The Proton Radius Puzzle

Gil Paz

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Introduction: The proton radius puzzle
Form Factors

Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

\[
\langle N(p_f)| \sum_q e_q \bar{q} \gamma^\mu q| N(p_i) \rangle = \bar{u}(p_f) \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu \right] u(p_i)
\]

Sachs electric and magnetic form factors

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \quad \text{and} \quad G_M(q^2) = F_1(q^2) + F_2(q^2)
\]

\[
G_E^p(0) = 1 \quad \text{and} \quad G_M^p(0) = \mu_p \approx 2.793
\]

The slope of $G_E^p$

\[
\langle r^2 \rangle_E^p = 6 \left. \frac{dG_E^p}{dq^2} \right|_{q^2=0}
\]

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

The proton magnetic radius

\[
\langle r^2 \rangle_M^p = \left. \frac{6}{G_M^p(0)} \frac{dG_M^p(q^2)}{dq^2} \right|_{q^2=0}
\]
The proton radius puzzle

- **Lamb shift in muonic hydrogen** [Pohl et al. *Nature* **466**, 213 (2010)]
  \[ r_E^p = 0.84184(67) \text{ fm} \]
  more recently \[ r_E^p = 0.84087(39) \text{ fm} \] [Antognini et al. *Science* **339**, 417 (2013)]

The proton radius puzzle

  \[ r^p_E = 0.84184(67) \text{ fm} \]
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- CODATA value [Mohr et al. RMP 80, 633 (2008)]
  \[ r^p_E = 0.87680(690) \text{ fm} \]
  more recently \[ r^p_E = 0.87510(610) \text{ fm} \] [Mohr et al. RMP 88, 035009 (2016)]

extracted mainly from (electronic) hydrogen
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5\(\sigma\) discrepancy!

This is the proton radius puzzle
What could be the reason for the discrepancy?

**Spectroscopy**
What could be the reason for the discrepancy?

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- Regular hydrogen spectroscopy
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- Regular hydrogen spectroscopy
  - Theory: very simple
  - Experiment: new measurements

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**Scattering**
- Electron proton scattering
  - Theory: How to extract $r_p$ from data? (Part 1 of this talk)
  - Experiment: New scattering experiments (e.g. PRad)
- Muon proton scattering
  - Theory: How to relate to spectroscopy? (Part 3 of this talk)
  - Experiment: New experiment called MUSE

**New Physics?**

---

Declaimer: I will mostly focus on work I am involved in

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Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic uncertainty in muonic hydrogen theory?
- Part 3: Connecting muon-proton scattering and muonic hydrogen
- Conclusions and outlook
Part 1: Proton radii from scattering
How to extract $r_P^E$ from scattering data?

- The form factors are non-perturbative objects.
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- **Nobody** knows the exact functional form of $G_E^p$ and $G_M^p$.
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- Should use model-independent $z$-expansion.
How to extract $r_E^p$ from scattering data?

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- The method for **meson** form factors, see e.g.
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Now it is used to extract $r_E^p$, $r_M^p$, $r_M^n$, $r_A$...
Example 1: $r_E^p$ in PDG 2018

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

\[ p \text{ CHARGE RADIUS} \]


