Double Heavy Baryons and Corrections to Heavy Quark-Diquark Symmetry

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in collaboration with ***Thomas Mehen***

arXiv: 1905.06965

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U.S. DEPARTMENT OF ENERGY

Outline

Introduction

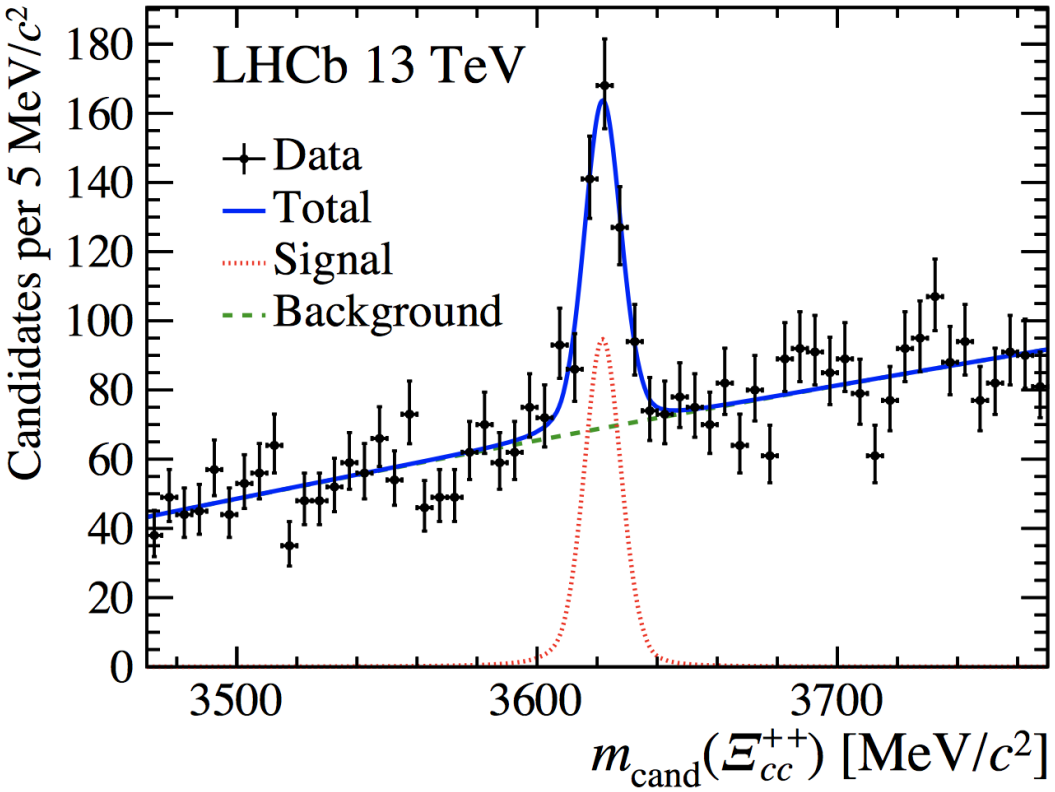
Formalism

Perturbative & Nonperturbative corrections to hyperfine splittings

Conclusions & Future Directions

Introduction

LHCb discovery of $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$

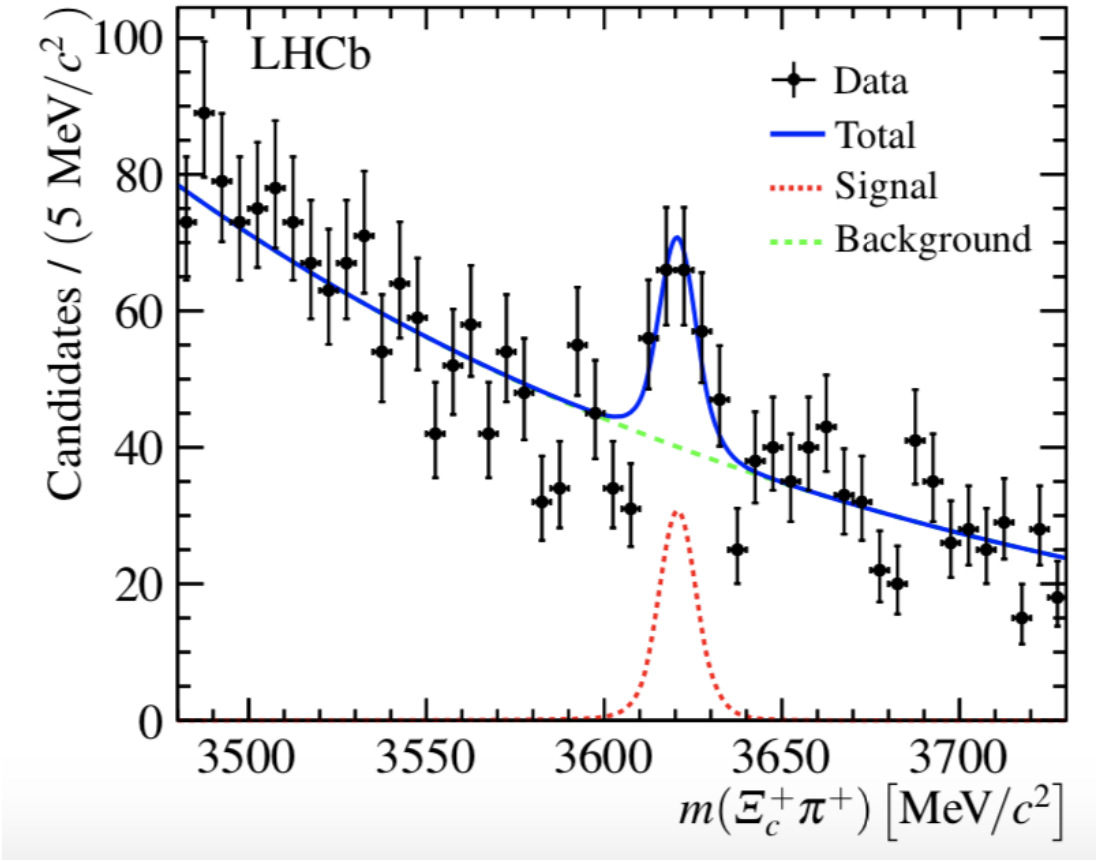


$$m_{\Xi_{cc}^{++}} = 3621.4 \pm 0.72 \pm 0.27 \pm 0.14\,\text{MeV}$$

*R. Aaij, et. al. (LHCb) , PRL **119** 112001 (2017)*

Introduction

LHCb discovery of $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$



$$m_{\Xi_{cc}^{++}} = 3620.6 \pm 1.5 \pm 0.4 \pm 0.3 \ \text{Mev}$$

*R. Aaij, et. al. (LHCb) , PRL **121** 162002 (2018)*

Introduction

- Heavy Mesons ($\bar{Q}q$):

  -- Hadrons with single **heavy** quark (anti-quark) and a light anti-quark (quark).

  -- Described in framework of heavy-quark effective theory (HQET) where operators scale in powers of $\Lambda_{\mathrm{QCD}}/m_Q$.

  -- The light quark orbit a static source of color in $\bar{3}$ representation.

- Double Heavy Baryons (QQq):

  -- Hadrons with **two heavy** quarks (anti-quark) and a light quark (anti-quark).

  -- Hierarchy of energy scales characterized by the **relative velocity** v of heavy quarks:

$$m_Q > m_Q v > m_Q v^2$$

  -- Described in framework of nonrelativistic QCD (NRQCD) with power counting of operators determined by velocity. *Bodwin, Braaten & Lepage (1994), Luke, Manohar & Rothstein (2000), Brambilla, Pineda & Soto (2000)*

  -- **Attractive** Coulomb force between the heavy quarks.

  -- Ground state is a **spin-1** diquark in $\bar{3}$ color representation.

