D=3 Matter Coupled to Chern-Simons Fields. Spontaneous Breaking of Scale Invariance, and Fermion-Boson Dual Mapping

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Work in collaboration with

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In this talk will discuss shortly:

1. Spontaneous Breaking of Scale Invariance in 3d field theories

2. AdS/CFT_3 Large N QFT in 3d and Higher Spin on AdS_4

3. Boson Fermion mapping in 3d

Simple models of spontaneous breaking of scale invariance

$$\int \mathrm{d}^3x \ \left[-\frac{1}{2}\vec{\phi}\cdot\partial^2\vec{\phi}+\frac{\mu^2}{2}(\vec{\phi})^2+\frac{\lambda}{4N}(\vec{\phi})^4+\frac{\eta}{6N^2}(\vec{\phi})^6\right]$$

and

$$\mathcal{S}=\int \mathrm{d}^3x\,\mathrm{d}^2\theta \ \left[\tfrac{1}{2}\bar{\mathrm{D}}\Phi\cdot\mathrm{D}\Phi+NU(\Phi^2/N)\right]$$

with

$$U(\Phi^2/N)=(\mu/N)\Phi^2+\tfrac{1}{2}(u/N^2)\Phi^4$$

Chern-Simons gauge field coupled to a U(N) scalar - light cone gauge

$$\mathcal{S}_{\rm CS}(\mathbf{A}) = -\frac{i\kappa}{4\pi}\epsilon_{\mu\nu\rho}\int \mathrm{d}^3x\, Tr\left[\mathbf{A}_\mu(x)\partial_\nu\mathbf{A}_\rho(x) + \frac{2}{3}\mathbf{A}_\mu(x)\mathbf{A}_\nu(x)\mathbf{A}_\rho(x)\right]$$

$$\mathcal{S}_{\rm Scalar} = \int \mathrm{d}^3x\left[(\mathbf{D}_\mu\phi(x))^\dagger\cdot\mathbf{D}_\mu\phi(x) + NV(\phi(x)^\dagger\cdot\phi(x)/N)\right].$$

and

$$\mathcal{S}(\psi,\bar{\psi},\mathbf{A}) = \mathcal{S}_{\rm CS}(\mathbf{A}) + \mathcal{S}_{\rm F}(\psi,\bar{\psi},\mathbf{A})$$

with $$\mathcal{S}_{\rm F}(\psi,\bar{\psi},\mathbf{A}) = -\int \mathrm{d}^3x\,\bar{\psi}(x)(\not{\mathbf{D}} + M_0)\psi(x)$$

In the $\mathbf{A}_3 = 0$ gauge,

Finnaly: On Fermion-Boson mapping in 3D

Two, very well understood, mechanisms for breaking scale invariance

(a) Explicit breaking of scale invariance, which is expressed at the quantum level by the anomaly in the trace of the energy momentum tensor, as the result of radiative corrections.

(b) Spontaneous breaking of scale invariance (e.g. Nambu–Jona-Lasinio as the relativistic version of the BCS theory)

In conventional quantum field theories the two mechanisms occur simultaneously, no massless Nambu Goldston boson (Dilaton).

Supersymmetric models in the large N limit

$O(N)$ invariant supersymmetric action (d=3):

$$\mathcal{S} = \int \mathrm{d}^3x\, \mathrm{d}^2\theta \ \left[\tfrac{1}{2}\bar{\mathrm{D}}\Phi \cdot \mathrm{D}\Phi + NU(\Phi^2/N)\right]$$

$O(N)$ vector: $\Phi(\theta, x) = \varphi + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F$

Components - for a generic super-potential:

$$\mathcal{S} = \int \mathrm{d}^3x \ \tfrac{1}{2}[-\bar{\psi}\partial\!\!\!/\psi + (\partial_\mu\varphi)^2 - (\bar{\psi}\cdot\psi)U'(\varphi^2/N)$$
$$- 2(\bar{\psi}\cdot\varphi)(\varphi\cdot\psi)U''(\varphi^2/N)/N + \varphi^2 U'^2(\varphi^2/N)]$$

The following are several phenomena that take place at $N \to \infty$:

(1) A supersymmetric ground state with $m_\psi = m_\phi \neq 0$ exists even in a renormalized **scale invariant** theory.

