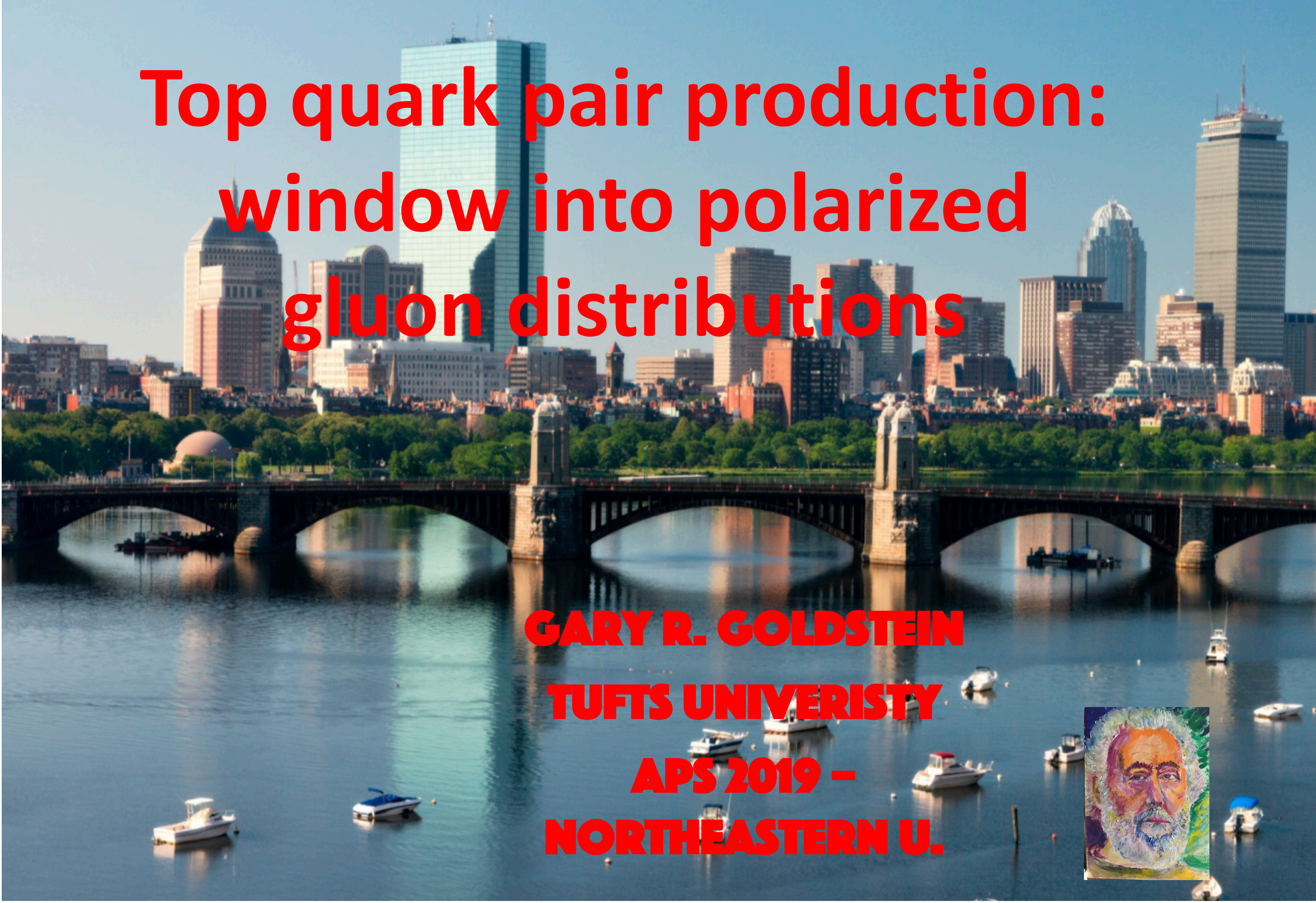

Top quark pair production: window into polarized gluon distributions

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APS 2019 – NORTHEASTERN U.

Gluon Distributions

Transversity

Top quarks

Gluon Transversity → Top Pair Spin Correlations

OUTLINE–top quarks←>gluons

- Gluon pdf's, Generalized PDF's (GPD) & Transverse MD's (TMD) in the process of establishing

  proton 3d holography
- Models for quark & gluon distributions– e.g. spectator "flexible parameterization"
- Electroproduction measurements
  - Gluon Distributions include Polarized Gluons!
    - Gluon Transversity recently seen
- Prolific t+tbar production via gluons in p+p
- top decay measures top polarization
- Gluon polarization?

  in p+p @ LHC.
- Inclusive reactions →

  Transverse momentum dependent pdf's ~ TMDs . .
- Top spin correlations & Observable quantities

Gluon GPDs

$$\frac{1}{\bar{P}^+}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)$$

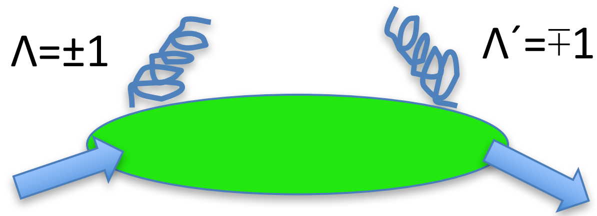
Even t-channel parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda)$$

Odd t-channel parity & Gluon helicity conserving

Must have 4 more Gluon helicity NONconserving

Extension to Gluon "Transversity"



$$
-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \mathbf{S} F^{+i}(-\tfrac{1}{2}z)\, F^{+j}(\tfrac{1}{2}z)\, |p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}
$$

$$
= \mathbf{S}\, \frac{1}{2P^+} \frac{P^+\Delta^j - \Delta^+P^j}{2mP^+}
$$

$$
\times \bar{u}(p',\lambda') \Bigg[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g \frac{P^+\Delta^i - \Delta^+P^i}{m^2}
$$

$$
+ E_T^g \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g \frac{\gamma^+P^i - P^+\gamma^i}{m} \Bigg] u(p,\lambda).
$$

4 GPDs: see M.Diehl, EPJC19, 485 (2001)

4 Gluon helicity NONconserving Double flip

Gluon "transversity" Double helicity flip *does not mix* with quark distributions

Transversity for on-shell gluons or photons : no |0> helicity

$$|+1)_{trans} = \{|+1> + |-1>\}/2 = |-1)_{trans}$$

$$|0\ )_{trans} = \{|+1> - |-1>\}/\sqrt{2}$$

$$\text{helicity}\ \ |\pm 1>= \{-/+\hat{x} - i\hat{y}\}/\sqrt{2}$$

$$\hat{x} = -|0)_{trans} = P_{parallel}$$

Linear polarization in the plane

$$\hat{y} = i\sqrt{2}\,|+1)_{trans} = P_{normal}$$

Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

Using Spectator Model
Many other models & recently

e.g. work in progress S.Liuti, GRG, Gonzalez-Hernandez, Poage (thesis)

How to Measure? What Processes? Long standing question.

M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998).

TMDs

P. Mulders, J. Rodrigues,PRD 63, 094021 (2001).
D. Boer, Few-Body Syst. (2017); C. Pisano, et al., JHEP 10, 024 (2013);
D. Boer, et al., PRL 106, 132001 (2011); . . . .