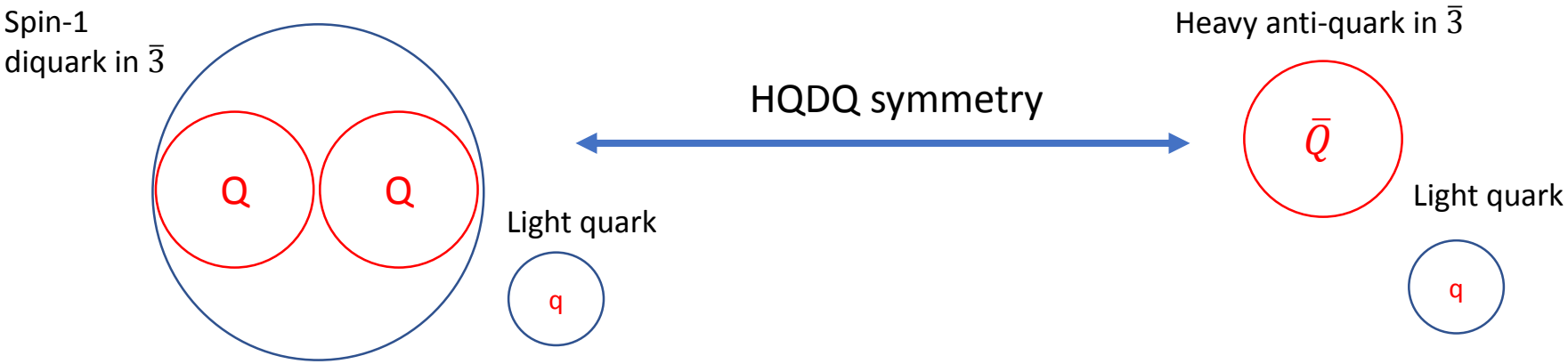
  -- Recently observed by the LHCb experiment $\Xi_{cc}^{++}(3621)$ *LHCb collaboration (2017)*

Introduction

- Heavy quark-diquark (HQDQ) symmetry

  -- Relates doubly heavy baryons and heavy anti-mesons.

  -- In $m_Q \to \infty$ limit, the light quark (q) in both hadrons orbit around static source of color in $\bar{3}$ representation.



Introduction

Effective theory for heavy quarks and diquarks

$$\mathcal{L} = h^\dagger\, iD_0\, h + \mathbf{T}^\dagger\, iD_0\, \mathbf{T} + \frac{g_s}{2m_Q} h^\dagger\, \boldsymbol{\sigma}\cdot\mathbf{B}\, h - \frac{ig_s}{2m_Q}\mathbf{T}^\dagger\cdot\mathbf{B}\times\mathbf{T} + \ldots.$$

Chromo-magnetic coupling of heavy quarks

Chromo-magnetic coupling of diquarks

M. Savage and M. Wise, PLB **248** (1990) S. Fleming and T. Mehen, PRD **73** (2006) N. Brambilla, A. Vairo, T. Rosch, PRD **72** (2005)

Hyperfine splittings

- Mass hyperfine splitting relation due to HQDQ symmetry

$$m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = \frac{3}{4}(m_{D^*} - m_D) \approx 106\,\mathrm{MeV}$$

$\Xi^*: S = 3/2$ $\quad D^*: S = 1$

$\Xi: S = 1/2$ $\quad D: S = 0$

Introduction

Effective theory for heavy quarks and diquarks

$$\mathcal{L} = h^\dagger\, iD_0\, h + \mathbf{T}^\dagger\, iD_0\, \mathbf{T} + \frac{g_s}{2m_Q} h^\dagger\, \boldsymbol{\sigma}\cdot\mathbf{B}\, h - \frac{ig_s}{2m_Q}\mathbf{T}^\dagger\cdot\mathbf{B}\times\mathbf{T} + \ldots.$$

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Corrections to chromo-magnetic coupling of diquarks

Corrections to HQDQ symmetry ??

Introduction

- **Brambilla, Vairo, & Rosch:** Corrections to HQDQ predictions for Hyperfine Splittings

$$M_{\Xi^*_{QQ}} - M_{\Xi_{QQ}} = \frac{3}{4}\left(M_{P^*_Q} - M_{P_Q}\right)\left[1 + \mathcal{O}\left(\alpha_s^2, \frac{\Lambda_{\rm QCD}}{m_Q}\right)\right]$$

N. Brambilla, A. Vairo, T. Rosch, PRD **72** (2005)

- **Mehen & Mohapatra:**

- Explicitly compute $\boldsymbol{\mathcal{O}\left(1/m_Q^2 \times \frac{\alpha_s}{r}\right)} \sim \boldsymbol{\mathcal{O}}(\alpha_s^2)$ perturbative correction.

