The global SMEFT fit at e+e- Higgs factories: what is the role of each measurement?

arXiv:1708.09079, 1708.08912;
+ paper in preparation

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proposals of future lepton colliders

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>beam polarisation</th>
<th>$\int L dt$ for Higgs</th>
<th>R&amp;D phase</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ILC</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.1 - 1 TeV</td>
<td>e-: 80%</td>
<td>2000 fb$^{-1}$ @ 250 GeV</td>
<td>TDR</td>
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<tr>
<td></td>
<td>e+: 30% (20%)</td>
<td>200 fb$^{-1}$ @ 350 GeV</td>
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<td></td>
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<td>4000 fb$^{-1}$ @ 500 GeV</td>
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<td>8000 fb$^{-1}$ @ 1 TeV</td>
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<td><strong>CLIC</strong></td>
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<tr>
<td>0.35 - 3 TeV</td>
<td>e-: 80%</td>
<td>500 fb$^{-1}$ @ 380 GeV</td>
<td>CDR</td>
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<tr>
<td></td>
<td>e+: 0%</td>
<td>1500 fb$^{-1}$ @ 1.4 TeV</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>2500 fb$^{-1}$ @ 3 TeV</td>
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<tr>
<td>90 - 240 GeV</td>
<td>e-: 0%</td>
<td>5600 fb$^{-1}$ @ 240 GeV</td>
<td>CDR</td>
</tr>
<tr>
<td></td>
<td>e+: 0%</td>
<td></td>
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<tr>
<td><strong>FCC-ee</strong></td>
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</tr>
<tr>
<td>90 - 365 GeV</td>
<td>e-: 0%</td>
<td>5000 fb$^{-1}$ @ 240 GeV</td>
<td>CDR</td>
</tr>
<tr>
<td></td>
<td>e+: 0%</td>
<td>1500 fb$^{-1}$ @ 365 GeV</td>
<td></td>
</tr>
</tbody>
</table>

common: Higgs factory with $O(10^6)$ Higgs events
two apparent important thresholds: $\sqrt{s} \sim 250$ GeV for ZH, 
$\sim 500$ GeV for ZHH and ttH

+ another threshold for t t-bar, important for vacuum stability
Higgs coupling determination — kappa formalism

1) recoil mass technique $\rightarrow$ inclusive $\sigma_{Zh}$

2) $\sigma_{Zh} \rightarrow K_Z \rightarrow \Gamma(h\rightarrow ZZ^*)$

3) W-fusion $\nu_e\nu_e h \rightarrow K_W \rightarrow \Gamma(h\rightarrow WW^*)$

4) total width $\Gamma_h = \Gamma(h\rightarrow ZZ^*)/BR(h\rightarrow ZZ^*)$

5) or $\Gamma_h = \Gamma(h\rightarrow WW^*)/BR(h\rightarrow WW^*)$

6) then all other couplings $BR(h\rightarrow XX) \star \Gamma_h \rightarrow K_X$
one question in kappa formalism:

\[
\frac{\sigma(e^+e^- \to Zh)}{SM} \quad \text{and} \quad \frac{\Gamma(h \to ZZ^*)}{SM} = \kappa_Z^2
\]

BSM territory: can deviations be represented by single \( \kappa_Z \)?
the answer is model dependent

$$\delta \mathcal{L} = (1 + \eta_Z) \frac{m_Z^2}{v} h Z_{\mu} Z^\mu + \zeta_Z \frac{h}{2v} Z_{\mu\nu} Z^{\mu\nu}$$

- BSM can induce new Lorentz structures in $hZZ$
- need a better, more theoretical sound framework

$$\sigma(e^+e^- \rightarrow Zh) = (SM) \cdot (1 + 2\eta_Z + (5.5)\zeta_Z)$$

$$\Gamma(h \rightarrow ZZ^*) = (SM) \cdot (1 + 2\eta_Z - (0.50)\zeta_Z)$$
new strategy: SM Effective Field Theory

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \Delta \mathcal{L} \]

\[ = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i \]

- a more model independent formalism
- most general effects from BSM represented
- respect SU(3)xSU(2)xU(1) gauge symmetries
- a consistent quantum field theory unifying BSM effects in Higgs, W/Z, top, 2-fermion physics
SM Effective Field Theory: some simplifications

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \Delta \mathcal{L} \]

\[ = \mathcal{L}_{\text{SM}} + \sum_{i} \frac{c_i}{\Lambda^{d_i - 4}} O_i \]

the new particle searches at LHC Run 2 suggest \( \Lambda > 500 \) GeV justify the analysis at dimension-6 operators

there are 84 of such operators for 1 fermion generation assuming B & L number conservation, there are 59

• there exists a smaller but complete set relevant to Higgs physics at e+e-
SM Effective Field Theory @ e+e-

(Barklow, Fujii, Jung, Peskin, JT, arXiv:1708.09079)

\[
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3
\]

\[
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu \nu} W^{a \mu \nu} + \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu \nu} B^{\mu \nu}
\]

\[
+ \frac{g^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^a_{\mu \nu} W^{b \rho \nu} \rho W^{c \mu \nu}
\]

\[
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\not{L} \gamma_\mu \not{L}) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D}^\mu \Phi)(\not{L} \gamma_\mu t^a \not{L})
\]

\[
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\not{e} \gamma_\mu \not{e})
\]

\[\Phi: \text{higgs field}\]

\[W, B: \text{SU(2), U(1) gauge}\]

\[L, e: \text{left/right electron}\]

- 10 operators modifying couplings for h/Z/W/γ
- in total, 23 parameters (see later slides)

next: highlight a few important implications
recap 1: absolute Higgs couplings (unique role of inclusive $\sigma_{Zh}$)

\[
\frac{c_H}{2 \nu^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi)
\]

\[
\frac{c_H}{2} \partial^\mu h \partial_\mu h \quad \rightarrow \quad \text{renormalize kinetic term of SM Higgs field}
\]

\[
h \quad \rightarrow \quad (1-c_H/2)h
\]

\[
\rightarrow \quad \text{shift all SM Higgs couplings by } -c_H/2
\]

- $c_H$ cannot be determined by any BR or ratio of couplings.