<table>
<thead>
<tr>
<th>VALUE (fm)</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
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<tr>
<td>0.8751 ± 0.0061</td>
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<td>RVUE</td>
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<tr>
<td>0.84087 ± 0.00026 ± 0.00029</td>
<td>ANTOGNINI 13</td>
<td>LSR</td>
<td>$\mu$-atom Lamb shift</td>
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<td>0.8335 ± 0.0095</td>
<td>Beyer 17</td>
<td>LSR</td>
<td>2S-4P transition in H</td>
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<tr>
<td>0.895 ± 0.014 ± 0.014</td>
<td>LEE 15</td>
<td>SPEC</td>
<td>Just 2010 Mainz data</td>
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<tr>
<td>0.916 ± 0.024</td>
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<td>World data, no Mainz</td>
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<td>MOHR 12</td>
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<tr>
<td>0.875 ± 0.008 ± 0.006</td>
<td>ZHAN 11</td>
<td>SPEC</td>
<td>Recoil polarimetry</td>
</tr>
<tr>
<td>0.879 ± 0.005 ± 0.007</td>
<td>BERNAUER 10</td>
<td>SPEC</td>
<td>$ep \rightarrow ep$ form factor reanalyzes old $ep$ data</td>
</tr>
<tr>
<td>0.912 ± 0.009 ± 0.007</td>
<td>BORISYUK 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.871 ± 0.009 ± 0.003</td>
<td>HILL 10</td>
<td>RVUE</td>
<td>2006 CODATA value</td>
</tr>
<tr>
<td>0.84184 ± 0.00036 ± 0.00056</td>
<td>POHL 10</td>
<td>LSR</td>
<td>See ANTOGNINI 13</td>
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<td>0.8768 ± 0.0069</td>
<td>MOHR 08</td>
<td>RVUE</td>
<td>2006 CODATA value</td>
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<tr>
<td>0.844 ± 0.008</td>
<td>BELUSHKIN 07</td>
<td></td>
<td>Dispersion analysis</td>
</tr>
<tr>
<td>0.897 ± 0.018</td>
<td>BLUNED 05</td>
<td>SICK 03 + 2$\gamma$ correction</td>
<td></td>
</tr>
<tr>
<td>0.8750 ± 0.0068</td>
<td>MOHR 05</td>
<td>RVUE</td>
<td>2002 CODATA value</td>
</tr>
<tr>
<td>0.895 ± 0.010 ± 0.013</td>
<td>SICK 03</td>
<td></td>
<td>$ep \rightarrow ep$ reanalysis</td>
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</table>

1 The Beyer 17 result is 3.3 combined standard deviations below the Mohr 16 (2014 CODATA) value. The experiment measures the 2S-4P transition in hydrogen and gets the proton radius and the Rydberg constant.

2 Authors also provide values for combinations of all available data.

[Hill, GP PRD 82 113005 (2010)]
[Lee, Arrington, Hill, PRD 92, 013013 (2015)]
Example 2: $r_p^M$ in PDG 2018

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

\[ p \text{ MAGNETIC RADIUS} \]

This is the rms magnetic radius, $\sqrt{\langle r_p^2 \rangle}$.

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<td>0.776±0.034±0.017</td>
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<td>0.914±0.035</td>
<td>LEE</td>
<td>15 SPEC</td>
<td>World data, no Mainz</td>
</tr>
<tr>
<td>0.87±0.02</td>
<td>EPSTEIN</td>
<td>14 SPEC</td>
<td>Using $e p$, $e n$, $\pi \pi$ data</td>
</tr>
<tr>
<td>0.867±0.009±0.018</td>
<td>ZHAN</td>
<td>11 SPEC</td>
<td>Recoil polarimetry</td>
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<td>0.777±0.013±0.010</td>
<td>BERNAUER</td>
<td>10 SPEC</td>
<td>$e p \rightarrow e p$ form factor</td>
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<tr>
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<td>BORISYUK</td>
<td>10</td>
<td>Reanalyzes old $e p \rightarrow e p$ data</td>
</tr>
<tr>
<td>0.854±0.005</td>
<td>BELUSHKIN</td>
<td>07</td>
<td>Dispersion analysis</td>
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1 Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD 90, 074027 (2014)]
[Lee, Arrington, Hill, PRD 92, 013013 (2015)]
Example 3: $r_n^M$ in PDG 2016

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

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<tr>
<td>0.864$^{+0.009}_{-0.008}$ OUR AVERAGE</td>
<td></td>
<td></td>
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<tr>
<td>0.89 $\pm$ 0.03</td>
<td>EPSTEIN 14</td>
<td>Using $ep$, $en$, $\pi\pi$ data</td>
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<td>0.862$^{+0.009}_{-0.008}$</td>
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Part 2: Hadronic uncertainty in muonic hydrogen theory?
The bottom line