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

- Argue that nonperturbative corrections scale as $O\left(\frac{\Lambda_{\rm QCD}^2}{m_Q^2}\right)$ not $O\left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)$

Bottom quark $\approx 1\%$
Charm quark $\approx 10\ \%$

T.Mehen and A. M, arXiv:1905.06965 [hep-ph]

Formalism

- Interested in calculating the LO and NLO chromo-magnetic coupling of diquarks.
- Use nonrelativistic QCD (NRQCD) formalism with velocity power counting rule for operators.
- NRQCD Lagrangian :

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left(iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m_Q} + \frac{g}{2m_Q}\,\sigma\cdot\mathbf{B} \right) \psi_{\mathbf{p}}$$
$$- \frac{1}{2}\sum_{\mathbf{p},\mathbf{q}} \frac{g_s^2}{(\mathbf{p}-\mathbf{q})^2} \psi^\dagger_{\mathbf{q}} T^A \psi_{\mathbf{p}} \psi^\dagger_{-\mathbf{q}} T^A \psi_{-\mathbf{p}} + \cdots ,$$

- Use color and spin Fierz identities to group the heavy quark field ψ in spin-1, $\bar{3}$ color and spin-0, 6 color representation.
- Composite diquark operators:

Spin-1, $\bar{3}$ color

$$T^i_{\mathbf{r}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{2}\,\epsilon^{ijk} (\psi_{-\mathbf{p}})_j \epsilon\, \sigma (\psi_{\mathbf{p}})_{\mathbf{k}}$$

Spin-0, 6 color

$$\Sigma^{(mn)}_{\mathbf{r}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{\sqrt{2}}\, d^{(mn)}_{ij} (\psi_{-\mathbf{p}})_i \epsilon^T (\psi_{\mathbf{p}})_j$$

Mehen & Fleming (2006)
Brambilla, Vairo & Rosch (2005)

Formalism

- NRQCD Lagrangian in terms of composite diquark operators :

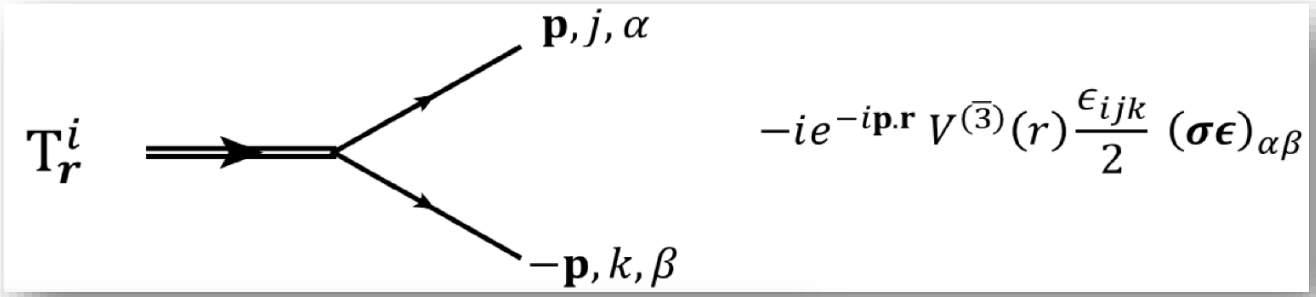
$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2}\int d^3\mathbf{r}\, V^{(\bar{\mathbf{3}})}(r)\left(\mathbf{T}^{i\dagger}_{\mathbf{r}} - \sum_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}}\epsilon_{ijk}\frac{1}{2}(\psi^{\dagger}_{\mathbf{q}})_j\sigma\epsilon(\psi^{\dagger}_{-\mathbf{q}})_{\mathbf{k}}\right)\left(\mathbf{T}^{i}_{\mathbf{r}} - \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{2}\epsilon_{ilm}(\psi_{-\mathbf{p}})_l\epsilon\sigma(\psi_{\mathbf{p}})_{\mathbf{m}}\right)$$

