ZZ Instantons and the Non-Perturbative Dual of $c = 1$ String Theory

Bruno Balthazar

arXiv:1907.07688 with V. A. Rodriguez and X. Yin

Harvard University

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Introduction

- In this talk I will review one of the oldest examples of a holographic duality, between a string theory in 1+1d and a matrix quantum mechanics.
- I will compute non-perturbative contributions to closed string scattering amplitudes.
- This will allow us to propose the exact non-perturbative quantum dual of $c = 1$ string theory.

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Worldsheet CFT consists of

- Timelike free boson, X^0

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Liouville CFT semiclassical action

$$S_L[\phi] = \frac{1}{4\pi} \int_\Sigma d^2\sigma \left((\partial_a \phi)^2 + 2R\phi + \mu e^{2\phi} \right)$$

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- Ghost CFT, b, c

$c = 1$ Worldsheet CFT

Spatial background

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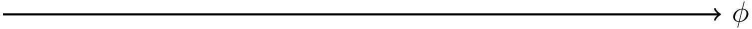

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$\longrightarrow \phi$

Weak Coupling — Strong Coupling

Theory is weakly coupled when $\phi \ll -1$ and strongly coupled when $\phi \gg 1$.

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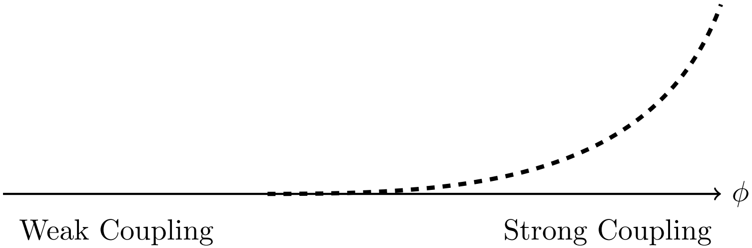

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$c = 1$ Worldsheet CFT

Let's look at the spatial background more closely

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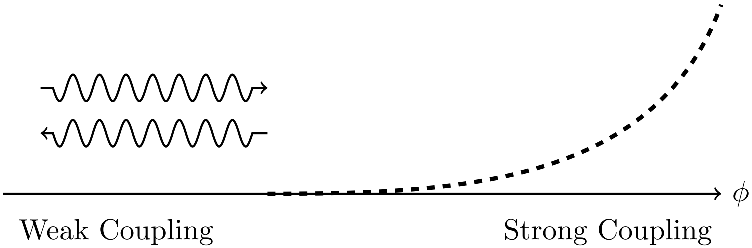

- Theory is weakly coupled when $\phi \ll -1$ and strongly coupled when $\phi \gg 1$.
- An exponential tachyonic potential 'cuts off' the strongly coupled region.
- States in the theory are scattering states.

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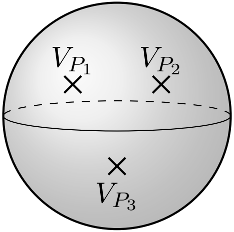

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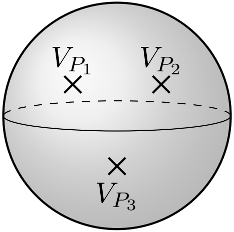


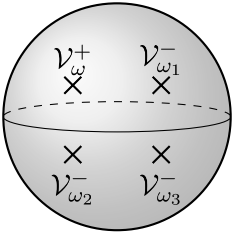
In/out closed strings:

$$\mathcal{V}_\omega^\pm = g_s : e^{\pm i\omega X^0} : V_{P=\frac{\omega}{2}}$$

Massless!

$c = 1$ Worldsheet CFT

$1 \to 3$ scattering amplitude:



$$S(\omega \to \omega_1, \omega_2, \omega_3) = \int d^2 z \left\langle \mathcal{V}_{\omega}^{+}(z, \bar{z}) \mathcal{V}_{\omega_1}^{-}(0) \mathcal{V}_{\omega_2}^{-}(1) \mathcal{V}_{\omega_3}^{-}(\infty) \right\rangle_{S_2}$$

$c = 1$ Worldsheet CFT

- Liouville CFT 4-point function

$$\left\langle V_{P=\frac{\omega}{2}}(z,\bar{z}) V_{P=\frac{\omega_1}{2}}(0) V_{P=\frac{\omega_2}{2}}(1) V_{P=\frac{\omega_3}{2}}(\infty) \right\rangle_{S_2,\text{Liouville}}$$

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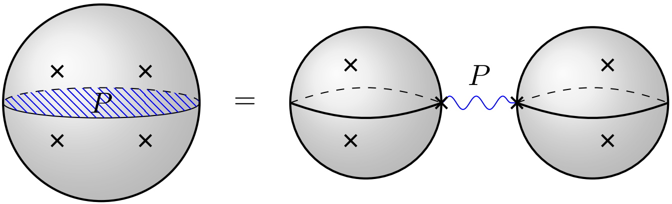

$$= \int_0^\infty \frac{dP}{\pi} C\left(\frac{\omega}{2}, \frac{\omega_1}{2}, P\right) C\left(\frac{\omega_2}{2}, \frac{\omega_3}{2}, P\right) F_P(z) F_P(\bar{z})$$

Computed numerically!

