Strings in Ramond-Ramond (RR) Backgrounds from the Neveu-Schwarz-Ramond (NSR) Formalism

Minjae Cho

Harvard University

(Based on **1811.00032** with S. Collier and X. Yin)

July 29, 2019

Overview

Goal

Describe type II superstring theory in Ramond-Ramond flux background

1. Type IIB supergravity
   - Description of backgrounds, spectrum, ...
   - Interesting backgrounds involving NS and RR fluxes
2. NSR formalism of string theory and its limitation
   - Pure NS backgrounds described by worldsheet CFT
   - RR flux?
3. NSR formalism of string field theory
   - Off-shell formulation
   - String field EOM $\rightarrow$ string backgrounds
   - Linearized EOM $\rightarrow$ string spectrum
4. Example: $AdS_3 \times S^3$ in mixed flux

IIB supergravity (10D)

- Ingredients in the IR (massless bosonic fields)
  - NS fields: metric $G_{\mu\nu}$, dilaton Φ, NS flux H_3
  - RR fields: fluxes F_1, F_3, F_5

IIB supergravity (10D)

- Ingredients in the IR (massless bosonic fields)
  - NS fields: metric $G_{\mu\nu}$, dilaton Φ, NS flux H_3
  - RR fields: fluxes F_1, F_3, F_5
- EOM at the lowest derivative order - consider Einstein equation with constant dilaton

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-2} G_{\mu\nu} T^{\alpha}_{\ \alpha}, \quad \text{with } T_{\mu\nu} \supset T^{q}_{\mu\nu} = F^2_{q,\mu\nu} - \frac{1}{2q} G_{\mu\nu} F^2_q$$

$\Rightarrow$ Fluxes source metric deformation.

IIB supergravity (10D)

- Ingredients in the IR (massless bosonic fields)
  - NS fields: metric $G_{\mu\nu}$, dilaton Φ, NS flux H_3
  - RR fields: fluxes F_1, F_3, F_5
- EOM at the lowest derivative order - consider Einstein equation with constant dilaton

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-2} G_{\mu\nu} T^{\alpha}_{\ \alpha}, \quad \text{with } T_{\mu\nu} \supset T^{q}_{\mu\nu} = F^2_{q,\mu\nu} - \frac{1}{2q} G_{\mu\nu} F^2_q$$

$\Rightarrow$ Fluxes source metric deformation.

- Solutions to EOM are backgrounds around which we can do physics.
- Spectrum around a given background is described by linearized solutions.

IIB supergravity (10D)

- Ingredients in the IR (massless bosonic fields)
  - NS fields: metric $G_{\mu\nu}$, dilaton Φ, NS flux H_3
  - RR fields: fluxes F_1, F_3, F_5
- EOM at the lowest derivative order - consider Einstein equation with constant dilaton

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-2} G_{\mu\nu} T^{\alpha}_{\ \alpha}, \quad \text{with } T_{\mu\nu} \supset T^{q}_{\mu\nu} = F^2_{q,\mu\nu} - \frac{1}{2q} G_{\mu\nu} F^2_q$$

$\Rightarrow$ Fluxes source metric deformation.

- Solutions to EOM are backgrounds around which we can do physics.
- Spectrum around a given background is described by linearized solutions.
- Example for today: $AdS_3 \times S^3 \times T^4$ with mixture of H_3 and F_3.

NSR formalism of string theory

- String theory: map from string worldsheet to target spacetime. There are redundancies in such a description, meaning that one has to fix a gauge. After fixing to superconformal gauge, the worldsheet theory becomes 2d $N = (1,1)$ CFT. String worldsheets are now generic Rieman surfaces of some genus with some punctures.

