

Revisit XEFT for the study of $X(3872)$

Lin Dai
Duke University

with
Thomas Mehen (DUKE)
Feng-Kun Guo (ITP, CAS)
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Outline

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◆ Review XEFT

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◆ XEFT Power Counting

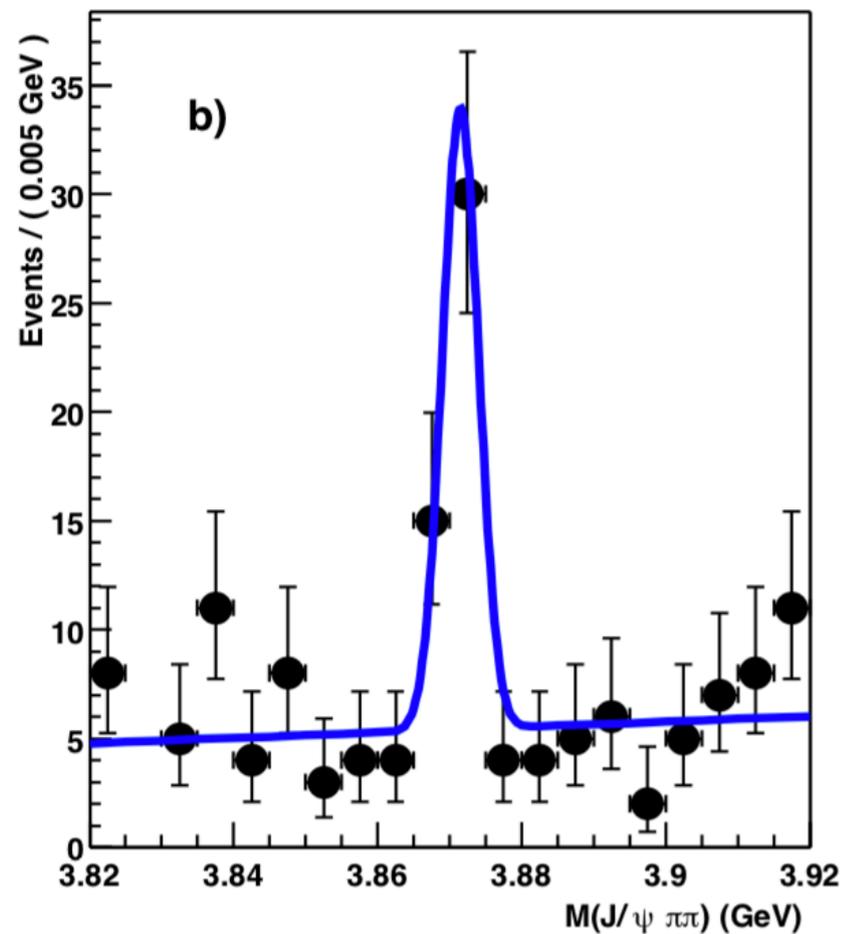
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- ◆ Review XEFT
- ◆ XEFT Power Counting
- ◆ NLO Calculations of X decay rate and Numerical Results

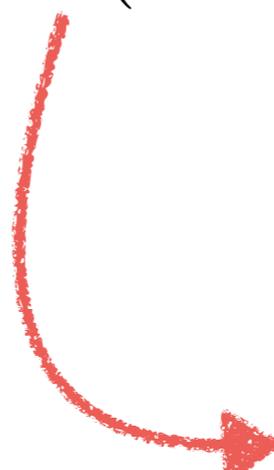
Outline

- ◆ Review XEFT
- ◆ XEFT Power Counting
- ◆ NLO Calculations of X decay rate and Numerical Results
- ◆ Some other things we want to add

X(3872)