(2) At a certain strength of the attractive force between $O(N)$ bosons and fermions, **massless** $O(N)$ singlets bound states are created.

(3) At the, above mentioned, critical value of the coupling constant, though $m_\psi = m_\phi \neq 0$ there is no explicit breaking of scale invariance $< \partial^\mu S_\mu > \sim < \tilde{T}^\nu_{\ \nu} > = 0$.

(4) The massless fermionic and bosonic $O(N)$ singlet bound states mentioned in (2) are the Goldston-bosons and fermion (Dilaton and Dilatino) of the spontaneously broken scale invariant theory.

Action density: $\mathcal{E} = \mathcal{S}_N/\text{volume}$:

$$\mathcal{E}/N = \tfrac{1}{2}M^2\varphi^2/N + \frac{1}{24\pi}(m - |M|)^2(m + 2|M|)$$

m is the boson mass, M is the fermion mass.

$\mathcal{E}$ is positive for all saddle points and has an absolute minimum at $m_\varphi \equiv m = |M| = m_\psi$ (a supersymmetric ground state).

Φ^4 super-potential in $d = 3$: phase structure

$$U(\Phi^2/N) = (\mu/N)\Phi^2 + \tfrac{1}{2}(u/N^2)\Phi^4$$

Gap equations (saddle point equations) reduce to

$$M = \mu - \mu_c + u\frac{\varphi^2}{N} - \frac{u}{4\pi}|M| \quad , \qquad M\varphi = 0$$

Note the special case:
$\mu - \mu_c \equiv \mu_R = 0$ in the $O(N)$ symmetric phase ($\varphi = 0$).

The gap equation is:

$$M = -\frac{u}{4\pi}|M|$$

E.g. The $\psi \cdot \varphi$ scattering amplitude

$T_{\psi\cdot\varphi,\psi\cdot\varphi}(p^2)$, in the limit $p^2 \longrightarrow 0$ satisfies:

$$T_{\psi\varphi,\psi\varphi}(p^2) \sim \frac{-i2u}{N}\left[1 + \frac{u}{4\pi}\frac{m_\psi}{|m_\psi|} - \frac{u}{8\pi}\frac{\not{p}}{|m_\psi|}\right]^{-1}$$
$$\longrightarrow i\frac{16\pi}{N}\frac{|m_\psi|}{\not{p}}$$

Namely, a masless $O(N)$ singlet, fermion-boson bound state Dilatino for $m_\psi < 0$ and $u \longrightarrow u_c$
If we slightly deviate from the critical coupling u_c, dilatino acquires a mass given by

$$m_{D_\psi} = 2\left(1 - \tfrac{u_c}{u}\right)|m_\psi|$$

Similarly, in the boson-boson scattering amplitude $T_{\varphi\cdot\varphi,\varphi\cdot\varphi}$ or fermion-fermion $T_{\psi\cdot\psi,\psi\cdot\psi}$ or fermion-fermion to boson-boson scattering amplitude $T_{\psi\cdot\psi,\varphi\cdot\varphi}$ one finds the Dilaton pole at

$$m^2_{D_\varphi} = 4\left(1 - \tfrac{u_c}{u}\right)^2 m^2_\varphi$$

The SUSY energy-momentum tensor in 3D ($\xi = \frac{1}{8}$ in 3D) reduces in the case of flat space to:

$$
\begin{aligned}
T_{\mu\nu} &= \partial_\mu\varphi\partial_\nu\varphi + \tfrac{i}{4}(\overline{\psi}\gamma_\mu\partial_\nu\psi + \overline{\psi}\gamma_\nu\partial_\mu\psi) \\
&- \eta_{\mu\nu}[\tfrac{1}{2}\partial_\alpha\varphi(x)\partial^\alpha\varphi(x) - \tfrac{\mu_0^2}{2}\varphi^2 \\
&\qquad - (u/N)\mu_0\left(\varphi^2\right)^2 - \tfrac{(u/N)^2}{2}\left(\varphi^2\right)^3] \\
&- \eta_{\mu\nu}\left(\tfrac{1}{2}\overline{\psi}i\partial\!\!\!/\psi - \tfrac{\mu_0}{2}\overline{\psi}\psi - \tfrac{(u/N)}{2}\varphi^2\left(\overline{\psi}\psi\right)\right) \\
&- \tfrac{1}{8}\left(\partial^2_{\mu\nu}\varphi^2 - \eta_{\mu\nu}\partial^2\varphi^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\langle p_2^a \,|T^{\mu\nu}|\, p_1^a\rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \\
- \eta^{\mu\nu} p_1 p_2 - \eta^{\mu\nu} m^2 \\
+ \tfrac{1}{4}\left(q^\mu q^\nu - \eta^{\mu\nu} q^2\right) \times \\
\times \left[1 - 8\int_0^1 dx x(1-x)\left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right] \times \\
\times \left[1 - \int_0^1 dx \left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]^{-1}
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\langle p_2^a \,|T^{\mu\nu}|\, p_1^a\rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \eta^{\mu\nu} p_1 p_2 - \eta^{\mu\nu} m^2 \\
+ \left(q^\mu q^\nu - \eta^{\mu\nu} q^2\right)\left(\tfrac{1}{4} - \tfrac{m^2}{q^2}\right)
\end{aligned}
$$

The trace of the energy-momentum tensor:

$$
\begin{aligned}
T^{\mu}_{\mu} &= 2p_1p_2 - 3p_1p_2 - 3m^2 \\
&+ \left(q^2 - 3q^2\right)\left(\tfrac{1}{4} - \tfrac{m^2}{q^2}\right) \\
&= -p_1p_2 - m^2 - \tfrac{q^2}{2} \\
&= -\tfrac{1}{2}\left((p_1 - p_2)^2 + 2p_1p_2\right) - m^2 = 0
\end{aligned}
$$

Used (here, Euclidean space) $p_1^2 = p_2^2 = -m^2$.

Energy momentum tensor in one particle fermionic state

$$\int d^3x e^{iqx} \left\langle p^a_{2_\psi} \left| T_{\mu\nu}(x) \right| p^a_{1_\psi} \right\rangle =$$
$$-\frac{1}{4}\overline{u}(p_2)\left[(p_{1\nu}\gamma_\mu + p_{1\mu}\gamma_\nu) + (p_{2\nu}\gamma_\mu + p_{2\mu}\gamma_\nu)\right.$$
$$\left. +2\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) m\right] u(p_1)$$

This expression is also traceless.

Scalar-Fermion thermal mass difference at finite temperature

$$m_A^2 - m_\psi^2 = u[\frac{m_\psi}{2\pi}(|m_\psi| - m_A) + \frac{m_\psi}{\beta\pi}\ln(\frac{1+e^{-\beta|m_\psi|}}{1-e^{-\beta m_A}})]$$

Clearly $m_\varphi^2 \neq m_\psi^2$ at $T \neq 0$

Dilatino mass:

$$M_\psi^D \approx 2\left(1 + \frac{u}{u_c}\frac{m_\psi}{|m_\psi|}\right) + \frac{u}{u_c}\frac{\delta}{m_\psi}$$

δ is the boson-fermion thermal mass difference.

Taking into account the gap equations, we get the following expression for the thermal expectation value of the energy-momentum trace

$$\left\langle \; T_{\mu} \;\; {}^{\mu} \right\rangle_{T} = N(m_{\psi}^{2} - m_{\varphi}^{2})\frac{\mu_{R}}{2u}$$

Supersymmetry is softly broken when the temperature is turned on but the vanishing of the trace of the energy momentum tensor is guaranteed at $\mu_R = 0$.

Chern-Simons gauge field coupled to a U(N) scalar - light cone gauge

William A. Bardeen and M. M.