Scattering:
- World $e - p$ data [Lee, Arrington, Hill ’15]
  \[ r_E^P = 0.918 \pm 0.024 \text{ fm} \]
- Mainz $e - p$ data [Lee, Arrington, Hill ’15]
  \[ r_E^P = 0.895 \pm 0.020 \text{ fm} \]
- Proton, neutron and $\pi$ data [Hill, GP ’10]
  \[ r_E^P = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \text{ fm} \]

Muonic hydrogen
- [Pohl et al. Nature 466, 213 (2010)]
  \[ r_E^P = 0.84184(67) \text{ fm} \]
- [Antognini et al. Science 339, 417 (2013)]
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The bottom line:
using $z$ expansion scattering disfavors muonic hydrogen

Is there a problem with muonic hydrogen theory?
Muonic hydrogen theory

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Potentially yes!

[Hill, GP PRL 107 160402 (2011)]

Muonic hydrogen measures $\Delta E$ and translates it to $r_p E$.


$\Delta E = 206.0573(45) - 5.2262(r_p E) + 0.0347(r_p E)^3$ meV


$\Delta E = 206.0336(15) - 5.2275(r_p E) + 0.0332(20)$ meV

Apart from $r_p E$ need two-photon exchange. 

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Muonic hydrogen theory

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- Apart from $r_E^p$ need two-photon exchange
Two-photon exchange for the proton

The proton two-photon interaction is given in terms of

\[ W^{\mu\nu}(p, q) = i \int d^4x \ e^{iqx} \langle p, s | T \{ J_{\text{e.m.}}^\mu(x) J_{\text{e.m.}}^\nu(0) \} | p, s \rangle \]
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Using translation invariance, and strong and EM interaction symmetries: parity and time reversal

$$W^{\mu\nu}(p, q) = \frac{1}{2M} \bar{u}_p(p, s) \left[ \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1 + \left( p^\mu - \frac{p \cdot q q^\mu}{q^2} \right) \left( p^\nu - \frac{p \cdot q q^\nu}{q^2} \right) W_2 ight.$$

$$\left. + \left( [\gamma^\nu, q] p^\mu - [\gamma^\mu, q] p^\nu + [\gamma^\mu, \gamma^\nu] p \cdot q \right) H_1 
+ \left( [\gamma^\nu, q] q^\mu - [\gamma^\mu, q] q^\nu + [\gamma^\mu, \gamma^\nu] q^2 \right) H_2 \right] u_p(p, s)$$
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\[ + \left( \left[ \gamma^\nu, q \right] p^\mu - \left[ \gamma^\mu, q \right] p^\nu + [\gamma^\mu, \gamma^\nu] p \cdot q \right) H_1 \]

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\[ \left. u_p(p, s) \right. \]

\[ W_1, W_2, H_1, H_2 \text{ depend on the variables } \nu = 2p \cdot q \text{ and } Q^2 = -q^2 \]
Two photon exchange

\[ W^{\mu\nu} \] contains huge amount of information

For muonic hydrogen we need mostly spin-independent parts: \( W_1, W_2 \)
Two photon exchange

- $W^{\mu\nu}$ contains huge amount of information
  - For muonic hydrogen we need mostly spin-independent parts: $W_1, W_2$
- Im $W_i$ is related to data: form factors and structure functions
Two photon exchange

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For muonic hydrogen we need mostly spin-independent parts: \( W_1, W_2 \)

\( \text{Im} \ W_i \text{ is related to data: form factors and structure functions} \)

Reconstruct \( W_i \) from the imaginary part using dispersion relations

\[
W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu' \frac{\text{Im} W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}
\]

\[
W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu' \frac{\text{Im} W_2(\nu', Q^2)}{\nu'^2 - \nu^2}
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- But $W_1$ requires subtraction... and $W_1(0, Q^2)$ is not well-constrained
$W_1(0, Q^2)$

- $W_1(0, Q^2)$ is calculable for small $Q^2$ using NRQED
  - The photon sees the proton “almost” like an elementary particle
  - Spin-0 calculated
  - Spin-2 calculated and spin-0 corrected

[Hill, GP, PRL 107 160402 (2011)]
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  [Hill, GP, PRL 107, 160402 (2011)]

