A Quadrillion Standard Models from F-theory

Jiahua Tian

[CHL$^+$19] Work with Mirjam Cvetič, James Halverson, Ling Lin and Muyang Liu

Northeastern Universty

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Main results

We constructed QUADRILLION (10^{15}) Standard Models via F-theory.

$$7.667 \times 10^{13} \lesssim N_{\mathrm{SM}}^{\mathrm{toric}} \lesssim 1.622 \times 10^{16}$$

Main results

- Construct F-theory geometry using a fibration of a 2D reflexive polyhedron F_{11}.
- Add consistent G_4-flux.
- Count the number of Standard Models.
- Some features

F-theory fibration

F-theory is a non-perturbative formulation of Type IIB compactifications with general [p, q] 7-branes backreacting on the geometry. [Wei18]

$$\text{IIB on } B_n \text{ with 7-branes} \rightarrow \text{F-theory on } \begin{array}{ccc} \pi : E_\tau & \rightarrow & \mathrm{CY}_{n+1} \\ & & \downarrow \\ & & B_n \end{array}$$

F-theory fibration

F-theory geometry can be encoded in the Weierstrass model:

$$y^2 = x^3 + fxz^4 + gz^6. \tag{1}$$

F-theory fibration

Discriminant locus (codimension 1):

$$S := \{\Delta = 4f^3 + 27g^2 = 0\} \subset B_n. \qquad (2)$$

E_τ is singular along S.

$\Delta = 0$ is the divisor wrapped by 7-branes.

F-theory fibration

Brane intersection $\to C_{IJ} = S_I \cap S_J$ (codimension 2)

$\mathrm{ord}_{C_{IJ}}(f, g, \Delta) \Rightarrow$ naive Kodaira type $\Rightarrow \mathfrak{h}_{IJ}$

$$\mathfrak{h}_{IJ} \to \mathfrak{g}_I \oplus \mathfrak{g}_J$$
$$\mathrm{adj}_{\mathfrak{h}_{IJ}} = \mathrm{adj}_{\mathfrak{g}_I} \oplus \mathrm{adj}_{\mathfrak{g}_J} \oplus \text{matters}$$

F-theory fibration

$P_{IJK} = S_I \cap S_J \cap S_K$ (codimension 3)

Naive Kodaira type: $\mathfrak{h}_{IJK}$.

Imagine all 7-branes are on top of each other, there is a "parent" coupling:

$$\mathrm{adj}_{\mathfrak{h}_{IJK}} \times \mathrm{adj}_{\mathfrak{h}_{IJK}} \times \mathrm{adj}_{\mathfrak{h}_{IJK}} \to 1. \tag{3}$$

Rotate a bit: $\mathfrak{h}_{IJK} \to \mathfrak{g}$.

Eq.(3) decomposes into [BHV09]:

$$\overline{R}_1 \times R_2 \times R_3 \to 1. \tag{4}$$

F-theory fibration

2D reflexive polyhedron F_{11}:

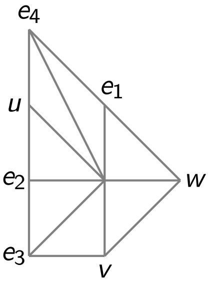

Figure: F_{11}

F-theory fibration

An elliptic CY4 Y_4 is a hypersurface defined by

$$
\begin{aligned}
P := & s_1\, e_1^2\, e_2^2\, e_3\, e_4^4\, u^3 + s_2\, e_1\, e_2^2\, e_3^2\, e_4^2\, u^2\, v + s_3\, e_2^2\, e_3^3\, u\, v^2 \\
& + s_5\, e_1^2\, e_2\, e_4^3\, u^2\, w + s_6\, e_1\, e_2\, e_3\, e_4\, u\, v\, w + s_9\, e_1\, v\, w^2 = 0.
\end{aligned} \tag{5}
$$

and s_i's are such that

$$
\begin{aligned}
&[s_1] = 3\,\overline{\mathcal{K}} - \mathcal{S}_7 - \mathcal{S}_9\,, \quad [s_2] = 2\,\overline{\mathcal{K}} - \mathcal{S}_9\,, \quad [s_6] = \overline{\mathcal{K}}\,, \\
&[s_3] = \overline{\mathcal{K}} + \mathcal{S}_7 - \mathcal{S}_9\,, \qquad [s_5] = 2\,\overline{\mathcal{K}} - \mathcal{S}_7\,, \quad [s_9] = \mathcal{S}_9\,.
\end{aligned} \tag{6}
$$

Choose $\mathcal{S}_7$ and $\mathcal{S}_9$ to ensure that the sections $[s_i]$ exist.

