WAYNE STATE UNIVERSITY

The Proton Radius Puzzle

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U.S. DEPARTMENT OF ENERGY Office of Science

SIMONS FOUNDATION

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2 \rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2 \rangle_M^p = \frac{6}{G_M^p(0)} \frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

The proton radius puzzle

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]

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- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
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- **5σ discrepancy!**
- This is the proton radius puzzle

What could be the reason for the discrepancy?

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New Physics?

- Declaimer: I will mostly focus on work I am involved in

Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic uncertainty in muonic hydrogen theory?
- Part 3: Connecting muon-proton scattering and muonic hydrogen
- Conclusions and outlook

Part 1: Proton radii from scattering

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- First applied to **baryon** form factors in
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- Now it is used to extract r_E^p, r_M^p, r_M^n, r_A...

Example 1: r_E^p in PDG 2018

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

p CHARGE RADIUS

See our 2014 edition (Chinese Physics **C38** 070001 (2014)) for values published before 2003.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.8335	±0.0095		[1] BEYER	17	LASR	2S-4P transition in H
0.895	±0.014	±0.014	[2] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, ep data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$ep \to ep$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old ep data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$ep \to ep$ reanalysis

[1] The BEYER 17 result is 3.3 combined standard deviations below the MOHR 16 (2014 CODATA) value. The experiment measures the 2S-4P transition in hydrogen and gets the proton radius and the Rydberg constant.

[2] Authors also provide values for combinations of all available data.

[Hill, GP PRD **82** 113005 (2010)]

[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

Example 2: r_M^p in PDG 2018

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)		*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	→	[1] LEE	15	SPEC	Just 2010 Mainz data
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.914±0.035	→	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	→	EPSTEIN	14		Using ep, en, $\pi\pi$ data
0.867±0.009±0.018		ZHAN	11	SPEC	Recoil polarimetry
0.777±0.013±0.010		BERNAUER	10	SPEC	$ep \to ep$ form factor
0.876±0.010±0.016		BORISYUK	10		Reanalyzes old $ep \to ep$ data
0.854±0.005		BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

Example 3: r_M^n in PDG 2016

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03 →	EPSTEIN	14	Using ep, en, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Part 2: Hadronic uncertainty in muonic hydrogen theory?

The bottom line

- Scattering:
  - World $e - p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz $e - p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.895 \pm 0.020$ fm
  - Proton, neutron and π data [Hill , GP '10]
    $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$
- Muonic hydrogen
  - [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - [Antognini et al. Science **339**, 417 (2013)]
    $r_E^p = 0.84087(39)$ fm
- The bottom line:
  using z expansion scattering disfavors muonic hydrogen
- Is there a problem with muonic hydrogen *theory*?

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- Muonic hydrogen measures ΔE and translates it to r_E^p

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  $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3$ meV

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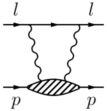
- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

- Apart from r_E^p need two-photon exchange



Two-photon exchange for the proton

- The proton two-photon interaction is given in terms of

$$W^{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\langle p,s|\, T\left\{J^{\mu}_{\rm e.m.}(x)J^{\nu}_{\rm e.m.}(0)\right\}|p,s\rangle$$

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- Using translation invariance, and strong and EM interaction symmetries: parity and time reversal

$$W^{\mu\nu}(p,q) = \frac{1}{2M}\bar{u}_p(p,s)\Bigg[\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(p^\mu - \frac{p\cdot q\, q^\mu}{q^2}\right)\left(p^\nu - \frac{p\cdot q\, q^\nu}{q^2}\right)W_2 + \left([\gamma^\nu,\not{q}]\, p^\mu - [\gamma^\mu,\not{q}]\, p^\nu + [\gamma^\mu,\gamma^\nu]\, p\cdot q\right)H_1 + \left([\gamma^\nu,\not{q}]\, q^\mu - [\gamma^\mu,\not{q}]\, q^\nu + [\gamma^\mu,\gamma^\nu]\, q^2\right)H_2\Bigg]u_p(p,s)$$

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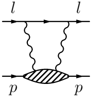
- W_1, W_2, H_1, H_2 depend on the variables $\nu = 2p\cdot q$ and $Q^2 = -q^2$