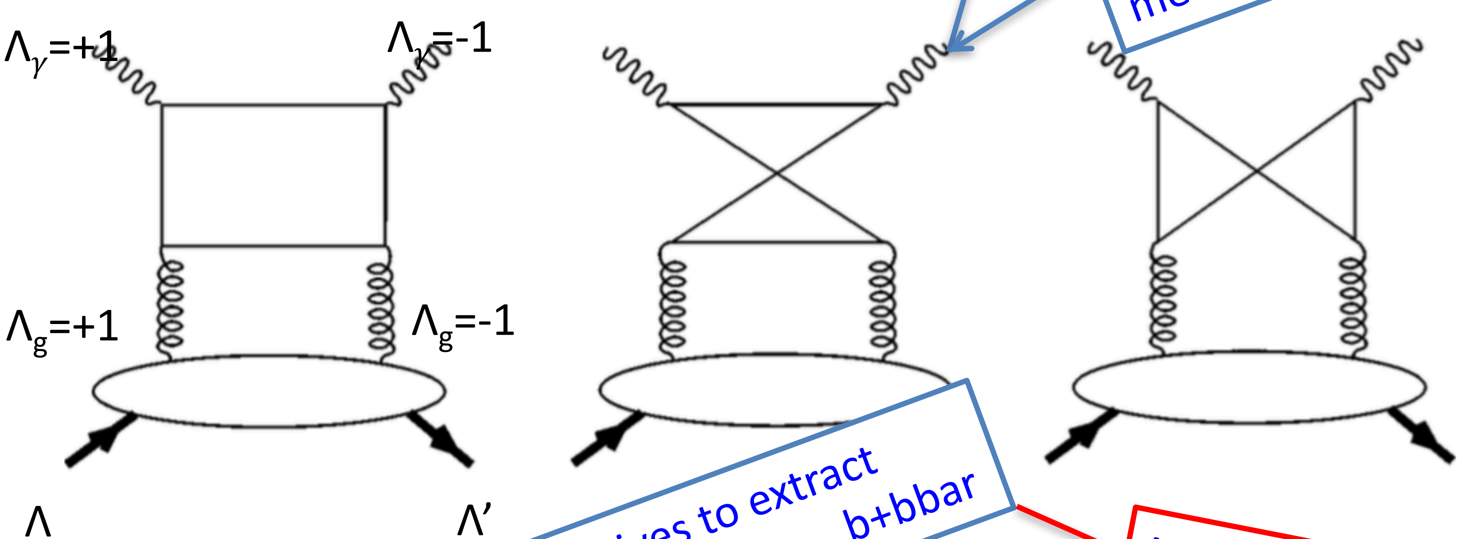
Helicity flip $A_{\Lambda',-1;\Lambda,+1}$ contributes to DVCS ~ α_S

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)} \; C'(x,\xi,Q^2)$$

See Hoodbhoy & Ji, PRD58, 054006 (1998)

Interference with Bethe-Heitler contains *cos*3φ modulation to distinguish from (leading twist) quark contribution

$\mu^- \mu^+$ alternative method to extract



other alternatives to extract J/Ψ production; jets . . . . b+bbar

b+bbar at EIC

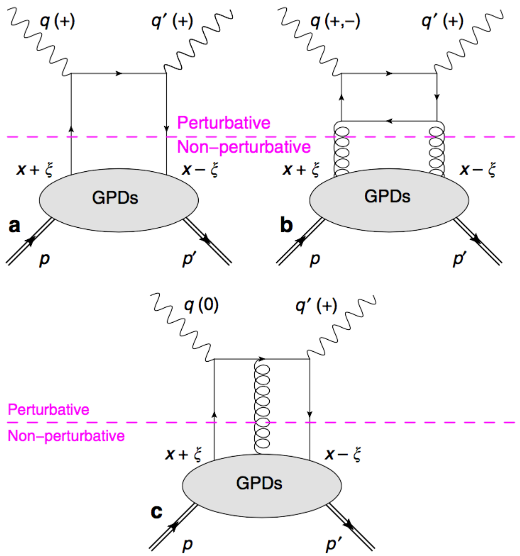
Gluon GPDs from DVCS – Jlab Hall A

Polarized & unpolarized beam measurements

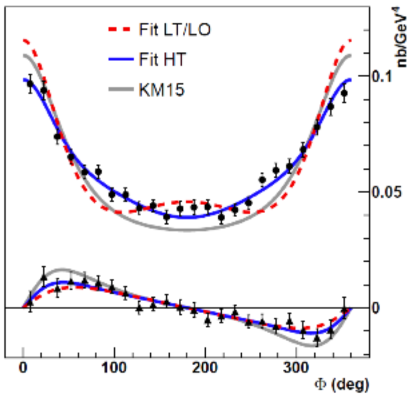
Evidence of gluon transversity

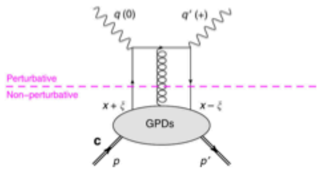
Fitting ϕ distribution requires F_{++} and both F_{+-} gluon transversity and F_{0+} higher twist

$$\frac{d^4\sigma(h)}{dQ^2 dx_B dt d\phi} = \frac{d^2\sigma_0}{dQ^2 dx_B} \times \left[\left|\mathcal{T}^{BH}\right|^2 + \left|\mathcal{T}^{DVCS}(h)\right|^2 - \mathcal{I}(h) \right]$$



A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3





Analysis of 6 GeV Hall A DVCS data on the proton.

Jefferson Lab

See Latifa Elouadrhiri talk at QCD Evolution 2018

LHC – many opportunities for studying gluons

p+p unpolarized → jets, hadrons, leptons

Interactions via g+g→ Q+Qbar +X

gluon TMDs in some kinematics

Extension to Gluon "Transversity"

c.f. TMDs $h_1^{\perp g}(x, p_T^2)$ Mulders & Rodrigues (2001), Gluon Boer-Mulders function
see D. Boer, Frascati talk (Nov.2016) & many references for measurements at EIC, RHIC, LHC

Gluon TMDs

$$\begin{pmatrix} G+\Delta G_L & \frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T - iG_T] & -e^{-2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -i\frac{|\boldsymbol{k}_T|e^{-3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} \\ \frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T + iG_T] & G-\Delta G_L & -i\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}\Delta H_T & -e^{-2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] \\ -e^{2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] & i\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}\Delta H_T & G-\Delta G_L & -\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T + iG_T] \\ i\frac{|\boldsymbol{k}_T|e^{3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} & -e^{2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T - iG_T] & G+\Delta G_L \end{pmatrix}$$

Mulders & Rodrigues, PRD63, 94021 (2001)

The matrix representation is also convenient to find the physical meaning of the distributions. Well known is *G* which measures the number of gluons with momentum (*x*,***k**T*) in a hadron. The functions *GL* (*GT*) represents the difference of the numbers of gluons with opposite circular polarizations in a longitudinally transversely polarized nucleon. The off- diagonal function *H* also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, *H* flips the polarization.

Corresponding GTMDs generalize GPDs & TMDs.

Unintegrated **models** connect all

Other notation $\Delta H_T^{\perp(1)}$, $h_1^{g\perp}$ **gluon Boer-Mulders function**

Unpolarized Nucleon → polarized gluon | factorization & evolution

Gluon TMDs

TMD Color gauge invariance

Small x gluons, Kharzeev, Kovchegov, Tuchin, saturation,

2 gluon distributions: WW vs. DP,

Saturation issues: McLerran-Venugopalan model, ColorGlassCondensate, . .?

See Mulders, et al.: small x DP is pure gauge link

$$\Gamma^{\mu\nu\,[\mathcal{U},\mathcal{U}']}(x,\boldsymbol{k}_T) \equiv \int \frac{d(\xi\cdot P)\,d^2\xi_T}{(P\cdot n)^2(2\pi)^3} e^{i(xP+k_T)\cdot\xi} \left\langle P \left| \mathrm{Tr}_c \left[F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]} \right] \right| P \right\rangle \Big|_{\xi\cdot n=0}.$$

Gauge link

$\xi=[0^+, \xi^-, \boldsymbol{\xi}_T]$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