F-theory fibration

F-theory on Y_4 has gauge symmetry $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.

Realizes the SM matter spectrum:

$$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}\,,\ \ (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}\,,\ \ (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}\,,\ \ (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}\,,\ \ (\mathbf{1},\mathbf{1})_{1}\,. \tag{7}$$

Realizes the SM Yukawa couplings.
[KMPO$^+$15, CL18]

F-theory fibration

Chiral spectrum requires G_4-flux [LW16]:

$$G_4(a, \omega) = aG_4^a + \pi^*\omega \wedge \sigma \tag{8}$$

where

$$\begin{aligned} G_4^a = & [e_4] \wedge ([e_4] + \pi^*[s_6]) \\ & + \frac{[e_1] \wedge \pi^*[s_9]}{2} + \frac{\pi^*[s_3] \wedge ([e_2] + 2[u])}{3}, \end{aligned} \tag{9}$$

and $\omega \in H^{1,1}(B_3)$ and σ is the Shioda divisor dual to $U(1)_Y$.

a is a rational number.

F-theory fibration

Quantization of G_4-flux requires:

$$G_4(a,\omega) + \frac{1}{2}c_2(Y_4) \in H^{2,2}(Y_4,\mathbb{Z}). \tag{10}$$

We check the integrality of the following integrals:

$$\begin{aligned} &\int_{Y_4}\left(G_4(a,\,\omega) + \frac{1}{2}c_2(Y_4)\right) \wedge \mathrm{PD}(\gamma_{\mathbf{R}})\,, \\ &\int_{Y_4}\left(G_4(a,\,\omega) + \frac{1}{2}c_2(Y_4)\right) \wedge \mathrm{PD}(D_1 \cdot D_2)\,. \end{aligned} \tag{11}$$

$\forall D_1,\ D_2$.

F-theory fibration

D3-tadpole cancellation requires [SVW96]:

$$n_{\mathrm{D3}} = \frac{\chi(Y_4)}{24} - \frac{1}{2}\int_{Y_4} G_4(a,\,\omega) \wedge G_4(a,\,\omega) \overset{!}{\in} \mathbb{Z}_{\geq 0}\,, \tag{12}$$

Masslessness of $U(1)_Y$ requires the D-term vanish [Gri11] :

$$\forall \eta \in H^{1,1}(B_3): \quad \int_{Y_4} G_4(a,\,\omega) \wedge \sigma \wedge \pi^* \eta \overset{!}{=} 0. \tag{13}$$

F-theory fibration

Finally we want:

$$\chi(\mathbf{R}) = \int_{\gamma_{\mathbf{R}}} G_4(a, \omega) = 3 \tag{14}$$

for all the SM matter representations.

Universally consistent fibrations

General method: The integrals on Y_4 can be reduced to integrals on B_3.

$$\text{Integrals on } Y_4 \Rightarrow \text{Intersection numbers on } B_3$$

We choose B_3 to be the toric varieties given by an FRST of the 3D reflexive polytope. We can compute the intersection numbers of this set of bases.

Universally consistent fibrations

We need a choice of $\mathcal{S}_7$ and $\mathcal{S}_9$.

Ideal strategy: Scan over different choices of $\mathcal{S}_7$ and $\mathcal{S}_9$ and the flux parameter (a, ω).

Computationally the scan is not feasible. The number of choices of $\mathcal{S}_7$ and $\mathcal{S}_9$ grows exponentially with $h^{1,1}(B_3)$. In our case $\max(h^{1,1}(B_3)) = 35$.

Universally consistent fibrations

Need suitable choices of $\mathcal{S}_7$ and $\mathcal{S}_9$.

There exists a clever choice!

We choose $\mathcal{S}_7 = \mathcal{S}_9 = \overline{\mathcal{K}}$.

In this case $\forall i$, $s_i = \overline{\mathcal{K}}$. No s_i factorizes when B_3 is a weak fano toric threefold associated to a reflexive polytope [Bat93].

$SU(3)$ sits along $s_9 = 0$, $SU(2)$ sits along $s_3 = 0$. No extra gauge sectors!

Universally consistent fibrations

Masslessness of $U(1)_Y$:

$$\forall \eta \in H^{1,1}(B_3): \quad \int_{B_3} \overline{\mathcal{K}} \wedge \eta \wedge (5\,\omega + a\,\overline{\mathcal{K}}) \stackrel{!}{=} 0\,. \tag{15}$$

We choose $\omega = -5\overline{\mathcal{K}}/a$.