$$B^{\pm} \rightarrow K^{\pm} X(3872)$$

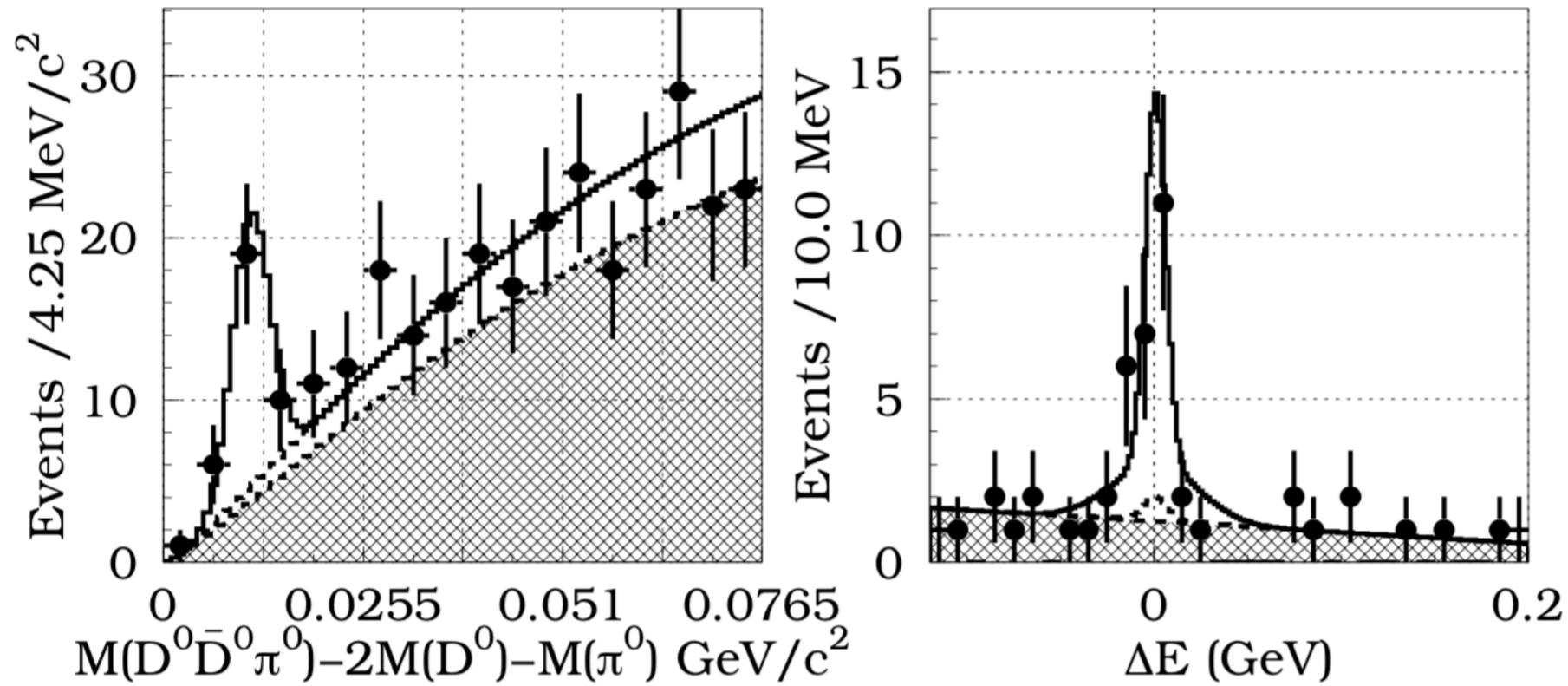

$$J/\psi \pi^{+} \pi^{-}$$

Belle Collaboration,
Phys.Rev.Lett. 91, 262001 (2003)

$$B_X = M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18) \text{MeV}$$

◆ Assume X(3872) a DD^* bound state in our analysis

X(3872)



Belle Collaboration,
Phys.Rev.Lett.97,
162002, (2006)

$$X \rightarrow D^0 \bar{D}^0 \pi^0$$

$$\frac{B(X(3872) \rightarrow D^0 \bar{D}^0 \pi^0)}{B(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 9.4^{+3.6}_{-4.3}$$

XEFT

◆ XEFT Lagrangian can be obtained from HH χ PT:

$$\mathcal{L}_0 = \frac{f^2}{8} \text{Tr} \partial^\mu \Sigma \partial_\mu \Sigma^\dagger + \frac{f^2 B_0}{4} \text{Tr} (m_q \Sigma + m_q \Sigma^\dagger) - \text{Tr} \bar{H}_a i v \cdot D_{ba} H_b + g \text{Tr} \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^\mu$$

◆ Heavy Mesons: $H_a = \frac{1 + \psi}{2} [P_a^{*\mu} \gamma_\mu - P_a \gamma_5]$

◆ Chiral Dynamics: $D_{ab}^\mu = \delta_{ab} \partial^\mu - V_{ab}^\mu$

$$\left\{ \begin{array}{l} V_{ab}^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) \\ A_{ab}^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) \end{array} \right.$$

◆ HQS, Chiral Sym.

XEFT

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◆ HQS, Chiral Sym.

◆ D, D* mass difference:

$$\mathcal{L}_m = \frac{\lambda_2}{4m_Q} \text{Tr} \bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}$$

XEFT

◆ Match $\text{HH}\chi\text{PT}$ to XEFT:

$$\begin{aligned}
 \mathcal{L}_{\text{XEFT}} = & \sum_{\phi=D,\bar{D}} \phi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M_{D^{*0}}} \right) \phi + \sum_{\phi=D,\bar{D}} D^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M_{D^0}} \right) D + \pi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M_{\pi^0}} + \delta \right) \pi \\
 & + \left[\frac{\bar{g}}{F_\pi} \frac{1}{\sqrt{2M_{\pi^0}}} \left(DD^\dagger \cdot \nabla \pi + \bar{D}^\dagger \bar{D} \cdot \nabla \pi^\dagger \right) + \text{H.c.} \right] \\
 & - \frac{C_0}{2} (\bar{D}D + D\bar{D})^\dagger \cdot (\bar{D}D + D\bar{D}) \\
 & + \left[\frac{C_2}{16} (\bar{D}D + D\bar{D})^\dagger \cdot \left(\bar{D}(\overleftarrow{\nabla})^2 D + D(\overleftarrow{\nabla})^2 \bar{D} \right) + \text{H.c.} \right] \\
 & + \left[\frac{B_1}{\sqrt{2}} \frac{1}{\sqrt{2M_{\pi^0}}} (\bar{D}D + D\bar{D})^\dagger \cdot D\bar{D}\nabla\pi + \text{H.c.} \right] \\
 & + \frac{C_\pi}{2M_{\pi^0}} \left(D^\dagger \pi^\dagger D\pi + \bar{D}^\dagger \pi^\dagger \bar{D}\pi \right) + C_{0D} D^\dagger \bar{D}^\dagger D\bar{D},
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 \end{aligned}$$