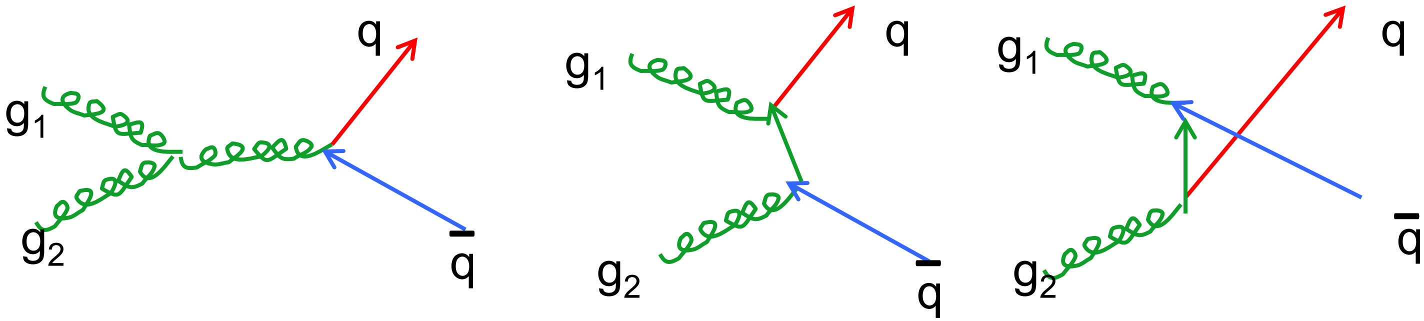
Can be forward light front pointing link +∞ FSI. Weizsacker-Williams

Or mixed light front pointing link -∞ ISI Dipole

For *U* and *U'* *have* [+ +] or [+ -] (& parity opposites)

Consider distributions vs. data at intermediate x

From p+p to gluon TMDs to quark pairs



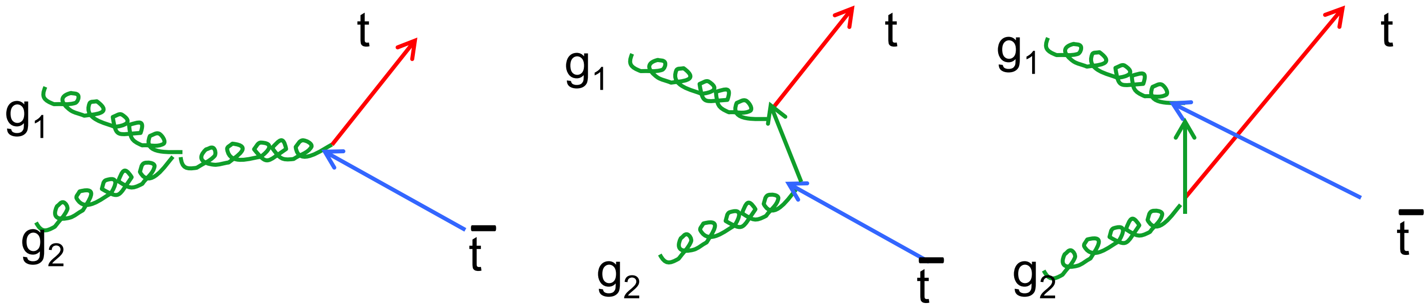
Form quarkonia & different possibilities for gg

Complications from f.s.i. & jets – hadronization

See Boer, Brodsky, Pisano, et al., . . .

Factorization and evolution

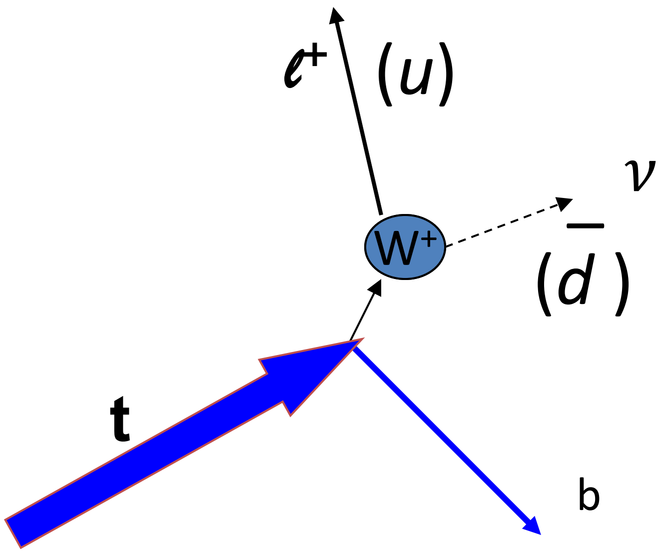
For Gluon fusion top production at LHC



- g_1 & g_2 carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$ & color 1, 8. . . & C=+ or -
- *t* & *t*-bar carry helicity λ_t , $\lambda_{tbar} = \pm \frac{1}{2}$ & color 1 or 8
- *t & tbar decay before hadronizing* => no toponia & large scale

How is top polarization determined?

Its decay is good analyzer for transverse polarization.



$$U_{\lambda_t,\lambda_t'} = \sum_{\lambda_b} B^*_{\lambda_b,\lambda_t'} B_{\lambda_b,\lambda_t'}$$

$$\propto (I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})_{\lambda_t,\lambda_t'} (p_b \cdot p_\nu)$$

Calculated in top rest frame

OR

$$U = (p_t - m_t S_t) \cdot p_{\bar{l}} \, (p_b \cdot p_\nu)$$

$$S_t = \left[\frac{\vec{p} \cdot \vec{P}_t}{m_t}, \vec{P}_t + \frac{(\vec{p} \cdot \vec{P}_t)\vec{P}_t}{m_t(E_t + m_t)} \right]$$

Covariant form in any frame

P_t = strength of top polarization

Dalitz & GRG, PLB287,225(1992); PRD45, 1531(1992)

$(I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})$ lepton or u-quark moves parallel to transverse polarization

What is known production of polarized tops?

Top Single Spin Asymmetry and Double Spin Correlations – Measurements

ATLAS PRD93, 012002 (2016) & ref. PRL114, 142001 (2015)

** SSA: B_1 or A_P =-0.035 +\- 0.040. (syst & stat)

*** Double: $C_{helicity}$ =0.315 +\- 0.07 vs. NLO QCD =0.31

(Bernreuther, et al., PRL 87,242002 (2001) QCD corrections but unpolarized gluons)

CMS PRL112, 182001 (2014): Different kinematics & selection criteria

** SSA: A_P=0.005+\- 0.01.

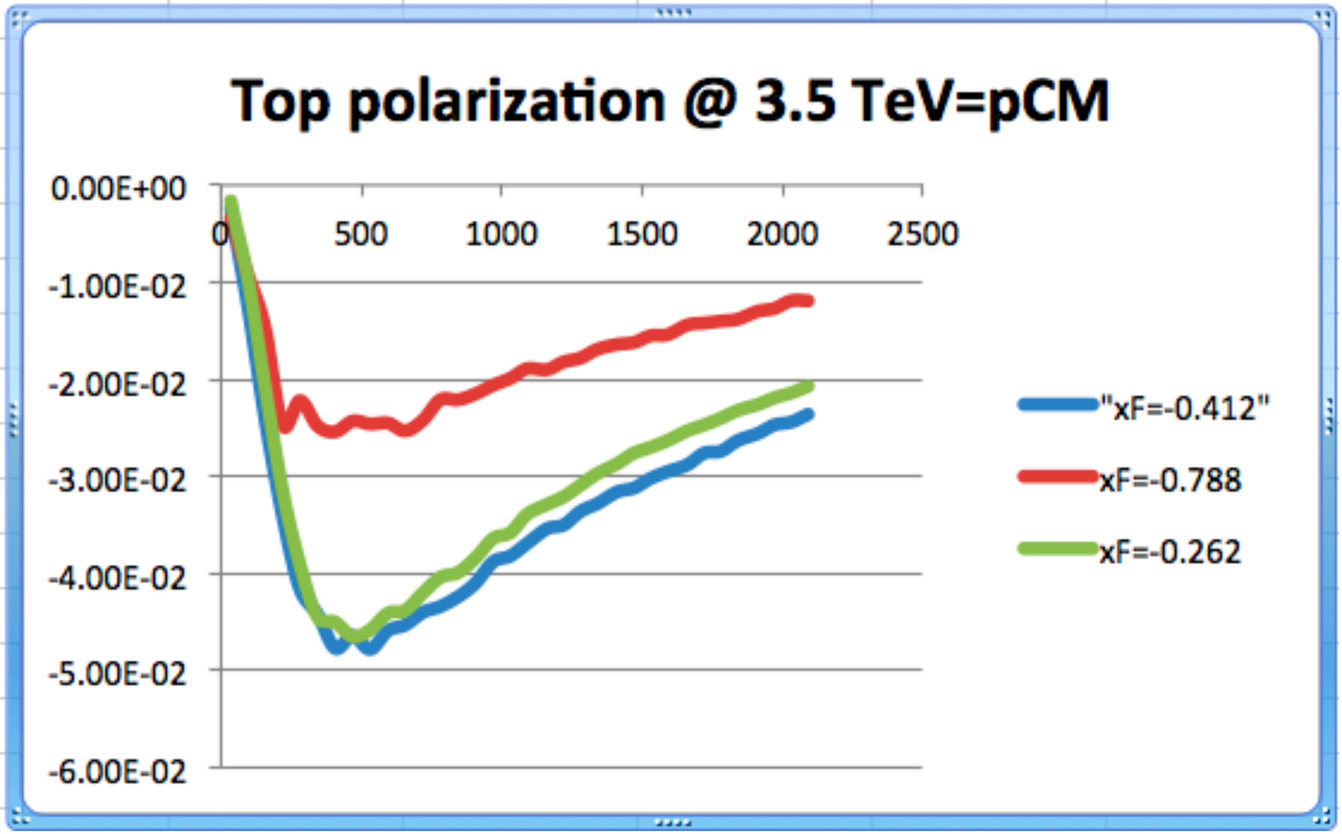
*** Double: $A_{\Delta\phi}=0.113$ +\- 0.01. vs. 0.110 +\- 0.001 (MC & QCD)