Dual Matrix Model

There is a dual description in terms of a suitable $N \to \infty$ limit of a $U(N)$ gauged matrix quantum mechanics.

$$H = \mathrm{Tr}\left(\frac{1}{2}P^2 + V(X)\right), \quad X \in \mathrm{Herm}_{N\times N}$$

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Restrict to $U(N)$ singlet states

$$X = \mathrm{diag}(\lambda_1, ..., \lambda_N), \quad \Psi(X) = \hat{\Psi}(\lambda_i)$$

$$\hat{H} = \sum_{i=1}^{N}\left(-\frac{1}{2}\frac{\partial^2}{\partial\lambda_i^2} + V(\lambda_i)\right) - \frac{1}{2}\sum_{i\neq j}^{N}\frac{1}{\lambda_i - \lambda_j}\frac{\partial}{\partial\lambda_i}$$

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The linear derivative term can be removed by a similarity transformation

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System of N non-relativistic non-interacting fermions!

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Consider

$$V(\lambda) = -\frac{1}{2}\lambda^2 + g\lambda^4$$

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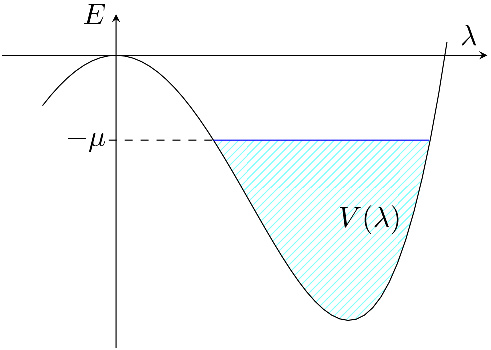

Now take the *double-scaling limit*, $g \to 0$ and $N \to \infty$ while keeping μ fixed.

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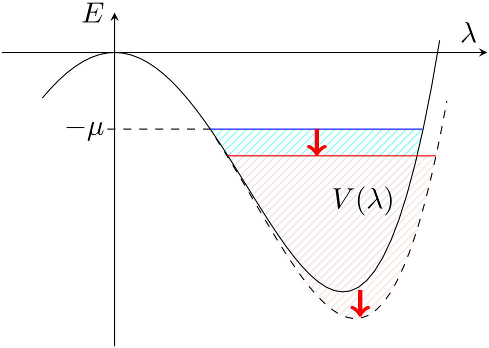

$g \downarrow$

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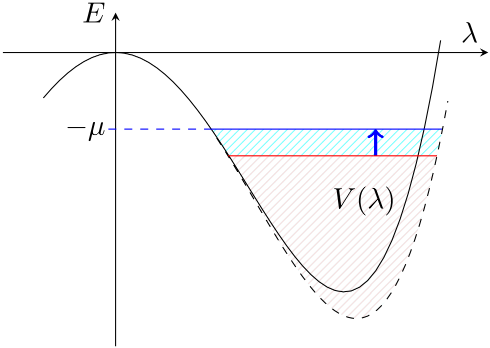

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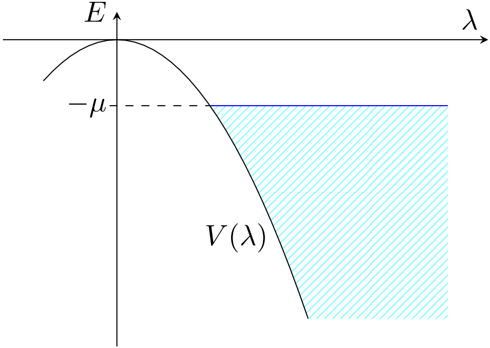
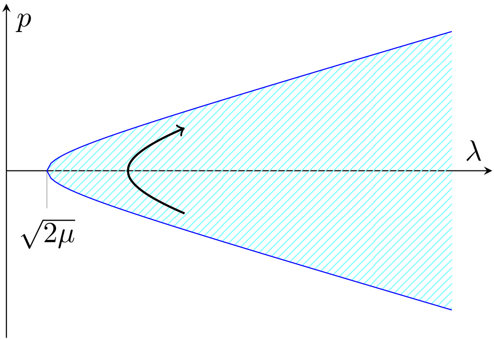


In the double-scaling limit, $V(\lambda) = -\frac{1}{2}\lambda^2$ and fermions fill energy levels up to a Fermi energy $-\mu$!