XEFT Power Counting

◆ P.C. parameter Q (dynamical mom of $X(3872)$):

$$\{p_D, p_{D^*}, p_\pi, \mu, \gamma_0\} = \mathcal{O}(Q)$$

$$\mu = \sqrt{\Delta^2 - M_{\pi^0}^2} \simeq 44\text{MeV} \quad \text{with} \quad \Delta \equiv M_{D^*} - M_D$$

$$\gamma_0 = \sqrt{2\mu_0 B_X}$$

$$\text{LOOP: } \int d^4p \sim Q^5$$

$$\text{Propagator: } \frac{1}{p^2 - \gamma^2} \sim Q^{-2}$$

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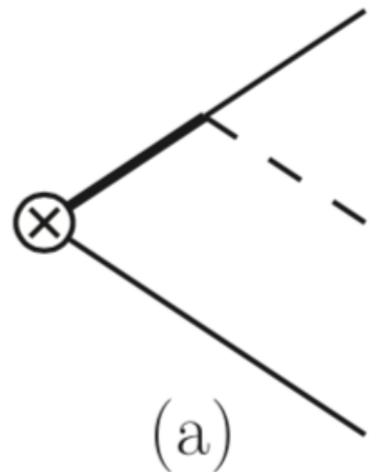
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◆ Dynamic pion: NR & Perturbative

$$D^* \rightarrow D + \pi \quad \text{with} \quad M_{D^*} - M_D - M_\pi \sim 7\text{MeV}$$



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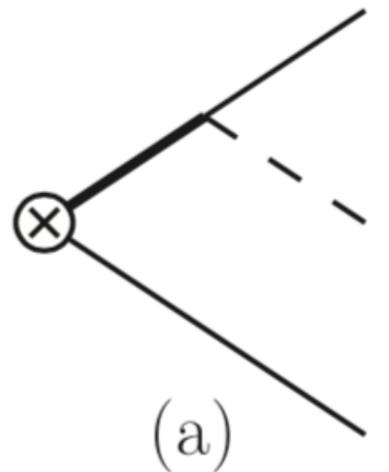
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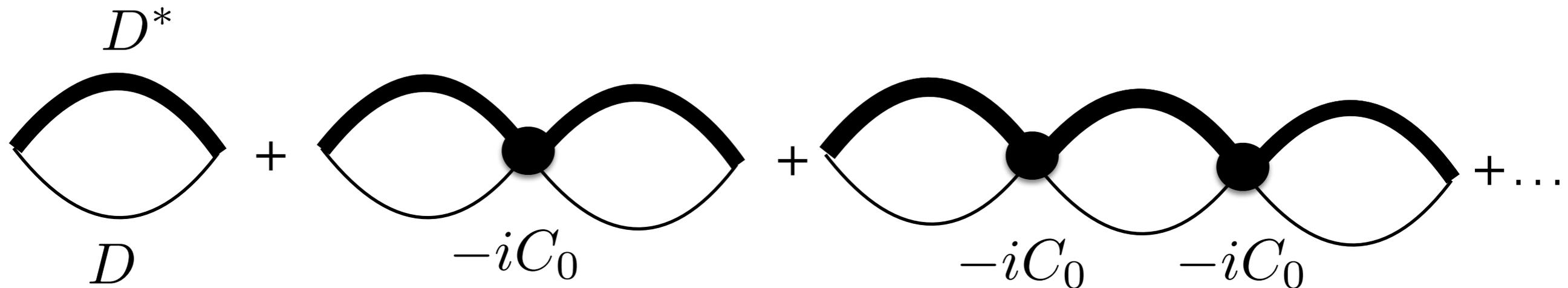
$$\frac{g^2 \mu_0 \mu}{8\pi F_\pi^2} \simeq \frac{1}{20} \cdots \frac{1}{10}$$



XEFT Power Counting

- ◆ C_0 treated non-perturbatively, generate X(3872) bound state

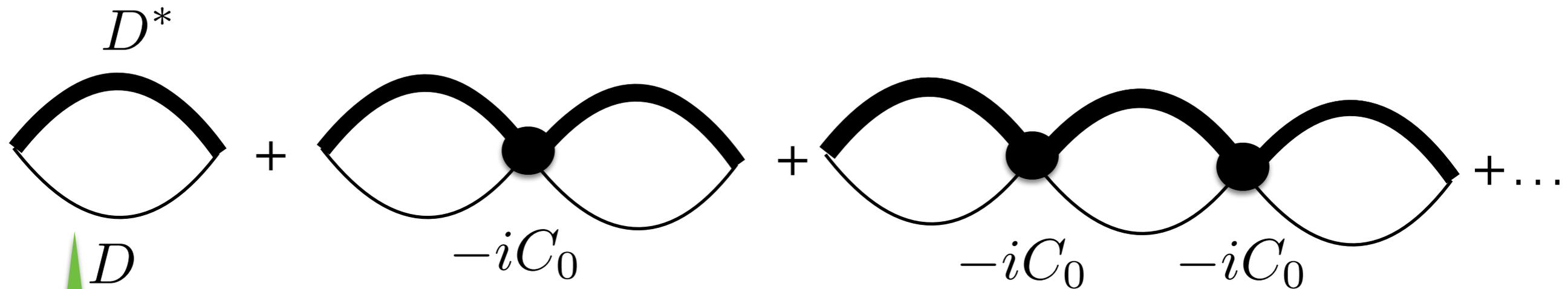
$$-\frac{C_0}{2} (\overline{D}D + D\overline{D})^\dagger \cdot (\overline{D}D + D\overline{D})$$



