Theoretical predictions for top-quark production processes

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- Soft-gluon corrections through three loops
- Top-pair production
- Single-top production
- Top associated production with $Z'$ or $H^-$

DPF2019

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Soft-gluon corrections

They are important for top-quark processes
They approximate known exact results at NLO and NNLO very well

\[ f_1(p_1) + f_2(p_2) \rightarrow t(p_t) + X \]

define \( s = (p_1 + p_2)^2 \), \( t = (p_1 - p_t)^2 \), \( u = (p_2 - p_t)^2 \) and \( s_4 = s + t + u - \sum m^2 \)

At partonic threshold \( s_4 \rightarrow 0 \)

Soft corrections \( \left[ \frac{\ln^k(s_4/m_t^2)}{s_4} \right]_+ \) with \( k \leq 2n - 1 \) for the order \( \alpha_s^n \) corrections

Resum these soft corrections for the double-differential cross section

At NNLL accuracy we need two-loop soft anomalous dimensions
At \( N^3LL \) accuracy we need three-loop soft anomalous dimensions

Finite-order expansions-no prescription needed

Approximate NNLO (aNNLO) and \( N^3LO (aN^3LO) \) predictions

for cross sections and differential distributions

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Soft-gluon Resummation

moments of the partonic cross section with moment variable $N$:
\[ \hat{\sigma}(N) = \int (ds_4/s) \ e^{-Ns_4/s} \hat{\sigma}(s_4) \]

factorized expression for the cross section in $4 - \epsilon$ dimensions
\[ \sigma^{f_1 f_2 \to tX}(N, \epsilon) = H^{f_1 f_2 \to tX}_{IL} (\alpha_s(\mu_R)) \ S^{f_1 f_2 \to tX}_{LI} \left( \frac{m_t}{N \mu_F}, \alpha_s(\mu_R) \right) \]
\[ \times \prod J_{\text{in}}(N, \mu_F, \epsilon) \prod J_{\text{out}}(N, \mu_F, \epsilon) \]

$H^{f_1 f_2 \to tX}_{IL}$ is hard function and $S^{f_1 f_2 \to tX}_{LI}$ is soft function

$S_{LI}$ satisfies the renormalization group equation
\[ \left( \mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s} \right) S_{LI} = -(\Gamma_S^\dagger)_{LK} S_{KI} - S_{LK}(\Gamma_S)_{KI} \]

Soft anomalous dimension $\Gamma_S$ controls the evolution of the soft function which gives the exponentiation of logarithms of $N$
Cusp anomalous dimension

A basic ingredient of soft anomalous dimensions

cusp angle \( \theta = \cosh^{-1}(p_i \cdot p_j / \sqrt{p_i^2 p_j^2}) \) and \( \Gamma_{\text{cusp}} = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \Gamma_{\text{cusp}}^{(n)} \)

One loop

\[
\Gamma_{\text{cusp}}^{(1)} = C_F (\theta \coth \theta - 1)
\]

In terms of \( \beta = \tanh(\theta/2) = \sqrt{1 - \frac{4m^2}{s}} \) we have \( \theta = \ln \left[ \frac{(1+\beta)}{(1-\beta)} \right] \) and

\[
\Gamma_{\text{cusp}}^{(1)} = C_F \left[ -\frac{(1 + \beta^2)}{2\beta} \ln \left( \frac{1 - \beta}{1 + \beta} \right) - 1 \right] = -C_F \left( L_\beta + 1 \right)
\]
Cusp anomalous dimension

Two loops

\[
\Gamma_{\text{cusp}}^{(2)} = \frac{K}{2} \Gamma_{\text{cusp}}^{(1)} + \frac{1}{2} C_F C_A \left\{ 1 + \zeta_2 + \theta^2 - \coth \theta \left[ \zeta_2 \theta + \theta^2 + \frac{\theta^3}{3} + \text{Li}_2 \left( 1 - e^{-2\theta} \right) \right] + \coth^2 \theta \left[ -\zeta_3 + \zeta_2 \theta + \frac{\theta^3}{3} + \theta \text{Li}_2 \left( e^{-2\theta} \right) + \text{Li}_3 \left( e^{-2\theta} \right) \right] \right\}
\]

where \( K = C_A \left( \frac{67}{18} - \zeta_2 \right) - 5n_f/9 \)