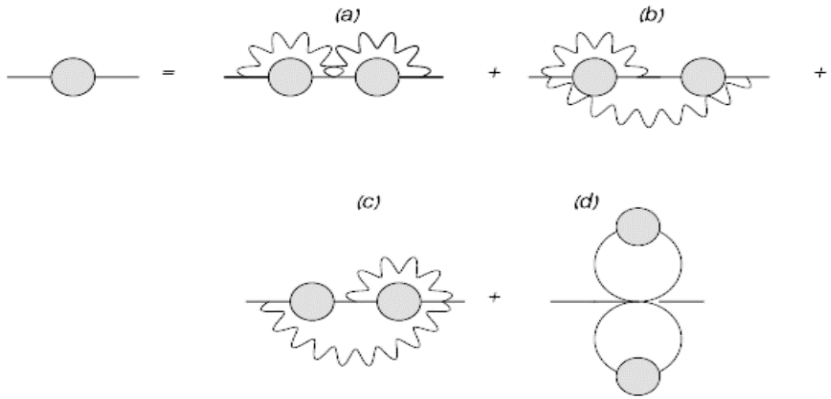
$$\mathcal{S}_{\rm CS}(\mathbf{A}) = -\frac{i\kappa}{4\pi}\epsilon_{\mu\nu\rho}\int \mathrm{d}^3x\, Tr\left[\mathbf{A}_\mu(x)\partial_\nu\mathbf{A}_\rho(x) + \frac{2}{3}\mathbf{A}_\mu(x)\mathbf{A}_\nu(x)\mathbf{A}_\rho(x)\right]$$

$$\mathcal{S}_{\rm Scalar} = \int \mathrm{d}^3x\left[(\mathbf{D}_\mu\phi(x))^\dagger\cdot\mathbf{D}_\mu\phi(x) + NV(\phi(x)^\dagger\cdot\phi(x)/N)\right].$$

in the light-cone gauge the action is linear in A^a_+

$$G_{+3}(p) = -G_{3+}(p) = \frac{4\pi i}{\kappa}\frac{1}{p^+} = 4\pi i \frac{\lambda}{N}\frac{1}{p^+}$$

$$NV(\phi^\dagger \cdot \phi/N) = \mu^2 \phi^\dagger \cdot \phi + \frac{1}{2}\frac{\lambda_4}{N}(\phi^\dagger \cdot \phi)^2 + \frac{1}{6}\frac{\lambda_6}{N^2}(\phi^\dagger \cdot \phi)^3$$



which sum up to

$$\Sigma^{(a,b,c)}(p,\lambda)_{ij} = 4\pi^2\lambda^2\delta_{ij}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3l}{(2\pi)^3}\frac{1}{(q^2+\Sigma(q))(l^2+\Sigma(l))}$$

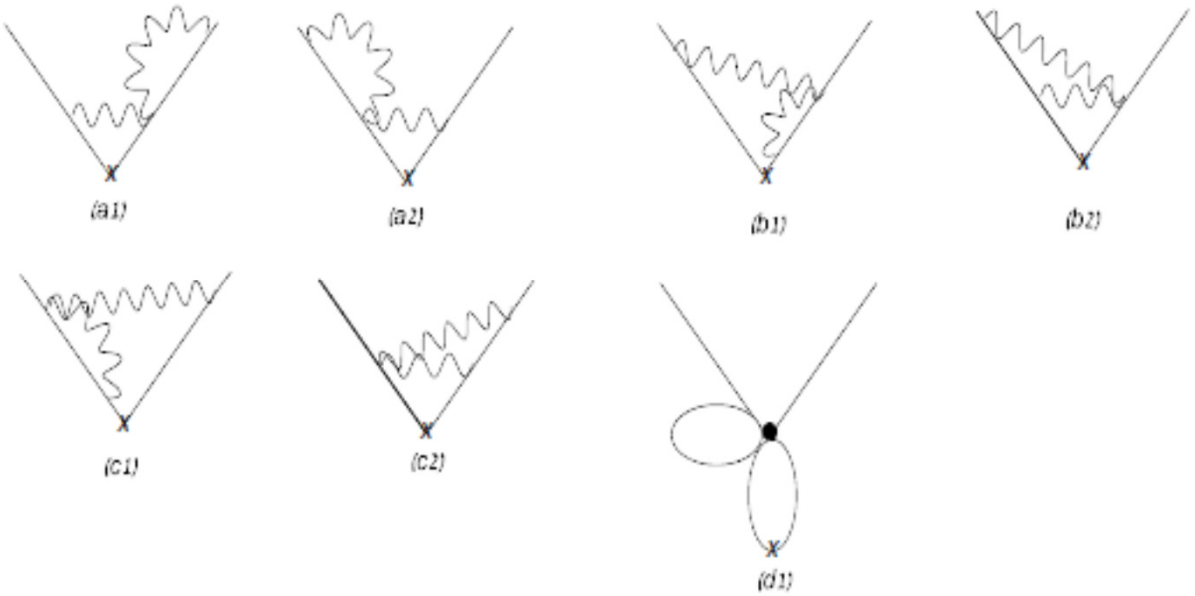
$$\Sigma^{(d)}(p,\lambda)_{ij} = \frac{1}{2}\lambda_6\delta_{ij}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3l}{(2\pi)^3}\frac{1}{(q^2+\Sigma(q))(l^2+\Sigma(l))}$$

$$\Sigma(p,\lambda,\lambda_6) = 4\pi^2\left(\lambda^2+\frac{\lambda_6}{8\pi^2}\right)\left\{\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{1}{(q^2+\Sigma(q))}\right\}^2$$