- Calculable in *large* $Q^2$ limit using Operator Product Expansion (OPE)
  The photon “sees” the quarks and gluons inside the proton
\( W_1(0, Q^2) \)

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    [J. C. Collins, NPB 149, 90 (1979)]
$W_1(0, Q^2)$

- $W_1(0, Q^2)$ is calculable for small $Q^2$ using NRQED
  The photon sees the proton “almost” like an elementary particle
  [Hill, GP, PRL 107 160402 (2011)]

- Calculable in \textit{large} $Q^2$ limit using Operator Product Expansion (OPE)
  The photon “sees” the quarks and gluons inside the proton
  - Spin-0 calculated in
    [J. C. Collins, NPB 149, 90 (1979)]
  - Spin-2 calculated and spin-0 corrected in
    [Hill, GP PRD 95, 094017 (2017)]
Two Photon Exchange: Modeling

- Simple modeling: use OPE for $Q^2 \geq 1 \text{ GeV}^2$
  - Model unknown $Q^4$: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500 \text{ MeV}$
  - Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500 \text{ MeV}$
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- How to connect the curves?
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- Left figure: Interpolating

Caveats: OPE valid for larger $Q^2$, $W$ different than interpolation
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Left figure: Interpolating
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- Left figure: Interpolating

- Right figure: Energy contribution proportional to area under curve

Gil Paz (Wayne State University)
The Proton Radius Puzzle
Two Photon Exchange: Modeling

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- Left figure: Interpolating

- Right figure: Energy contribution proportional to area under curve
  - Energy contribution: $\delta E(2S)^{W_1(0,Q^2)} \in [-0.046\text{ meV}, -0.021\text{ meV}]$
  - To explain the puzzle need this to be $\sim -0.3\text{ meV}$
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  - Caveats: OPE valid for larger $Q^2$, $W_1$ different than interpolation
Two Photon Exchange: Other approaches

- Similar results found by other groups

![Graph showing data points with references]

[34] K. Pachucki, PRA 60, 3593 (1999).
[Fig. 8] Hill, GP PRD 95, 094017 (2017).
Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen
  can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Solon, PRD 87 053017 (2013)]
  [Dye, Gonderinger, GP, PRD 94 013006 (2016)]
Part 3: Connecting muon-proton scattering and muonic hydrogen
Muonic hydrogen:
Muon momentum $\sim m_\mu c \alpha \sim 1 \text{ MeV} \ll m_\mu, m_p$
Both proton and muon non-relativistic
Muonic hydrogen:
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Both proton and muon non-relativistic

MUSE:
Muon momentum $\sim m_\mu \sim 100$ MeV
Muon is relativistic, proton is still non-relativistic
Muonic hydrogen:
Muon momentum $\sim m_\mu c \alpha \sim 1$ MeV $\ll m_\mu, m_p$
Both proton and muon non-relativistic

MUSE:
Muon momentum $\sim m_\mu \sim 100$ MeV
Muon is relativistic, proton is still non-relativistic

QED-NRQED effective theory:
- Use QED for muon alone
- Use NRQED for proton alone
- Use contact terms for combined muon-proton interaction
  $m_\mu/m_p \sim 0.1$ as expansion parameter

A new effective field theory suggested in
[Hill, Lee, GP, Solon, PRD 87 053017 (2013)]
QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$

[Dye, Gonderinger, GP, PRD 94 013006 (2016)]
QED-NRQED Effective Theory

Example: TPE at the lowest order in $1/m_p$
[Dye, Gonderinger, GP, PRD 94 013006 (2016)]

\[ d\sigma \frac{d\Omega}{d\theta} = \frac{Z^2 \alpha^2 4E^2}{\bar{q}^4} \left(1 - \nu^2 \sin^2 \frac{\theta}{2}\right) \left[1 + \frac{Z\alpha\pi \nu \sin \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2}\right)}{1 - \nu^2 \sin^2 \theta}\right] \]

$Z = 1$, $E =$ muon energy, $\nu = |\vec{p}|/E$, $q = \vec{p}' - \vec{p}$, $\theta$ scattering angle
QED-NRQED Effective Theory