NSR formalism of string theory

- String theory: map from string worldsheet to target spacetime. There are redundancies in such a description, meaning that one has to fix a gauge. After fixing to superconformal gauge, the worldsheet theory becomes 2d $N = (1,1)$ CFT. String worldsheets are now generic Rieman surfaces of some genus with some punctures.
- Contents of worldsheet CFT:
  $c = 15$ $N = (1,1)$ matter CFT + bc-ghost CFT + $\beta\gamma$-ghost CFT.
  ** Fermions on the worldsheet can be periodic (R-sector) or anti-periodic (NS-sector) on a cylinder.

NSR formalism of string theory

- String theory: map from string worldsheet to target spacetime. There are redundancies in such a description, meaning that one has to fix a gauge. After fixing to superconformal gauge, the worldsheet theory becomes 2d $N = (1,1)$ CFT. String worldsheets are now generic Rieman surfaces of some genus with some punctures.
- Contents of worldsheet CFT:
  $c = 15$ $N = (1,1)$ matter CFT + bc-ghost CFT + $\beta\gamma$-ghost CFT.
  ** Fermions on the worldsheet can be periodic (R-sector) or anti-periodic (NS-sector) on a cylinder.
- As a result of gauge fixing, we get BRST charge Q_B. Physical on-shell states (which are states in the worldsheet CFT) are given by BRST cohomology: $Q_B|\psi\rangle = 0$, $|\psi\rangle \sim |\psi\rangle + Q_B|\delta\rangle$.

NSR formalism of string theory

- String theory: map from string worldsheet to target spacetime. There are redundancies in such a description, meaning that one has to fix a gauge. After fixing to superconformal gauge, the worldsheet theory becomes 2d $N = (1, 1)$ CFT. String worldsheets are now generic Rieman surfaces of some genus with some punctures.
- Contents of worldsheet CFT:
  $c = 15$ $N = (1, 1)$ matter CFT + bc-ghost CFT + $\beta\gamma$-ghost CFT.
  ** Fermions on the worldsheet can be periodic (R-sector) or anti-periodic (NS-sector) on a cylinder.
- As a result of gauge fixing, we get BRST charge Q_B. Physical on-shell states (which are states in the worldsheet CFT) are given by BRST cohomology: $Q_B|\psi\rangle = 0$, $|\psi\rangle \sim |\psi\rangle + Q_B|\delta\rangle$.
- String amplitudes (on-shell) are schematically given by

$$\mathcal{A}(\psi_1, ..., \psi_N) = \sum_{topologies} g_s^{-\chi} \int [dt_i]\langle\psi_1, ..., \psi_N\rangle_{t_i}^{CFT}.$$

Need for string field theory (SFT)

- Different matter CFTs correspond to different backgrounds (actually these always have zero RR fluxes). But can backgrounds with RR fluxes be described by some local worldsheet CFT? Not in any obvious way. Rather, we'd like to have an analogue of Einstein field equation in general relativity so that we can try to look for solutions describing backgrounds. Such field equations are most manifest in off-shell formalism. Therefore, we want to have an off-shell formulation of string theory: superstring field theory [Sen et al 15].

Need for string field theory (SFT)

- Different matter CFTs correspond to different backgrounds (actually these always have zero RR fluxes). But can backgrounds with RR fluxes be described by some local worldsheet CFT? Not in any obvious way. Rather, we'd like to have an analogue of Einstein field equation in general relativity so that we can try to look for solutions describing backgrounds. Such field equations are most manifest in off-shell formalism. Therefore, we want to have an off-shell formulation of string theory: superstring field theory [Sen et al 15].
- But we need a reference point where we can express off-shell fields as states of worldsheet theory. So we start with some pure NS background described by a worldsheet CFT, and then write equations of motions for off-shell fields written in the form of states in the CFT.