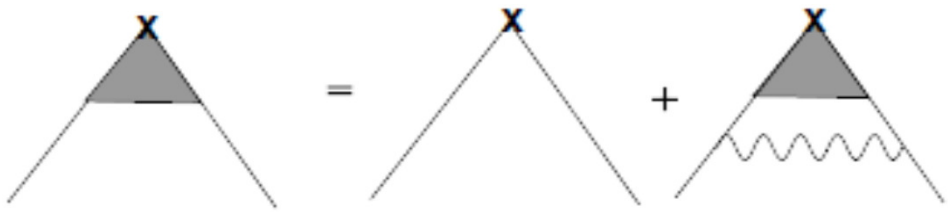
$$\Sigma(p, \lambda, \mu, \lambda_4 \lambda_6) = \frac{1}{4}\left(\lambda^2 + \frac{\lambda_6}{8\pi^2}\right)|\Sigma| - \lambda_{4R}\frac{\sqrt{|\Sigma|}}{4\pi} + \mu_R^2$$

$$\Sigma = \frac{1}{4}\left(\lambda^2 + \frac{\lambda_6}{8\pi^2}\right)|\Sigma|$$

(a) $\boldsymbol{\Sigma = M^2 = 0}$ (b) $\boldsymbol{\Sigma = M^2 \neq 0}$ if $\lambda^2 + \frac{\lambda_6}{8\pi^2} = 4$



$$V^{(a-d)}(p,k_3) = V = -8\pi^2\left(\lambda^2 + \frac{\lambda_6}{8\pi^2}\right)\int \frac{d^3l}{(2\pi)^3}\left(\frac{1}{l^2+\Sigma}\right)\int \frac{d^3q}{(2\pi)^3}$$
$$\left(\frac{1}{(l+k)^2+\Sigma}\right)\left(\frac{1}{q^2+\Sigma}\right)$$



$$V(p^2, k_3) = 1 + i4\pi\lambda k_3 \int \frac{d^3l}{(2\pi)^3} V(l^2, k_3) \frac{(l+p)^+}{(l-p)^+} \frac{1}{l^2+\Sigma} \frac{1}{(l+k)^2+\Sigma}$$

$$V(p^2, k_3) = C \exp\left\{ i\lambda k_3 \int dx (p^2 + x(1-x)k_3^2 + M^2)^{-1/2} \right\}$$

$$= 2 \exp\left\{ i\lambda k_3 \int_0^1 dx (p^2 + x(1-x)k_3^2 + M^2)^{-1/2} \right\}$$

$$\left\{ 1 + \exp[i\lambda k_3 \int_0^1 (x(1-x)k_3^2 + M^2)^{-\frac{1}{2}}] \right\}^{-1}$$