A_{c1c2} = -0.021 +\- 0.03 vs -0.078 +\- 0.001

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}(1 + B_1\cos\theta_1 + B_2\cos\theta_2 - C_{\text{helicity}}\cos\theta_1\cdot\cos\theta_2)$$

θ_1 θ_2 decay product angles w.r.t. t+tbar CM

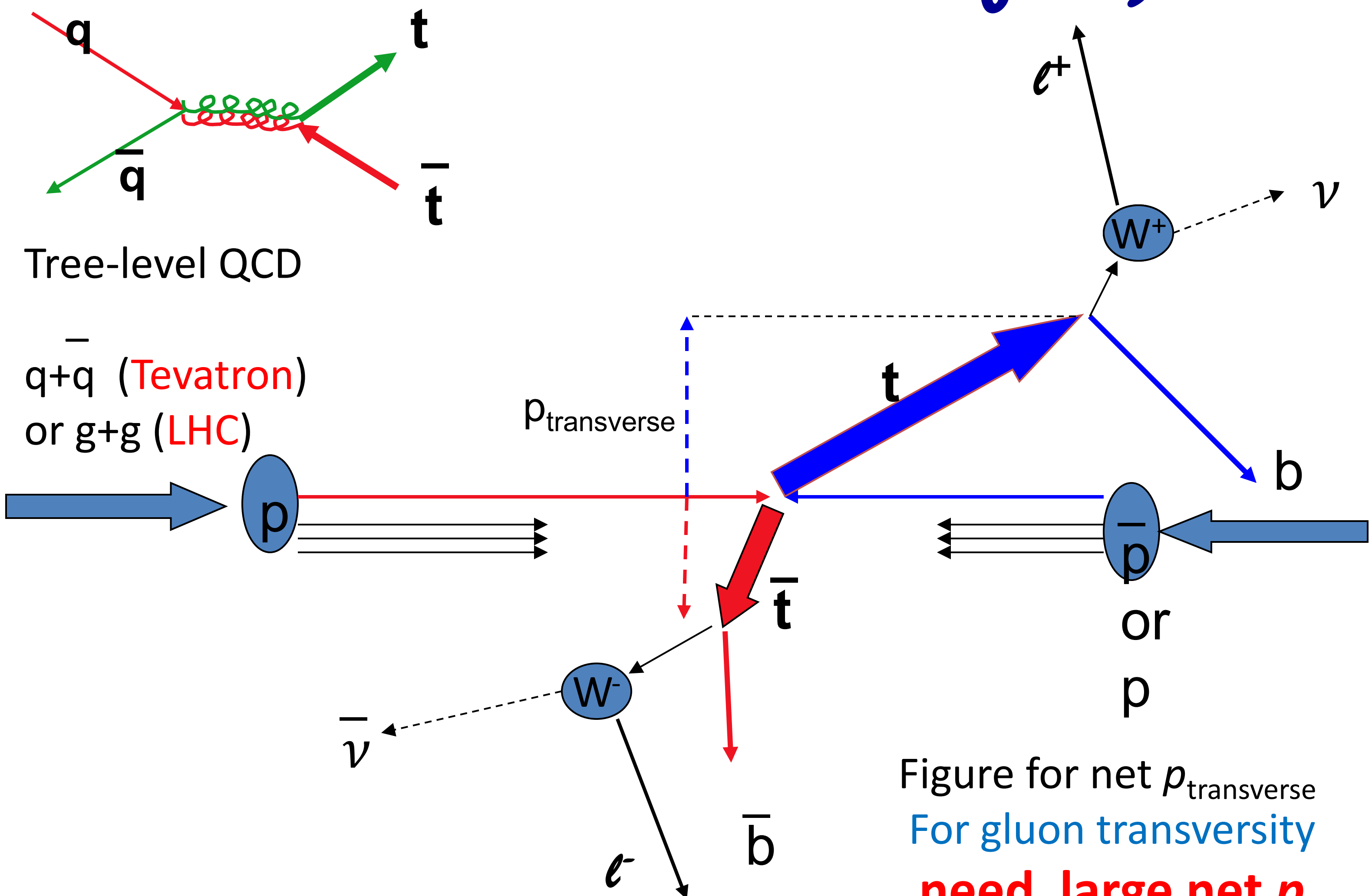
Direct measure of hard process- top polarization
Top decays weakly before hadronizing
⇒ decay "self-analyzing"



Contributions to order α_S
Imaginary Part (Dharmaratna & GRG 1990,1996; arXiv:hep-ph/0001187)

Analyze t--> W^+ b

Dilepton events or lepton+hadron jets or all Hadron jets)



Tree-level QCD

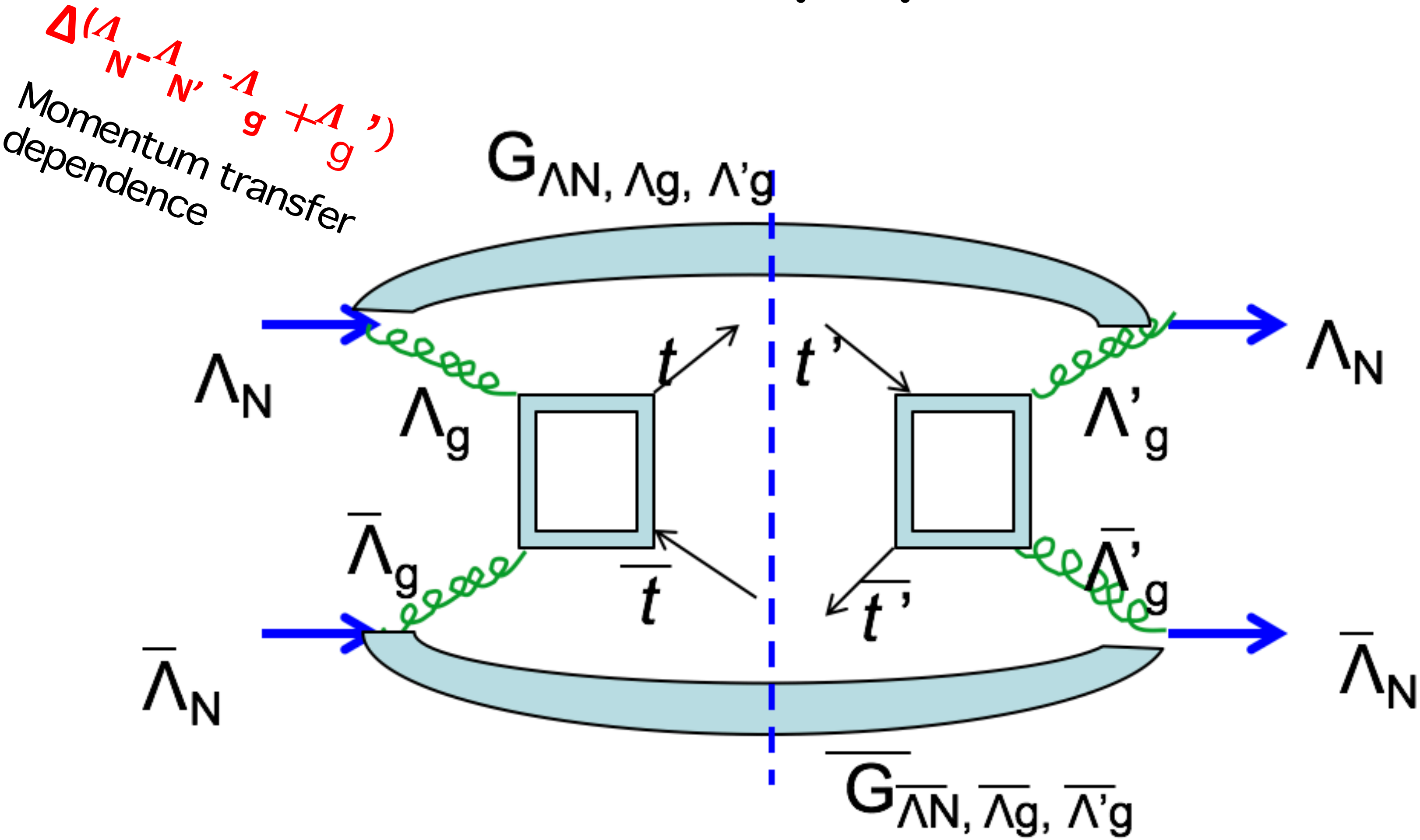
q+q̄ (Tevatron)
or g+g (LHC)