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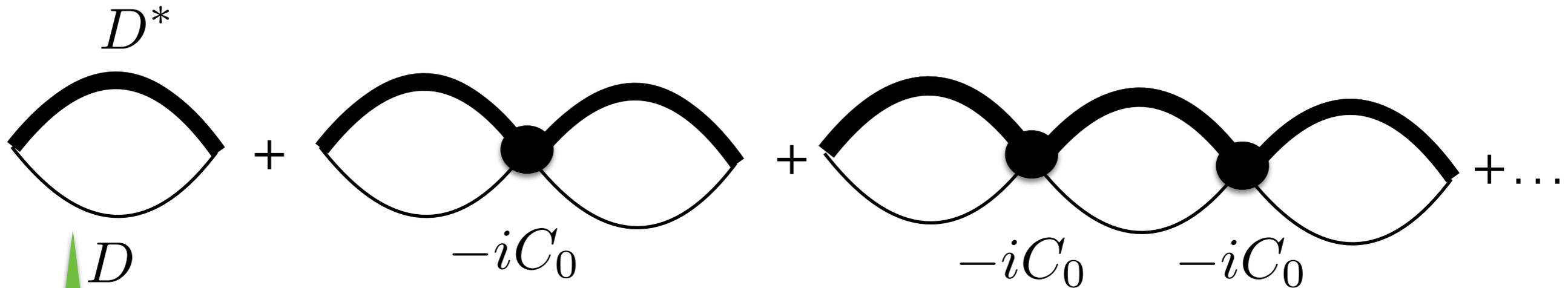


$$\Sigma_0(-B_X) \sim Q^5 Q^{-4} = Q^1$$

XEFT Power Counting

- ◆ C_0 treated non-perturbatively, generate X(3872) bound state

$$-\frac{C_0}{2} (\overline{D}D + D\overline{D})^\dagger \cdot (\overline{D}D + D\overline{D})$$



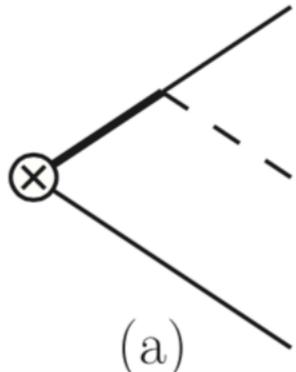
$$\Sigma_0(-B_X) \sim Q^5 Q^{-4} = Q^1$$

- ◆ Pole at B_X

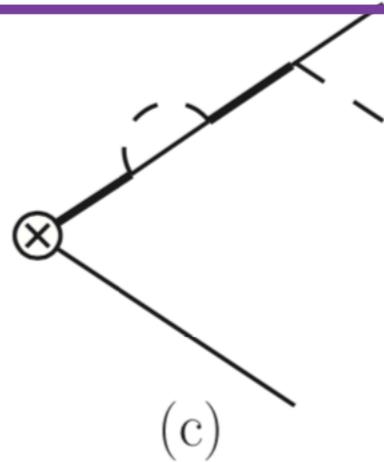
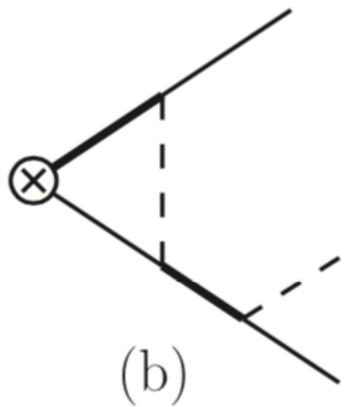
$$1 + C_0 \Sigma_0(-B_X) = 0 \rightarrow C_0 \sim Q^{-1}$$

XEFT Power Counting

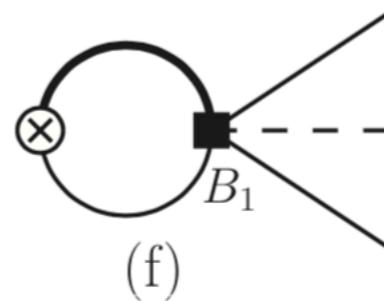
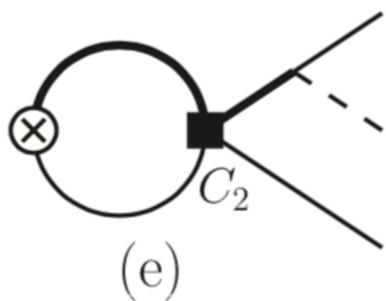
$$\mathbf{D}D^\dagger \cdot \nabla \pi$$



$$\mathcal{O}(Q/Q^2) = \mathcal{O}(Q^{-1})$$



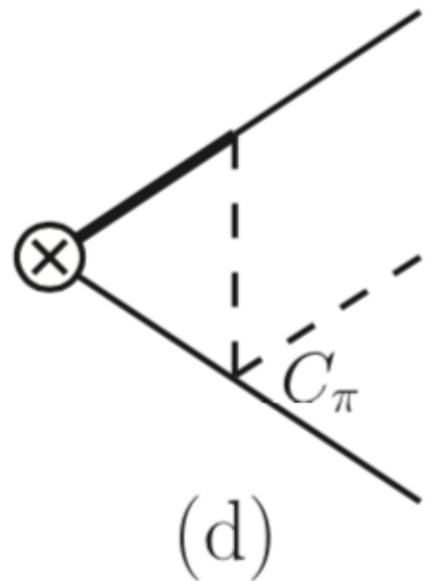
$$(b): \mathcal{O}(Q^3 Q^{-8} Q^5) = \mathcal{O}(Q^0)$$