- $c_H$ has to rely on inclusive cross section of $e^+e^- \rightarrow Zh$, enabled by recoil mass technique at $e^+e^-$. 
recap 2: Higgs couplings are related to W-/Z- couplings (EWPOs)

\[ i \frac{c_{HL}}{v^2} (\Phi^+ \overleftrightarrow{D} \mu \Phi)(L \gamma_\mu L) \]

\[ + (c'_L, c_{HE}) \]

- Higgs coupling helped by EWPOs at Z-pole: \( A_{LR}, \Gamma_i \)
- \( Z \) coupling helped by Higgs meas. at high \( \sqrt{s} \): \( \delta \sigma \sim s/m^2_Z \)
recap 2: Higgs couplings are related to W-/Z- couplings (TGCs)

\[ \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu\nu}B^{\mu\nu} \]

+ \( (c_{WW}, c_{BB}) \)

- Longitudinal modes of W/Z are from Higgs fields
- Higgs coupling helped by meas. of TGCs in e+e- -> WW
recap 3: Higgs couplings are related to themselves

\[ \Delta \mathcal{L}_h = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - (1 + \eta_h) \overline{\lambda} v h^3 + \frac{\theta_h}{v} h \partial_\mu h \partial^\mu h \\
+ (1 + \eta_W) \frac{2m_W^2}{v} W^+_\mu W^-\mu h + (1 + \eta_{WW}) \frac{m_W^2}{v^2} W^+_\mu W^-\mu h^2 \\
+ (1 + \eta_Z) \frac{m_Z^2}{v} Z_\mu Z^\mu h + \frac{1}{2} (1 + \eta_{ZZ}) \frac{m_Z^2}{v^2} Z_\mu Z^\mu h^2 \\
+ \zeta_W \hat{W}_{\mu\nu}^+ \hat{W}^{-\mu\nu} \left( \frac{h}{v} + \frac{1}{2} \frac{h^2}{v^2} \right) + \frac{1}{2} \zeta_Z \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} \left( \frac{h}{v} + \frac{1}{2} \frac{h^2}{v^2} \right) \\
+ \frac{1}{2} \zeta_A \hat{A}_{\mu\nu} \hat{A}^{\mu\nu} \left( \frac{h}{v} + \frac{1}{2} \frac{h^2}{v^2} \right) + \zeta_{AZ} \hat{A}_{\mu\nu} \hat{Z}^{\mu\nu} \left( \frac{h}{v} + \frac{1}{2} \frac{h^2}{v^2} \right). \]

(SM structure: kappa like) \hspace{2cm} (Anomalous: new Lorentz structure)

\[ \eta_h = \delta \overline{\lambda} + \delta v - \frac{3}{2} c_H + c_6 \]

\[ \eta_W = 2 \delta m_W - \delta v - \frac{1}{2} c_H \]

\[ \eta_{WW} = 2 \delta m_W - 2 \delta v - c_H \]

\[ \eta_Z = 2 \delta m_Z - \delta v - \frac{1}{2} c_H - c_T \]

\[ \eta_{ZZ} = 2 \delta m_Z - 2 \delta v - c_H - 5c_T \]

\[ \theta_h = c_H \]

\[ \zeta_W = \delta Z_W = (8c_{WW}) \]

\[ \zeta_Z = \delta Z_Z = c_w^2 (8c_{WW}) + 2s_w^2 (8c_{WB}) + s_w^4 / c_w^2 (8c_{BB}) \]

\[ \zeta_A = \delta Z_A = s_w^2 \left( (8c_{WW}) - 2(8c_{WB}) + (8c_{BB}) \right) \]

\[ \zeta_{AZ} = \delta Z_{AZ} = s_w c_w \left( (8c_{WW}) - (1 - s_w^2)(8c_{WB}) - s_w^2 (8c_{BB}) \right) \]

• \( hZZ/hWW/h\gamma Z/h\gamma\gamma \) highly related: SU(2)xU(1) gauge symmetries
Recap 3: Higgs couplings are related to themselves (synergy w/ LHC)

two measurements from LHC (model independent)

\[ R_{\gamma\gamma} = \frac{BR(h \to \gamma\gamma)}{BR(h \to ZZ^*)} \quad R_{\gamma Z} = \frac{BR(h \to \gamma Z)}{BR(h \to ZZ^*)} \]

\[ \delta\Gamma(h \to \gamma\gamma) = 528 \delta Z_A - c_H + \ldots \]

\[ \delta\Gamma(h \to Z\gamma) = 290 \delta Z_{AZ} - c_H + \ldots \]

\[ \delta\Gamma(h \to ZZ^*) = -0.50\delta Z_Z - c_H + \ldots \]

- Loop induced $h\to\gamma\gamma/\gamma Z$ depend strongly on $c_{WW}/c_{WB}/c_{BB}$

- $h\to\gamma\gamma/\gamma Z$ at LHC can nicely help higgs couplings at e+e-
SM-like hVV

\[ \eta_W = -\frac{1}{2} c_H \]

\[ \eta_Z = -\frac{1}{2} c_H - c_T \]

custodial symmetry is broken by \( c_T \rightarrow \) constrained by EWPOs

anomalous hVV

\[ \zeta_W = (8c_{WW}) \]
\[ \zeta_Z = c_w^2(8c_{WW}) + 2s_w^2(8c_{WB}) + (s_w^4/c_w^2)(8c_{BB}) \]

- hWW/hZZ ratio can be determined to <0.1%
- very important for physics case of any 250 GeV e+e-
- hWW can be determined as precisely as hZZ at 250 GeV; hence precision total width & other couplings
## SMEFT fit: typical difference with kappa fit

ILC250: \( \int \mathcal{L} dt = 2\text{ ab}^{-1} @ 250\text{ GeV} \)

<table>
<thead>
<tr>
<th>coupling ( \Delta g/g )</th>
<th>kappa-fit</th>
<th>EFT-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>hZZ</td>
<td>0.38%</td>
<td>0.50%</td>
</tr>
<tr>
<td>hWW</td>
<td>1.8%</td>
<td>0.50%</td>
</tr>
<tr>
<td>hbb</td>
<td>1.8%</td>
<td>0.99%</td>
</tr>
<tr>
<td>( \Gamma_h )</td>
<td>3.9%</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

(definition for higgs coupling precision: \( 1/2 \) of partial width precision)
recap 4: role of beam polarizations

\[ P(e^-,e^+) \]

(\(-1,+1\)) \[ \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \sin^2 \theta_w \right) \quad g \sin \theta_w \quad \frac{g}{\cos \theta_w} (c_{HL} + c'_{HL}) \]

(+1,-1) \[ \frac{g}{\cos \theta_w} (-\sin^2 \theta_w) \quad g \sin \theta_w \quad \frac{g}{\cos \theta_w} (c_{HE}) \]

- sensitive to different couplings -> lift degeneracy
- \( A_{LR} \) in \( \sigma_{ZH} \) -> improve \( c_{WW} \), \( c_{HL} + c'_{HL} \) and \( c_{HE} \)
- large cancellation in \((+1,-1)\) -> weaker dependence on \( c_{WW} \)
recap 4: role of beam polarizations (e+e- -> Zh)

\[ \delta \sigma_L = - c_H + 7.7(8c_{WW}) + \ldots \]
\[ \delta \sigma_R = - c_H + 0.6(8c_{WW}) + \ldots \]
\[ \delta \sigma_0 = - c_H + 4.6(8c_{WW}) + \ldots \]