Example: TPE at the lowest order in $1/m_p$

[Dye, Gonderinger, GP, PRD 94 013006 (2016)]

\[\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2 (1 - v^2 \sin^2 \frac{\theta}{2})}{\bar{q}^4} \left[ 1 + \frac{Z\alpha\pi v \sin \frac{\theta}{2} (1 - \sin \frac{\theta}{2})}{1 - v^2 \sin^2 \theta} \right] \]

$Z = 1$, $E =$ muon energy, $v = |\vec{p}|/E$, $q = p' - p$, $\theta$ scattering angle

Same result as scattering relativistic lepton off static $1/r$ potential

reproduced in [Itzykson, Zuber, “Quantum Field Theory”]

Gil Paz  (Wayne State University)
QED-NRQED Effective Theory

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\]

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- QED-NRQED result

- Same result as scattering relativistic lepton off static $1/r$ potential
  reproduced in [Itzykson, Zuber, “Quantum Field Theory”]

- Same result as $m_p \to \infty$ of “point particle proton” QED scattering
  (For $m_p \to \infty$ only proton charge is relevant)
**QED-NRQED Effective Theory beyond $m_p \to \infty$ limit**

- QED-NRQED allows to calculate $1/m_p$ corrections

\[
\mathcal{L} = \psi^\dagger \left\{ iD_t + \frac{D^2}{2M} + c_F e \frac{\sigma \cdot B}{2M} + c_D e \frac{\nabla \cdot E}{8M^2} + ic_S e \frac{\sigma \cdot (D \times E - E \times D)}{8M^2} \right\} \psi + \ldots
\]

\[Z = F_1(0), \quad c_F = F_1(0) + F_2(0), \quad c_D = F_1(0) + 2F_2(0) + 8M^2 F_1'(0)\]
QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections

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\mathcal{L} = \psi^\dagger \left\{ iD_t \frac{D^2}{2M} + c_F e \frac{\sigma \cdot B}{2M} + c_D e \frac{\nabla \cdot E}{8M^2} + ic_S e \frac{\sigma \cdot (D \times E - E \times D)}{8M^2} \right\} \psi + \ldots
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$$

- Example: one photon exchange $\mu + p \to \mu + p$:

QED-NRQED = $1/m_p$ expansion of form factors

[Dye, Gonderinger, GP, PRD 94 013006 (2016)]
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\]

- Example: one photon exchange \( \mu + p \to \mu + p \):

\[
\begin{align*}
\begin{array}{ccc}
\hline
\text{\( k \)} & \text{\( k' \)} & \text{\( p \)} & \text{\( p' \)} \\
\hline
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{ccc}
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\hline
\end{array}
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- QED-NRQED = \( 1/m_p \) expansion of form factors

[Dye, Gonderinger, GP, PRD 94 013006 (2016)]

- To connect to muonic hydrogen we need

QED-NRQED contact interactions

QED-NRQED contact interactions

- At $1/M^2$ we have two possible contact interactions

$$\mathcal{L}_{\ell \psi} = \frac{b_1}{M^2} \psi^\dagger \psi \bar{\ell} \gamma^0 \ell + \frac{b_2}{M^2} \psi^\dagger \sigma^i \psi \bar{\ell} \gamma^i \gamma^5 \ell + \mathcal{O}(1/M^3)$$

[Hill, Lee, GP, Solon, PRD 87 053017 (2013)]

- We need to determine the Wilson coefficients $b_1$ and $b_2$

- Calculate $\ell + p \rightarrow \ell + p$ off-shell forward scattering at $\mathcal{O}(Z^2\alpha^2)$ and power $1/M^2$ in effective and full theory