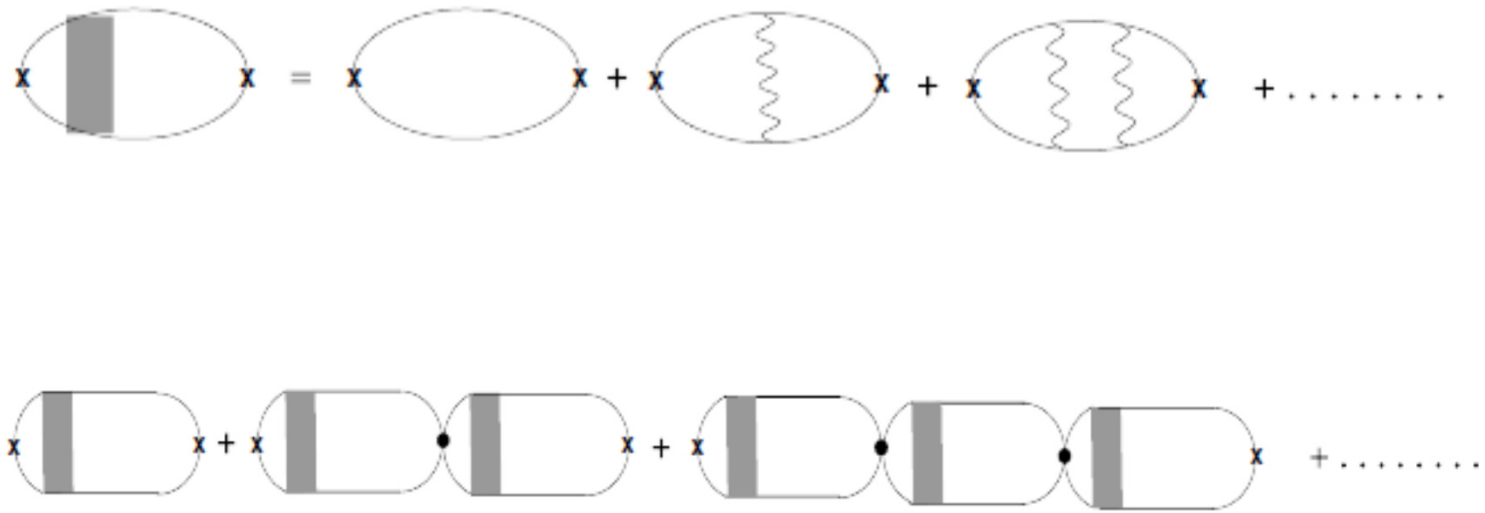


Figure 6. Full planar "bubble graph".

Figure 7. Full planar vertex.

$$\lambda_4^{\text{eff}} = \lambda_{4R} - 2\pi M\left(\lambda^2 + \frac{\lambda_6}{8\pi^2}\right) = -8\pi M$$

$\langle J_0 J_0 \rangle$ correlator and the dilaton

$$\langle J_0(k)\ J_0(-k)\rangle = \frac{3N}{2\pi}\left(\frac{M}{1-\lambda^2}\right)\frac{1}{k^2} = \frac{f_D^2}{k^2}$$

where

$$f_D = \sqrt{\frac{3NM}{2\pi(1-\lambda^2)}}$$

the effective Lagrangian of the dilaton

In terms of the dilaton field $D(x)$ (where $J_0(x) = f_D D(x)$)

$$\mathcal{L} = \frac{1}{2}\partial_\mu D \cdot \partial_\mu D - g_D(\phi^\dagger \cdot \phi)D$$

where $g_D = -\frac{M^{3/2}}{\sqrt{N}}\sqrt{(96\pi)/(1-\lambda^2)}$

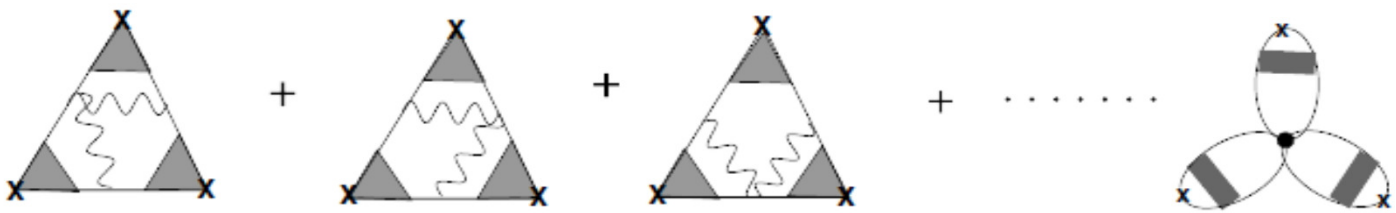


Figure 8. $\langle J_0(k)J_0(k')\ J_0(-k-k')\rangle$.