Figure for net $p_{transverse}$

For gluon transversity

need large net $p_{transverse}$ to access transversity

For inclusive p+p→t+tbar+X



Gluon linear polarization with like and unlike t-tbar helicities

(work in progress S.Liuti, GRG, Gonzalez-Hernandez, Poage (thesis))

$F \sim G_{XX}+G_{YY}$, $H \sim G_{XX}-G_{YY}$ or linear polarization

$\rho_{t',\bar{t}';t,\bar{t}}$	$\bar{F}F$	$\bar{H}H$	$\bar{F}H$	$\bar{H}F$
++;++	$\gamma^{-2}(1+\beta^2(1+\sin^4\theta))$	$\gamma^{-2}(-1+\beta^2(1+\sin^4\theta))$	$-2\frac{\beta^2}{\gamma^2}\sin^2\theta$	$-2\frac{\beta^2}{\gamma^2}\sin^2\theta$
+-;+-	$\beta^2\sin^2\theta(2-\sin^2\theta))$	$-\beta^2\sin^4\theta$	0	0

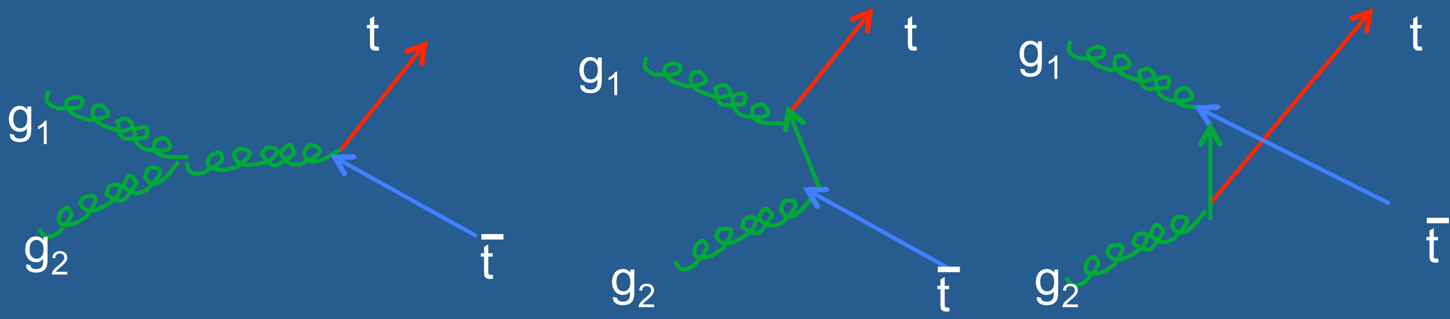
Density matrix & dσ/dΩ

$$\rho_{t',\bar{t}';t,\bar{t}} = \sum_{\Lambda_{g1},\Lambda_{g2},\Lambda'_{g1},\Lambda'_{g2}} \sum_{\Lambda_{N2},\Lambda_{N1}} G^{(2)}_{\Lambda_{N2},\Lambda_{g2},\Lambda'_{g2}} G^{(1)}_{\Lambda_{N1},\Lambda_{g1},\Lambda'_{g1}} A^{*}_{\Lambda'_{g1},\Lambda'_{g2};t',\bar{t}'} A_{\Lambda_{g1},\Lambda_{g2};t,\bar{t}}.$$

- Gluon (or quark) spin correlations are transmitted to the decay products
- Correlations between decay product spins & parent top quark spins (easiest in *t* rest frame)
- The gluon fusion mechanism gives rise to higher order angular correlations ($\sin^4\theta$) vs. q+qbar mechanism
- G.R.Goldstein,"Spin Correlations in Top Quark Production and the Top Quark Mass "in Proceedings of the 12th International Symposium on High Energy Spin Physics, Amsterdam, ed.:C.W. deJager et al., World Scientific, Singapore, 1997, p. 328.

R.H. Dalitz, G.R. Goldstein and R. Marshall, "Heavy Quark Spin Correlations in e+e- annihilations", Phys. Lett. B215, 783 (1988);
R.H. Dalitz, G.R. Goldstein and R. Marshall, "On the Helicity of Charm Jets", Zeits.f. Phys. C42, 441 (1989).

(Color gauge invariance)



g_1, g_2 carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$

t, *t*-bar carry helicity $\lambda_t\ \lambda_{tbar} = \pm \frac{1}{2}$

OR transversity 1 or 0

OR transversity $\pm 1/2$

Introduced in:

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass" in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

R.H. Dalitz, G.R. Goldstein and R. Marshall, "Heavy Quark Spin Correlations in e+e- annihilations", Phys. Lett. B215, 783 (1988);

R.H. Dalitz, G.R. Goldstein and R. Marshall, "On the Helicity of Charm Jets", Zeits.f. Phys. C42, 441 (1989).

q+q-bar → t + t-bar
Dilepton channel

- The light quark-antiquark annihilation mechanism gives rise to the **angular distribution between opposite charge lepton pairs**, more information than $C_{helicity}$ or $A_{c1\ c2}$

$$
\begin{aligned}
W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4}\{1 + [\sin^2\theta([p^2+m^2](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2-m^2](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}})\\
-2mp\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}) + ([p^2-m^2]\\
+[p^2+m^2]\cos^2\theta)(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}]/[(p^2+m^2)+(p^2-m^2)cos^2\theta]\}\\
= \frac{1}{4} + \frac{1}{4}\{(2-\beta^2)\sin^2\theta(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + \beta^2(\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}\\
+[\beta^2+(2-\beta^2)\cos^2\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}\\
-\frac{2}{\gamma}\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}})\Big\}/[(2-\beta^2)+\beta^2\cos^2\theta]
\end{aligned}
$$

m =top mass, θ = t production angle in q+q-bar CM
p= light quark 3-momentum in CM
Unit vectors p-hat are anti-lepton$^+$ and lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

