes

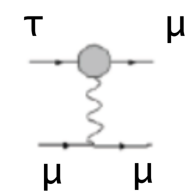
Celis, Cirigliano, E.P.'14

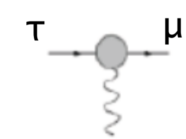
- Two handles:
  - ➢ Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$

Benchmark







- ρ (770) resonance ($J^{PC}=1^{--}$): cut in the $\pi^+\pi^-$ invariant mass:

$$587 \text{ MeV} \le \sqrt{s} \le 962 \text{ MeV}$$

- f_0 (980) resonance ($J^{PC}=0^{++}$): cut in the $\pi^+\pi^-$ invariant mass:

$$906 \text{ MeV} \le \sqrt{s} \le 1065 \text{ MeV}$$

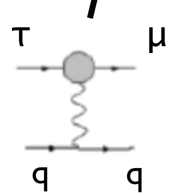
2.5 Model discriminating power of Tau processes

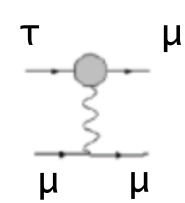
Celis, Cirigliano, E.P.'14

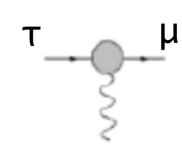
- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

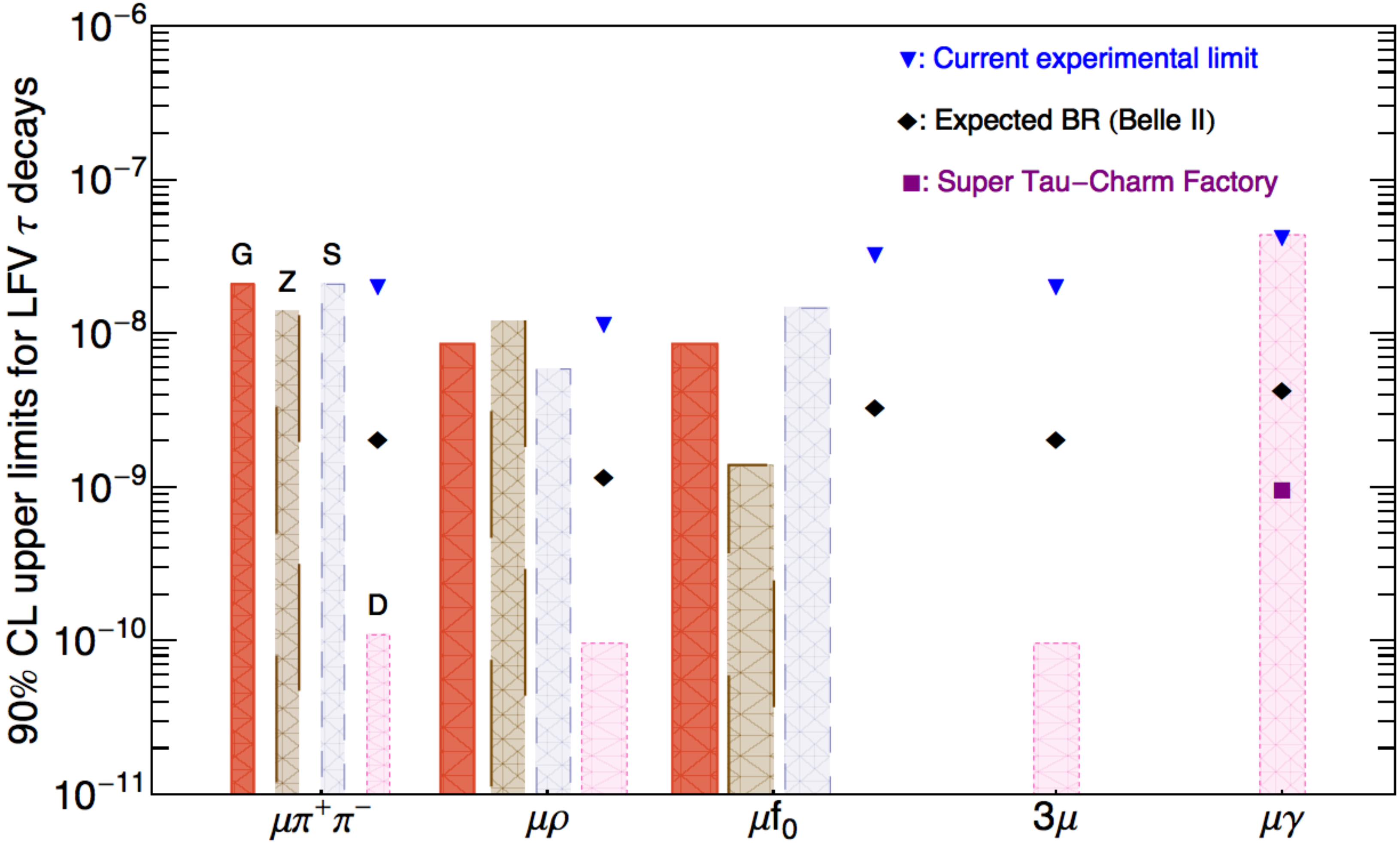
Benchmark







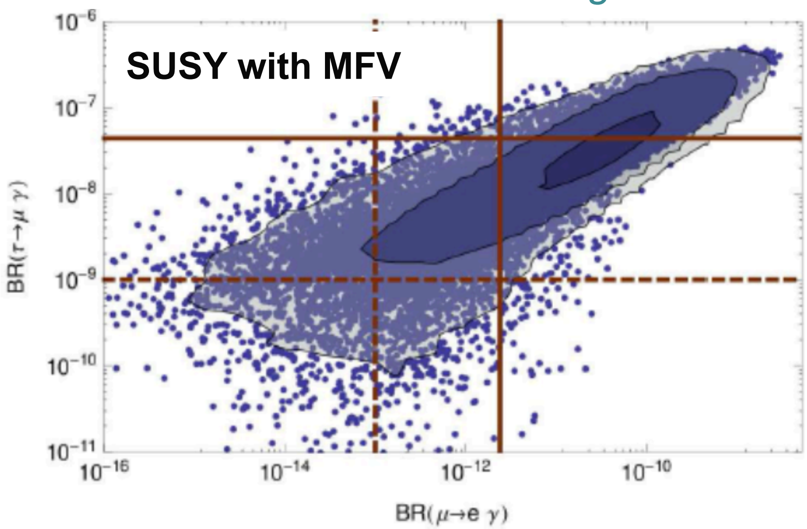
4.2 Prospects:



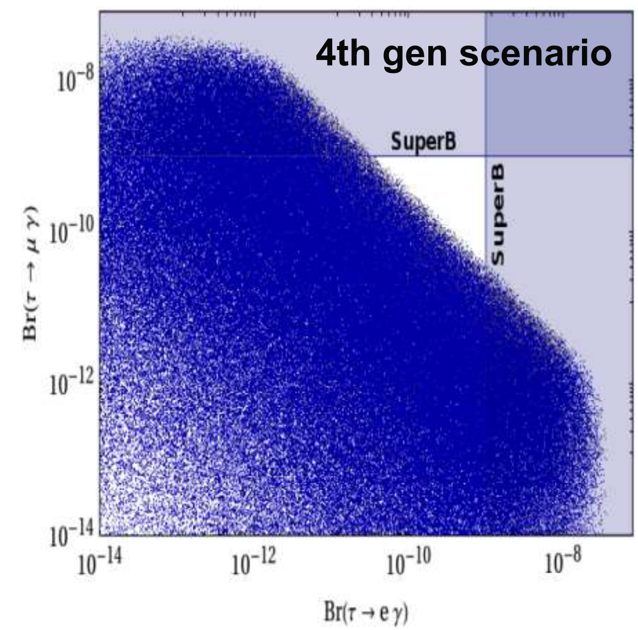
2.5 Model discriminating power of Tau processes