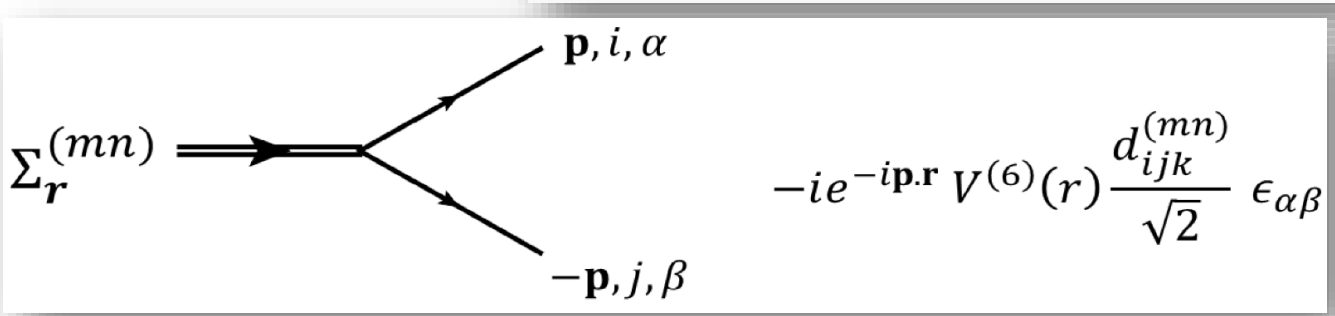
$$+\frac{1}{2}\int d^3\mathbf{r}\, V^{(\mathbf{6})}(r)\left(\Sigma^{(mn)\dagger}_{\mathbf{r}} - \sum_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}}\frac{1}{\sqrt{2}}d^{(mn)}_{ij}(\psi_{\mathbf{q}})_i\epsilon(\psi_{-\mathbf{q}})_j\right)\left(\Sigma^{(mn)}_{\mathbf{r}} - \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{\sqrt{2}}d^{(mn)}_{ij}(\psi_{-\mathbf{p}})_i\epsilon^T(\psi_{\mathbf{p}})_j\right).$$

Anti-triplet potential $V^{(\bar{3})} = -\frac{2}{3}\frac{\alpha_s}{r}$

Sextet potential $V^{(6)} = \frac{1}{3}\frac{\alpha_s}{r}$

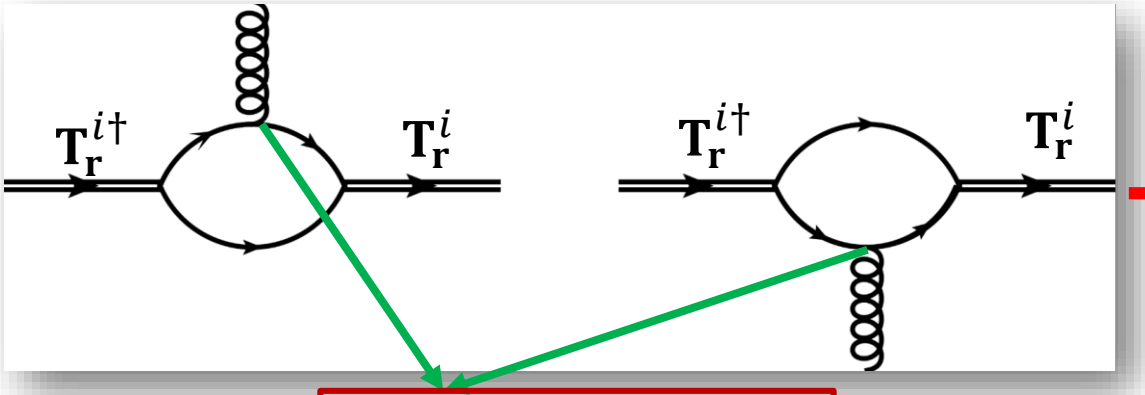
- Coupling of diquark to heavy quarks :





Chromo-magnetic coupling

- $\mathcal{O}(v^2)$ diagrams contributing to the chromo-magnetic coupling of diquarks:



$\boldsymbol{\sigma}.\boldsymbol{B}$ quark coupling

Diquark chromomagnetic interaction

- Leading order chromo-magnetic Lagrangian of diquarks :

$$\mathcal{L}_{\sigma.\mathbf{B}} = \frac{g}{2m_Q}\int d^3\mathbf{r}\, i\, \mathbf{T}_{\mathbf{r}}^{i\,\dagger}\cdot\mathbf{B}^c\,\bar{T}_{ij}^{c}\times\mathbf{T}_{\mathbf{r}}^{j}.$$