The dilaton self interaction can be now defined in the effective Lagrangian

$$\mathcal{L}_3 D = g_{3D}(\partial_\mu D \cdot \partial_\mu D)D \qquad g_{3D} = -\sqrt{\frac{6\pi}{NM(1-\lambda^2)^3}}$$

3D field theories with Chern-Simons term for large N in the Weyl gauge

M. M and Jean Zinn-Justin

$$\mathcal{S}(\psi, \bar{\psi}, \mathbf{A}) = \mathcal{S}_{\mathrm{CS}}(\mathbf{A}) + \mathcal{S}_{\mathrm{F}}(\psi, \bar{\psi}, \mathbf{A})$$

We now add to the Chern-Simons action, quantized in the $\mathbf{A}_3 = 0$ gauge, a $\mathrm{U}(N)$ gauge-invariant action for an N-component spinor field ψ,

$$\mathcal{S}_{\mathrm{CS}}(\mathbf{A}) = \frac{N}{ig}\mathrm{CS}_3(\mathbf{A}) = \frac{N}{ig}\int \mathrm{d}^3x\, \mathrm{tr}\,[\mathbf{A}_2(x)\partial_3\mathbf{A}_1(x) - \mathbf{A}_1(x)\partial_3\mathbf{A}_2(x)]$$

with $$\mathcal{S}_{\mathrm{F}}(\psi, \bar{\psi}, \mathbf{A}) = -\int \mathrm{d}^3x\, \bar{\psi}(x)(\not{\mathbf{D}} + M_0)\psi(x)$$

gauge field propagator $$\tilde{\Delta}^{ab}_{\alpha\beta}(p) = \epsilon_{\alpha\beta}\delta^{ab}\frac{g}{2N}\frac{1}{p_3}$$

The free energy density $$W = \frac{1}{NV} \ln(\mathcal{Z}/\mathcal{Z}_0)$$

Is exactly calculable at large N

Gauge-invariant observables

No summation

$$R_\alpha(x) = \frac{1}{N}\bar{\psi}_\alpha(x) \cdot \psi_\alpha(x)$$

$$\langle R \rangle = \langle R_1 + R_2 \rangle = 2\langle \rho_1 \rangle = 2M\Omega_1(M) + g\Omega_1^2(M).$$

$$= g\frac{\Lambda^2}{16\pi^2} + \frac{\Lambda M}{2\pi}\left(1 - \frac{g}{4\pi}\right) - \frac{M^2}{2\pi}\left(1 - \frac{g}{8\pi}\right)$$

Connected R correlation function at zero momentum

$$\langle \tilde{R}(0)\tilde{R}(0)\rangle_{\rm c} = 2\Omega_1(M) - \frac{4M^2\Omega_2(M)}{1-2gM\Omega_2(M)} = \frac{\Lambda}{2\pi} - \frac{M}{\pi}\frac{(1-g/8\pi)}{(1-g/4\pi)}$$

$$\langle \tilde{R}(0)\tilde{R}(0)\tilde{R}(0)\rangle_{\rm c} = -\frac{1}{\pi}\frac{(1-g/8\pi)}{(1-g/4\pi)^2}$$

The $\langle(\bar{\psi}\psi)\psi\bar{\psi}\rangle$ vertex function

$$W^{(1,2)}(x;y,z) = \langle\bar{\psi}(x)\cdot\psi(x)\psi(y)\cdot\bar{\psi}(z)\rangle_{\mathrm{c}}$$

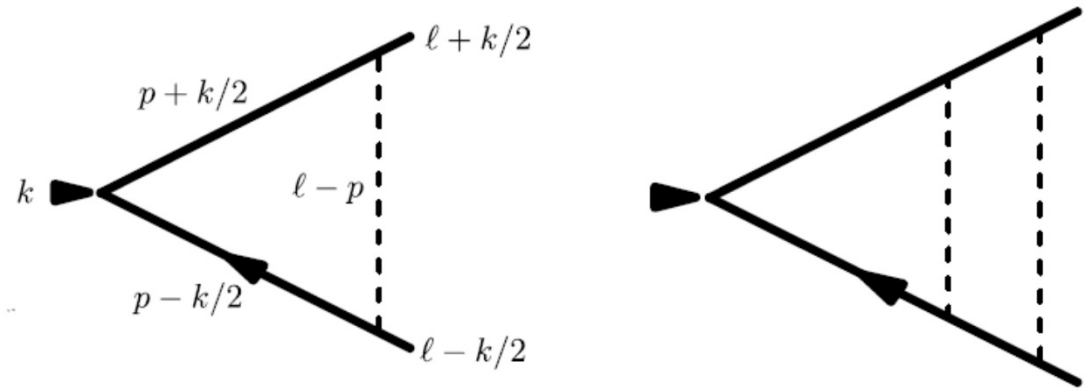


Figure 4. One-loop and two-loop contributions to the $\langle(\bar{\psi}\psi)\psi\bar{\psi}\rangle$ vertex function with dressed propagator. Dotted lines represent gauge fields.