Dual Matrix Model

Semiclassical description in phase space



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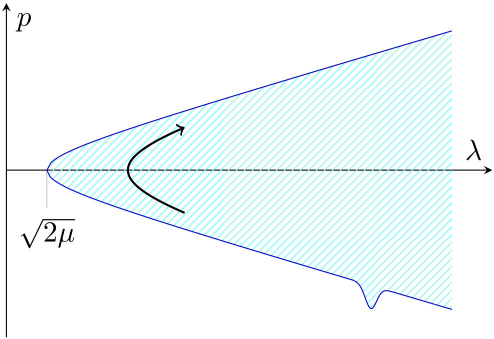

Closed strings $\leftrightarrow$ Collective excitations of the Fermi density

Dual Matrix Model

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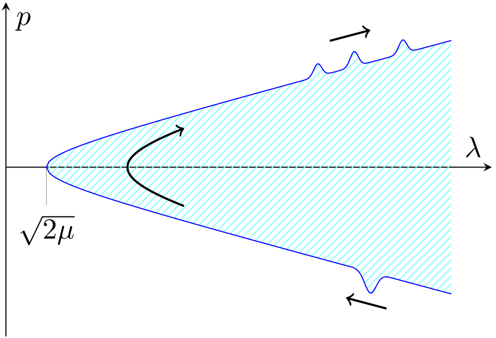

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String coupling $g_s \leftrightarrow 1/(2\pi\mu)$

Dual Matrix Model

Fluctuation of the Fermi density is given by a collection of particle-hole pairs, with total energy ω.

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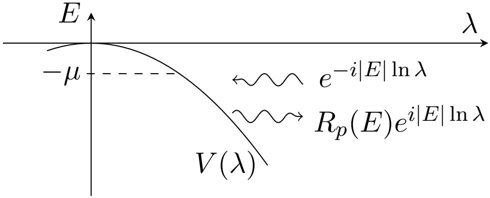
Let $R_p(E)$ and $R_h(E)$ be the reflection phases of a particle and hole wavefunctions, respectively.



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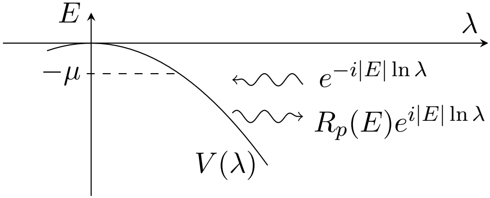

$1 \to 1$ scattering amplitude: [Moore, Plesser, Ramgoolam '92]

$$\mathcal{A}(\omega \to \omega) = \int_0^\omega dx R_p(-\mu+\omega-x)R_h(-\mu-x)$$

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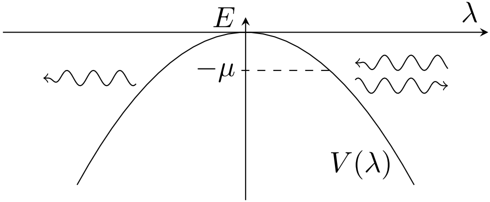
- Perturbative series in $1/\mu \leftrightarrow$ string perturbation theory.
- $R_p(E), R_h(E)$ have non-perturbative corrections in $2\pi\mu = 1/g_s$ (tunneling!).



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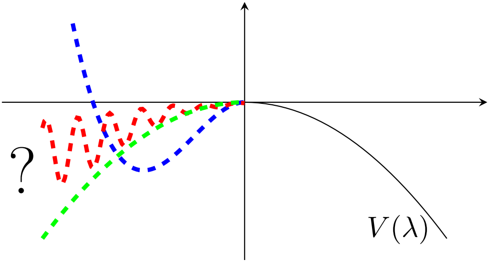
Potentials $V(\lambda)$ that are equal to $V(\lambda) = -\frac{1}{2}\lambda^2$ for $\lambda > 0$ give reflection phases $R_p(E)$ and $R_h(E)$ that agree in perturbation theory, but differ *non-perturbatively*.



