A Quadrillion Standard Models from F-theory

Jiahua Tian


Northeastern University

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Main results

We constructed QUADRILLION ($10^{15}$) Standard Models via F-theory.

$$7.667 \times 10^{13} \lesssim N_{\text{SM}}^{\text{toric}} \lesssim 1.622 \times 10^{16}$$
Main results

- Construct F-theory geometry using a fibration of a 2D reflexive polyhedron $F_{11}$.
- Add consistent $G_4$-flux.
- Count the number of Standard Models.
- Some features
F-theory is a non-perturbative formulation of Type IIB compactifications with general \([p, q]\) 7-branes backreacting on the geometry. [Wei18]

\[
\text{IIB on } B_n \text{ with 7-branes } \rightarrow \text{ F-theory on } \pi : E_T \rightarrow \text{CY}_{n+1} \\
\downarrow \\
B_n
\]
F-theory geometry can be encoded in the Weierstrass model:

\[ y^2 = x^3 + f x z^4 + g z^6. \]
Discriminant locus (codimension 1):

\[ S := \{ \Delta = 4f^3 + 27g^2 = 0 \} \subset B_n. \]  

\( E_\tau \) is singular along \( S \).

\( \Delta = 0 \) is the divisor wrapped by 7-branes.
F-theory fibration

Brane intersection \( \rightarrow C_{IJ} = S_I \cap S_J \) (codimension 2)

\( \text{ord}_{C_{IJ}}(f, g, \Delta) \Rightarrow \text{naive Kodaira type} \Rightarrow h_{IJ} \)

\[ h_{IJ} \rightarrow g_I \oplus g_J \]

\[ \text{adj}_{h_{IJ}} = \text{adj}_{g_I} \oplus \text{adj}_{g_J} \oplus \text{matters} \]
F-theory fibration

\[ P_{IJK} = S_I \cap S_J \cap S_K \text{ (codimension 3)} \]

Naive Kodaira type: \( h_{IJK} \).

Imagine all 7-branes are on top of each other, there is a “parent” coupling:

\[
\text{adj}_{h_{IJK}} \times \text{adj}_{h_{IJK}} \times \text{adj}_{h_{IJK}} \rightarrow 1. \tag{3}
\]

Rotate a bit: \( h_{IJK} \rightarrow g \).

Eq.(3) decomposes into [BHV09]:

\[
\overline{R}_1 \times R_2 \times R_3 \rightarrow 1. \tag{4}
\]
F-theory fibration

2D reflexive polyhedron $F_{11}$:

Figure: $F_{11}$
An elliptic CY4 $Y_4$ is a hypersurface defined by

$$P := s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^3 u v^2 + s_5 e_1^2 e_2 e_3^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2 = 0. \quad (5)$$

and $s_i$’s are such that

$$[s_1] = 3 \overline{K} - S_7 - S_9, \quad [s_2] = 2 \overline{K} - S_9, \quad [s_6] = \overline{K},$$

$$[s_3] = \overline{K} + S_7 - S_9, \quad [s_5] = 2 \overline{K} - S_7, \quad [s_9] = S_9. \quad (6)$$

Choose $S_7$ and $S_9$ to ensure that the sections $[s_i]$ exist.
F-theory fibration

F-theory on $Y_4$ has gauge symmetry $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.

Realizes the SM matter spectrum:

$$(3, 2)^{1\over 6}, (1, 2)^{-{1\over 2}}, (\bar{3}, 1)^{-2\over 3}, (\bar{3}, 1)^{1\over 3}, (1, 1)_1.$$  \hspace{1cm} (7)

Realizes the SM Yukawa couplings.

[KMPO⁺15, CL18]
F-theory fibration

Chiral spectrum requires $G_4$-flux [LW16]:

$$G_4(a, \omega) = aG_4^a + \pi^* \omega \wedge \sigma$$  \hspace{1cm} (8)

where

$$G_4^a = [e_4] \wedge ([e_4] + \pi^*[s_6])$$
$$+ \frac{[e_1] \wedge \pi^*[s_9]}{2} + \frac{\pi^*[s_3] \wedge ([e_2] + 2[u])}{3},$$  \hspace{1cm} (9)

and $\omega \in H^{1,1}(B_3)$ and $\sigma$ is the Shioda divisor dual to $U(1)_Y$.

$a$ is a rational number.
Quantization of $G_4$-flux requires:

$$G_4(a, \omega) + \frac{1}{2} c_2(Y_4) \in H^{2,2}(Y_4, \mathbb{Z}).$$\hspace{1cm}(10)$$

We check the integrality of the following integrals:

$$\int_{Y_4} \left( G_4(a, \omega) + \frac{1}{2} c_2(Y_4) \right) \wedge \text{PD}(\gamma_R),$$

$$\int_{Y_4} \left( G_4(a, \omega) + \frac{1}{2} c_2(Y_4) \right) \wedge \text{PD}(D_1 \cdot D_2).$$\hspace{1cm}(11)$$

$\forall D_1, D_2.$
F-theory fibration

D3-tadpole cancellation requires [SVW96]:

\[ n_{D3} = \frac{\chi(Y_4)}{24} - \frac{1}{2} \int_{Y_4} G_4(a, \omega) \wedge G_4(a, \omega) \notin \mathbb{Z}_{\geq 0}, \quad (12) \]

Masslessness of \( U(1)_Y \) requires the D-term vanish [Gri11]:

\[ \forall \eta \in H^{1,1}(B_3) : \int_{Y_4} G_4(a, \omega) \wedge \sigma \wedge \pi^* \eta \nexists 0. \quad (13) \]
Finally we want:

$$\chi(R) = \int_{\gamma_R} G_4(a, \omega) = 3$$  \hspace{1cm} (14)$$

for all the SM matter representations.
Universally consistent fibrations

General method: The integrals on $Y_4$ can be reduced to integrals on $B_3$.