Two photon exchange



- $W^{\mu\nu}$ contains huge amount of information

  For muonic hydrogen we need mostly spin-independent parts: W_1, W_2

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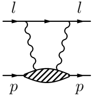

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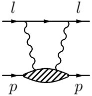

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  For muonic hydrogen we need mostly spin-independent parts: W_1, W_2
- Im W_i is related to data: form factors and structure functions
- Reconstruct W_i from the imaginary part using dispersion relations

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im} W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im} W_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

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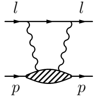

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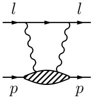
- But W_1 requires subtraction... and $W_1(0, Q^2)$ is not well-constrained

$W_1(0, Q^2)$



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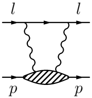

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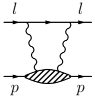

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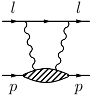

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    [J. C. Collins, NPB **149**, 90 (1979)]
  - Spin-2 calculated and spin-0 corrected in
    [Hill, GP PRD **95**, 094017 (2017)]

Two Photon Exchange: Modeling

- Simple modeling: use OPE for $Q^2 \geq 1$ GeV^2
- Model unknown Q^4: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV

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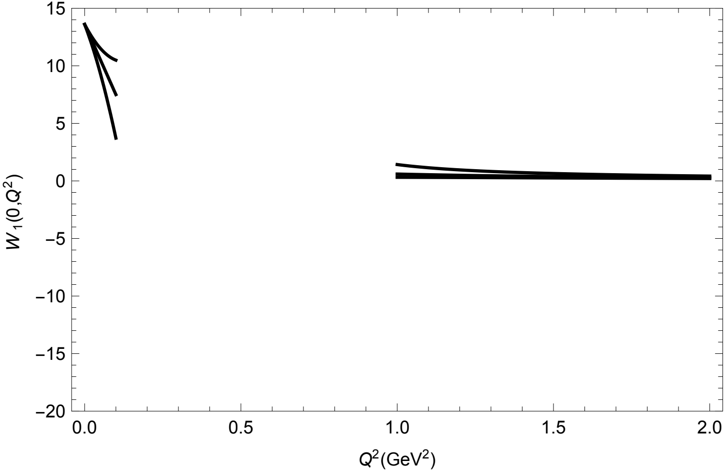
- How to connect the curves?



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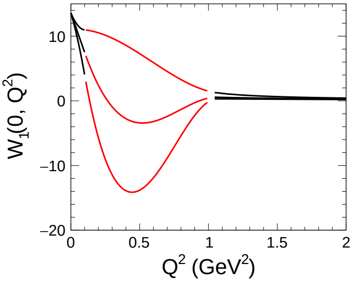

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- Simple modeling: use OPE for $Q^2 \geq 1$ GeV^2
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- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV
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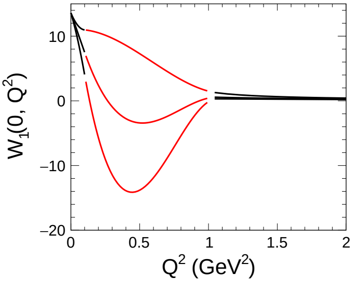

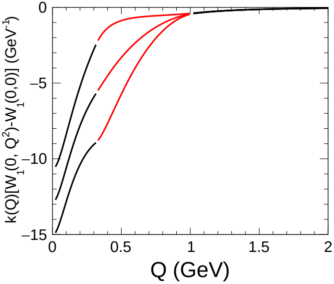

- Right figure: Energy contribution proportional to area under curve

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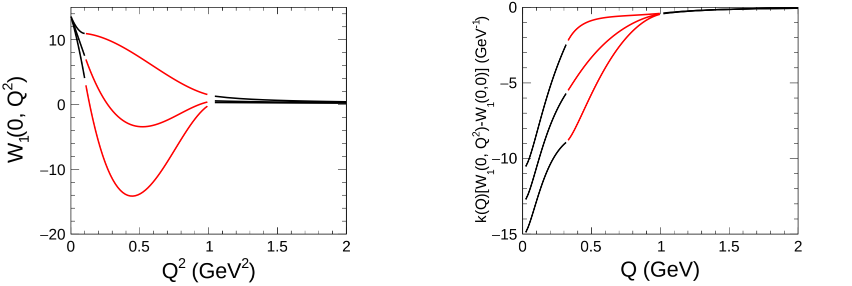