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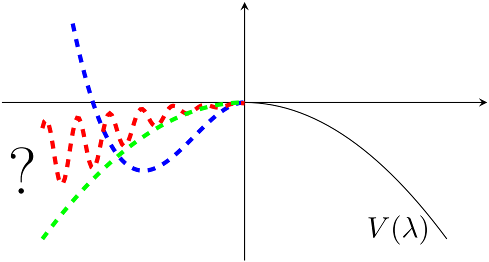

What is the quantum state dual to the closed string vacuum?

Dual Matrix Model

- Our goal is to understand what is the non-perturbative dual of $c = 1$ string theory.
- This requires computing non-perturbative effects on the worldsheet. [Green, Gutperle '97]

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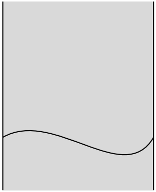
- This requires computing non-perturbative effects on the worldsheet. [Green, Gutperle '97]
- We will study closed string scattering in the background of instanton solutions of $c = 1$ string theory - ZZ instantons.

ZZ Instantons



Unitary conformal boundary conditions of Liouville theory on the strip [Fateev, Zamolodchikov^2, Teschner '00; Zamolodchikov^2 '01]

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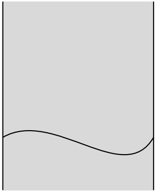

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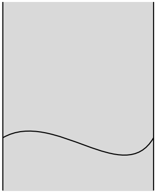

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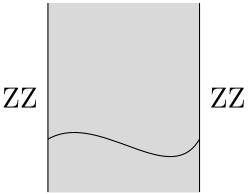

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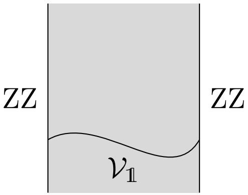

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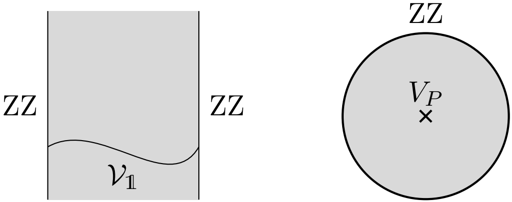
- Hilbert space on the strip with ZZ boundary condition:

$$\mathcal{H}_{\mathrm{ZZ}} = \mathcal{V}_{\mathbb{1}}$$

ZZ Instantons



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- Hilbert space on the strip with ZZ boundary condition:
$$\mathcal{H}_{\mathrm{ZZ}} = \mathcal{V}_{\mathbb{1}}$$
- Bulk disk 1-point $\Psi_{\mathrm{ZZ}}(P)$ is known.

ZZ Instantons

ZZ instanton boundary conditions

- ZZ boundary condition in Liouville
- Dirichlet boundary condition in X^0, labelled by collective coordinate x^0

ZZ Instantons

ZZ instanton boundary conditions

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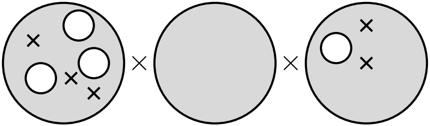
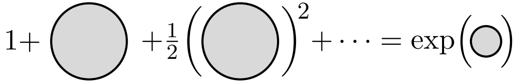
Generic closed string amplitude in ZZ instanton background

$$\int dx^0 \sum$$



ZZ Instantons

Disconnected empty disc diagrams factor out [Polchinski '94]



ZZ Instantons

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$$1 + \bigcirc + \frac{1}{2}\left(\bigcirc\right)^2 + \cdots = \exp\left(\circ\right)$$

Where

$$\exp\left(\circ\right) = \exp\left(-S_{\mathrm{ZZ}}\right) = e^{-\frac{1}{g_s}}$$

ZZ Instantons

$1 \to n$ scattering at order $e^{-\frac{1}{g_s}}$

$$S^{\text{inst},(0)}(\omega_1 \to \omega_2, ..., \omega_n) = e^{-\frac{1}{g_s}} \int_{-\infty}^{\infty} dx^0 \left\langle \mathcal{V}_1^+ \right\rangle_{\text{Z}Z,x^0}^{D^2} \left\langle \mathcal{V}_2^- \right\rangle_{\text{Z}Z,x^0}^{D^2} \cdots \left\langle \mathcal{V}_{n+1}^- \right\rangle_{\text{Z}Z,x^0}^{D^2}$$

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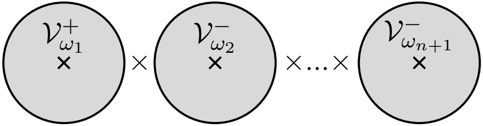

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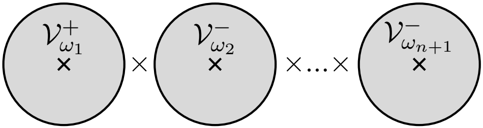


Non-perturbatively there is loss of unitarity.