See G.R.Goldstein, ``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

$g_1+g_2 \rightarrow t + t\text{-bar}$

Spin correlations – dilepton channel

Correlations expressed as a weighting factor first ***for unpolarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$
\begin{aligned}
W(\theta, p, p_{\bar l}, p_l) = \frac{1}{4} - \frac{1}{4}\{&[p^4 \sin^4\theta + m^4](\hat p_{\bar l})_x(\hat p_l)_{\bar x} + [p^2(p^2-2m^2)\sin^4\theta - m^4](\hat p_{\bar l})_y(\hat p_l)_{\bar y} \\
&+ [p^4\sin^4\theta - 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)](\hat p_{\bar l})_z(\hat p_l)_{\bar z} \\
&+ 2mp^2\sqrt{p^2-m^2}\cos\theta\sin^3\theta[(\hat p_{\bar l})_x(\hat p_l)_{\bar z} - (\hat p_{\bar l})_z(\hat p_l)_{\bar x}]\} \\
&/ \left[p^2(2m^2-p^2)\sin^4\theta + 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)\right] \qquad (20) \\
= \frac{1}{4} - \frac{1}{4}\{&[(1-\beta^2)^2 + \sin^4\theta)](\hat p_{\bar l})_x(\hat p_l)_{\bar x} \\
&+[-(1-\beta^2)^2 - (1-2\beta^2)\sin^4\theta](\hat p_{\bar l})_y(\hat p_l)_{\bar y} \\
&+[(1-\beta^4) - 2\beta^2\sin^2\theta + \sin^4\theta](\hat p_{\bar l})_z(\hat p_l)_{\bar z} \\
&+2\frac{\beta}{\gamma}\sin^3\theta\cos\theta[(\hat p_{\bar l})_x(\hat p_l)_{\bar z} - (\hat p_{\bar l})_z(\hat p_l)_{\bar x}]\Big\} \\
&/ \left[(1-\beta^4) + 2\beta^2\sin^2\theta + (1-2\beta^2)\sin^4\theta\right] \qquad (21)
\end{aligned}
$$

m =top mass, θ = t production angle in g+g CM; *p*= gluon 3-momentum in CM
p-hat's are lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

Use these to test SM vs. BSM – Integrated version agrees – with big errors -- GRG *in process* – see also Mahlon & Parke

See GG& Liuti, 1710.01683; 2024742 (APS-DPF 2017)

$g_1+g_2 \rightarrow t + t\text{-}bar$
Spin correlations

Correlations expressed as a weighting factor first ***for polarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

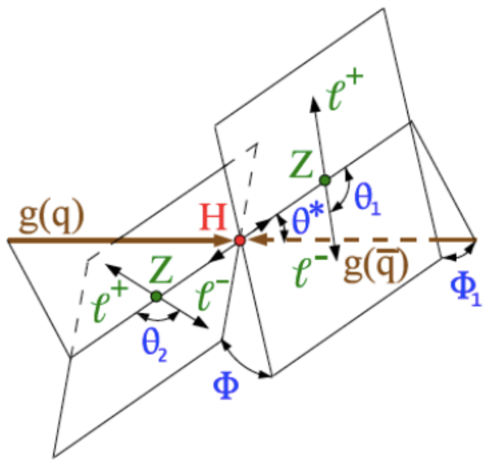
$$
\begin{aligned}
W^{(LP,LP)}(\theta, p, p_{\bar{l}}, p_l) = -\frac{1}{4} + \frac{1}{4}\Big\{ & [(1-\beta^4) + \beta^2 \sin^2\theta(-2 + (2-\beta^2)\sin^2\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} \\
& + [(1-\beta^4) + \beta^2 \sin^2\theta(2 - \beta^2 \sin^2\theta)](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
& + [-(1-\beta^2)^2 + \beta^2(2-\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
& - 4\frac{\beta^2}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}] \Big\} \\
& / \left[(1-\beta^2)^2 + \beta^4 \sin^4\theta\right]
\end{aligned}
$$

Crucial measurements $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} = W_{xx}$, $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} = W_{xz}$, ... **Weighting tensor**

- **Use these to compare with unpolarized to extract the Gluon transversity**
- **or linear polarizations G_{xx}-G_{yy}**
- **Careful about Frames:**
- **Collider LAB, $t + \bar{t}$ pair CM, separate *t* & *t*-bar rest, $W^{+/-}$ rest frames**

Planes & azimuthal correlations

- Similar to tests of p+p → VV+H



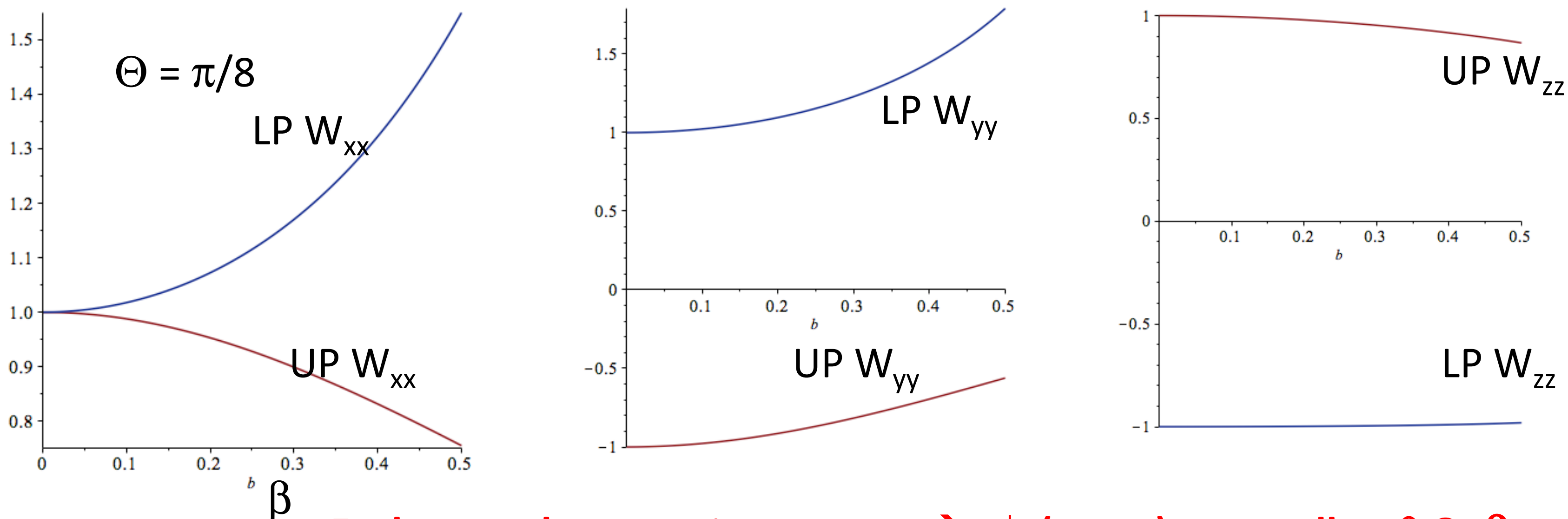
From K. Kaadze talk Wed. morning

t+tbar CM in lab with p_T defines plane
In t rest frame $p_{\text{lepton or quark}}$
& tbar rest frame with lepton or quark
Correlations between both

Comparing lepton directional correlations

Weighting tensor for lepton$^+$ lepton$^-$ when $\theta=\pi/8$

or lepton$^+$ d-quark *or* u-quark lepton$^-$



Each event has μ^- μ^+ momenta → $p^\pm$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

G(UP) W(θ,p,$\bar{p}$ l,pl) +G(LP) W^{LP} (θ,p,$\bar{p}$ l,pl)