\( \sqrt{s} = 250 \text{ GeV} \)

\( (8c_{WW}) \sim 0.16\% \) from other meas.

contribution from
almost cancels out

up to a difference in Z/γ propagator suppressed by \( \frac{m_Z^2}{s} \)

why?
### Recap 4: Role of Beam Polarizations (Overall Effects)

**ILC250: 2 ab\(^{-1}\)  **  **FCCee240: 5 ab\(^{-1}\)**

<table>
<thead>
<tr>
<th>Coupling</th>
<th>2/ab-250</th>
<th>+4/ab-500</th>
<th>5/ab-250</th>
<th>+ 1.5/ab-350</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H ZZ)</td>
<td>0.50</td>
<td>0.35</td>
<td>0.41</td>
<td>0.34</td>
</tr>
<tr>
<td>(H WW)</td>
<td>0.50</td>
<td>0.35</td>
<td>0.42</td>
<td>0.35</td>
</tr>
<tr>
<td>(H bb)</td>
<td>0.99</td>
<td>0.59</td>
<td>0.72</td>
<td>0.62</td>
</tr>
<tr>
<td>(H \tau \tau)</td>
<td>1.1</td>
<td>0.75</td>
<td>0.81</td>
<td>0.71</td>
</tr>
<tr>
<td>(H gg)</td>
<td>1.6</td>
<td>0.96</td>
<td>1.1</td>
<td>0.96</td>
</tr>
<tr>
<td>(H cc)</td>
<td>1.8</td>
<td>1.2</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>(H \gamma \gamma)</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(H \gamma Z)</td>
<td>9.1</td>
<td>6.6</td>
<td>9.5</td>
<td>8.1</td>
</tr>
<tr>
<td>(H \mu \mu)</td>
<td>4.0</td>
<td>3.8</td>
<td>3.8</td>
<td>3.7</td>
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<tr>
<td>(H tt)</td>
<td>-</td>
<td>6.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(H HH)</td>
<td>-</td>
<td>27</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\Gamma_{tot})</td>
<td>2.3</td>
<td>1.6</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>(\Gamma_{inv})</td>
<td>0.36</td>
<td>0.32</td>
<td>0.34</td>
<td>0.30</td>
</tr>
<tr>
<td>(\Gamma_{other})</td>
<td>1.6</td>
<td>1.2</td>
<td>1.1</td>
<td>0.94</td>
</tr>
</tbody>
</table>

- 250 GeV e+e-: power of 2 ab\(^{-1}\) polarized \(\approx\) 5 ab\(^{-1}\) unpolarized

(arXiv:1903.01629)
SM Effective Field Theory: full formalism (23 pars.)

\[
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3
\]
\[
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W_{\mu \nu}^a W^{a \mu \nu} + \frac{4 g g' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W_{\mu \nu}^a B^{\mu \nu}
\]
\[
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W_{\mu \nu}^a W^{b \rho \nu} W_{c \rho \mu}
\]
\[
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\not{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D}^\mu \Phi)(\not{L} \gamma_\mu t^a L)
\]
\[
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\not{e} \gamma_\mu \not{e}).
\]

10 operators (h,W,Z,γ): c_H, c_T, c_6, c_{WW}, c_{WB}, c_{BB}, c_{3W}, c_{HL}, c'_{HL}, c_{HE}

+ 4 SM parameters: g, g', v, λ

+ 5 operators modifying h couplings to b, c, τ, μ, g

+ 2 operators for contact interactions with quarks

+ 2 parameters for h->invisible and exotic
strategy to determine all the 23 parameters at e+e-

Electroweak Precision Observables (9) +
Triple Gauge boson Couplings (3) +
Higgs observables at LHC & e+e- (3+12x2)

• at e+e-, all the 23 parameters can be determined simultaneously

(details in backup)
precisions at Higgs factories: complementarity with LHC

#qualitative:
model independence, hcc coupling

#quantitative (<~1%):
hZZ, hWW, hbb, h\(\tau\)\(\tau\)
h\(\rightarrow\)invisible/exotic

#synergy:
h\(\gamma\)\(\gamma\), h\(\gamma\)Z, h\(\mu\)\(\mu\), htt, \(\lambda\)

Model Independent EFT Fit

- HL-LHC + ILC250
- HL-LHC + ILC250 + ILC500

dark/light: \(S1^*/S2^*\)

Precision of Higgs boson couplings [%]

LCC Physics WG

(\(\times 1/3\))

(\(\times 1/2\))

(\(\times 1/10\))

(arXiv:1903.01629)
role of each measurement: more transparent understanding

(Peskin, JT, paper in preparation)

why not following $1/\sqrt{L}$? why so different for $hZZ/hbb/hcc$?
role of each measurement: more transparent understanding

(Peskin, JT, paper in preparation)

to a very good approximation, every EFT coefficient and Higgs coupling can be expressed directly by a set of input observables

for example: unpolarized e+e- at 250 GeV

\[
\delta g_{hZZ} = \frac{1}{2} \delta \sigma_{Zh} + 6.4 \delta \Gamma_l + 5.3 \delta g_{Z, eff} - 0.015 \delta R_{\gamma Z} - 2.4 \delta \kappa_{A, eff} + 8.9 \delta m_h + 0.098 \delta A_l + \cdots
\]

\[
\delta X = \frac{\Delta X}{X}
\]

\[
\begin{align*}
\sigma_{Zh} & : \text{cross section of e+e- -> Zh} \\
A_l, \Gamma_l & : A_{LR} \text{ and } \Gamma(Z->\ell\ell) \text{ at Z-pole} \\
g_{Z, eff}, \kappa_{A, eff} & : \text{Triple Gauge Couplings} \\
R_{\gamma Z} & : \text{BR}(h->\gamma Z) / \text{BR}(h->ZZ^*) \\
m_h & : \text{Higgs mass}
\end{align*}
\]
role of each measurement: more transparent understanding

(Peskin, JT, paper in preparation)

for example: unpolarized e+e- at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab-1

\[
\delta g_{hZZ} = \frac{1}{2} \delta \sigma_{Zh} + 6.4 \delta \Gamma_l + 5.3 \delta g_{Z,\text{eff}} - 0.015 \delta R_{\gamma Z} - 2.4 \delta \kappa_{A,\text{eff}} + 8.9 \delta m_h + 0.098 \delta A_l + \ldots
\]

\[
= 41 \oplus 66 \oplus 30 \oplus 23 \oplus 14 \oplus 11 \oplus 8.7 \oplus \ldots \times 10^{-4}
\]