- Depending on the UV model different correlations between the BRs

Blankerburg et al.'12



Buras et al.'10

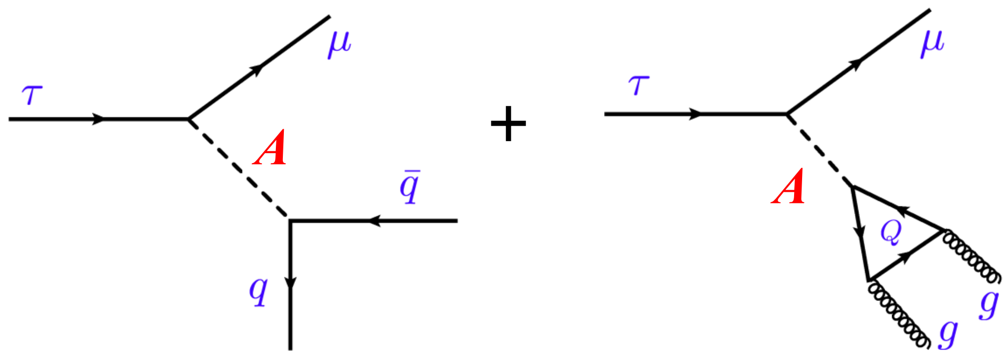


➡ Interesting to study to determine the underlying dynamics of NP

4. CP-odd Higgs with LFV

4.1 Constraints from $\tau \to 1P$

- Tree level Higgs exchange



- L_Y ⇨ $$\boxed{\mathcal{L}^A_{eff} \simeq -\frac{A}{v}\left(\sum_{q=u,d,s} y_q^A\, m_q\, \bar{q} i\gamma_5\, q - \sum_{q=c,b,t} y_q^A\, \frac{\alpha_s}{8\pi}\, G^a_{\mu\nu}\, \widetilde{G}^a_{\mu\nu}\right)}$$

$$\widetilde{G}^a_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\alpha\beta}\, G^a_{\alpha\beta}$$

- Mediate only one pseudoscalar meson ⇨ very characteristic!

4.1 Constraints from $\tau \to 1P$

- Tree level Higgs exchange
  - η, η'

$$\Gamma\left(\tau \to \ell\eta^{(\prime)}\right) = \frac{\bar{\beta}\left(m_\tau^2 - m_\eta^2\right)\left(|Y^A_{\mu\tau}|^2 + |Y^A_{\tau\mu}|^2\right)}{256\,\pi\, M_A^4\, v^2\, m_\tau}\left[(y_u^A + y_d^A)h^q_{\eta'} + \sqrt{2}y_s^A h^s_{\eta'} - \sqrt{2}a_{\eta'}\sum_{q=c,b,t} y_q^A\right]^2$$

with the decay constants :

$$\langle\eta^{(\prime)}(p)|\bar{q}\,\gamma_5\, q|0\rangle = -\frac{i}{2\sqrt{2}m_q}\, h^q_{\eta^{(\prime)}} \qquad \langle\eta^{(\prime)}(p)|\bar{s}\,\gamma_5\, s|0\rangle = -\frac{i}{2m_s}\, h^s_{\eta^{(\prime)}}$$

$$\langle\eta^{(\prime)}(p)|\frac{\alpha_s}{4\pi}\, G_a^{\mu\nu}\widetilde{G}^a_{\mu\nu}|0\rangle = a_{\eta^{(\prime)}}$$

- π : $$\Gamma(\tau \to \ell\pi^0) = \frac{f_\pi^2\, m_\pi^4\, m_\tau}{256\pi M_A^4\, v^2}\left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2\right)\left(y_u^A - y_d^A\right)^2$$