Need for string field theory (SFT)

- Different matter CFTs correspond to different backgrounds (actually these always have zero RR fluxes). But can backgrounds with RR fluxes be described by some local worldsheet CFT? Not in any obvious way. Rather, we'd like to have an analogue of Einstein field equation in general relativity so that we can try to look for solutions describing backgrounds. Such field equations are most manifest in off-shell formalism. Therefore, we want to have an off-shell formulation of string theory: superstring field theory [Sen et al 15].
- But we need a reference point where we can express off-shell fields as states of worldsheet theory. So we start with some pure NS background described by a worldsheet CFT, and then write equations of motions for off-shell fields written in the form of states in the CFT.
- Strategy: start with worldsheet CFT $\rightarrow$ define off-shell fields as CFT states $\rightarrow$ acquire EOM for off-shell fields $\rightarrow$ solutions will give string backgrounds and linearized solutions around a solution will give string spectrum in that background.

String field theory (bosonic)

- On-shell fields were Q_B cohomology. Thus, it is natural to lift this condition to have off-shell fields. String fields are ALL states of worldsheet CFT satisfying $(L_0 - \bar{L}_0)|\psi\rangle = (b_0 - \bar{b}_0)|\psi\rangle = 0$.

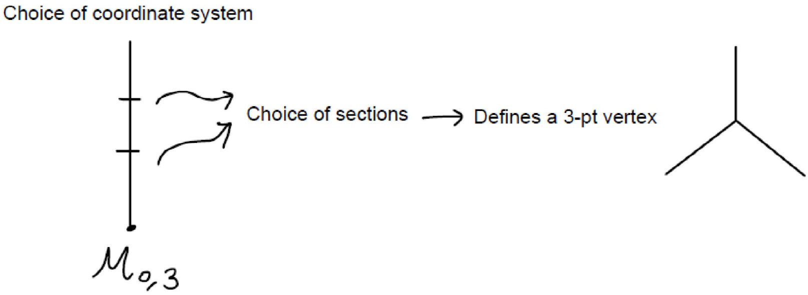
String field theory (bosonic)

- On-shell fields were Q_B cohomology. Thus, it is natural to lift this condition to have off-shell fields. String fields are ALL states of worldsheet CFT satisfying $(L_0 - \bar{L}_0)|\psi\rangle = (b_0 - \bar{b}_0)|\psi\rangle = 0$.
- Off-shell amplitudes can be defined analogously to the on-shell amplitudes as moduli integration of 2d CFT correlation functions. But due to off-shell nature of the states, different choices of coordinate system around punctures give different results for amplitudes. How is this consistent?

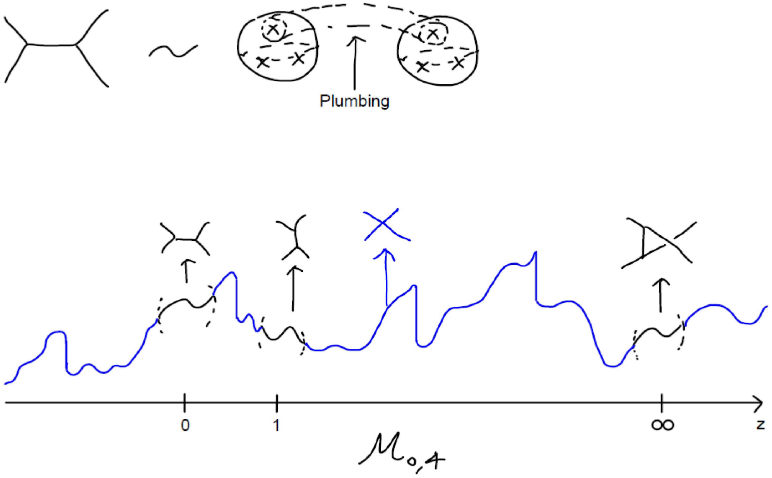
String field theory (bosonic)