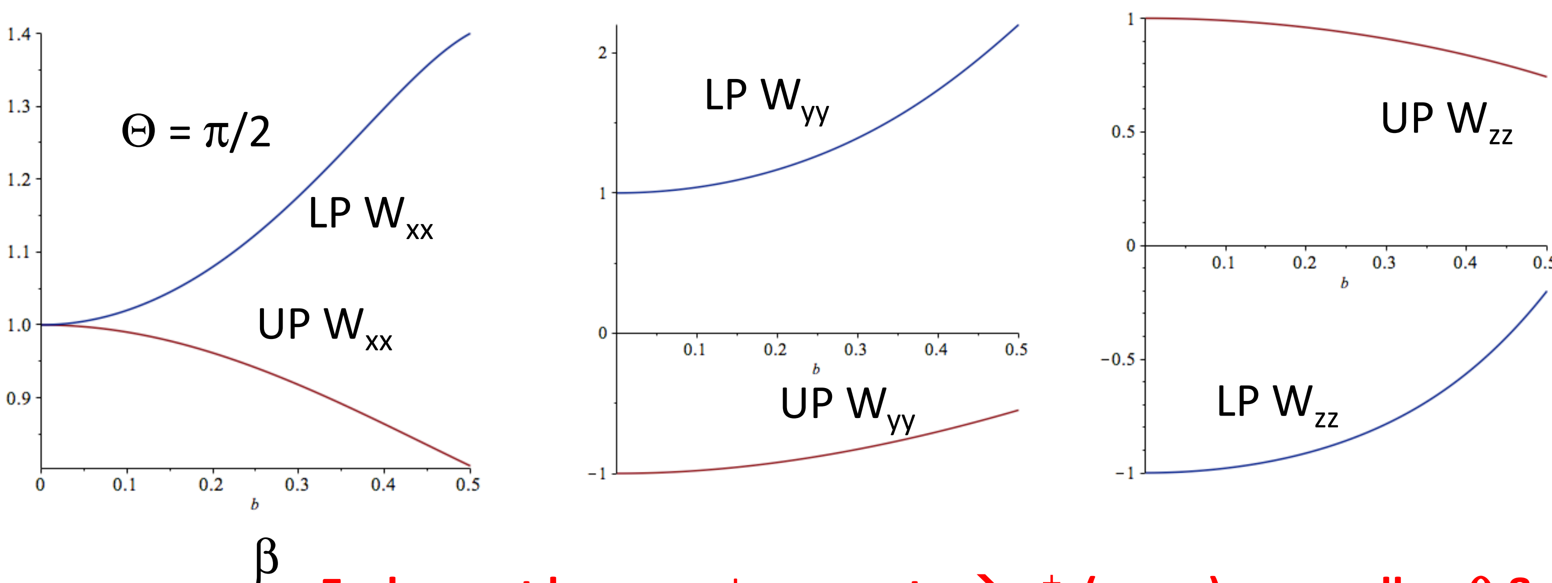
Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Can Diagonalize (with $W_{XY,\,YX}$) to obtain positive ellipsoidal weighting

Comparing lepton directional correlations

Weighting factors for lepton$^+$ lepton$^-$ when $\theta=\pi/2$

W_{xz}=0 for the off-diagonal



Each event has μ^- μ^+ momenta → $p^\pm$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

G(UP) W(θ,p,p¯l,pl) +G(LP) W^{LP} (θ,p,p¯l,pl)

Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Diagonalize (with $W_{XY,\ YX}$) to obtain positive ellipsoidal weighting

Separating polarized gluons

* Each event has μ^- μ^+ momenta → $p^{\pm}$ (x, y, z) in t & tbar rest frame
* t+tbar CM determines θ direction as well as β for t & tbar
* Probability for given event configuration is given by

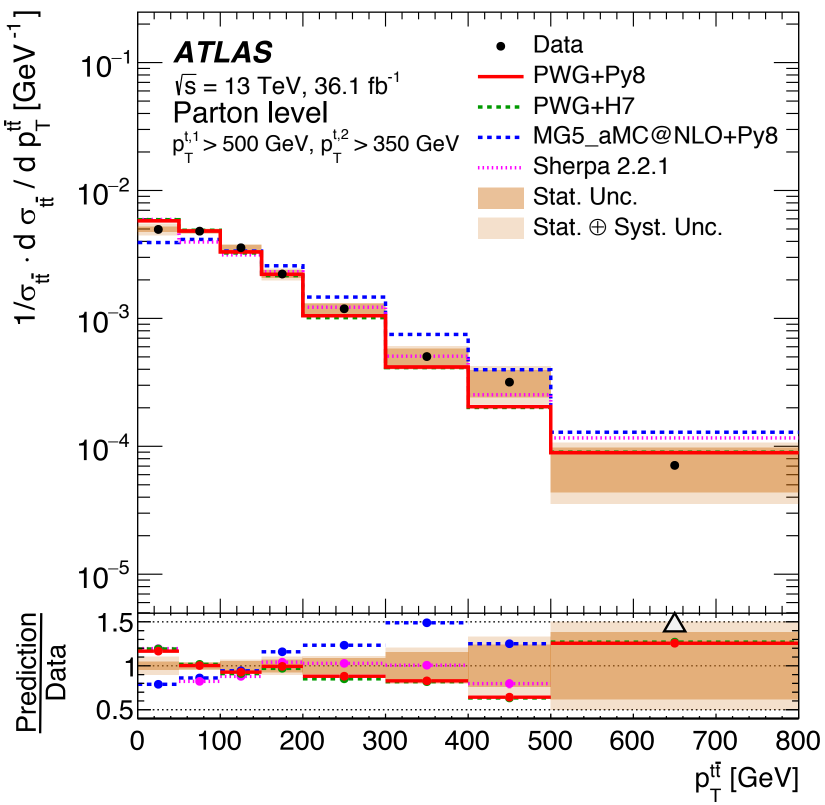
$G(UP)\, W^{UP}(\theta, p, \bar{p}l, pl) + G(LP)\, W^{LP}(\theta, p, \bar{p}l, pl)$

(ignoring light quarks)

- Quite distinct! x & y components are _
- aligned for LP, anti-aligned for UP
- G's convoluted with W's all gluon k_T & k_T satisfying
- measured $p_t + p_{anti\text{-}t}$ <-> large transverse momenta : transversity

Large transverse momentum

- t-tbar inclusive at 13 TeV



How are top pair polarizations measured at LHC?

- Purely hadronic events ⊃ 6 particles/jets (b,u,d +$\bar{b}$,$\bar{u}$,d). Combinatorics!
- Dilepton events leave unknown ν & $\bar{\nu}$ momenta (b,e^{+} ν +$\bar{b}$,$\bar{u}$,d). Clean, lower dσ
- Single lepton events ⊃ one ν missing
  - Most promising: with H.Beauchemin(ATLAS), M. Yampolskaya, T. Lachance
- What is t or $\bar{t}$ momentum?
- Measuring e or μ and b-jet fixes t to an ellipse

Finding top momentum

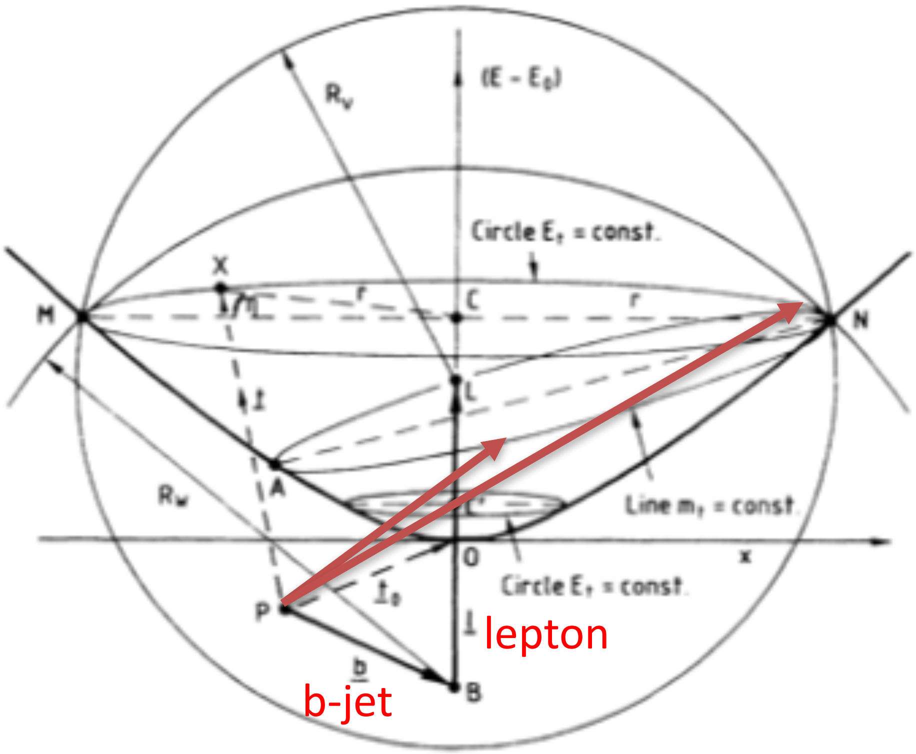


FIG. 5. Momentum vectors **b** and $\bar{l}$ observed in the laboratory frame for bottom quark and lepton, and the construction for locating all top-quark momenta **t** such that these three vectors can correspond to the decay sequence $t \to bW^+$, $W^+ \to \bar{l}^+ \nu_l$ for a given top-quark mass m_t.