- Right figure: Energy contribution proportional to area under curve
- Energy contribution: $\delta E(2\mathrm{S})^{W_1(0,Q^2)} \in [-0.046\,\text{meV},\ -0.021\,\text{meV}]$
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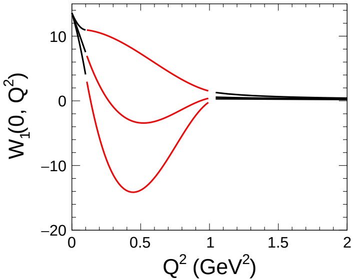

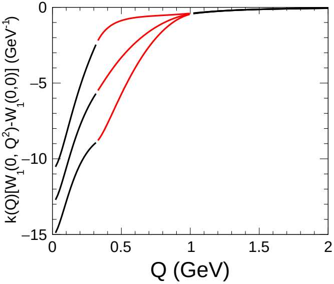

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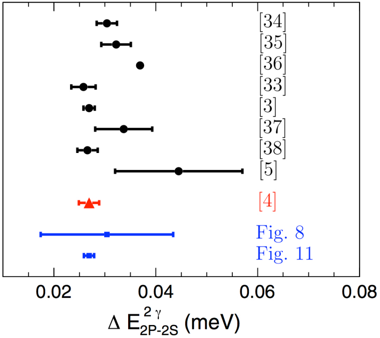
- Caveats: OPE valid for larger Q^2, W_1 different than interpolation

Two Photon Exchange: Other approaches

- Similar results found by other groups



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[Fig. 8] Hill, GP PRD 95, 094017 (2017).

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]
  [Dye, Gonderinger, GP, PRD **94** 013006 (2016)]

Part 3: Connecting muon-proton scattering and muonic hydrogen

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV $\ll m_\mu, m_p$

  Both proton and muon non-relativistic

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- QED-NRQED effective theory:
- Use QED for muon alone
- Use NRQED for proton alone
- Use contact terms for combined muon-proton interaction

  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in
  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Dye, Gonderinger, GP, PRD **94** 013006 (2016)]

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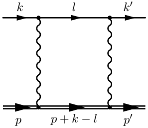

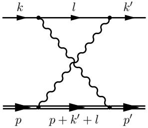

- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1 + \frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\theta}\right]$$

$Z = 1$, $E =$ muon energy, $v = |\vec{p}|/E$, $q = p' - p, \theta$ scattering angle

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  [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)]
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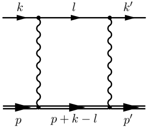

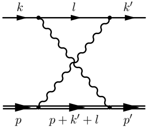

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- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering
  (For $m_p \to \infty$ only proton charge is relevant)

QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections

$$\mathcal{L} = \psi^\dagger \left\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + c_F e \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}}{2M} + c_D e \frac{[\boldsymbol{\nabla} \cdot \boldsymbol{E}]}{8M^2} + ic_S e \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D})}{8M^2} \right\} \psi + \cdots$$

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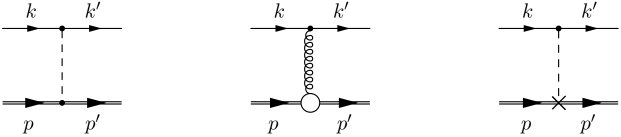

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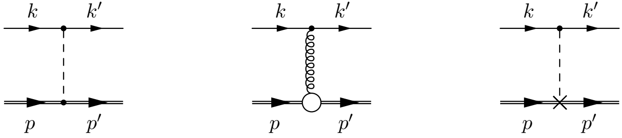


QED-NRQED $= 1/m_p$ expansion of form factors
[Dye, Gonderinger, GP, PRD **94** 013006 (2016)]

- To connect to muonic hydrogen we need QED-NRQED contact interactions
[Dye, Gonderinger, GP, arXiv:1812.05056 (hep-ph)]

QED-NRQED contact interactions

- At $1/M^2$ we have two possible contact interactions

$$\mathcal{L}_{\ell\psi} = \frac{b_1}{M^2}\psi^\dagger\psi\,\bar{\ell}\gamma^0\ell + \frac{b_2}{M^2}\psi^\dagger\sigma^i\psi\,\bar{\ell}\gamma^i\gamma^5\ell + \mathcal{O}\left(1/M^3\right)$$