- On-shell fields were Q_B cohomology. Thus, it is natural to lift this condition to have off-shell fields. String fields are ALL states of worldsheet CFT satisfying $(L_0 - \bar{L}_0)|\psi\rangle = (b_0 - \bar{b}_0)|\psi\rangle = 0$.
- Off-shell amplitudes can be defined analogously to the on-shell amplitudes as moduli integration of 2d CFT correlation functions. But due to off-shell nature of the states, different choices of coordinate system around punctures give different results for amplitudes. How is this consistent?
- We are doing field theory, so as long as such ambiguities amount to field redefinitions, all on-shell objects are free from any ambiguities. This was explicitly shown to be the case by Sen in the case of type II superstring field theory [Sen 14].
- So we can make choices of coordinate system around punctures, as long as they are consistent with plumbing construction of higher point/genus amplitudes from lower point/genus ones.

3-point vertex



4-point vertex



String field EOM (classical)

- We have defined vertices, meaning that we can write down the classical action by considering genus zero vertices. By varying it, we arrive at classical string field EOM:

$$Q_B|\Psi\rangle + \sum_{n=2}^{\infty} \frac{1}{n!}\mathcal{G}|[\Psi^n]\rangle = 0.$$

where string bracket is defined by

$$\langle A_0|\frac{c_0 - \bar{c}_0}{2}|[A_1 ... A_N]\rangle = \{A_0 A_1 ... A_N\} = \text{string vertex of } A_0, ..., A_N.$$

($\mathcal{G}$ acts as identity on NS sector and as zero mode of picture changing operator (PCO) on R sector. Explicitly, writing $\beta = e^{-\phi}\partial\xi,\ \gamma = e^{\phi}\eta$, PCO is given by $Q_B \cdot \xi$)

String field EOM (classical)

- We have defined vertices, meaning that we can write down the classical action by considering genus zero vertices. By varying it, we arrive at classical string field EOM:

$$Q_B|\Psi\rangle + \sum_{n=2}^{\infty} \frac{1}{n!}\mathcal{G}|[\Psi^n]\rangle = 0.$$

  where string bracket is defined by

$$\langle A_0|\frac{c_0 - \bar{c}_0}{2}|[A_1 \ldots A_N]\rangle = \{A_0 A_1 \ldots A_N\} = \text{string vertex of } A_0, \ldots, A_N.$$

  ($\mathcal{G}$ acts as identity on NS sector and as zero mode of picture changing operator (PCO) on R sector. Explicitly, writing $\beta = e^{-\phi}\partial\xi,\ \gamma = e^{\phi}\eta$, PCO is given by $Q_B \cdot \xi$)
- Solutions $|\Psi_0\rangle$ to EOM give backgrounds for string theory. The pure NS background given by worldsheet CFT we began with is simply given by $|\Psi_0\rangle = 0$.

Lineariezd equations for spectrum

- Spectrum around a given background $|\Psi_0\rangle$ is given by linearized solutions around it: plug into EOM $|\Psi\rangle = |\Psi_0\rangle + |\Phi\rangle$ and keep linear terms in $|\Phi\rangle$:

$$(Q_B + \mathcal{G}K)|\Phi\rangle = \hat{Q}_B|\Phi\rangle = 0, \quad \text{where } K|A\rangle = \sum_{n=1}^{\infty} \frac{1}{n!}|[\Psi_0^n A]\rangle.$$

Lineariezd equations for spectrum

- Spectrum around a given background $|\Psi_0\rangle$ is given by linearized solutions around it: plug into EOM $|\Psi\rangle = |\Psi_0\rangle + |\Phi\rangle$ and keep linear terms in $|\Phi\rangle$:

$$(Q_B + \mathcal{G}K)|\Phi\rangle = \hat{Q}_B|\Phi\rangle = 0, \quad \text{where } K|A\rangle = \sum_{n=1}^{\infty} \frac{1}{n!}|[\Psi_0^n A]\rangle.$$

- Roughly speaking, K takes into account background insertions. One can prove that $\hat{Q}_B^2 = 0$, so the spectrum is given by $\hat{Q}_B$ cohomology.