Similar for others

XEFT Power Counting

The diagram: $\mathcal{O}(QQ^{-6}Q^5) = \mathcal{O}(Q^0)$

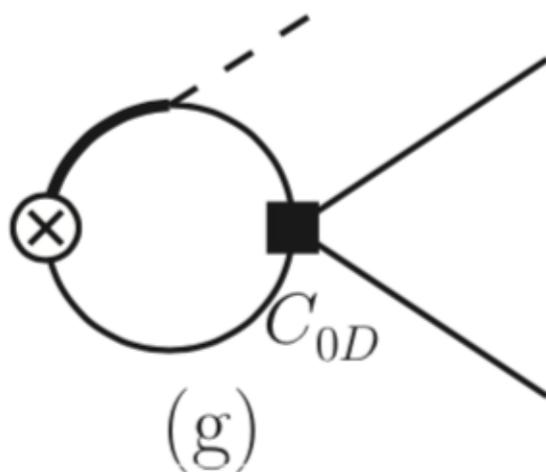


Num. HH χ PT: $i\mathcal{A}_{h_0, h_1} = i\frac{2}{3} (6h_0 + h_1) \frac{M_\pi^2}{F_\pi^2} \sim 0.65$

Match to $\frac{C_\pi}{2M_{\pi^0}} (D^\dagger \pi^\dagger D \pi + \bar{D}^\dagger \pi^\dagger \bar{D} \pi) \rightarrow C_\pi = \mathcal{O}(Q^0)$

FK. Guo, C. Hanhart, S. Krewald, Phys. Lett. B666, 251–255 (2008)

FK. Guo, C. Hanhart, Ulf-G. Meissner Eur. Phys. J. A40, 171–179 (2009)



The diagram: $\mathcal{O}(QQ^{-6}Q^5) = \mathcal{O}(Q^0)$

To estimate C_{0D} , assume there is a bound state near $D\bar{D}$ threshold:

$$C_{0D} \sim 1 \text{ fm}^2$$

* C_0 treated non-perturbatively, generate X(3872) bound state
 $-\frac{C_0}{2} (\bar{D}D + D\bar{D})^\dagger \cdot (\bar{D}D + D\bar{D})$

 $\Sigma_0(-B_X) \sim Q^2 Q^{-4} = Q^1$
 * Pole at M_X
 $1 + C_0 \Sigma_0(-B_X) = 0 \rightarrow C_0 \sim Q^{-1}$

Decay Rate of $X \rightarrow D\bar{D}\pi$

$$\begin{aligned}
 \frac{d\Gamma_{\text{NLO}}}{dp_D^2 dp_{\bar{D}}^2} &= \frac{d\Gamma_{\text{LO}}}{dp_D^2 dp_{\bar{D}}^2} \left(1 + \frac{\bar{g}^2 \mu_0 \gamma}{3\pi F_\pi^2} \left(\frac{4\gamma^2 - \mu^2}{4\gamma^2 + \mu^2} \right) + C_2 (\Lambda_{\text{PDS}}) \frac{\mu_0 \gamma (\gamma - \Lambda_{\text{PDS}})^2}{\pi} \right) \\
 &- \frac{\bar{g}\gamma}{8\sqrt{2}\pi^3 F_\pi} \left(\sqrt{2} \frac{\bar{g}\mu_0}{F_\pi} C_2 (\Lambda_{\text{PDS}}) - B_1 (\Lambda_{\text{PDS}}) \right) (\Lambda_{\text{PDS}} - \gamma) (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \\
 &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[(\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{p_D^2 \tilde{I}_1^{(2)}(p_D)}{P_D^2 + \gamma^2} + \frac{p_{\bar{D}}^2 \tilde{I}_1^{(2)}(p_{\bar{D}})}{P_{\bar{D}}^2 + \gamma^2} \right) \right] \\
 &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[\vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{\vec{p}_D \cdot \vec{\epsilon}_X \vec{p}_D \cdot \vec{p}_\pi}{p_D^2 + \gamma^2} \left(I(p_D) - 2I^{(1)}(p_D) + I_0^{(2)}(p_D) \right) \right. \right. \\
 &\left. \left. + \frac{\vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \vec{p}_{\bar{D}} \cdot \vec{p}_\pi}{p_{\bar{D}}^2 + \gamma^2} \left(I(p_{\bar{D}}) - 2I^{(1)}(p_{\bar{D}}) + I_0^{(2)}(p_{\bar{D}}) \right) \right) \right] \\
 &+ \frac{C_\pi \bar{g}^2 \gamma}{16\pi^2 F_\pi^2 M_{\pi^0} \mu_0} \vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\vec{p}_D \cdot \vec{\epsilon}_X \left(I^{(1)}(p_D) - I(p_D) \right) + \vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \left(I^{(1)}(p_{\bar{D}}) - I(p_{\bar{D}}) \right) \right) \\
 &+ \frac{C_{0D} \bar{g}^2 \gamma}{4\pi^2 F_\pi^2 \mu_0} (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 I(p_\pi) \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right).
 \end{aligned}$$