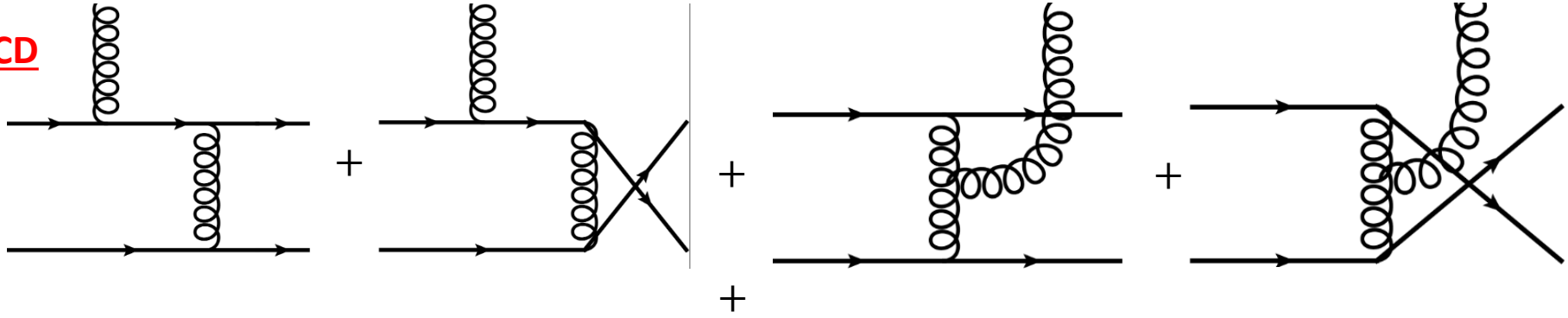
Mehen & Fleming (2006)
Brambilla, Vairo & Rosch (2005)

- $\mathcal{O}(1/m_Q)$ correction to heavy spin symmetry.
- Responsible for hyperfine mass splittings in the ground state of doubly heavy baryons.

Perturbative correction

- Corrections to chromo-magnetic couplings of diquarks scaling with **powers of** α_s.
- Perturbative corrections from Effective 5-point operator: **matching** diagrams in QCD and NRQCD.

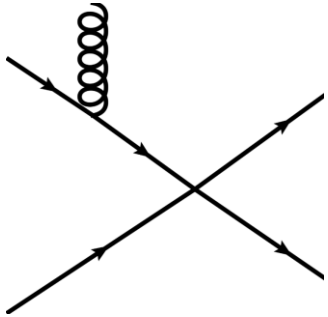
QCD



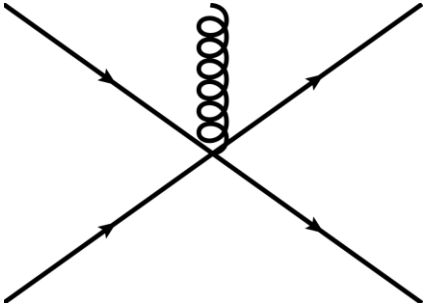
6 more with gluon attached to each external quark

NRQCD

=



+ 3 more with gluon attached to each external quark +



Effective 5-point operator

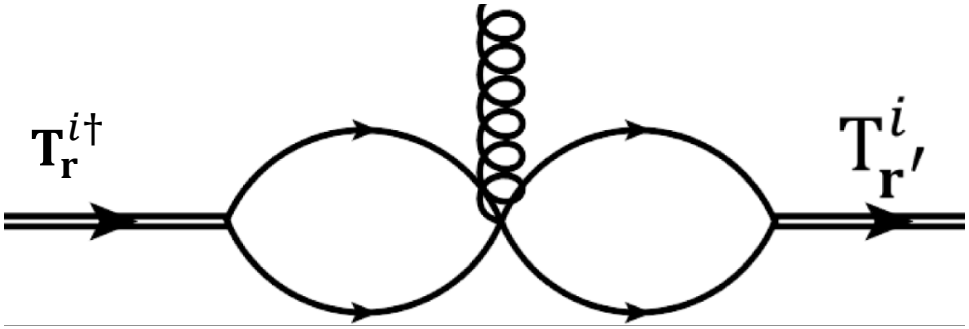
AM & Mehen (2019)

Perturbative correction

- Effective Lagrnagian for 5-point contact operator:

$$\mathcal{L}_{eff} = -\frac{g^3}{2}\frac{1}{2m_Q}\sum_{\mathbf{P_1},\mathbf{P_2},\mathbf{P_3},\mathbf{P_4}} \psi^\dagger_{\mathbf{P_4}} T^a \psi_{\mathbf{P_2}} \psi^\dagger_{\mathbf{P_3}} \left(T^a T^c + T^c T^a\right) \frac{\sigma \cdot \mathbf{B}^c}{2m_Q} \psi_{\mathbf{P_1}} \frac{1}{(\mathbf{P_4} - \mathbf{P_2})^2}$$

- 5-point contact operator contributing to the chromo-magnetic coupling of diquarks:



$\mathcal{O}(v^4)$ diagram in velocity power counting of NRQCD

- Corrections to chromo-magnetic coupling of diquarks from 2-loop diagram:

$$\mathcal{L}'_{\sigma.\mathbf{B}} = \frac{ig\alpha_s}{6m_Q^2}\int d^3\mathbf{r}\ \mathbf{T}^{i\,\dagger}_{\mathbf{r}} \cdot \frac{1}{r}\,\mathbf{B}^c\,\bar{T}^c_{ij} \times \mathbf{T}^j_{\mathbf{r}}.$$

- Next-to-leading order (NLO) Lagrangian: $\boldsymbol{\mathcal{O}\left(1/m_Q^2 \times \alpha_s/r\right)}$ corrections to heavy spin-symmetry.