Backgrounds with RR flux

- So far, none of backgrounds with RR fluxes was realized as a worldsheet CFT in NSR formalism. But solutions to string field EOM may take any combination of NS and RR sector fields. Therefore, let's start with pure NS background described by a worldsheet CFT, and find a solution to string field EOM perturbatively in RR flux μ.

Backgrounds with RR flux

- So far, none of backgrounds with RR fluxes was realized as a worldsheet CFT in NSR formalism. But solutions to string field EOM may take any combination of NS and RR sector fields. Therefore, let's start with pure NS background described by a worldsheet CFT, and find a solution to string field EOM perturbatively in RR flux μ.
- The form of the solution we are looking for is then:

$$\Psi_0 = \sum_{n=2}^{\infty} \mu^n V_{NS,n} + \sum_{n=1}^{\infty} \mu^n V_{R,n}.$$

We plug this into EOM and solve perturbatively in μ:

$$\mu : \; Q_B V_{R,1} = 0, \qquad \mu^2 : \; Q_B V_{NS,2} + \frac{1}{2} \left[V_{R,1}^{\otimes 2} \right] = 0$$

First equation means that first order RR deformation should be on-shell. The second equation shows that RR deformation sources NS deformation, which is exactly what Einstein equation already told us!

Solving for the spectrum perturbatively

- Linearized solution up to order μ^n can be iteratively acquired as

$$\Phi_0 = \phi_n, \quad \Phi_{l+1} = -\frac{b_0^+}{L_0^+}(1 - P_\phi)\mathcal{G}K\Phi_l + \phi_n \text{ for } 0 \leq l \leq n-1$$

where $b_0^+ = b_0 + \bar{b}_0, L_0^+ = L_0 + \bar{L}_0$, and ϕ_n satisfies

$$P_\phi \phi_n = \phi_n, \quad Q_B \phi_n = -P_\phi GK\Phi_{n-1} + \mathcal{O}(\mu^{n+1}).$$

P_ϕ projects to operators carrying the weight of ϕ_n. In practice, one needs the weight only up to order μ^{n-1}.

Solving for the spectrum perturbatively

- Linearized solution up to order μ^n can be iteratively acquired as

$$\Phi_0 = \phi_n, \quad \Phi_{l+1} = -\frac{b_0^+}{L_0^+}(1 - P_\phi)\mathcal{G}K\Phi_l + \phi_n \text{ for } 0 \leq l \leq n-1$$

where $b_0^+ = b_0 + \bar{b}_0$, $L_0^+ = L_0 + \bar{L}_0$, and ϕ_n satisfies

$$P_\phi \phi_n = \phi_n, \quad Q_B \phi_n = -P_\phi GK\Phi_{n-1} + \mathcal{O}(\mu^{n+1}).$$

P_ϕ projects to operators carrying the weight of ϕ_n. In practice, one needs the weight only up to order μ^{n-1}.

- To first few orders,

$$Q_B\phi_0 = 0, \quad Q_B\phi_1 = -\mu P_0 \mathcal{G}[V_{R,1}\phi_0] \;\; (P_0 \text{ projects to weight zero}),$$

$$Q_B\phi_2 = -P_{\phi_1}\mathcal{G}K\left(1 - \frac{b_0^+}{L_0^+}(1 - P_{\phi_1})\mathcal{G}K\right)\phi_2 + \mathcal{O}(\mu^3).$$

Example: $AdS_3 \times S^3 \times T^4$

- Supergravity background:

$$ds^2 = R^2(ds^2_{AdS_3} + ds^2_{S^3}) + ds^2_{T^4}$$
$$H_3 = 2qR^2(w_{AdS_3} + w_{S^3}), \quad F_3 = 2\sqrt{1-q^2}R^2(w_{AdS_3} + w_{S^3}),$$

where $q = 1 - \frac{\mu^2}{2} + \mathcal{O}(\mu^3)$, $qR^2 = \alpha' k$, $k \in \mathbb{N}$.