Decay Rate of $X \rightarrow D\bar{D}\pi$

$$\begin{aligned}
 \frac{d\Gamma_{\text{NLO}}}{dp_D^2 dp_{\bar{D}}^2} = & \frac{d\Gamma_{\text{LO}}}{dp_D^2 dp_{\bar{D}}^2} \left(1 + \frac{\bar{g}^2 \mu_0 \gamma}{3\pi F_\pi^2} \left(\frac{4\gamma^2 - \mu^2}{4\gamma^2 + \mu^2} \right) + C_2 (\Lambda_{\text{PDS}}) \frac{\mu_0 \gamma (\gamma - \Lambda_{\text{PDS}})^2}{\pi} \right) \\
 & - \frac{\bar{g}\gamma}{8\sqrt{2}\pi^3 F_\pi} \left(\sqrt{2} \frac{\bar{g}\mu_0}{F_\pi} C_2 (\Lambda_{\text{PDS}}) - B_1 (\Lambda_{\text{PDS}}) \right) (\Lambda_{\text{PDS}} - \gamma) (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \\
 & + \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[(\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{p_D^2 \tilde{I}_1^{(2)}(p_D)}{P_D^2 + \gamma^2} + \frac{p_{\bar{D}}^2 \tilde{I}_1^{(2)}(p_{\bar{D}})}{P_{\bar{D}}^2 + \gamma^2} \right) \right] \\
 & + \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[\vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{\vec{p}_D \cdot \vec{\epsilon}_X \vec{p}_D \cdot \vec{p}_\pi}{p_D^2 + \gamma^2} \left(I(p_D) - 2I^{(1)}(p_D) + I_0^{(2)}(p_D) \right) \right. \right. \\
 & \left. \left. + \frac{\vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \vec{p}_{\bar{D}} \cdot \vec{p}_\pi}{p_{\bar{D}}^2 + \gamma^2} \left(I(p_{\bar{D}}) - 2I^{(1)}(p_{\bar{D}}) + I_0^{(2)}(p_{\bar{D}}) \right) \right) \right] \\
 & + \frac{C_\pi \bar{g}^2 \gamma}{16\pi^2 F_\pi^2 M_{\pi^0} \mu_0} \vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\vec{p}_D \cdot \vec{\epsilon}_X \left(I^{(1)}(p_D) - I(p_D) \right) + \vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \left(I^{(1)}(p_{\bar{D}}) - I(p_{\bar{D}}) \right) \right) \\
 & + \frac{C_{0D} \bar{g}^2 \gamma}{4\pi^2 F_\pi^2 \mu_0} (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 I(p_\pi) \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right).
 \end{aligned}$$

New Contribution

Decay Rate of $X \rightarrow D\bar{D}\pi$

$$C_2(\Lambda_{\text{PDS}}) = \frac{2\pi r_0}{\mu_0} \frac{1}{2(\Lambda_{\text{PDS}} - \gamma)^2}$$

$$\begin{aligned} \frac{d\Gamma_{\text{NLO}}}{dp_D^2 dp_{\bar{D}}^2} &= \frac{d\Gamma_{\text{LO}}}{dp_D^2 dp_{\bar{D}}^2} \left(1 + \frac{\bar{g}^2 \mu_0 \gamma}{3\pi F_\pi^2} \left(\frac{4\gamma^2 - \mu^2}{4\gamma^2 + \mu^2} \right) + C_2(\Lambda_{\text{PDS}}) \frac{\mu_0 \gamma (\gamma - \Lambda_{\text{PDS}})^2}{\pi} \right) \\ &- \frac{\bar{g}\gamma}{8\sqrt{2}\pi^3 F_\pi} \left(\sqrt{2} \frac{\bar{g}\mu_0}{F_\pi} C_2(\Lambda_{\text{PDS}}) - B_1(\Lambda_{\text{PDS}}) \right) (\Lambda_{\text{PDS}} - \gamma) (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \\ &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[(\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{p_D^2 \tilde{I}_1^{(2)}(p_D)}{P_D^2 + \gamma^2} + \frac{p_{\bar{D}}^2 \tilde{I}_1^{(2)}(p_{\bar{D}})}{P_{\bar{D}}^2 + \gamma^2} \right) \right] \\ &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[\vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{\vec{p}_D \cdot \vec{\epsilon}_X \vec{p}_D \cdot \vec{p}_\pi}{p_D^2 + \gamma^2} \left(I(p_D) - 2I^{(1)}(p_D) + I_0^{(2)}(p_D) \right) \right. \right. \\ &\left. \left. + \frac{\vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \vec{p}_{\bar{D}} \cdot \vec{p}_\pi}{p_{\bar{D}}^2 + \gamma^2} \left(I(p_{\bar{D}}) - 2I^{(1)}(p_{\bar{D}}) + I_0^{(2)}(p_{\bar{D}}) \right) \right) \right] \end{aligned}$$

$$\begin{aligned} &+ \frac{C_\pi \bar{g}^2 \gamma}{16\pi^2 F_\pi^2 M_{\pi^0} \mu_0} \vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\vec{p}_D \cdot \vec{\epsilon}_X \left(I^{(1)}(p_D) - I(p_D) \right) + \vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \left(I^{(1)}(p_{\bar{D}}) - I(p_{\bar{D}}) \right) \right) \\ &+ \frac{C_{0D} \bar{g}^2 \gamma}{4\pi^2 F_\pi^2 \mu_0} (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 I(p_\pi) \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right). \end{aligned}$$