4.2 Results

- $\tau \longrightarrow \mu P$

Process	BR 90% CL	$M_A = 200$ GeV	$M_A = 500$ GeV	$M_A = 700$ GeV
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	$Z < 0.018$	$Z < 0.040$	$Z < 0.055$
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	$Z < 0.28$	$Z < 0.60$	$Z < 0.85$
$^{(*)}$ $\tau \to \mu\pi$	$< 11 \times 10^{-8}$	$Z < 41$	$Z < 257$	$Z < 503$
$^{(*)}$ $\tau \to \mu\eta$	$< 6.5 \times 10^{-8}$	$Z < 0.52$	$Z < 3.3$	$Z < 6.4$
$^{(*)}$ $\tau \to \mu\eta'$	$< 13 \times 10^{-8}$	$Z < 1.1$	$Z < 7.2$	$Z < 14.1$
$\tau \to \mu\pi^+\pi^-$	$< 2.1 \times 10^{-8}$	$Z < 0.25$	$Z < 0.54$	$Z < 0.75$
$\tau \to \mu\rho$	$< 1.2 \times 10^{-8}$	$Z < 0.20$	$Z < 0.44$	$Z < 0.62$

BaBar'06'10 , Belle'10'11'13

$$Z = \sqrt{\left|Y^A_{\mu\tau}\right|^2 + \left|Y^A_{\tau\mu}\right|^2}$$

(*) : No contribution from effective dipole operator or CP-even Higgs

N.B.: Diagonal couplings $\left|y^A_f\right| = 1$

4.2 Results

- $\tau \longrightarrow eP$

Process	BR 90% CL	$M_A = 200$ GeV	$M_A = 500$ GeV	$M_A = 700$ GeV
$\tau \to e\gamma$	$< 3.3 \times 10^8$	$Z < 0.016$	$Z < 0.034$	$Z < 0.05$
$\tau \to eee$	$< 2.7 \times 10^8$	$Z < 0.14$	$Z < 0.30$	$Z < 0.42$
$^{(*)}$ $\tau \to e\pi$	$< 8 \times 10^8$	$Z < 35$	$Z < 219$	$Z < 430$
$^{(*)}$ $\tau \to e\eta$	$< 9.2 \times 10^8$	$Z < 0.6$	$Z < 3.9$	$Z < 7.6$
$^{(*)}$ $\tau \to e\eta'$	$< 16 \times 10^8$	$Z < 1.3$	$Z < 8$	$Z < 15.6$
$\tau \to e\pi^+\pi^-$	$< 2.3 \times 10^8$	$Z < 0.26$	$Z < 0.56$	$Z < 0.80$
$\tau \to e\rho$	$< 1.8 \times 10^8$	$Z < 0.25$	$Z < 0.54$	$Z < 0.76$

BaBar'06'10 , Belle'10'11'13

$$\boldsymbol{Z = \sqrt{\left|Y^A_{e\tau}\right|^2 + \left|Y^A_{\tau e}\right|^2}}$$

(*) : No contribution from effective dipole operator or CP-even Higgs

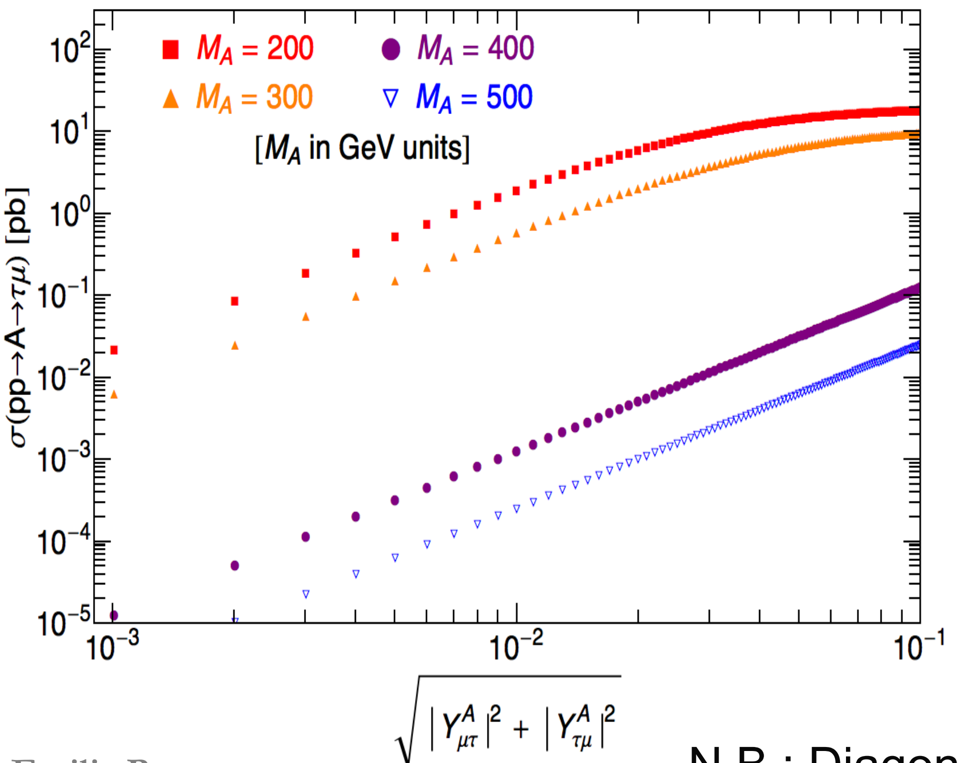
N.B.: Diagonal couplings $\left|y^A_f\right| = 1$

