Compens. Fields
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Compensator fields in dimensional reduction and compactification without truncation

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Plan

1. In dimensional reduction of gauge and/or gravity theories:
   - what are compensator fields?
   - why do we need them?

   — explained nicely by Douglas & Torroba (`hep-th/0805.3700`).

2. What about compactification without truncation, retaining the full KK tower:
   - do we still introduce compensators "by hand?"
   - or are they "already there?"

Motivation: very little literature on KK without truncation, despite naive expectation that this would have been extensively studied long ago.

We'll stick to U(1) gauge theory in this talk for simplicity.

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- Dim reduction = compactification truncated to zero modes

- Consider D-dim U(1) gauge theory

$$S = \int d^D x \sqrt{-g_D} \left(-\frac{1}{4} F^{MN} F_{MN}\right)$$

compactified on $\mathcal{Y}_{D-d}$, and let $A_M = (A_\mu, A_m)$.

- Gauge theory on $\mathcal{Y}_{D-d}$ alone has EOM $0 = \nabla^m_{\mathcal{Y}} F_{mn}$ with physical soln space $A_m = A_m(u; y)$ parametrized by u^I.

- In dim red theory, promote u^I to d-dim *fields* $u^I(x)$, and set $A_M = \big(A_\mu(x), A_m((u(x); y)\big)$ in D-dim action. Then,

$$S_{\text{dim red}} = \int d^d x \sqrt{-g_d} \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + G_{IJ} \partial^\mu u^I \partial_\nu u^J\right),$$

where $G_{IJ}(u) = \int_{\mathcal{Y}} d^{D-d} y \sqrt{g_{D-d}} g^{mn} \partial_I A_m \partial_J A_n$.

- *Problem*: $G_{IJ}(u)$ is **not** inv under $A_m(u; y) \to A_m(u; y) + \partial_m v(y)$.

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- *Resolution.* The correct moduli space metric is

$$G_{IJ}(u) = \int_{\mathcal{Y}} d^{D-d}y \sqrt{g_{D-d}}\, g^{mn} \delta_I A_m \delta_J A_n,$$

where

$$\delta_I A_m(u;y) = \partial_I A_m(u;y) - \Omega_{Im}(u;y)$$

is **gauge-invariant** and satisfies

$$\nabla^m_{\mathcal{Y}} \delta_I A_m(u;y) = 0 \qquad \text{(harmonic gauge condition)}.$$

- The **compensator** $\Omega_{Im}(u;y)$ projects out the gauge-variant part of $\partial_I A_m$ leaving a harmonic result independent of gauge choice.

- From the perspective of the dimensionally reduced theory, the compensator $\Omega_{Im}(u;y)$ is a **non-dynamical Lagrange multiplier.**

Geometry of the full space of gauge connections

- Let $\mathcal{A}$ denote the full space of connections $A_m(y)$ on $\mathcal{Y}$ (distinguishing different gauge choices).
- $\mathcal{A}$ is a fiber bundle with
  - fiber $\mathcal{G}$ — the space of gauge transformations,
  - base $\mathcal{A}/\mathcal{G}$ — the physical space of gauge connections mod gauge transfs.
- Convenient to parametrize with
  - base coords u^I parametrizing a fiducial rep $A_m(u; y)$ of each gauge orbit,
  - fiber coords v^ℓ parametrizing other reps $A_m(u, v; y) = A_m(u; y) + \partial_m v(y)$, where we expand $v(y) = v^\ell Y_\ell(y)$ in Laplace eigenfns $\nabla^2_{\mathcal{Y}} Y_\ell = -{m_\ell}^2 Y_\ell$.
- Let $\Lambda = (I, \ell)$. Then, the metric $G_{\Lambda\Lambda'} = \int_{\mathcal{Y}} d^{D-d}y \sqrt{g_{D-d}}\, g^{mn} \partial_\Lambda A_m \partial_{\Lambda'} A_n$ on the full space of connections is also of fiber bundle form

$$ds^2_{\mathcal{A}} = G_{IJ}(u) du^I du^J + G_{\ell\ell'} \left(dv^\ell + \Omega^\ell{}_I du^I\right)\left(dv^{\ell'} + \Omega^{\ell'}{}_J du^J\right).$$

- The compensator $\Omega^\ell{}_I$ is simply the **connection on the fiber bundle $\mathcal{A}$**.

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$$ds^2_{\mathcal{A}} = G_{IJ}(u)du^I du^J + G_{\ell\ell'}\big(dv^\ell + \Omega^\ell{}_I du^I\big)\big(dv^{\ell'} + \Omega^{\ell'}{}_J du^J\big).$$

we have

$$G_{\ell\ell'} = \int_{\mathcal{Y}} d^{D-d}y\sqrt{g_{D-d}}\,g^{mn}\partial_m Y_\ell \partial_n Y_{\ell'} = m_\ell{}^2\delta_{\ell\ell'} \quad \text{(no sum)},$$

$$m_\ell{}^2\Omega^\ell{}_I = \int_{\mathcal{Y}} d^{D-d}y\sqrt{g_{D-d}}\,g^{mn}\partial_I A_m \partial_n Y_\ell,$$

$$G_{IJ} + G_{\ell\ell'}\Omega^\ell{}_I\Omega^{\ell'}{}_J = \int_{\mathcal{Y}} d^{D-d}y\sqrt{g_{D-d}}\,g^{mn}\partial_I A_m \partial_J A_n,$$

where $\nabla^2_{\mathcal{Y}} Y_\ell = -m_\ell{}^2 Y_\ell$.