<table>
<thead>
<tr>
<th>\sigma_{Zh}</th>
<th>EWPOs</th>
<th>TGCs</th>
<th>BR(h-&gt;\gamma Z)</th>
<th>Higgs mass</th>
</tr>
</thead>
</table>

importance hierarchy
**role of each measurement: more transparent understanding**

(Peskin, JT, paper in preparation)

for example: unpolarized $e^+e^-$ at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab$^{-1}$

$$\delta g_{hbb} = \frac{1}{2} \delta B_{bb} - \frac{1}{2} \delta B_{WW} + \frac{1}{2} \delta \sigma_{Zh} - 5.79 \delta \Gamma_l - 0.016 \delta \Gamma_{\gamma Z} + \cdots$$

$$= 28 \oplus 91 \oplus 41 \oplus 59 \oplus 32 \oplus \cdots \times 10^{-4}$$

\[\begin{array}{l}
\text{BR}(h\rightarrow WW)\\
\sigma_{Zh} \quad \text{EWPOs}\\
\text{BR}(h\rightarrow bb) \quad \text{BR}(h\rightarrow \gamma Z)
\end{array}\]
role of each measurement: more transparent understanding

(Peskin, JT, paper in preparation)

for example: unpolarized e+e- at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab-1

\[
\delta g_{hcc} = \frac{1}{2} \delta B_{cc} - \frac{1}{2} \delta B_{WW} + \frac{1}{2} \delta \sigma_Z - 5.79 \delta \Gamma_l - 0.016 \delta \Gamma_{\gamma Z} + \cdots
\]

\[
= 160 \oplus 91 \oplus 41 \oplus 59 \oplus 32 \oplus \cdots \times 10^{-4}
\]

| BR(h->cc) | BR(h->WW) | \sigma_Z | EWPOs | BR(h->\gamma Z) |

stay tuned
summary

• the capabilities of a $e^+e^-$ are best represented in SMEFT formalism

• EWPOs and TGCs are related to Higgs couplings, hence important

• Higgs measurements from (HL-)LHC are helpful for $e^+e^-$

• beam polarizations play an important role

• all proposed Higgs factories can deliver 1% or below precision for many Higgs couplings already at $\sqrt{s} = 250$ GeV

• let’s go for at least one of them
backup
Higgs self-coupling: indirect determination

\[ \delta\sigma \] could receive contributions from many other sources

\[ \delta h \sim 500\% \] at 250GeV only; Gu, et al, arXiv:1711.03978

\[ \delta h \sim 50\% + 350/500\text{GeV}; \] Peskin, JT, paper in preparation

• open: what if we include other NLO effects as well?
(vi) Higgs self-coupling

- Direct probe of the Higgs potential
- Large deviation (> 20%) motivated by electroweak baryogenesis, could be ~100%
- $\sqrt{s} \geq 500$ GeV, $e^+e^- \rightarrow ZHH$
- $\sqrt{s} \geq 1$ TeV, $e^+e^- \rightarrow \bar{\nu}\nu HH$ (WW-fusion)

**ILC**

<table>
<thead>
<tr>
<th>$\Delta \lambda_{HHH}/\lambda_{HHH}$</th>
<th>500 GeV</th>
<th>+ 1 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>H20</td>
<td>27%</td>
<td>10%</td>
</tr>
</tbody>
</table>

**CLIC**

<table>
<thead>
<tr>
<th>1.5 TeV</th>
<th>+3 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>36%</td>
<td>10%</td>
</tr>
</tbody>
</table>
Higgs self-coupling: when $\lambda_{HHH} \neq \lambda_{SM}$?

- Constructive interference in ZHH, while destructive in $\nu\nu HH$ (& LHC) $\rightarrow$ complementarity between ILC & LHC, between $\sqrt{s} \sim 500$ GeV and $>1$ TeV
- If $\lambda_{HHH} / \lambda_{SM} = 2$, Higgs self-coupling can be measured to $\sim 15\%$ using ZHH at 500 GeV $e^+e^-$

Duerig, Tian, et al, paper in preparation
role of each measurement: more transparent understanding

(Peskin, JT, paper in preparation)

for example: \textbf{polarized} e+e- at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab$^{-1}$

\[
\delta g_{hZZ} = \frac{1}{2} \delta \sigma_{Zh}^R + 0.35 \delta A_l + 3.6 \delta \kappa_{A,\text{eff}} - 4.0 \delta g_{Z,\text{eff}} - 1.8 \delta \Gamma_l + 8.9 \delta m_h + \ldots
\]

\[
= 51 \oplus 31 \oplus 20 \oplus 17 \oplus 18 \oplus 11 \oplus \ldots \times 10^{-4}
\]

\begin{align*}
\sigma_{Zh} & \quad A_{LR} & \quad TGCs & \quad TGCs & \quad \Gamma_l & \quad \text{Higgs mass} \\
\text{importance hierarchy}
\end{align*}
strategy to determine all the 23 parameters

- \( m_W \) and \( a(m_Z) \rightarrow g, g' \);
- \( G_F \rightarrow v; m_h \rightarrow \lambda; m_Z \rightarrow c_T \);
- \( A_l \) and \( \Gamma_l \rightarrow c_{HL} + c_{HL}', c_{HE} \);
- \( \Gamma_W \) and \( \Gamma_Z \rightarrow c_W, c_Z \);
- \( g_1Z \rightarrow c_{HL}' \); \( K_\gamma \rightarrow c_{WB} \); \( K_\lambda \rightarrow c_{3W} \);
- \( BR(h\rightarrow\gamma\gamma) \) and \( BR(h\rightarrow\gamma Z) \rightarrow c_{BB}, c_{WW} \);
- \( \sigma_{ZH} \rightarrow c_H \); \( \sigma_{ZHH} \rightarrow c_6 \);
- \( BR(h\rightarrow bb/cc/gg/\mu\mu/\tau\tau) \rightarrow y_b, y_c, c_g, y_\mu, y_\tau \);
- \( BR(h\rightarrow\text{invisible}) \) and \( BR(h\rightarrow\text{other}) \);
- \( c_{WW} \) is helped by \( A_{LR} \) in \( \sigma_{ZH} \), angular meas., W-fusion;
- \( c_{HL}/c_{HL}'/c_{HE} \) are helped by \( A_{LR} \) in \( \sigma_{ZH} \).
recap 4: role of beam polarizations (improving EWPOs)

- a Higgs factory is meantime a Z factory
- $\sim 10^8$ Z produced @ ILC250 + beam polarizations
- improve $A_i$ by a factor of 10
Table 4: Deviations from the Standard Model predictions for the Higgs boson couplings, in $\%$, for the set of new physics models described in the text. As in Table 1, the effective couplings $g(hWW)$ and $g(hZZ)$ are defined as proportional to the square roots of the corresponding partial widths.