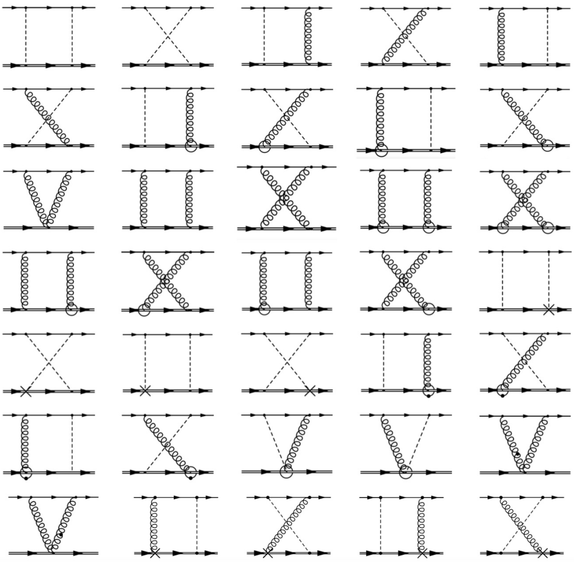
[Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- We need to determine the Wilson coefficients b_1 and b_2

- Calculate $\ell + p \to \ell + p$ *off-shell* forward scattering at $\mathcal{O}(Z^2\alpha^2)$ and power $1/M^2$ in effective and full theory
Matching in both Feynman and Coulomb gauges
[Dye, Gonderinger, GP, arXiv:1812.05056 (hep-ph)]

QED-NRQED calculation

[Dye, Gonderinger, GP, arXiv:1812.05056 (hep-ph)]



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- What happens for a real proton?

Matching calculation: Real proton

- The IR singularities match QED-NRQED, b_1, b_2 are determined by

$$\Bigg[Z^2\left(\frac{2m\pi}{\lambda^3}-\frac{2m^2\pi}{M\lambda^3}+\frac{2m^3\pi}{M^2\lambda^3}-\frac{5\pi}{4M\lambda}+\frac{3m\pi}{4M^2\lambda}-\frac{2}{3mM}+\frac{4\log(m/\lambda)}{mM}\right)$$

$$+c_F^2\frac{m\pi}{M^2\lambda}-c_D Z\frac{m\pi}{2M^2\lambda}\Bigg]+\frac{b_1(\alpha^2 Q_\ell^2)^{-1}}{M^2}=$$

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$$\left.\left.+\frac{\log(m/\lambda)}{M^2}-\frac{\log(2\Lambda/m)}{2M^2}-\frac{1}{12M^2}\right)+c_S Z\left(-\frac{\log(m/\lambda)}{M^2}-\frac{3\log(2\Lambda/m)}{2M^2}+\frac{13}{12M^2}\right)\right]$$

$$+\frac{b_2(\alpha^2Q_\ell^2)^{-1}}{M^2}=\frac{8}{3\pi}\int_0^\infty dQ\,Q^3\int_{-1}^1 dx\sqrt{1-x^2}\,\frac{1}{\left(Q^2+\lambda^2\right)^2\left(Q^2+4m^2x^2\right)}\times$$

$$\left[(2Q^2+x^2Q^2+6m^2x^2)H_1(2iMQx,Q^2)+\left(3ixQ^3+2iQxm^2+2iQx^3m^2\right)H_2(2iMQx,Q^2)\right]$$

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&+ F_2(0)^2\left[\frac{17}{24} - \frac{1}{2}\log\left(\frac{M}{2\Lambda}\right) + \frac{3}{2}\log\left(\frac{Q}{M}\right)\right] \\
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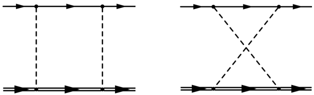

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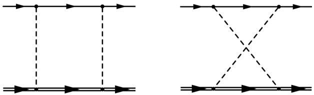

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- Usually direct and crossed with even powers of M have opposite signs:

$$\left(\pm l^0 - \frac{\vec{l}^{\,2}}{2M} + i\epsilon\right)^{-1} = \pm\frac{1}{l^0} + \frac{\vec{l}^{\,2}}{2(l^0)^2 M} \pm \frac{(\vec{l}^{\,2})^2}{4(l^0)^3 M^2} + \mathcal{O}\left(\frac{1}{M^3}\right)$$