Example: $AdS_3 \times S^3 \times T^4$

- Supergravity background:

$$ds^2 = R^2(ds^2_{AdS_3} + ds^2_{S^3}) + ds^2_{T^4}$$
$$H_3 = 2qR^2(w_{AdS_3} + w_{S^3}), \quad F_3 = 2\sqrt{1-q^2}R^2(w_{AdS_3} + w_{S^3}),$$

  where $q = 1 - \frac{\mu^2}{2} + \mathcal{O}(\mu^3)$, $qR^2 = \alpha' k$, $k \in \mathbb{N}$.
- $q = 1$ is pure NS background with exact worldsheet CFT description given by $SL(2,\mathbb{R})_{k+2} \oplus SU(2)_{k-2} \oplus U(1)^4 \oplus 10$ free fermions. This CFT is where SFT will be formulated.

Example: $AdS_3 \times S^3 \times T^4$

- Supergravity background:

$$ds^2 = R^2(ds^2_{AdS_3} + ds^2_{S^3}) + ds^2_{T^4}$$
$$H_3 = 2qR^2(w_{AdS_3} + w_{S^3}), \quad F_3 = 2\sqrt{1-q^2}R^2(w_{AdS_3} + w_{S^3}),$$

  where $q = 1 - \frac{\mu^2}{2} + \mathcal{O}(\mu^3)$, $qR^2 = \alpha' k$, $k \in \mathbb{N}$.
- $q = 1$ is pure NS background with exact worldsheet CFT description given by $SL(2,\mathbb{R})_{k+2} \oplus SU(2)_{k-2} \oplus U(1)^4 \oplus 10$ free fermions. This CFT is where SFT will be formulated.
- To get mixed flux solution, we deform by RR operator at leading order: $V_{R,1} \sim S_+^{\alpha\alpha'}\Theta_+\tilde{S}_+^{\beta\beta'}\tilde{\Theta}_+ \left(V^{sl}_{j=-\frac{1}{2}}\right)_{\alpha\beta}\left(V^{su}_{j'=\frac{1}{2}}\right)_{\alpha'\beta'}$. (This is the only Q_B-closed RR sector operator preserving $AdS_3 \times S^3$-isometry)

Example: $AdS_3 \times S^3 \times T^4$

- Supergravity background:

$$ds^2 = R^2(ds^2_{AdS_3} + ds^2_{S^3}) + ds^2_{T^4}$$
$$H_3 = 2qR^2(w_{AdS_3} + w_{S^3}), \quad F_3 = 2\sqrt{1-q^2}R^2(w_{AdS_3} + w_{S^3}),$$

  where $q = 1 - \frac{\mu^2}{2} + \mathcal{O}(\mu^3)$, $qR^2 = \alpha' k$, $k \in \mathbb{N}$.

- $q = 1$ is pure NS background with exact worldsheet CFT description given by $SL(2,\mathbb{R})_{k+2} \oplus SU(2)_{k-2} \oplus U(1)^4 \oplus 10$ free fermions. This CFT is where SFT will be formulated.
- To get mixed flux solution, we deform by RR operator at leading order: $V_{R,1} \sim S_+^{\alpha\alpha'}\Theta_+\tilde{S}_+^{\beta\beta'}\tilde{\Theta}_+\left(V^{sl}_{j=-\frac{1}{2}}\right)_{\alpha\beta}\left(V^{su}_{j'=\frac{1}{2}}\right)_{\alpha'\beta'}$. (This is the only Q_B-closed RR sector operator preserving $AdS_3 \times S^3$-isometry)
- At order μ^2, we have $V_{NS,2} = -\frac{1}{2}\frac{b_0^+}{L_0^+}(1-P_0)[V_{R,1}^{\otimes 2}]$. This corresponds to metric deformation needed to solve the Einstein equation.