So, the metric $ds^2_{\mathcal{A}}$ on the space $\mathcal{A}$ of gauge connections (including the compensator Ω) **is determined,** given a metric on $\mathcal{Y}$ and coordinate chart u^I, v^ℓ on $\mathcal{A}$.

In this context, the compensator is "already there" and does not need to be introduced "by hand."

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For compactification of D-dimensional U(1) gauge theory on a compact manifold $\mathcal{Y}$ without trunction, we promote u^I, v^ℓ of the last slide to fields $u^I(x), v^\ell(x)$ and write $A_M(x,y) = (A_\mu, A_m) = \big(A^\ell{}_\mu(x) Y_\ell(y), A_m(u(x), v(x); y)\big)$. Then,

$$S = \int d^D x \sqrt{-g_D} \big(-\frac{1}{4} F^{MN} F_{MN}\big)$$

$$= \int d^d x \sqrt{-g_d} \big(\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{charged scalar}} + \mathcal{L}_{\text{neutral scalar}}\big),$$

where

$$\mathcal{L}_{\text{gauge}} = \sum_\ell \big(-\frac{1}{4} F^{\ell\mu\nu} F^\ell{}_{\mu\nu}\big),$$

$$\mathcal{L}_{\text{charged scalar}} = -\frac{1}{2} \sum_{\ell \neq 0} m_\ell{}^2 D^\mu v^\ell(x) D_\mu v^\ell(x),$$

$$\mathcal{L}_{\text{neutral scalar}} = -\frac{1}{2} G_{IJ}(u) \partial^\mu u^I(x) \partial_\mu u^J(x) - V\big(u(x)\big),$$

with

$$D_\mu v^\ell = \partial_\mu v^\ell - \Omega^\ell{}_I \partial_\mu u^I + A^\ell{}_\mu \quad \text{and} \quad V(u) = \int_{\mathcal{Y}} d^{D-d} y \sqrt{g_{D-d}} \frac{1}{4} F^{mn}(u;y) F_{mn}(u;y).$$

- For $\ell \neq 0$, the vector $A^\ell{}_\mu(x)$ eats $\partial_\mu v^\ell(x) - \Omega^\ell{}_I \partial_\mu u^I$ to become massive.
- The scalars u^I deforming away from flat $F_{mn}(u) = 0$ are massive due to $V(u)$.

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The low energy effective field theory below the compactification scale is exactly of the dim red form discussed earlier,

$$\mathcal{L}_{\mathrm{EFT}} = -\frac{1}{4} F^{0\mu\nu} F^{0}{}_{\mu\nu} - G^{\mathrm{flat}}_{IJ}(u)\partial^{\mu} u^{I} \partial_{\mu} u^{J},$$

where in the first term only $\ell = 0$ contributes, and in the second term "flat" denotes the restriction to flat deformations u^I with $F_{mn}(u) = 0$.
Here, as above,

$$G_{IJ} = \int_{\mathcal{Y}} d^{D-d}y \sqrt{g_{D-d}}\, g^{mn} \delta_I A_m \delta_J A_n,$$

with

$$\delta_I A_m = \partial_I A_m - \Omega^{\ell}{}_I(u) \partial_m Y_{\ell}.$$

The compensator $\Omega_{Im} = \Omega^{\ell}{}_I(u)\partial_m Y_{\ell}(y)$ of the dim red EFT has contributions due to arbitrary D-dimensional gauge transformations $v(x, y) = v^{\ell}(x) Y_{\ell}(y)$, not just those with $\ell = 0$.

Conclusions

In the context of D-dimensional U(1) gauge theory compactified on a manifold $\mathcal{Y}_{D-d}$, we have seen that:

- compensators introduced "by hand" as Lagrange multipliers ensure D-dimensional gauge invariance of the dimensional reduction ansatz and of the resulting d-dimensional scalar kinetic terms;
- the full space of gauge connections on $\mathcal{Y}_{D-d}$ is a fiber bundle, and the compensators can be interpreted as a preferred connection on this bundle;
- as such, the compensators arise naturally in the untruncated theory and need not be included by hand;
- in the full untruncated theory, the KK gauge bosons eat the vertical moduli, and the scalar potential lifts those horizontal moduli corresponding to non-flat deformations of the gauge field;
- the effective field theory below the compactification scale exactly agrees with that of the dimensional reduction ansatz.

Generalizing to gravity and Yang-Mills, the story is analogous, but the KK is expansion is not as "clean." For example, $v^\ell \partial_m Y_\ell$ of the U(1) theory generalizes to invariant 1-forms $g^{-1}\partial_m g$ and $(\partial_m g)g^{-1}$ of Yang-Mills, but with no correspondingly simple global expansion in Laplace eigenfunctions.