Three loops (very long expression for \( C^{(3)} \))

\[
\Gamma_{\text{cusp}}^{(3)} = C^{(3)} + K'^{(3)} \Gamma_{\text{cusp}}^{(1)} + K \left[ \Gamma_{\text{cusp}}^{(2)} - \frac{K}{2} \Gamma_{\text{cusp}}^{(1)} \right]
\]

For \( n_f = 5 \)

\[
\Gamma_{\text{cusp}}^{(3) \text{approx}} (\beta) = 0.092 \beta^2 + 2.803 \Gamma_{\text{cusp}}^{(1)} (\beta)
\]

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Cusp anomalous dimension

Case where one line is massive and one is massless: simpler expressions

If eikonal line $i$ represents a massive quark and eikonal line $j$ a massless quark, then

$$\Gamma^{(1)}_c = C_F \left[ \ln \left( \frac{2p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right]$$

$$\Gamma^{(2)}_c = C_F \frac{K}{2} \left[ \ln \left( \frac{2p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{1}{4} C_F C_A (1 - \zeta_3)$$

$$\Gamma^{(3)}_c = C_F K'(3) \left[ \ln \left( \frac{2p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right] + C_F C_A \frac{K}{4} (1 - \zeta_3)$$

$$+ C_F C_A^2 \left[ -\frac{1}{4} + \frac{3}{8} \zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8} \zeta_2 \zeta_3 + \frac{9}{16} \zeta_5 \right]$$

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Top-antitop pair production

At one loop for $q \bar{q} \rightarrow t \bar{t}$

$$
\Gamma_{q \bar{q}}^{(1)}_{11} = \Gamma_{cusp}^{(1)}, \quad \Gamma_{q \bar{q}}^{(1)}_{12} = \frac{C_F}{C_A} \ln \left( \frac{t_1}{u_1} \right), \quad \Gamma_{q \bar{q}}^{(1)}_{21} = 2 \ln \left( \frac{t_1}{u_1} \right)
$$

$$
\Gamma_{q \bar{q}}^{(1)}_{22} = \left( 1 - \frac{C_A}{2C_F} \right) \Gamma_{cusp}^{(1)} + 4C_F \ln \left( \frac{t_1}{u_1} \right) - \frac{C_A}{2} \left[ 1 + \ln \left( \frac{s m_t^2 t_1^2}{u_1^4} \right) \right]
$$

At two loops for $q \bar{q} \rightarrow t \bar{t}$

$$
\Gamma_{q \bar{q}}^{(2)}_{11} = \Gamma_{cusp}^{(2)}, \quad \Gamma_{q \bar{q}}^{(2)}_{12} = \left( \frac{K}{2} - \frac{C_A}{2} N_2l \right) \Gamma_{q \bar{q}}^{(1)}_{12}, \quad \Gamma_{q \bar{q}}^{(2)}_{21} = \left( \frac{K}{2} + \frac{C_A}{2} N_2l \right) \Gamma_{q \bar{q}}^{(1)}_{21}
$$

$$
\Gamma_{q \bar{q}}^{(2)}_{22} = \frac{K}{2} \Gamma_{q \bar{q}}^{(1)}_{22} + \left( 1 - \frac{C_A}{2C_F} \right) \left( \Gamma_{cusp}^{(2)} - \frac{K}{2} \Gamma_{cusp}^{(1)} \right)
$$

3 × 3 matrix for $gg \rightarrow t \bar{t}$

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Top-antitop pair production at $aN^3$LO with NNLL accuracy