---

### benchmark BSM models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\bar{b}\bar{b}$</th>
<th>$c\bar{c}$</th>
<th>$gg$</th>
<th>$WW$</th>
<th>$\tau\tau$</th>
<th>$ZZ$</th>
<th>$\gamma\gamma$</th>
<th>$\mu\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MSSM [34]</td>
<td>+4.8</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.2</td>
<td>+0.4</td>
<td>-0.5</td>
<td>+0.1</td>
<td>+0.3</td>
</tr>
<tr>
<td>2 Type II 2HD [36]</td>
<td>+10.1</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>+9.8</td>
<td>0.0</td>
<td>+0.1</td>
<td>+9.8</td>
</tr>
<tr>
<td>3 Type X 2HD [36]</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>+7.8</td>
<td>0.0</td>
<td>0.0</td>
<td>+7.8</td>
</tr>
<tr>
<td>4 Type Y 2HD [36]</td>
<td>+10.1</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>-0.2</td>
<td>0.0</td>
<td>0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td>5 Composite Higgs [38]</td>
<td>-6.4</td>
<td>-6.4</td>
<td>-6.4</td>
<td>-2.1</td>
<td>-6.4</td>
<td>-2.1</td>
<td>-2.1</td>
<td>-6.4</td>
</tr>
<tr>
<td>6 Little Higgs w. T-parity [39]</td>
<td>0.0</td>
<td>0.0</td>
<td>-6.1</td>
<td>-2.5</td>
<td>0.0</td>
<td>-2.5</td>
<td>-1.5</td>
<td>0.0</td>
</tr>
<tr>
<td>7 Little Higgs w. T-parity [40]</td>
<td>-7.8</td>
<td>-4.6</td>
<td>-3.5</td>
<td>-1.5</td>
<td>-7.8</td>
<td>-1.5</td>
<td>-1.0</td>
<td>-7.8</td>
</tr>
<tr>
<td>8 Higgs-Radion [41]</td>
<td>-1.5</td>
<td>-1.5</td>
<td>10.</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.0</td>
<td>-1.5</td>
</tr>
<tr>
<td>9 Higgs Singlet [42]</td>
<td>-3.5</td>
<td>-3.5</td>
<td>-3.5</td>
<td>-3.5</td>
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<td>-3.5</td>
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</tbody>
</table>

$\rightarrow$ quantitative assessment for models discrimination
model parameters (chosen as escaping direct search at HL-LHC)

- a PMSSM model with b squarks at 3.4 TeV, gluino at 4 TeV
- a Type II 2 Higgs doublet model with $m_A = 600$ GeV, $\tan\beta = 7$
- a Type X 2 Higgs doublet model with $m_A = 450$ GeV, $\tan\beta = 6$
- a Type Y 2 Higgs doublet model with $m_A = 600$ GeV, $\tan\beta = 7$
- a composite Higgs model MCHM5 with $f = 1.2$ TeV, $m_T = 1.7$ TeV
- a Little Higgs model with T-parity with $f = 785$ GeV, $m_T = 2$ TeV
- A Little Higgs model with couplings to 1st and 2nd generation with $f = 1.2$ TeV, $m_T = 1.7$ TeV
- A Higgs-radion mixing model with $m_r = 500$ GeV
- a model with a Higgs singlet at 2.8 TeV creating a Higgs portal to dark matter and large $\lambda$ for electroweak baryogenesis
BSM benchmark models discrimination at ILC250

**ILC250, S2**

**EFT interpretation**

<table>
<thead>
<tr>
<th>Model</th>
<th>SM</th>
<th>pMSSM</th>
<th>2HDM-II</th>
<th>2HDM-X</th>
<th>2HDM-Y</th>
<th>Composite</th>
<th>LHT-6</th>
<th>LHT-7</th>
<th>Radion</th>
<th>Singlet</th>
</tr>
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<tbody>
<tr>
<td>SM</td>
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<td>9.7</td>
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<td>5.4</td>
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<td>2HDM-Y</td>
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<td>Singlet</td>
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effect of improvement from TGC, vvH, ZH at 500GeV

<table>
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<tr>
<th>Model</th>
<th>SM</th>
<th>pMSSM</th>
<th>2HDM-II</th>
<th>2HDM-X</th>
<th>2HDM-Y</th>
<th>Composite</th>
<th>LHT-6</th>
<th>LHT-7</th>
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<th>Singlet</th>
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<td></td>
<td></td>
<td></td>
<td>10.0</td>
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</tbody>
</table>

ILC500, S2*

EFT interpretation

model discrimination in σ
λ_{hhh} determination in SMEFT
$\frac{\sigma_{Zhh}}{\sigma_{SM}} - 1 = 0.565c_6 - 3.58c_H + 16.0(8c_{WW}) + 8.40(8c_{WB}) + 1.26(8c_{BB})$
$- 6.48c_T - 65.1c'_{HL} + 61.1c_{HL} + 52.6c_{HE}$,

$$c_6 = \frac{1}{0.565} \left[ \frac{\sigma_{Zhh}}{\sigma_{SM}} - 1 - \sum_i a_i c_i \right]$$

$$\Delta c_6 = \frac{1}{0.565} \left[ \left( \frac{\Delta \sigma_{Zhh}}{\sigma_{SM}} \right)^2 + \sum_{i,j} a_i a_j (V_c)_{ij} \right]^{1/2}$$

Given the full ILC program of 2 ab$^{-1}$ at 250 GeV and 4 ab$^{-1}$ at 500 GeV

$$\left[ \sum_{i,j} a_i a_j (V_c)_{ij} \right]^{1/2} = 0.04 \quad \ll \quad \frac{\Delta \sigma_{Zhh}}{\sigma_{SM}} = 0.168$$

(systematic error)  (statistical error)
(ii-5) WW-fusion channel & Higgs total width $\Gamma_H$

$$\Gamma_H = \frac{\Gamma_{HZZ}}{\text{Br}(H \rightarrow ZZ^*)} \propto \frac{g_{HZZ}^2}{\text{Br}(H \rightarrow ZZ^*)}$$

$$\Gamma_H = \frac{\Gamma_{HWW}}{\text{Br}(H \rightarrow WW^*)} \propto \frac{g_{HWW}^2}{\text{Br}(H \rightarrow WW^*)}$$

$\rightarrow$ Br(H→ZZ*) very small

$\rightarrow$ better option!

\[\begin{align*}
\text{e}^+\text{e}^- &\rightarrow \nu\bar{\nu}H @ 500 \text{GeV} \\
\int L & = 500 \text{ fb}^{-1} \\
P(e^+,e^-) & = (-0.8,+0.3)
\end{align*}\]