New Contribution

Decay Rate of $X \rightarrow D\bar{D}\pi$

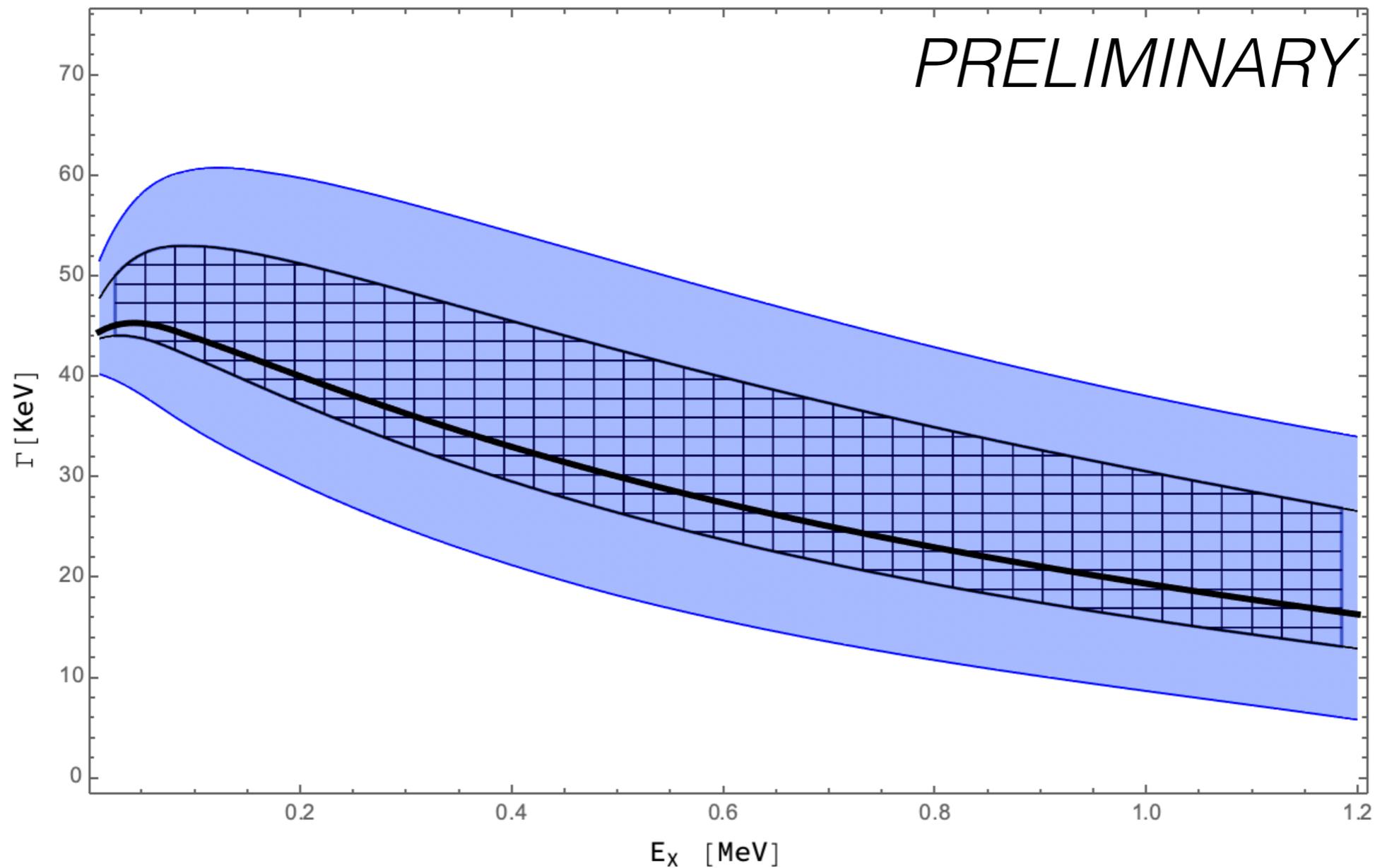
$$\left(\frac{\bar{g}\mu_0}{F_\pi} C_2(\Lambda_{\text{PDS}}) + B_1(\Lambda_{\text{PDS}}) \right) (\Lambda_{\text{PDS}} - \gamma) = \pm r_0^3$$

$$C_2(\Lambda_{\text{PDS}}) = \frac{2\pi r_0}{\mu_0} \frac{1}{2(\Lambda_{\text{PDS}} - \gamma)^2}$$