Integrals on $Y_4 \Rightarrow$ Intersection numbers on $B_3$

We choose $B_3$ to be the toric varieties given by an FRST of the 3D reflexive polytope. We can compute the intersection numbers of this set of bases.
Universally consistent fibrations

We need a choice of $S_7$ and $S_9$.

Ideal strategy: Scan over different choices of $S_7$ and $S_9$ and the flux parameter $(a, \omega)$.

Computationally the scan is not feasible. The number of choices of $S_7$ and $S_9$ grows exponentially with $h^{1,1}(B_3)$. In our case $\max(h^{1,1}(B_3)) = 35$. 
Universally consistent fibrations

Need suitable choices of $S_7$ and $S_9$.

There exists a clever choice!

We choose $S_7 = S_9 = \overline{K}$.

In this case $\forall i$, $s_i = \overline{K}$. No $s_i$ factorizes when $B_3$ is a weak fano toric threefold associated to a reflexive polytope [Bat93].

$SU(3)$ sits along $s_9 = 0$, $SU(2)$ sits along $s_3 = 0$. No extra gauge sectors!
Universally consistent fibrations

Masslessness of $U(1)_Y$:

$$\forall \eta \in H^{1,1}(B_3) : \int_{B_3} \overline{\mathcal{K}} \wedge \eta \wedge (5\omega + a\overline{\mathcal{K}}) = 0. \quad (15)$$

We choose $\omega = -5\overline{\mathcal{K}}/a$. 
Universally consistent fibrations

Three families:

\[ \chi(R) = -\frac{a}{5} \int_{B_3} \kappa \wedge \kappa \wedge \kappa =: -\frac{a}{5} \kappa^3. \] (16)

Three families \( \Rightarrow a = -15/\kappa^3. \)

So we have \( (a, \omega) = \left( -\frac{15}{\kappa^3}, \frac{\kappa^3}{3} \kappa \right). \)
Universally consistent fibrations

Flux quantization

There are several non-manifestly integer quantities need to be checked:

\[ \int_{B_3} \frac{c_2(B) \wedge k}{2}, \quad \frac{k^3}{2}, \]

and

\[ \int_{B_3} \frac{\alpha \wedge (c_2(B_3) + k^2)}{2} \text{ with } \alpha \in H^{1,1}(B_3, \mathbb{Z}), \]

For our base \( B_3 \) it is known that \( \int_{B_3} c_2(B_3) \wedge k = 24 \) and that \( c_2(B_3) + k^2 \) is an even class. We only need to check whether \( k^3 \) is even.
Universally consistent fibrations

All the above checks are reduced to:

D3-tadpole cancellation

\[ n_{D3} = 12 + \frac{5}{8} \mathcal{K}^3 - \frac{45}{2 \mathcal{K}^3}. \]  

(18)

Observation: If \( n_{D3} \in \mathbb{Z}_{\geq 0}, \mathcal{K}^3 \) must be even. Thus flux quantization conditions are simultaneously satisfied.

We have \( \mathcal{K}^3 \in \{2, 6, 10, 18, 30, 90 \ldots \}. \)
Universally consistent fibrations

Now all the consistency conditions are fulfilled and we have obtained SM gauge group and matters. Start to count!
4319 3D reflexive polytopes

Different FRST $\rightarrow$ Inequivalent $B_3$

There are 708 polytopes satisfy $n_{D3} \in \mathbb{Z}_{\geq 0}$, we call this set $S$. So we have

$$N_{SM}^{\text{toric}} = \sum_{\Delta \in S} N_{\text{FRST}}(\Delta).$$

(19)

We need to count the number of FRST. [HT17, CHKN17]
Count geometries

There are 237 polytopes with less than 15 integral points. Those are easy. One line command in SageMath solves the counting problem.

There are in total 41430 FRST.

The other 471 polytopes are hard.
Strategy: Counting the number of FRT of each facet, then take the product.

One drawback: The product triangulation is not guaranteed to be a regular triangulation.
Count geometries

A statistical result

We tested $1.3 \times 10^4$ random samples out of $O(10^9)$ such product triangulations of polytope $\Delta_8$, roughly $\frac{2}{3}$ of them are regular.

(Why $\Delta_8$? Because it has the largest number of integral points so it dominates the ensemble. For $\Delta_8$, $\bar{K}^3 = 6$.)

We believe the factor $\frac{2}{3}$ can be applied to other polytopes which we can only compute the product of FRT.
Count geometries

Combine the above results, we estimate the number of Standard Models:

\[ 7.667 \times 10^{13} \lesssim N_{\text{SM}}^{\text{toric}} \lesssim 1.622 \times 10^{16} \]
Some features

Feature 1:

\[ g_3^2 = g_2^2 = \frac{5}{3} g_Y^2 = \frac{2}{\text{vol}(\mathcal{K})} \]  \hspace{1cm} (20)

MSSM gauge coupling unification at compactification scale for free!

Feature 2:

Gauge divisors in $\overline{K}_B \rightarrow$ Gravity cannot be decoupled

Feature 3:

$D3$-branes give rise to $U(1)$'s that are cosmologically relevant. (No reason that $D3$-branes are stabilized near the gauge divisors in which case the jointly charged matters decouple.)
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References I


Timo Weigand, *TASI Lectures on F-theory*. 