Duerig, et al., arXiv:1403.7734
very different at Ecm=250 GeV

\[ \rho = -34\% \text{ correlation (larger if unpolarized)} \]

between \( \sigma_{WH} \times \text{BR}(H\rightarrow bb) \) and \( \sigma_{ZH} \times \text{BR}(H\rightarrow bb) \)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HL-LHC</td>
<td>$^{+60}_{-50}%$ (50%)</td>
<td>52%</td>
<td>46%</td>
<td>50%</td>
</tr>
<tr>
<td>HE-LHC</td>
<td>10-20% (n.a.)</td>
<td>n.a.</td>
<td>41%</td>
<td>50%</td>
</tr>
<tr>
<td>ILC$_{250}$</td>
<td>-</td>
<td>-</td>
<td>28%</td>
<td>49%</td>
</tr>
<tr>
<td>ILC$_{350}$</td>
<td>-</td>
<td>-</td>
<td>28%</td>
<td>47%</td>
</tr>
<tr>
<td>ILC$_{500}$</td>
<td>27% (27%)</td>
<td>27%</td>
<td>26%</td>
<td>37%</td>
</tr>
<tr>
<td>CLIC$_{380}$</td>
<td>-</td>
<td>-</td>
<td>45%</td>
<td>50%</td>
</tr>
<tr>
<td>CLIC$_{1500}$</td>
<td>36% (36%)</td>
<td>36%</td>
<td>40%</td>
<td>49%</td>
</tr>
<tr>
<td>CLIC$_{3000}$</td>
<td>$^{+11}_{-7}%$ (n.a.)</td>
<td>n.a.</td>
<td>35%</td>
<td>49%</td>
</tr>
<tr>
<td>FCC-ee$_{240}$</td>
<td>-</td>
<td>-</td>
<td>19%</td>
<td>48%</td>
</tr>
<tr>
<td>FCC-ee$_{365}$</td>
<td>-</td>
<td>-</td>
<td>19%</td>
<td>34%</td>
</tr>
<tr>
<td>FCC-ee/eh/hh</td>
<td>5% (5%)</td>
<td>6%</td>
<td>18%</td>
<td>25%</td>
</tr>
<tr>
<td>CEPC</td>
<td>-</td>
<td>-</td>
<td>17%</td>
<td>49%</td>
</tr>
</tbody>
</table>
ECFA Higgs @ FC WG
Impact of TGCs

Model Independent EFT Fit

Impact of improved TGC precisions
- HL-LHC \oplus ILC250
- HL-LHC \oplus ILC250, TGCs from LEP

Precision of Higgs boson couplings [%]

- Z
- W
- b
- \tau
- g
- c
- \Gamma_{inv}
- \Gamma_{h}
- \gamma
- Z_{\gamma}
- \mu

\times \frac{1}{3}
\times \frac{1}{2}
Precision of Higgs boson couplings [%]

- **HL-LHC** $\oplus e^+e^- 5 \text{ ab}^{-1} 250$ GeV unpolarised
- ... $\oplus e^+e^- 1.5 \text{ ab}^{-1} 350$ GeV unpolarised
- **HL-LHC** $\oplus e^+e^- 2 \text{ ab}^{-1} 250$ GeV polarised
- ... $\oplus e^+e^- 4 \text{ ab}^{-1} 500$ GeV polarised

dark/light: current / improved EWPO

Model Independent EFT Fit

LCC Physics WG
(ii-5) Top-Yukawa coupling

- largest Yukawa coupling; crucial role in theory
- non-relativistic tt-bar bound state correction: enhancement by ~2 at 500 GeV
- Higgs CP measurement

\[
\Delta g_{ttH} / g_{ttH} \begin{array}{|c|c|c|}
\hline
& 500 \text{ GeV} & +1 \text{ TeV} \\
\hline
\text{Snowmass} & 7.8\% & 2.0\% \\
\text{H20} & 6.3\% & 1.5\% \\
\hline
\end{array}
\]

With QCD Correction
No QCD Correction

Yonamine, et al., PRD84, 014033;
Top-Yukawa coupling

Scaled to values at 500 GeV

1 ab$^{-1}$ @ 500 GeV, $P(e_-,e^+)$ = (-0.8,0.3)

$\sigma_{tH} = 0.485$ fb

$\delta y_t/y_t = 9.9\%$

Y. Sudo
λ runs < 0? top mass precision crucial for vacuum stability

at e+e- : top-pair threshold scan, much lower theory error

\[ \Delta m_t(\text{MS-bar}) \sim 50 \text{ MeV} \ (\Delta m_H=14\text{MeV}) \]
simplifications of our analysis

- at tree level, and to linear order in D-6 coefficients
- ignore some possible D-6 corrections involving light leptons, e.g. 4-fermion operators
- avoid using observables that involve contact interactions that include quark currents (see more later)
- ignore the effects of CP-violating operators

\[
\Delta \mathcal{L}_{CP} = + \frac{g^2 \tilde{c}_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu\nu} \tilde{W}^{a\mu\nu} + \frac{4gg' \tilde{c}_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu\nu} \tilde{B}^{\mu\nu}
+ \frac{g' \tilde{c}_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} \tilde{B}^{\mu\nu} + \frac{g^3 \tilde{c}_3W}{m_W^2} \epsilon_{abc} W^a_{\mu\nu} W^{b\nu}_{\rho} \tilde{W}^{c\rho\mu}
\]
on-shell renormalization

- D-6 operators modify the SM expressions for precision electroweak observables, thus shift the appropriate values for the SM couplings $\rightarrow g, g', v, \lambda$ free parameters

- D-6 operators also renormalize the kinetic terms of the SM fields $\rightarrow$ rescale the boson fields

\[
\mathcal{L} = -\frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} \cdot (1 - \delta Z_W) - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} \cdot (1 - \delta Z_Z) - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} \cdot (1 - \delta Z_A) + \frac{1}{2} (\partial_{\mu}h)(\partial^{\mu}h) \cdot (1 - \delta Z_h),
\]

with

\[
\delta Z_W = (8c_{WW}) \\
\delta Z_Z = c_w^2(8c_{WW}) + 2s_w^2(8c_{WB}) + s_w^4/c_w^2(8c_{BB}) \\
\delta Z_A = s_w^2\left((8c_{WW}) - 2(8c_{WB}) + (8c_{BB})\right) \\
\delta Z_h = -c_H.
\]

\[
\Delta \mathcal{L} = \frac{1}{2} \delta Z_{AZ} A_{\mu\nu} Z^{\mu\nu}, \quad \delta Z_{AZ} = s_w c_w \left( (8c_{WW}) - \left(1 - \frac{s_w^2}{c_w^2}\right)(8c_{WB}) - \frac{s_w^2}{c_w^2}(8c_{BB}) \right)
\]
systematic errors included in the global fit