$pp \rightarrow t\bar{t}$ at LHC energies $aN^3$LO $m_t=172.5$ GeV

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Top $p_T$ and rapidity distributions in $t\bar{t}$ production

Top quark $p_T$ distribution at LHC

$\mu = m_t/2$, $m_t$, $2m_t$

$m_t = 172.5$ GeV

Top rapidity distribution at LHC

$\mu = m_t/2$, $m_t$, $2m_t$

$m_t = 172.5$ GeV

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Top $p_T$ distributions in $t\bar{t}$ production

Top quark $p_T$ distribution at 13 TeV LHC $\text{aN}^3\text{LO}$ $m_t=172.5$ GeV

Normalized top $p_T$ distribution at the LHC $S^{1/2}=13$ TeV

Top quark $p_T$ distribution at LHC $S^{1/2}=8$ TeV $m_t=172.5$ GeV

Normalized top $p_T$ distribution at the LHC $S^{1/2}=8$ TeV

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Top rapidity distributions in $t\bar{t}$ production

Top rapidity distribution at 13 TeV LHC $aN^3LO$

Normalized top rapidity distribution at LHC $S^{1/2}=13\text{ TeV}$

Normalized top rapidity distribution at LHC $S^{1/2}=8\text{ TeV}$

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Top double-differential distributions in $t\bar{t}$ production

Top-quark double-differential distribution at 8 TeV LHC

$m_t=172.5$ GeV $\ Y=0$ $\ \mu=m_t$

Top-quark double-differential distribution at 13 TeV LHC

$m_t=172.5$ GeV $\ Y=0$ $\ \mu=m_t$

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Single-top production

Single-top cross sections \( m_t = 172.5 \text{ GeV} \)

\[ \sigma (\text{pb}) \]

\[ S^{1/2} \text{ (TeV)} \]

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Single-top $t$-channel production

At one loop

$$\Gamma^{t (1)}_{S11} = C_F \left[ \ln \left( \frac{t(t - m_t^2)}{m_t s^{3/2}} \right) - \frac{1}{2} \right], \quad \Gamma^{t (1)}_{S12} = \frac{C_F}{2N} \ln \left( \frac{u(u - m_t^2)}{s(s - m_t^2)} \right), \quad \Gamma^{t (1)}_{S21} = \ln \left( \frac{u(u - m_t^2)}{s(s - m_t^2)} \right)$$

$$\Gamma^{t (1)}_{S22} = C_F \left[ \ln \left( \frac{t(t - m_t^2)}{m_t s^{3/2}} \right) - \frac{1}{2} \right] - \frac{1}{N} \ln \left( \frac{u(u - m_t^2)}{s(s - m_t^2)} \right) + \frac{N}{2} \ln \left( \frac{u(u - m_t^2)}{t(t - m_t^2)} \right)$$

At two loops

$$\Gamma^{t (2)}_{S11} = K'(2) \Gamma^{t (1)}_{S11} + \frac{1}{4} C_F C_A (1 - \zeta_3), \quad \Gamma^{t (2)}_{S12} = K'(2) \Gamma^{t (1)}_{S12}$$

$$\Gamma^{t (2)}_{S21} = K'(2) \Gamma^{t (1)}_{S21}, \quad \Gamma^{t (2)}_{S22} = K'(2) \Gamma^{t (1)}_{S22} + \frac{1}{4} C_F C_A (1 - \zeta_3)$$

At three loops

$$\Gamma^{t (3)}_{S11} = K'(3) \Gamma^{t (1)}_{S11} + \frac{1}{2} K^{(2)} C_A (1 - \zeta_3) + C^{(3)}$$
$t$-channel production at aNNLO with NNLL accuracy

Single-top $t$-channel aNNLO cross sections $m_t = 172.5$ GeV
Top $p_T$ distributions in $t$-channel production at the LHC

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Single-top $s$-channel production

At one loop

$\Gamma^s_{S\,11} = C_F \left[ \ln \left( \frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right]$ ,