Universally consistent fibrations

Three families:

$$\chi(\mathbf{R}) = -\frac{a}{5}\int_{B_3} \overline{\mathcal{K}} \wedge \overline{\mathcal{K}} \wedge \overline{\mathcal{K}} =: -\frac{a}{5}\overline{\mathcal{K}}^3 \,. \tag{16}$$

Three families $\Rightarrow$ $a = -15/\overline{\mathcal{K}}^3$.

So we have $(a,\omega) = (\frac{-15}{\overline{\mathcal{K}}^3}, \frac{\overline{\mathcal{K}}^3}{3}\overline{\mathcal{K}})$.

Universally consistent fibrations

Flux quantization

There are several non-manifestly integer quantities need to be checked:

$$\int_{B_3} \frac{c_2(B) \wedge \overline{\mathcal{K}}}{2}\,, \quad \frac{\overline{\mathcal{K}}^3}{2}\,, \tag{17}$$

$$\text{and} \quad \int_{B_3} \frac{\alpha \wedge (c_2(B_3) + \overline{\mathcal{K}}^2)}{2} \text{ with } \alpha \in H^{1,1}(B_3, \mathbb{Z})\,,$$

For our base B_3 it is known that $\int_{B_3} c_2(B_3) \wedge \overline{\mathcal{K}} = 24$ and that $c_2(B_3) + \overline{\mathcal{K}}^2$ is an *even* class. We only need to check whether $\overline{\mathcal{K}}^3$ is even.

Universally consistent fibrations

All the above checks are reduced to:

D3-tadpole cancellation

$$n_{\mathrm{D3}} = 12 + \frac{5}{8}\,\overline{\mathcal{K}}^3 - \frac{45}{2\,\overline{\mathcal{K}}^3}\,. \tag{18}$$

Observation: If $n_{\mathrm{D3}} \in \mathbb{Z}_{\geq 0}$, $\overline{\mathcal{K}}^3$ must be even. Thus flux quantization conditions are simultaneously satisfied.

We have $\overline{\mathcal{K}}^3 \in \{2, 6, 10, 18, 30, 90 \ldots\}$.

Universally consistent fibrations

Now all the consistency conditions are fulfilled and we have obtained SM gauge group and matters. Start to count!

Count geometries

4319 3D reflexive polytopes

Different FRST $\rightarrow$ Inequivalent B_3

There are 708 polytopes satisfy $n_{\mathrm{D3}} \in \mathbb{Z}_{\geq 0}$, we call this set S. So we have

$$N_{\mathrm{SM}}^{\mathrm{toric}} = \sum_{\Delta \in S} N_{\mathrm{FRST}}(\Delta)\,. \tag{19}$$

We need to count the number of FRST. [HT17, CHKN17]

Count geometries

There are 237 polytopes with less than 15 integral points. Those are easy. One line command in `SageMath` solves the counting problem.

There are in total 41430 FRST.

The other 471 polytopes are hard.

Count geometries

Strategy: Counting the number of FRT of each facet, then take the product.

One drawback: The product triangulation is not guaranteed to be a regular triangulation.

Count geometries

A statistical result

We tested 1.3×10^4 random samples out of $\mathcal{O}(10^9)$ such product triangulations of polytope Δ_8, roughly $\frac{2}{3}$ of them are regular.

(Why Δ_8? Because it has the largest number of integral points so it dominates the ensemble. For Δ_8, $\overline{\mathcal{K}}^3 = 6$.)

We believe the factor $\frac{2}{3}$ can be applied to other polytopes which we can only compute the product of FRT.

Count geometries

Combine the above results, we estimate the number of Standard Models:

$$7.667 \times 10^{13} \lesssim N_{\mathrm{SM}}^{\mathrm{toric}} \lesssim 1.622 \times 10^{16}$$

Some features

Feature 1:

$$g_3^2 = g_2^2 = \frac{5}{3} g_Y^2 = \frac{2}{\mathrm{vol}(\overline{\mathcal{K}})} \tag{20}$$

MSSM gauge coupling unification at compactification scale for free!

Feature 2:
Gauge divisors in $\overline{K}_B$ $\rightarrow$ Gravity cannot be decoupled

Feature 3:
$D3$-branes give rise to $U(1)$'s that are cosmologically relevant.
(No reason that $D3$-branes are stabilized near the gauge divisors in which case the jointly charged matters decouple.)

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