Matching in both Feynman and Coulomb gauges

QED-NRQED calculation

Matching calculation: toy example NR point particle

\[ \mathcal{L}_{\ell \psi} = \frac{b_1}{M^2} \psi \gamma^0 \ell + \frac{b_2}{M^2} \sigma^i \psi \gamma^i \gamma^5 \ell + O \left( \frac{1}{M^3} \right) \]
Matching calculation: toy example NR point particle

\[ \mathcal{L}_{\ell\psi} = \frac{b_1}{M^2} \psi^\dagger \psi \bar{\ell} \gamma^0 \ell + \frac{b_2}{M^2} \psi^\dagger \sigma^i \psi \bar{\ell} \gamma^i \gamma^5 \ell + \mathcal{O} \left( \frac{1}{M^3} \right) \]

- Toy example: non-relativistic point particle

\[ b_{1 \cdot p.p.}^p = 0, \quad b_{2 \cdot p.p.}^p = Q_f^2 Z^2 \alpha^2 \left[ \frac{16}{3} + \log \left( \frac{M}{2\Lambda} \right) \right] \]

Surprisingly \( b_1 = 0 \) at \( \mathcal{O}(Z^2 \alpha^2) \)
Matching calculation: toy example NR point particle

\[ \mathcal{L}_{\ell\psi} = \frac{b_1}{M^2} \bar{\psi} \gamma^0 \ell + \frac{b_2}{M^2} \bar{\psi} \sigma_i \gamma^5 \ell + \mathcal{O}(1/M^3) \]

- Toy example: non-relativistic point particle

\[
 b_{p.p.}^1 = 0, \quad b_{p.p.}^2 = Q_i^2 Z^2 \alpha^2 \left[ \frac{16}{3} + \log \left( \frac{M}{2\Lambda} \right) \right]
\]

*Surprisingly* \( b_1 = 0 \) at \( \mathcal{O}(Z^2 \alpha^2) \)

- What happens for a real proton?
Matching calculation: Real proton

- The IR singularities match QED-NRQED, \( b_1, b_2 \) are determined by

\[
Z^2 \left( \frac{2m\pi}{\lambda^3} - \frac{2m^2\pi}{M\lambda^3} + \frac{2m^3\pi}{M^2\lambda^3} - \frac{5\pi}{4M\lambda} + \frac{3m\pi}{4M^2\lambda} - \frac{2}{3mM} + \frac{4\log(m/\lambda)}{mM} \right) \\
+ c_F^2 \frac{m\pi}{M^2\lambda} - c_D Z \frac{m\pi}{2M^2\lambda} \right] + \frac{b_1(\alpha^2 Q^2)}{M^2} = \\
\frac{2m}{\pi M} \int_0^\infty dQ Q^3 \int_{-1}^1 dx \sqrt{1-x^2} \frac{(1-4x^2)W_1(2iMQx, Q^2) + (1-x^2) M^2 W_2(2iMQx, Q^2)}{(Q^2 + \lambda^2)^2 (Q^2 + 4m^2x^2)}
\]
Matching calculation: Real proton

- The IR singularities match QED-NRQED, \( b_1, b_2 \) are determined by

\[
\left[ Z^2 \left( \frac{2m\pi}{\lambda^3} - \frac{2m^2\pi}{M\lambda^3} + \frac{2m^3\pi}{M^2\lambda^3} - \frac{5\pi}{4M\lambda} + \frac{3m\pi}{4M^2\lambda} - \frac{2}{3mM} + \frac{4\log(m/\lambda)}{mM} \right) + \frac{c_F^2 m\pi}{M^2\lambda} - \frac{c_D Z m\pi}{2M^2\lambda} \right] + \frac{b_1(\alpha^2 Q^2)^{-1}}{M^2} =
\]

\[
= \frac{2}{\pi} \frac{m}{M} \int_0^\infty dQ \ Q^3 \int_{-1}^1 dx \sqrt{1 - x^2} \frac{(1 - 4x^2) W_1(2iMQx, Q^2) + (1 - x^2) M^2 W_2(2iMQx, Q^2)}{(Q^2 + \lambda^2)^2 (Q^2 + 4m^2x^2)}
\]