- 0.1% from theory computations
- 0.1% from luminosity
- 0.1% from beam polarizations
- 0.1% $\oplus 0.3%/\sqrt{L/250}$ from b-tagging and analysis

improvement factors in S2

- 10% from better jet-clustering algorithm
- 20% from better flavor-tagging algorithm
- 20% from including more signal channels in $h\to WW^*$
- $x10$ better for $A_{LR}$ using $e^+e^-\to \gamma Z$ at ILC250
expected meas. for direct observables

estimates at ILC by full simulation

-90% $e^-$, +30% $e^+$ polarization:

<table>
<thead>
<tr>
<th></th>
<th>250 GeV $Zh$</th>
<th>250 GeV $\nu\bar{\nu}h$</th>
<th>350 GeV $Zh$</th>
<th>350 GeV $\nu\bar{\nu}h$</th>
<th>500 GeV $Zh$</th>
<th>500 GeV $\nu\bar{\nu}h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ [50–53]</td>
<td>2.0</td>
<td>1.8</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h \to invis.$ [54, 55]</td>
<td>0.86</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h \to bb$ [56–59]</td>
<td>1.3</td>
<td>8.1</td>
<td>1.5</td>
<td>1.8</td>
<td>2.5</td>
<td>0.93</td>
</tr>
<tr>
<td>$h \to c\bar{c}$ [56, 57]</td>
<td>8.3</td>
<td>11</td>
<td>19</td>
<td>18</td>
<td>8.8</td>
<td></td>
</tr>
<tr>
<td>$h \to gg$ [56, 57]</td>
<td>7.0</td>
<td>8.4</td>
<td>7.7</td>
<td>15</td>
<td>5.8</td>
<td></td>
</tr>
<tr>
<td>$h \to WW$ [59–61]</td>
<td>4.6</td>
<td>5.6 *</td>
<td>5.7 *</td>
<td>7.7</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>$h \to \tau\tau$ [63]</td>
<td>3.2</td>
<td>4.0 *</td>
<td>16 *</td>
<td>6.1</td>
<td>9.8</td>
<td></td>
</tr>
<tr>
<td>$h \to ZZ$ [2]</td>
<td>18</td>
<td>25 *</td>
<td>20 *</td>
<td>35 *</td>
<td>12 *</td>
<td></td>
</tr>
<tr>
<td>$h \to \gamma\gamma$ [64]</td>
<td>34 *</td>
<td>39 *</td>
<td>45 *</td>
<td>47</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$h \to \mu\mu$ [65, 66]</td>
<td>72 *</td>
<td>87 *</td>
<td>160 *</td>
<td>120 *</td>
<td>100 *</td>
<td></td>
</tr>
<tr>
<td>$a$ [27]</td>
<td>7.6</td>
<td>2.7 *</td>
<td></td>
<td>4.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>2.7</td>
<td>0.69 *</td>
<td></td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(a, b)$</td>
<td>-99.17</td>
<td>-95.6 *</td>
<td></td>
<td>-84.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(arXiv: 1708.08912; numbers are in %, for nominal $\int Ldt = 250$ fb$^{-1}$)
EFT input from TGCs in $e^+e^- \rightarrow W^+W^-$

<table>
<thead>
<tr>
<th></th>
<th>250 GeV $W^+W^-$</th>
<th>350 GeV $W^+W^-$</th>
<th>500 GeV $W^+W^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{1Z}$</td>
<td>0.062 *</td>
<td>0.033 *</td>
<td>0.025</td>
</tr>
<tr>
<td>$\kappa_A$</td>
<td>0.096 *</td>
<td>0.049 *</td>
<td>0.034</td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.077 *</td>
<td>0.047 *</td>
<td>0.037</td>
</tr>
<tr>
<td>$\rho(g_{1Z}, \kappa_A)$</td>
<td>63.4 *</td>
<td>63.4 *</td>
<td>63.4</td>
</tr>
<tr>
<td>$\rho(g_{1Z}, \lambda_A)$</td>
<td>47.7 *</td>
<td>47.7 *</td>
<td>47.7</td>
</tr>
<tr>
<td>$\rho(\kappa_A, \lambda_A)$</td>
<td>35.4 *</td>
<td>35.4 *</td>
<td>35.4</td>
</tr>
</tbody>
</table>

(arXiv: 1708.08912; numbers are in %, for nominal $\int L dt = 500\text{ fb}^{-1}$ shared equally by left-/right- polarized data)
### EFT input: EWPOs

<table>
<thead>
<tr>
<th>Observable</th>
<th>current value</th>
<th>current $\sigma$</th>
<th>future $\sigma$</th>
<th>SM best fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{-1}(m_Z^2)$</td>
<td>128.9220</td>
<td>0.0178</td>
<td></td>
<td>(same)</td>
</tr>
<tr>
<td>$G_F \ (10^{-10} \text{ GeV}^{-2})$</td>
<td>1166378.7</td>
<td>0.6</td>
<td></td>
<td>(same)</td>
</tr>
<tr>
<td>$m_W \ (\text{MeV})$</td>
<td>80385</td>
<td>15</td>
<td>5</td>
<td>80361</td>
</tr>
<tr>
<td>$m_Z \ (\text{MeV})$</td>
<td>91187.6</td>
<td>2.1</td>
<td></td>
<td>91188.0</td>
</tr>
<tr>
<td>$m_h \ (\text{MeV})$</td>
<td>125090</td>
<td>240</td>
<td>15</td>
<td>125110</td>
</tr>
<tr>
<td>$A_\ell$</td>
<td>0.14696</td>
<td>0.0013</td>
<td></td>
<td>0.147937</td>
</tr>
<tr>
<td>$\Gamma_\ell \ (\text{MeV})$</td>
<td>83.984</td>
<td>0.086</td>
<td></td>
<td>83.995</td>
</tr>
<tr>
<td>$\Gamma_Z \ (\text{MeV})$</td>
<td>2495.2</td>
<td>2.3</td>
<td></td>
<td>2494.3</td>
</tr>
<tr>
<td>$\Gamma_W \ (\text{MeV})$</td>
<td>2085</td>
<td>42</td>
<td>2</td>
<td>2088.8</td>
</tr>
</tbody>
</table>
EFT input: EWPOs (7)

\[ \alpha(m_Z), G_F, m_W, m_Z, m_h, A_{LR}(\ell), \Gamma(Z \rightarrow \ell^+\ell^-) \]

\[ \delta e = \delta(4\pi\alpha(m_Z^2))^{1/2} = s_w^2\delta g + c_w^2\delta g' + \frac{1}{2}\delta Z_A \]

\[ \delta G_F = -2\delta v + 2c'_{HL} \]

\[ \delta m_W = \delta g + \delta v + \frac{1}{2}\delta Z_W \]