AM & Mehen (2019)

Perturbative correction

- In $m_Q \to \infty$, ground state spatial wavefunction of diquarks in doubly heavy baryons:

$$\phi(\mathbf{r}) = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$

Bohr Radius:
$\boldsymbol{a_0 = 3/\alpha_s m_Q}$

- Chromo-magnetic coupling of diquarks from 2-loop diagram:

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Suppression from Heavy quark mass

Enhancement from Coloumb exchange

$$\left\langle \frac{1}{r} \right\rangle = \frac{\alpha_s m_Q}{3}$$

AM & Mehen (2019)

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$$\left\langle \frac{1}{r} \right\rangle = \frac{\alpha_s m_Q}{3}$$

Suppression from Heavy quark mass

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Overall $\boldsymbol{\mathcal{O}(\alpha_s^2/m_Q)}$ order chromo-magnetic coupling of diquarks !!!

AM & Mehen (2019)

Pertubative Correction

- HQDQ symmetry relates mass hyperfine splittings of ground state double heavy baryons and heavy anti-mesons.
- Correction to HQDQ symmetry due to the $\mathcal{O}(\alpha_s^2/m_Q)$ order chromo-magnetic coupling of diquark from effective 5-point operator.
- Relation between mass hyperfine splittings:

$$m_{P^*} - m_P = \frac{4}{3}(m_{\Sigma^*} - m_\Sigma)\left(1 + \frac{\alpha_s}{12 m_Q}\left\langle \frac{1}{r} \right\rangle\right),$$
$$= \frac{4}{3}(m_{\Sigma^*} - m_\Sigma)\left(1 + \frac{\alpha_s^2}{9}\right)$$

Hyperfine splitting in heavy meson

- Correction to hyperfine splitting of diquarks from $\mathcal{O}(\alpha_s^2/m_Q)$ coupling :

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

$(\alpha_s(m_c) \approx 0.3, \alpha_s(m_b) \approx 0.1)$

AM & Mehen (2019)

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- Correction to hyperfine splitting of diquarks from $\mathcal{O}(\alpha_s^2/m_Q)$ coupling :

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

Small correction to hyperfine splittings of doubly charm / bottom quark systems !!

AM & Mehen (2019)

Small correction to HQDQ symmetry!!

Non-Perturbative Corrections

- Corrections to chromo-magnetic couplings scaling with **powers of Λ_{QCD}/m_Q**.
- Spin Symmetry breaking HQET(-like) Lagrangians for heavy quarks and diquarks

Heavy quarks:

$$\frac{g}{2m_Q}\sum_{\mathbf{P}}\psi^{\dagger}_{\mathbf{P}}\,\boldsymbol{\sigma}\cdot\mathrm{B}\,\psi_{\mathbf{P}} + i\frac{g}{8m_Q^2}\sum_{\mathbf{P}}\psi^{\dagger}_{\mathbf{P}}\,\boldsymbol{\sigma}\cdot(\mathrm{D}\times\mathrm{E}-\mathrm{E}\times\mathrm{D})\,\psi_{\mathbf{P}}$$

$\mathcal{O}(\Lambda^2_{QCD}/m^2_Q)$ term in Heavy quark effective theory

T.Mehen and A. M, arXiv:1905.06965 [hep-ph]

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Coupling to different background field
But similar color and spin structure

$$\mathbf{B}\rightarrow i(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})/4m_Q$$

T.Mehen and A. M, arXiv:1905.06965 [hep-ph]

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$$\mathbf{B}\rightarrow i(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})/4m_Q$$