Spectrum of pulsating strings

- We study spectrum of pulsating strings in mixed flux. Classically, it's a solution where the radius of the string oscillates in the radial direction of AdS_3. At pure NS point, corresponding vertex operator is

$$\phi_0 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0,j',n}, \ \ V_{j_0,j',n} \sim \psi^-\tilde{\psi}^-(J_{-1})^n(\tilde{J}_{-1})^n V^{sl}_{j_0,j_0,j_0}V^{su}_{j',j',j'}V_{T^4}.$$

  Here, j_0 is $SL(2)$ quantum number which is real and continuous, while j' is $SU(2)$ quantum number which is a nonnegative half integer. J_{-1} gives oscillator modes and ψ^- is $SL(2)$ fermionic oscillator.

Spectrum of pulsating strings

- We study spectrum of pulsating strings in mixed flux. Classically, it's a solution where the radius of the string oscillates in the radial direction of AdS_3. At pure NS point, corresponding vertex operator is

$$\phi_0 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0,j',n}, \;\; V_{j_0,j',n} \sim \psi^-\tilde{\psi}^-(J_{-1})^n(\tilde{J}_{-1})^n V^{sl}_{j_0,j_0,j_0}V^{su}_{j',j',j'}V_{T^4}.$$

  Here, j_0 is $SL(2)$ quantum number which is real and continuous, while j' is $SU(2)$ quantum number which is a nonnegative half integer. J_{-1} gives oscillator modes and ψ^- is $SL(2)$ fermionic oscillator.
- On-shell condition $Q_B\phi_0 = 0$ gives $-\frac{j_0(j_0-1)}{k} + n + \frac{j'(j'+1)}{k} + h_{T^4} = 0$. We want to study how turning on RR flux modifies this dispersion relation. Since j' and n are discrete labels, only $SL(2)$ quantum number j_0 will change: $j = j_0 + \delta j$. This corresponds to change in AdS_3 mass/energy.

Spectrum of pulsating strings

- We study spectrum of pulsating strings in mixed flux. Classically, it's a solution where the radius of the string oscillates in the radial direction of AdS_3. At pure NS point, corresponding vertex operator is

$$\phi_0 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0,j',n}, \;\; V_{j_0,j',n} \sim \psi^-\tilde{\psi}^-(J_{-1})^n(\tilde{J}_{-1})^n V^{sl}_{j_0,j_0,j_0} V^{su}_{j',j',j'} V_{T^4}.$$

  Here, j_0 is $SL(2)$ quantum number which is real and continuous, while j' is $SU(2)$ quantum number which is a nonnegative half integer. J_{-1} gives oscillator modes and ψ^- is $SL(2)$ fermionic oscillator.
- On-shell condition $Q_B\phi_0 = 0$ gives $-\frac{j_0(j_0-1)}{k} + n + \frac{j'(j'+1)}{k} + h_{T^4} = 0$. We want to study how turning on RR flux modifies this dispersion relation. Since j' and n are discrete labels, only $SL(2)$ quantum number j_0 will change: $j = j_0 + \delta j$. This corresponds to change in AdS_3 mass/energy.
- At order μ, due to $P\mathcal{G}[V_R\phi_0] = 0$, we take the same solution as zeroth order solution.

Spectrum of pulsating strings (cont'd)

- At order μ^2, we take the solution of the form

$$\phi_2 = c\tilde{c}e^{-\phi-\tilde{\phi}} V_{j_0+\mu^2 j_2, j', n} + \text{(ghosts, descendants)}.$$

Our goal is to get j_2, which is shift in $SL(2)$ quantum number due to RR flux. Plugging this into EOM, we get

$$\frac{2j_2(2j_0-1)}{k} = \mathcal{A}(\phi_0, \phi_0, V_{R,1}, V_{R,1}),$$

where RHS is the usual on-shell 4pt amplitude. This is a new result valid even at finite AdS_3 radius, which was not obtainable in any of previous formalisms in the prescence of RR flux.