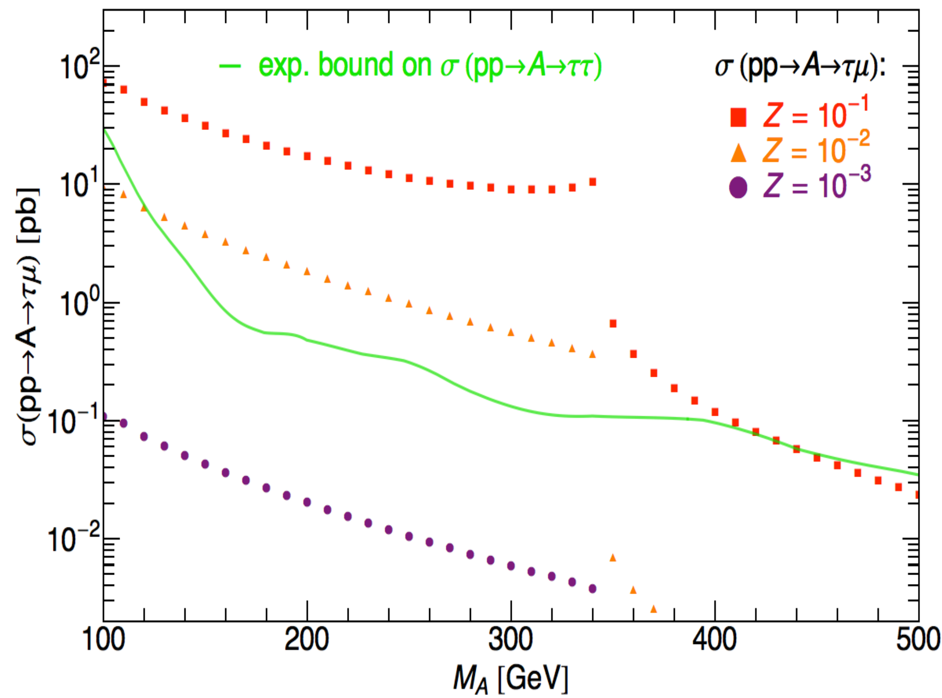
4.3 Prospects at LHC

see also e.g., Assamagan, Deandrea, Delsart'03
Davidson & Verdier'12
Arana-Catania, Arganda, Herrero'14

- Decay width : $\boxed{\Gamma(A \to \tau^+\mu^- + \tau^-\mu^+) \equiv \Gamma(A \to \tau\mu) = \dfrac{M_A\left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2\right)}{8\pi}}$

  Assumption : only SM channels ($A \to gg, b\bar{b}, c\bar{c}, \tau\tau...$) are important
- Large BR for $\boxed{A \to \tau\mu}$ can be expected since A does not couple to WW, ZZ at tree level. Results :





N.B.: Diagonal couplings $\left|y^A_f\right| = 1$