Update on Semileptonic Decays via Lattice QCD

Outline:
1. Physics motivation
2. Lattice techniques
3. Preliminary results
4. Outlook
Thanks to my close collaborators

• Carleton DeTar
• Aida El-Khadra
• Elvira Gamiz
• Zech Gelzer
• Steve Gottlieb
• Andreas Kronfeld
• Jim Simone

Note: Focus today on work by the Fermilab Lattice and MILC collaborations
See plenary talk by Andreas Kronfeld on Monday for more information
and an overview of what other groups are doing.
Physics overview
Flavor physics and lattice QCD
Two complementary goals

1. Determine CKM quark-mixing matrix elements
   » Study tree-level decays dominated by Standard-Model processes

2. Constrain possible scenarios for physics beyond the Standard Model
   » Study rare processes sensitive to contributions from new physics
Flavor physics and lattice QCD
CKM matrix elements

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
\pi \to \ell \nu & K \to \ell \nu & B \to \ell \nu \\
V_{cd} & V_{cs} & V_{cb} \\
D \to \ell \nu & D_s \to \ell \nu & B \to D \ell \nu \\
V_{td} & V_{ts} & V_{tb} \\
\langle B_d | \bar{B} \rangle & \langle B_s | \bar{B}_s \rangle & B \to D^* \ell \nu \\
B \to \pi \ell \ell & B \to K \ell \ell & B \to K \ell \ell
\end{pmatrix}
\]

Columns orthogonal?

= today’s main focus

More context, especially with pointers to the impressive experimental context in plenary talks from Monday morning
Form factors are necessary to extract CKM matrix elements
Flavor physics and the Standard Model
Rare loop-induced decays of heavy mesons

\[ \frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{tb}V_{tf}^*|^2 \times \begin{cases} \left| f_+(q^2) \right|^2 \\ \left| f_0(q^2) \right|^2 \\ \left| f_T(q^2) \right|^2 \end{cases} \]

Phase space, kinematics  CKM matrix element  Form factors/QCD matrix elements

\[ \rightarrow \text{Form factors are necessary to extract CKM matrix elements} \]
Lattice QCD
Computing form factors directly from QCD
Example: D to \( \pi \)

**Slogan:** \((\text{form factors}) \propto (\text{matrix elements})\)

\[
f_\mathcal{J} = (\text{known factors}) \times \langle D | \mathcal{J} | \pi \rangle
\]

- **Kinematics**
- **Compute via three-point functions in lattice QCD**

Create a pion  \( \text{Current J} \)  Destroy a D meson

\(\text{time} = 0\)  \(\text{time} = t\)  \(\text{time} = T\)
QCD on the lattice
Calculate the full non-perturbative path integral using Monte Carlo

Extract energies and amplitudes (= matrix elements) from a fit
Some published results: Leptonic decay constants
MILC (2+1+1)-flavor HISQ ensembles

- **B- and D-meson leptonic decay constants from four-flavor lattice QCD**
  PRD 98, 074512 Fermilab Lattice and MILC Collaborations (1712.09262)
  - Most precise determination heavy decay constants $f_B$ and $f_D$ to date: < 1%
- The work described today uses the same gauge configurations, generated by the MILC collaboration

### HISQ u/d/s/c and physical-mass b

<table>
<thead>
<tr>
<th>$f_{B+}$ (MeV)</th>
<th>$f_{B_s}$ (MeV)</th>
</tr>
</thead>
<tbody>
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<td>175 185 195 205 215 225 235 245 255</td>
<td></td>
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Fermilab/MILC 18
HPQCD 17 (pseudoscalar current)
ETM 16
HPQCD 13 (NRQCD b)

RBC/UKQCD 14
HPQCD 12 (NRQCD b)
HPQCD 11 (HISQ b)
Fermilab/MILC 11 (Clover b)

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Our goal is to build on this success and compute
  » Form factors for B-meson decays at the 1% level
  » From factors for D-meson decays at less than 1% level
Preliminary Results
Overview of lattice ensembles
MILC (2+1+1)-flavor HISQ ensembles

\[ N_{\text{config}} = 1000 \]

Filled = “Fermilab” b quark
Open = HISQ “b” quark
Overview of lattice ensembles
MILC (2+1+1)-flavor HISQ ensembles

» Several different lattice spacings
» Several light quark masses, including the physical point

Filled = “Fermilab” b quark
Open = HISQ b quark

» Nonrelativistic b quark, based on HQET
» Good statistics on many ensembles
» $B_s \rightarrow K$, $B \rightarrow K$
» Analysis led by Zech Gelzer

» Fully relativistic b quark
» Improved lattice artifacts
» Advantageous renormalization properties
» Running is ongoing
» Analysis by WJ
Preliminary Results
Fermilab b quark
Preliminary results

$B_s \to K$

Valence: HISQ u/d/s + Fermilab b quark

$w_0^{-1/2} f_1/\rho_1$

$w_0 f_T/\rho_T$

at leading-order in EFT description

$f \propto \frac{1}{E_K + \text{shift}}$
Preliminary results

**B → K**

**Valence:** HISQ u/d + Fermilab b quark

\[ f \propto \frac{1}{E_K + \text{shift}} \]

at leading-order in EFT description
Preliminary results

Continuum limit: $B_S \rightarrow K$

Valence: HISQ u/d/s + Fermilab b quark

- Take the “chiral” and continuum limit using EFT
- Extend results to full kinematic range using model-independent the “z-expansion”
- Look for results this fall
Preliminary Results
Heavy HISQ b
Preliminary results

$D \rightarrow \pi$: HISQ form factors, physical-mass ensembles

Valence: HISQ $u/d/c$ + heavy HISQ "b"
Preliminary results: Pion dispersion relation
Physical masses for light quarks
Three lattice spacings 0.088, 0.12, and 0.15 fm
Preliminary results
Comparing heavy quarks

Noise / Signal (%) for three-point functions

HISQ correlators shows good statistical precision versus Fermilab b

Caveat: Fermilab b at physical \( m_b \)
Heavy HISQ h at \( 1.4 \times m_c \) (more to follow at smaller \( a \)...)
Summary
Summary and take-home points

• Overall goals:
  - Form factors for B-meson decays at the 1% level
  - Form factors for D-meson decays at less than 1% level

• What will make this possible?
  - Large statistics, large volumes, and small lattice artifacts with HISQ (2+1+1)-flavor ensembles generated by the MILC collaboration
  - Physical-mass light (u/d) quarks
  - Physical-mass b-quarks

• These ensembles have already yielded the most precise measurements of B- and D-meson decay constants

• Timeline for form factors?
  - Fall 2019: papers on arXiv using Fermilab b
  - Results for HISQ b to follow…

• Questions?
Conformal mapping $q^2 \mapsto |z| \leq 1$ exploits analytic structure in complex plane to extend chiral-continuum form factors to low $q^2$.

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_\pm = (M_B \pm M_P)^2$

$$f(q^2) = \frac{1}{1 - \frac{q^2}{M^2}} \sum_n a_n z^n(q^2)$$

- $t_0^{\text{opt}}$ minimizes $|z|$ in physical region $\Rightarrow |z| \leq \{0.30, 0.15\}$ for $P = \{\pi, K\}$
- Smallness of $|z|$ controls truncation
- Unitarity guarantees convergence