Spectrum of pulsating strings (cont'd)

- At order μ^2, we take the solution of the form

$$\phi_2 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0+\mu^2 j_2, j', n} + (\text{ghosts, descendants}).$$

  Our goal is to get j_2, which is shift in $SL(2)$ quantum number due to RR flux. Plugging this into EOM, we get

$$\frac{2j_2(2j_0-1)}{k} = \mathcal{A}(\phi_0, \phi_0, V_{R,1}, V_{R,1}),$$

  where RHS is the usual on-shell 4pt amplitude. This is a new result valid even at finite AdS_3 radius, which was not obtainable in any of previous formalisms in the prescence of RR flux.

- First non-trivial check is BPS spectrum. They correspond to $n = 0, h_{T^4} = 0, j_0 = j' + 1$. Indeed we get $j_2 = 0$, agreeing with the expectation that supersymmetry protects these operators.

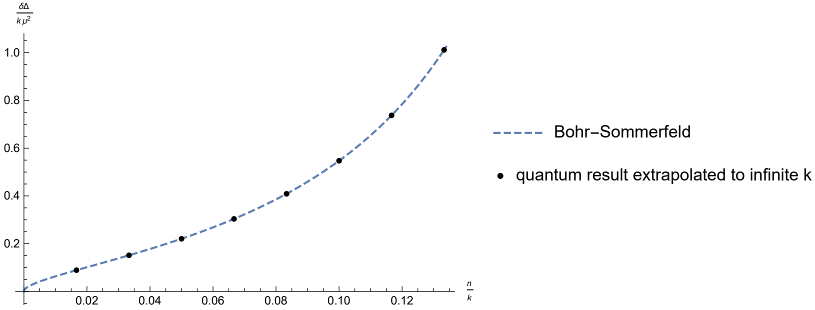
Spectrum of pulsating strings (cont'd)

- Sample results for anomalous weight $\delta h = -\frac{\alpha' \delta m^2}{4} = \frac{\mu^2 j_2(2j_0-1)}{k}$ with $n = 1, h_{T^4} = 0$:

$k \backslash j'$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
7	4.51353	7.7253			
8	2.61214	3.18173	5.03926	38.0435	
9	1.97318	2.21068	2.76008	4.25035	15.9923

Spectrum of pulsating strings (cont'd)

- Comparison with Bohr-Sommerfeld quantization in the semi-classical limit for AdS_3 energy $\delta\Delta \simeq \sqrt{\frac{k}{n}}\delta h$, with $j' = 0, h_{T^4} = 0$:



Conclusion

- Thanks to type II superstring field theory base on NSR formalism, we now have a well-defined procedure to describe perturbative strings in backgrounds with RR fluxes.

Conclusion

- Thanks to type II superstring field theory base on NSR formalism, we now have a well-defined procedure to describe perturbative strings in backgrounds with RR fluxes.
- Finding the solutions at least perturbatively in RR flux is straightforward and similarly we can find the string spectrum in such backgrounds.

Conclusion

- Thanks to type II superstring field theory base on NSR formalism, we now have a well-defined procedure to describe perturbative strings in backgrounds with RR fluxes.
- Finding the solutions at least perturbatively in RR flux is straightforward and similarly we can find the string spectrum in such backgrounds.
- This was just a very small (and straightforward) application of closed superstring field theory, which is now established as a consistent formalism. It's time to think about more interesting applications!

Conclusion

- Thanks to type II superstring field theory base on NSR formalism, we now have a well-defined procedure to describe perturbative strings in backgrounds with RR fluxes.
- Finding the solutions at least perturbatively in RR flux is straightforward and similarly we can find the string spectrum in such backgrounds.
- This was just a very small (and straightforward) application of closed superstring field theory, which is now established as a consistent formalism. It's time to think about more interesting applications!

THANK YOU!