$$\begin{aligned} \frac{d\Gamma_{\text{NLO}}}{dp_D^2 dp_{\bar{D}}^2} &= \frac{d\Gamma_{\text{LO}}}{dp_D^2 dp_{\bar{D}}^2} \left(1 + \frac{\bar{g}^2 \mu_0 \gamma}{3\pi F_\pi^2} \left(\frac{4\gamma^2 - \mu^2}{4\gamma^2 + \mu^2} \right) + C_2(\Lambda_{\text{PDS}}) \frac{\mu_0 \gamma (\gamma - \Lambda_{\text{PDS}})^2}{\pi} \right) \\ &- \frac{\bar{g}\gamma}{8\sqrt{2}\pi^3 F_\pi} \left(\sqrt{2} \frac{\bar{g}\mu_0}{F_\pi} C_2(\Lambda_{\text{PDS}}) - B_1(\Lambda_{\text{PDS}}) \right) (\Lambda_{\text{PDS}} - \gamma) (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \\ &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[(\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{p_D^2 \tilde{I}_1^{(2)}(p_D)}{P_D^2 + \gamma^2} + \frac{p_{\bar{D}}^2 \tilde{I}_1^{(2)}(p_{\bar{D}})}{P_{\bar{D}}^2 + \gamma^2} \right) \right] \\ &+ \frac{1}{8\pi^2} \frac{\bar{g}^4}{F_\pi^4} \frac{\gamma}{M_{\pi^0}} \left[\vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\frac{\vec{p}_D \cdot \vec{\epsilon}_X \vec{p}_D \cdot \vec{p}_\pi}{p_D^2 + \gamma^2} \left(I(p_D) - 2I^{(1)}(p_D) + I_0^{(2)}(p_D) \right) \right. \right. \\ &\left. \left. + \frac{\vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \vec{p}_{\bar{D}} \cdot \vec{p}_\pi}{p_{\bar{D}}^2 + \gamma^2} \left(I(p_{\bar{D}}) - 2I^{(1)}(p_{\bar{D}}) + I_0^{(2)}(p_{\bar{D}}) \right) \right) \right] \\ &+ \frac{C_\pi \bar{g}^2 \gamma}{16\pi^2 F_\pi^2 M_{\pi^0} \mu_0} \vec{p}_\pi \cdot \vec{\epsilon}_X \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right) \left(\vec{p}_D \cdot \vec{\epsilon}_X \left(I^{(1)}(p_D) - I(p_D) \right) + \vec{p}_{\bar{D}} \cdot \vec{\epsilon}_X \left(I^{(1)}(p_{\bar{D}}) - I(p_{\bar{D}}) \right) \right) \\ &+ \frac{C_{0D} \bar{g}^2 \gamma}{4\pi^2 F_\pi^2 \mu_0} (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 I(p_\pi) \left(\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right). \end{aligned}$$

New Contribution

Decay Rate of $X \rightarrow D\bar{D}\pi$



Parameters: $0 \leq r_0 \leq \frac{1}{100} \text{MeV}^{-1}$, $C_\pi = (4.1 \pm 0.7) \times 10^{-3} \text{MeV}^{-1}$, $C_{0D} = \pm 1 \text{fm}$

NLO Correction: $C_\pi \rightarrow \leq 1\%$, $C_{0D} \rightarrow 20\%$

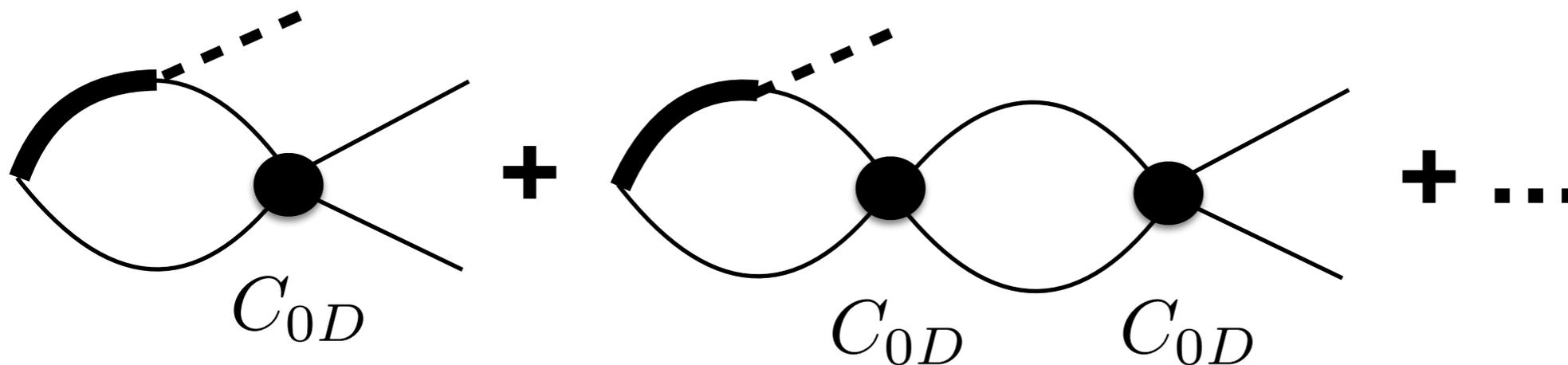
Work in Progress

- ◆ Add Charged channel $X \rightarrow D^+D^-\pi$

$$g_0 = \sqrt{\frac{2\pi\gamma_n}{\mu_{DD^*}^2}} \cos \theta, \quad g_+ = \sqrt{\frac{2\pi\gamma_c}{\mu_{DD^*}^2}} \sin \theta$$

T. Mehen,
Phys. Rev. D **92**, 034019

- ◆ Final State Rescattering



Summary

- ◆ Added πD and DD scattering terms, which are needed by symmetry and power counting
- ◆ Did NLO calculations, C_π correction is small, C_{OD} can be very large

THANKS