Diquarks:

$$i\frac{g}{2m_Q}\int d^3\mathbf{r}\,\mathbf{T}^{i\,\dagger}_{\mathbf{r}}\cdot\mathbf{B}^c\,\bar{T}^c_{ij}\times\mathbf{T}^j_{\mathbf{r}}-\frac{g}{8m_Q^2}\int d^3\mathbf{r}\,\mathbf{T}^{i\,\dagger}_{\mathbf{r}}\cdot(\mathbf{D}\times\mathbf{E}^c-\mathbf{D}\times\mathbf{E}^c)\,\bar{T}^c_{ij}\times\mathbf{T}^j_{\mathbf{r}}.$$

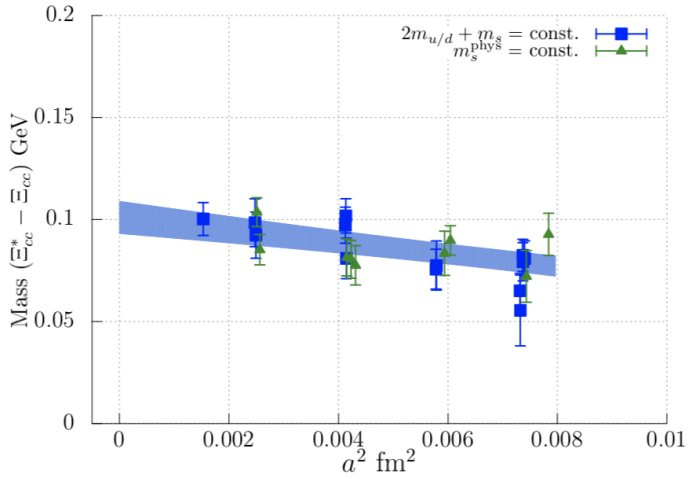
- **3/4 prediction** still valid: coefficient of the two Lagrangians same after

$$\sigma^i_{\alpha\beta}\rightarrow i\epsilon^{ijk},T^a\rightarrow\bar{T}^a$$

- $\mathcal{O}(\Lambda^3_{QCD}/m^3_Q)$ terms in HQET: many with different color structures: will break 3/4
- **Nonperturbative Corrections scale as $\mathcal{O}(\Lambda^2_{QCD}/m^2_Q)$.**

T.Mehen and A. M, arXiv:1905.06965 [hep-ph]

Comparision with Lattice



100 MeV (lat.) vs. 106 MeV (HQDQ)

6% vs. $\Lambda_c/m_c \sim 0.3$

$\Xi_{cc}^* - \Xi_{cc}$

N.Mathur, M. Padamanth, PRD **99** (2019) 031501

$\Xi_{bb}^* - \Xi_{bb}$ $\quad 34.6(2.5)(7.4)\,\text{MeV}$ vs. $34\,\text{MeV}$

Z.S. Brown, et. al., PRD 90(2014) 094507

$$\Lambda_{\text{QCD}}/m_B \sim 0.1$$

$\Omega_{cc}^* - \Omega_{cc}/(D_s^{*0} - D_s^0[Expt]) = 0.58(4)$ Z.S. Brown, et. al., PRD 90(2014) 094507

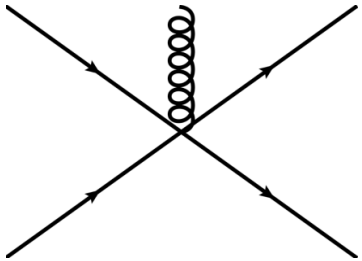
$\Omega_{bb}^* - \Omega_{bb}/(B_s^{*0} - B_s^0[Expt]) = 0.74(12)$ Sara Collins, 2019 QWG Talk

Adapted from T. Mehen's talk

Conclusions

- Heavy quark-diquark symmetry: $\boldsymbol{\bar{Q}q \leftrightarrow QQq}$

- Perturbative Corrections to HQDQ symmetry from effective **5-point contact operator** scales as $\boldsymbol{\mathcal{O}(\alpha_s^2)}$.



$(\alpha_s(m_c) \approx 0.3, \alpha_s(m_b) \approx 0.1)$

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

- **Small** corrections to HQDQ symmetry from $\mathcal{O}(\alpha_s^2)$ coupling.

- Non-perturbative Corrections to HQDQ symmetry scales as $\boldsymbol{\mathcal{O}(\Lambda_{QCD}^2/m_Q^2)}$ and not as $\boldsymbol{\mathcal{O}(\Lambda_{QCD}/m_Q)}$.

Bottom quark $\approx 1\%$
Charm quark $\approx 10\ \%$

Future Directions

- RG evolution coupling constant of effective 5–point operator due to one loop ?? .

Thank You!!