Mass gap and critical coupling

For $g \neq 4\pi$, the fermion mass, solution of the gap equation, is

$$M = \frac{m}{1 - g/4\pi}\,.$$

where M is the fermion physical mass

For the special value $m = 0$ or $M_0 = M_c$:

$$M = \frac{g}{4\pi}|M|$$

But !

$$\langle \tilde{R}(k)\tilde{R}(-k)\rangle_c \underset{g\to 4\pi}{\sim} -\frac{1}{(4\pi - g)}\frac{k}{\arctan(k/2M)}$$

Adding a deformation to the Chern-Simons fermion action

$$\mathcal{S}_\sigma = \int \mathrm{d}^3x \left[-\sigma(x)\bar{\psi}(x)\cdot\psi(x) + \frac{N}{3g_\sigma}\sigma^3(x) - N\mathcal{R}\sigma(x) \right]$$

Gap equation:

$$\left[1 - \frac{(g-g_\sigma)}{2\pi}\left(1-\frac{g}{8\pi}\right)\right] M^2 + \left(1-\frac{g}{4\pi}\right)\left[\frac{(g-g_\sigma)}{2\pi}\Lambda - 2M_0\right] M$$
$$- g_\sigma\mathcal{R} + M_0^2 - \frac{g}{2\pi}M_0\Lambda + g(g-g_\sigma)\frac{\Lambda^2}{16\pi^2} = 0\,,$$

The gap equation then reads

$$\left[1-\frac{(g-g_\sigma)}{2\pi}\left(1-\frac{g}{8\pi}\right)\right]M^2-2\left(1-\frac{g}{4\pi}\right)mM+m^2\left(1-\eta\frac{g_\sigma}{4\pi}\right)=0$$

Finally, the gap equation is satisfied for any value of $M \geq 0$ if the coefficient of M^2 also vanishes, that is, for

$$\left(\frac{g-g_\sigma}{2\pi}\right)\left(1-\frac{g}{8\pi}\right)=1 \;\Leftrightarrow\; g_\sigma=-\frac{(4\pi-g)^2}{8\pi-g}\,.$$

The dilaton effective action

$$\varsigma(x) = f_D^{-1} D(x)$$

$$\langle \tilde{\varsigma}(k)\ \tilde{\varsigma}(-k) \rangle \underset{k\to 0}{\sim} \frac{f_D^2}{k^2}\,, \quad f_D = \sqrt{24\pi}\sqrt{\frac{1-g/4\pi}{2-g/4\pi}}\sqrt{M}\,.$$

$$\mathcal{S}(D) = \frac{1}{2}\int \mathrm{d}^3x\, \partial_\mu D(x)\partial_\mu D(x) F(D(x)) + \mathcal{O}(\text{higher order terms in } \partial_\mu)\,.$$

Spontaneously broken scale invariance in boson and fermion theories

The condition we found for the existence of a massive phase

Fermion +CS

$$\left(\frac{g-g_\sigma}{2\pi}\right)\left(1-\frac{g}{8\pi}\right)=1$$

In the boson theory, the existence of a massive ground state requires

Boson +CS

$$\lambda_b^2+\frac{\lambda_6}{8\pi^2}=4$$

using the mapping between the fermion and the boson theories

O. Aharony, S.Giombi, G.Gur-Ari, J. Maldacena and R. Yacoby
and S. Jain, S. Minwalla and S. Yokoyama

$$\lambda_b=\frac{g}{4\pi}-1\,,\quad \lambda_6=8\pi^2\left(1-\frac{g}{4\pi}\right)^2\left(3-4\frac{g}{g_\sigma}\right)$$

We find that the bosonic and the fermionic conditions are copies of each other

. . . . . .and finally an open problem

It would be interesting to explore the implications for the bulk four dimensional AdS dual description of the massive phase.

This is an open problem whose solution is not known at this point. In particular it is unknown whether the bulk theory is just a modification of Vassiliev's theory or whether new fields are required.

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Thanks

The End