ZZ Instantons

Non-perturbative dual matrix model:

ZZ Instantons

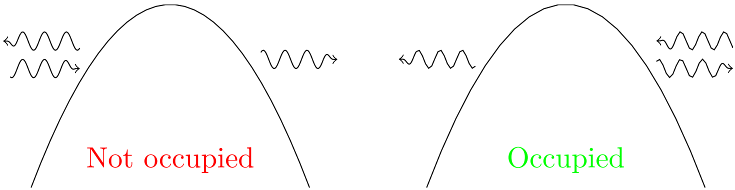
Non-perturbative dual matrix model:

- $V(\lambda) = -\frac{\lambda^2}{2}$ for all λ

ZZ Instantons

Non-perturbative dual matrix model:

- $V(\lambda) = -\frac{\lambda^2}{2}$ for all λ
- The closed string vacuum has only states with no incoming flux from the left occupied.



ZZ Instantons

$1 \to 1$ scattering at order $e^{-\frac{1}{g_s}} g_s$

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$$S^{\text{inst},(1)}(\omega_1 \to \omega_2) = e^{-\frac{1}{g_s}} \int_{-\infty}^{\infty} dx^0 \left(\langle \mathcal{V}_1 \mathcal{V}_2 \rangle^{D^2}_{\text{Z}Z,x^0} + \langle \mathcal{V}_1 \rangle^{D^2}_{\text{Z}Z,x^0} \langle \mathcal{V}_2 \rangle^{A^2}_{\text{Z}Z,x^0} + \langle \mathcal{V}_2 \rangle^{D^2}_{\text{Z}Z,x^0} \langle \mathcal{V}_1 \rangle^{A^2}_{\text{Z}Z,x^0} \right)$$

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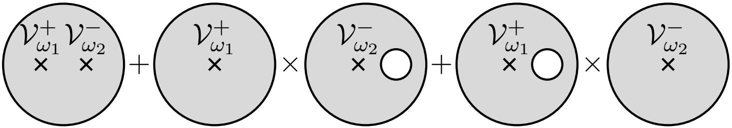

ZZ Instantons

E.g. disk bulk 2-point

$$\int_0^1 dy \left\langle \mathcal{V}^+_{\omega_1}(yi)\mathcal{V}^-_{\omega_2}(i)\right\rangle^{D^2}_{\mathrm{ZZ},x^0}$$

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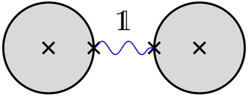
The Liouville disk bulk 2-point



$$= \Psi_{\mathrm{ZZ}}\left(\tfrac{\omega_1}{2}\right)\Psi_{\mathrm{ZZ}}\left(\tfrac{\omega_2}{2}\right)F^{\mathrm{Bdry}}_{\Delta=0}\left(\Delta_1,\Delta_2|y\right)$$

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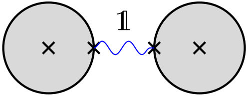

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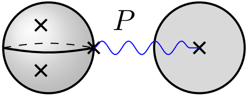
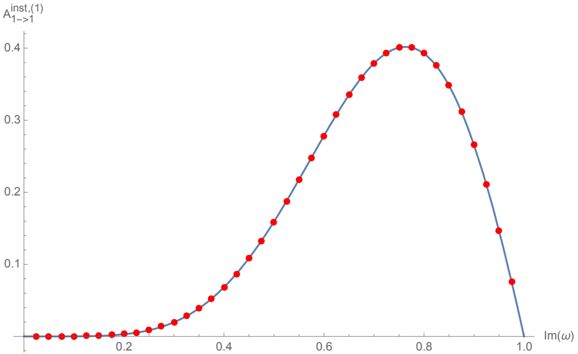


$$= \int dP\,\Psi_{\mathrm{ZZ}}(P)\,C\left(\frac{\omega_1}{2},\frac{\omega_2}{2},P\right)F^{\mathrm{Bulk}}_{\Delta_P}\left(\Delta_1,\Delta_2|y\right)$$

ZZ Instantons



Worldsheet
Matrix Model

Summary & Outlook

- We have computed non-perturbative effects to closed string scattering due to ZZ instantons, and proposed the non-perturbative matrix model dual.
- It would be interesting to compute scattering amplitudes including ZZ branes (not ZZ instanton!).
- What is the matrix model dual of FZZT branes?