\[ \delta m_Z = c_w^2\delta g + s_w^2\delta g' + \delta v - \frac{1}{2}c_T + \frac{1}{2}\delta Z_Z \]

\[ \delta m_h = \frac{1}{2}\delta \tilde{\lambda} + \delta v + \frac{1}{2}\delta Z_h \]

\[ (\delta X = \Delta X/X) \]

\[ \bar{\lambda} = \lambda(1 + \frac{3}{2}c_6) \]

\[ s_w^2 = \sin^2 \theta_w = \frac{g'^2}{g^2 + g'^2} \]

\[ c_w^2 = \cos^2 \theta_w = \frac{g^2}{g^2 + g'^2} \]
EFT input: EWPOs (7)

\[ \alpha(m_Z), G_F, m_W, m_Z, m_h, A_{LR}(\ell), \Gamma(Z \rightarrow \ell^+\ell^-) \]

\[ \delta \Gamma_\ell = \delta m_Z + 2 \frac{g_L^2 \delta g_L + g_R^2 \delta g_R}{g_L^2 + g_R^2} \]

\[ \delta A_\ell = \frac{4 g_L^2 g_R^2 (\delta g_L - \delta g_R)}{g_L^4 - g_R^4} \]

\[ g_L = \frac{g}{c_w} \left[ \left( -\frac{1}{2} + s_w^2 \right)(1 + \frac{1}{2} \delta Z_Z) - \frac{1}{2} (c_{HL} + c'_{HL}) - s_w c_w \delta Z_{AZ} \right] \]

\[ g_R = \frac{g}{c_w} \left[ (s_w^2)(1 + \frac{1}{2} \delta Z_Z) - \frac{1}{2} c_{HE} - s_w c_w \delta Z_{AZ} \right] \]

\[ \text{CHL} + \text{C'}_{HL}, \text{CHE} \]
EFT input: TGC (3)

\[
\Delta \mathcal{L}_{TGC} = i g_V \left\{ V^\mu (\hat{W}_\mu^- W^+ + \hat{W}_\mu^+ W^-) + \kappa_V W_\mu^+ W_\nu^- \hat{V}^{\mu \nu} + \frac{\lambda_V}{m_W^2} \hat{W}_\mu^- \hat{W}_\rho^- \hat{W}_\rho^+ \hat{V}^{\mu \nu} \right\}
\]

\[
g_Z = g c_w \left( 1 + \frac{1}{2} \delta Z_Z + \frac{s_w}{c_w} \delta Z_{AZ} \right)
\]

\[
\kappa_A = 1 + (8c_W B)
\]

\[
\lambda_A = -6g^2 c_{3W}
\]
EFT input: TGC (3)

\[ \delta g_{Z, eff} = \delta g_Z + \frac{1}{c_w^2}((c_w^2 - s_w^2)\delta g_L + s_w^2 \delta g_R - 2\delta g_W) \]

\[ \delta \kappa_{A, eff} = (c_w^2 - s_w^2)(\delta g_L - \delta g_R) + 2(\delta e - \delta g_W) + (8c_{WB}) \]

\[ \delta \lambda_{A, eff} = -6g^2c_{3W} \]

\[ g_W = g \left( 1 + c'_{HL} + \frac{1}{2} \delta Z_W \right) \]
EFT input: \( \frac{\text{BR}(h \rightarrow \gamma\gamma)}{\text{BR}(h \rightarrow ZZ^*)}, \frac{\text{BR}(h \rightarrow \gamma Z)}{\text{BR}(h \rightarrow ZZ^*)} \)

\[ \delta \Gamma(h \rightarrow \gamma\gamma) = 528 \delta Z_A - c_H + 4 \delta e + 4.2 \delta m_h - 1.3 \delta m_W - 2 \delta v \]

\[ \delta \Gamma(h \rightarrow Z\gamma) = 290 \delta Z_{AZ} - c_H - 2(1 - 3s_W^2)\delta g + 6c_w^2\delta g' + \delta Z_A + \delta Z_Z \\
+ 9.6 \delta m_h - 6.5 \delta m_Z - 2 \delta v \]

\[ \delta \Gamma(h \rightarrow ZZ^*) = 2\eta_Z - 2 \delta v - 13.8 \delta m_Z + 15.6 \delta m_h - 0.50 \delta Z_Z - 1.02 C_Z + 1.18 \delta \Gamma_Z \]

\[ \delta Z_A = s_w^2 \left( (8c_{WW}) - 2(8c_{WB}) + (8c_{BB}) \right) \quad \delta Z_{AZ} = s_w c_w \left( (8c_{WW}) - \left( 1 - \frac{s_w^2}{c_w^2} \right)(8c_{WB}) - \frac{s_w^2}{c_w^2} (8c_{BB}) \right) \]
EFT coefficients

10: $c_H$, $c_T$, $c_6$, $c_{WW}$, $c_{WB}$, $c_{BB}$, $c_{3W}$, $c_{HL}$, $c'_{HL}$, $c_{HE}$

+ 4: $g$, $g'$, $v$, $\lambda$

can already be determined, except $c_6$, $c_H$

$\rightarrow$ Higgs observables @ e+e-
EFT input: $\sigma(e^+e^- \rightarrow Zh), \sigma(e^+e^- \rightarrow Zhh)$

- $c_H$ has to be determined by inclusive $\sigma_{Zh}$ measurement
- $c_6$ has to be determined by double Higgs measurement

EFT input: $\text{BR}(h \rightarrow XX)$

\[ \Delta \mathcal{L} = -c_{\tau \Phi} \frac{y_{\tau}}{y^2} (\Phi^\dagger \Phi) \bar{L}_3 \cdot \Phi \tau_R + h.c. \]
\[ \delta \mathcal{L} = A_h^{-1} G_{\mu\nu} G^{\mu\nu} \]

- $h$ couplings to $b, c, \tau, \mu, g$
- $\Gamma(h \rightarrow \text{invisible}),$ total decay width

note: beam polarizations provide several independent (redundant) set of $\sigma,\sigma \times \text{BR}$ input, which are powerful to test EFT validity
two more parameters: $C_W$, $C_Z$ for $\Gamma(h\rightarrow WW^*)$ and $\Gamma(h\rightarrow ZZ^*)$

$$\Gamma/(SM) = 1 + 2\eta W - 2\delta v - 11.7\delta m_W + 13.6\delta m_h$$
$$-0.75\zeta_W - 0.88C_W + 1.06\delta\Gamma_W,$$

$$C_W = \sum_x c'_X N_x / \sum_x N_x,$$

(c'$_X$: contact interactions)

EFT input: $$\Gamma_W = \frac{g^2 m_W}{48\pi} \left( \sum_x N_x \right) \cdot \left( 1 + 2\delta g + \delta m_W + \delta Z_W + 2C_W \right)$$

(similar for Z)