$\Gamma^s_{S\,12} = \frac{C_F}{2N} \ln \left( \frac{t(t - m_t^2)}{u(u - m_t^2)} \right)$ ,

$\Gamma^s_{S\,21} = \ln \left( \frac{t(t - m_t^2)}{u(u - m_t^2)} \right)$

$\Gamma^s_{S\,22} = C_F \left[ \ln \left( \frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] - \frac{1}{N} \ln \left( \frac{t(t - m_t^2)}{u(u - m_t^2)} \right) + \frac{N}{2} \ln \left( \frac{t(t - m_t^2)}{s(s - m_t^2)} \right)$

At two loops

$\Gamma^s_{S\,11} = K'(2) \Gamma^s_{S\,11} + \frac{1}{4} C_F C_A (1 - \zeta_3)$ ,

$\Gamma^s_{S\,12} = K'(2) \Gamma^s_{S\,12}$

$\Gamma^s_{S\,21} = K'(2) \Gamma^s_{S\,21}$ ,

$\Gamma^s_{S\,22} = K'(2) \Gamma^s_{S\,22} + \frac{1}{4} C_F C_A (1 - \zeta_3)$

At three loops

$\Gamma^s_{11} = K'(3) \Gamma^s_{11} + \frac{1}{2} K(2) C_A (1 - \zeta_3) + C(3)$
s-channel production at aNNLO with NNLL accuracy

Single-top s-channel aNNLO cross sections $m_t=172.5$ GeV

![Graph showing single-top s-channel aNNLO cross sections](image_url)
Associated $tW$ production

At one loop

$$\Gamma^{tW(1)}_S = C_F \left[ \ln \left( \frac{m_t^2 - t}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln \left( \frac{u - m_t^2}{t - m_t^2} \right)$$

At two loops

$$\Gamma^{tW(2)}_S = K''(2) \Gamma^{tW(1)}_S + \frac{1}{4} C_F C_A (1 - \zeta_3)$$

At three loops

$$\Gamma^{tW(3)}_S = K''(3) \Gamma^{tW(1)}_S + \frac{1}{2} K''(2) C_A (1 - \zeta_3) + C^{(3)}$$
\( tW \) production at aN\(^3\)LO with NNLL accuracy

\( tW^+ + \bar{t}W^- \) aN\(^3\)LO cross section \( m_t = 172.5 \) GeV

\( \sigma \) (pb) vs. \( S^{1/2} \) (TeV)

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Top quark $p_T$ and rapidity distributions in $tW$ production

Top $p_T$ distribution in $tW$ production at LHC $aN^3LO \ m_t=173.3$ GeV

Top rapidity distribution in $tW$ production at LHC $aN^3LO \ m_t=173.3$ GeV
$tZ'$ production
(with Marco Guzzi)

via anomalous couplings

$gu \rightarrow tZ'$ in pp collisions $m_{Z'}=1, 3, 5, 8$ TeV $k_{tZ'}/\Lambda = 0.01/m_t$

with initial-state top

$gt \rightarrow tZ'$ in pp collisions $m_{Z'}=1, 3, 5, 8$ TeV $g_{Z'}=1$

(Results for $tZ$ and $t\gamma$ production via anomalous couplings covered in talk by Matthew Forslund)

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Leading-order cross section for $bg \to tH^- \propto \alpha \alpha_s (m_b^2 \tan^2 \beta + m_t^2 \cot^2 \beta)$

$\tan \beta = v_2 / v_1$ ratio of vevs of two Higgs doublets

$bg \to tH^-$ at LHC $aN^3 \text{LO}$ $\tan \beta = 30$ $\mu = m_H$
Summary

- soft-gluon corrections at three loops
- $\bar{t}t$ production at aN$^3$LO
- $t$-channel and $s$-channel single top at aNNLO
- $tW$ production at aN$^3$LO
- excellent agreement with collider data
- $tZ'$ production in various models
- $tH^-$ at aN$^3$LO
- high-order corrections are very significant