\[
\left[ \frac{c_F Z}{3M\lambda} - \frac{2m\pi}{3M^2\lambda} + \frac{2\log(2\Lambda/\lambda)}{M^2} + \frac{\log(2\Lambda/m)}{M^2} - \frac{16}{3M^2} \right] + \frac{c_s Z}{3M^2\lambda} \left( - \frac{\log(m/\lambda)}{M^2} - \frac{3\log(2\Lambda/m)}{2M^2} + \frac{13}{12M^2} \right)\]

\[
+ \frac{b_2(\alpha^2 Q^2)^{-1}}{M^2} = \frac{8}{3\pi} \int_0^\infty dQ \ Q^3 \int_{-1}^1 dx \sqrt{1 - x^2} \frac{1}{(Q^2 + \lambda^2)^2 (Q^2 + 4m^2x^2)} \times
\]

\[
(2Q^2 + x^2 Q^2 + 6m^2x^2) H_1(2iMQx, Q^2) + \left( 3ixQ^3 + 2iQxm^2 + 2iQx^3m^2 \right) H_2(2iMQx, Q^2)
\]
Matching calculation: Real proton

\[ \mathcal{L}_{\ell\psi} = \frac{b_1}{M^2} \bar{\psi} \gamma^0 \ell + \frac{b_2}{M^2} \bar{\psi} \gamma^i \gamma^5 \ell + O(1/M^3) \]

- Given hadronic tensor can find explicit expression for \( b_1 \) and \( b_2 \)
Matching calculation: Real proton

\[ \mathcal{L}_{\ell\psi} = \frac{b_1}{M^2} \psi^\dagger \psi \bar{\ell} \gamma^0 \ell + \frac{b_2}{M^2} \psi^\dagger \sigma^i \psi \bar{\ell} \gamma^i \gamma^5 \ell + \mathcal{O} \left( \frac{1}{M^3} \right) \]

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- Full theory contributions to \( b_1 \) and \( b_2 \) from \( F_1(0) \), \( F_2(0) \) and \( M^2 F'_1(0) \)
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\[ b_1(\alpha^2 Q_\ell^2)^{-1} = 0 + \text{possibly non } F_1(0), F_2(0), M^2 F'_1(0) \text{ terms} \]

\[ b_2(\alpha^2 Q_\ell^2)^{-1} = F_1(0)^2 \left[ \frac{16}{3} + \log \left( \frac{M}{2\Lambda} \right) \right] + F_1(0) F_2(0) \cdot \frac{16}{3} + \]

\[ + \ F_2(0)^2 \left[ \frac{17}{24} - \frac{1}{2} \log \left( \frac{M}{2\Lambda} \right) + \frac{3}{2} \log \left( \frac{Q}{M} \right) \right] + \text{non } F_1(0), F_2(0), M^2 F'_1(0) \text{ terms} \]

\( \log(Q/M) \) is a UV divergence regulated when using the full \( F_2 \)
Matching calculation: Real proton

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\[ + F_2(0)^2 \left[ \frac{17}{24} - \frac{1}{2} \log \left( \frac{M}{2\Lambda} \right) + \frac{3}{2} \log \left( \frac{Q}{M} \right) \right] \]

\[ + \text{non } F_1(0), F_2(0), M^2 F'_1(0) \text{ terms.} \]

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- Surprisingly, \textit{again} no contribution to \( b_1 \)
Why is $b_1 = 0$? EFT

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Why is \( b_1 = 0 \)? EFT

- Surprisingly, *again* no contribution to \( b_1 \). Why?
- EFT side:

\[
\frac{I_{D,C}^{m,0}}{-i(4\pi)^2} = \int \frac{d^4 l}{(2\pi)^4} \frac{\{m, m - l^0\}}{(l^2 - 2m l^0 + i\epsilon)(l^2 - \lambda^2 + i\epsilon)^2(\pm l^0 - \frac{\vec{l}^2}{2M} + i\epsilon)}
\]
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\]