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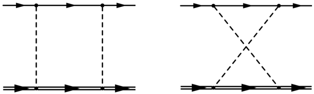

$$\frac{I_{D,C}^{m,0}}{-i(4\pi)^2} = \int \frac{d^4 l}{(2\pi)^4} \frac{\{m, m - l^0\}}{(l^2 - 2ml^0 + i\epsilon)(l^2 - \lambda^2 + i\epsilon)^2(\pm l^0 - \frac{\vec{l}^{\,2}}{2M} + i\epsilon)}$$

- Usually direct and crossed with even powers of M have opposite signs:

$$(\pm l^0 - \frac{\vec{l}^{\,2}}{2M} + i\epsilon)^{-1} = \pm\frac{1}{l^0} + \frac{\vec{l}^{\,2}}{2(l^0)^2 M} \pm \frac{(\vec{l}^{\,2})^2}{4(l^0)^3 M^2} + \mathcal{O}\left(\frac{1}{M^3}\right)$$

- Direct and crossed diagrams usually appear as a sum for spin-independent terms and cancel each other

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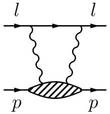

$$i\mathcal{M}_{\text{Full}} = -Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu(\not{k} - \not{l} + m)\gamma_\nu u}{(k-l)^2 - m^2} \left(\frac{1}{l^2 - \lambda^2}\right)^2 W^{\mu\nu}(p,l).$$

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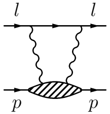

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- In the limit $m \to 0 \Rightarrow k \to 0$

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- Translation invariance implies $W^{\mu\nu}(p, l) = W^{\nu\mu}(p, -l)$
- Full spin-independent amplitude vanishes for $m \to 0$

The bottom line

$$\mathcal{L}_{\ell\psi} = \frac{b_1}{M^2}\psi^\dagger\psi\,\bar{\ell}\gamma^0\ell + \frac{b_2}{M^2}\psi^\dagger\sigma^i\psi\,\bar{\ell}\gamma^i\gamma^5\ell + \mathcal{O}\left(1/M^3\right)$$

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- MUSE experiment is much less sensitive to such effects but extraction of the proton charge radius will be more robust

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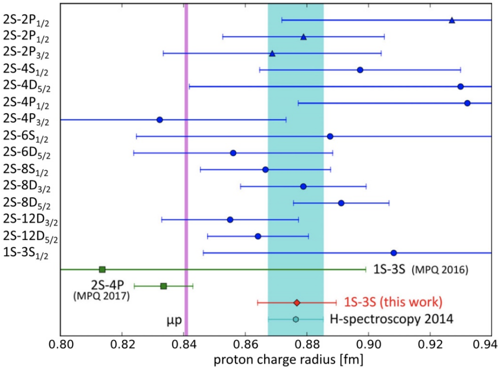

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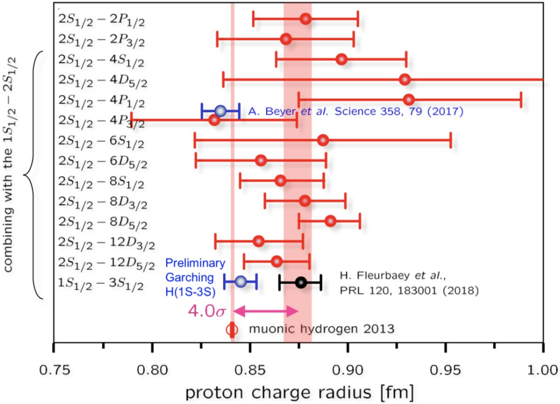

[Arthur Matveev talk at PRP 2018]

Conclusions

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- Proton radius puzzle: $> 5\sigma$ discrepancy between
- r_E^p from muonic hydrogen
- r_E^p from hydrogen and $e - p$ scattering

- After more than 9 years the origin is still not clear

1) Is it a problem with the electronic extraction?
2) Is it a hadronic uncertainty?
3) Is it new physics?

- Motivates a reevaluation of our understanding of the proton

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- Thank you!