- Usually direct and crossed with even powers of $M$ have opposite signs:

\[
(\pm l^0 - \frac{\vec{l}^2}{2M} + i\epsilon)^{-1} = \pm \frac{1}{l^0} + \frac{\vec{l}^2}{2(l^0)^2M} \pm \frac{(\vec{l}^2)^2}{4(l^0)^3M^2} + \mathcal{O}\left(\frac{1}{M^3}\right)
\]
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$$\frac{I_{D,C}^{m,0}}{-i(4\pi)^2} = \int \frac{d^4 l}{(2\pi)^4} \left\{ m, m - l^0 \right\} \frac{(l^2 - 2ml^0 + i\epsilon)(l^2 - \lambda^2 + i\epsilon)^2(\pm l^0 - \frac{\vec{l}^2}{2M} + i\epsilon)}{4(l^0)^3M^2} + O \left( \frac{1}{M^3} \right)$$

- Usually direct and crossed with even powers of $M$ have opposite signs:

$$\frac{1}{l^0 + \frac{\vec{l}^2}{2M} + i\epsilon} = \pm \frac{1}{l^0} + \frac{\vec{l}^2}{2(l^0)^2M} \pm \frac{(\vec{l}^2)^2}{4(l^0)^3M^2} + O \left( \frac{1}{M^3} \right)$$

- Direct and crossed diagrams usually appear as a sum for spin-independent terms and cancel each other
Why is $b_1 = 0$? Full theory

Surprisingly, *again* no contribution to $b_1$. Why?
Why is $b_1 = 0$? Full theory

- Surprisingly, *again* no contribution to $b_1$. Why?
- Full theory side:

\[
\begin{align*}
  i M_{\text{Full}} &= -Q_{\ell}^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu(k - l + m)\gamma_\nu u}{(k - l)^2 - m^2} \left( \frac{1}{l^2 - \lambda^2} \right)^2 W^{\mu\nu}(p, l).
\end{align*}
\]

where $k = (m, \vec{0})$
Why is $b_1 = 0$? Full theory

- Surprisingly, *again* no contribution to $b_1$. Why?
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\[
\begin{split}
i\mathcal{M}_{\text{Full}} &= -Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu (k - l + m)\gamma_\nu u}{(k - l)^2 - m^2} \left( \frac{1}{l^2 - \lambda^2} \right)^2 W^{\mu\nu}(p, l).
\end{split}
\]

where $k = (m, \vec{0})$

- In the limit $m \to 0 \Rightarrow k \to 0$

\[
\begin{split}
i\mathcal{M}_{\text{Full}} \bigg|_{m \to 0} &= -Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu (-l)\gamma_\nu u}{l^2} \left( \frac{1}{l^2 - \lambda^2} \right)^2 W^{\mu\nu}(p, l).
\end{split}
\]

- Translation invariance implies $W^{\mu\nu}(p, l) = W^{\nu\mu}(p, -l)$

- Full spin-independent amplitude vanishes for $m \to 0$
The bottom line

\[ \mathcal{L}_{\ell\psi} = \frac{b_1}{M^2} \bar{\psi} \gamma^0 \psi \ell + \frac{b_2}{M^2} \bar{\psi} \sigma_i \psi \ell \gamma^i \gamma^5 \ell + \mathcal{O}(1/M^3) \]

- Surprisingly \( b_1 = 0 \) at \( \mathcal{O}(Z^2\alpha^2) \)
The bottom line

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- Proton radius puzzle: $>5\sigma$ discrepancy between
  - $r_E^p$ from muonic hydrogen
  - $r_E^p$ from hydrogen and $e-p$ scattering

- After more than 9 years the origin is still not clear
  1) Is it a problem with the electronic extraction?
  2) Is it a hadronic uncertainty?
  3) Is it new physics?

- Motivates a reevaluation of our understanding of the proton
Conclusions

- Presented three topics:

  1) Extraction of proton radii from scattering: Use the $z$ expansion for form factors. Studies disfavor the muonic hydrogen value.
  2) The first full and correct evaluation of large $Q^2$ behavior of forward virtual Compton tensor. Can improve the modeling of two photon exchange effects.
  3) Direct connection between muon-proton scattering and muonic hydrogen using a new effective field theory: QED-NRQED. Surprising result for the spin-independent matching coefficient.

The proton radius puzzle is still puzzling...

Thank you!
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Thank you!

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