Dalitz & GG, PRD45,1531(1992)

- top leptonic decay in lab
- → momenta in lab
- b-jet & lepton measured ⇒
- Ellipse of t-vectors determined
- Boost to rest frame
- helicity preserved
- Lepton direction correlated

Summary

- Gluon GPDs & TMDs (from spectator & Regge R✖Dq)
- *Helicity* conserving & Helicity flip →gluon *Transversity*
- Electroproduction & DVCS → gluon transversity GPDs
- pp→gluons→t+tbar + X
- Measurements? Single Top polarization
- **t+tbar spin correlations** via lepton decays or hadron jets

  – To Do List

- **More phenomenology to come**
- **Parton showers & jets**
- **Care about evolution, factorization, power counting, . . .**

Collaborators: Gluons

Simonetta Liuti[2], Osvaldo Gonzalez Hernandez[3], Jon Poage[1]

- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- GRG, Liuti, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
- J.Poage, Tufts U. dissertation (2016)
- GRG & Liuti, Hernandez, PoS QCDEV2017, 037 (2017)

Collaborators: Tops

Richard Dalitz,

Discussions: Krzysztof Sliwa, Hugo Beauchemin Tufts and Atlas

- Dalitz, R.H., and GRG, Phys. Rev. D45, 1531 (1992); Phys.Lett.B287, 225 (1992);
- GRG, Sliwa, K., Dalitz, R.H., Phys. Rev. D47, 967 (1993).

Collaborators: Transversity

Micheal J. Moravcsik,

- GRG & M.J. Moravcsik, Ann. Phys. 98, 128 (1976); ibid. 142, 219 (1982);
- Ibid. 195, 213 (1989).

See also K. Chen, GRG, R.L. Jaffe, X.-D. Ji, Nucl Phys B 445 (1995) 380-396.

Thank you!

Backup Slides

Construct helicity flip amps Spectator Model, then GPDs

$$
\begin{aligned}
A_{++,+-} &= \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1-\xi)\frac{E_T^g+\tilde{E}_T^g}{2}\right)\\
A_{-+,--} &= \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1+\xi)\frac{E_T^g-\tilde{E}_T^g}{2}\right)\\
A_{++,--} &= +e^{-i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\left(H_T^g + \frac{t_0-t}{M^2}\tilde{H}_T^g - \frac{\xi^2}{1-\xi^2}E_T^g + \frac{\xi}{1-\xi^2}\tilde{E}_T^g\right)\\
A_{-+,+-} &= -e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}_T^g,
\end{aligned}
$$

Compare to spectator model results

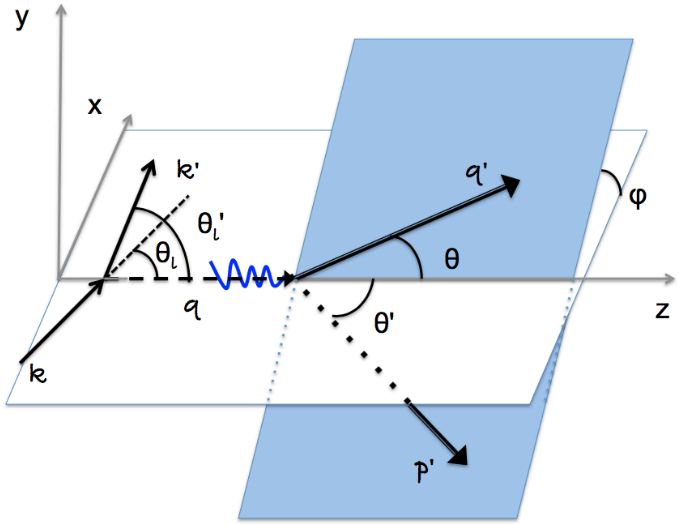
$$\tilde{H}_T^g = 0$$

$$(1-X)A^0_{-+.--} = (1-X')A^0_{++,+-}$$

$$\tilde{E}_T^g = 0.$$

As in Hoodbhoy & Ji, PRD58, 054006 (1998)

Measuring Gluon GPDs in Nucleons



DVCS

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}.$$

For unpolarized e+p→e'+γ+p' cross section depends on azimuthal angle ϕ.
cos3ϕ modulation in interference dσ measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^gF_2 - E_T^gF_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi$$

$\mathcal{H}_T^{\,g} \sim \int dx\, H^g_T/(x-\xi)(x+\xi)$ CFF's

But $\mathcal{H}_T^{\,g}$ ~ may need EIC

See Diehl, *et al*. PLB411, 193 (1997);
Diehl, EPJC25, 223 (2002);
Belitsky, Mueller, PLB486, 369 (2000).

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3

Evidence of gluon transversity

Fitting ϕ distribution requires
F_{++} and both F_{+-} gluon transversity
and F_{0+} higher twist

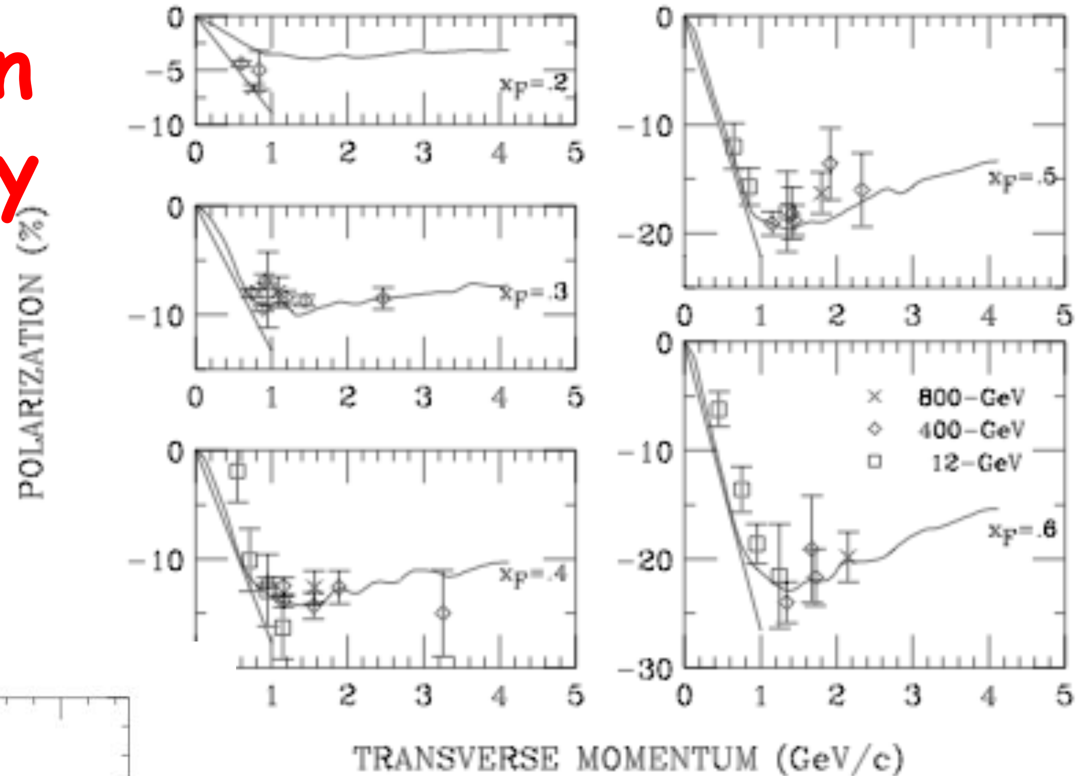
Table 2 Results of the cross-section fits

Fit description	**LO/LT**	**Higher twist**	**NLO**
Helicity states	**++**	**++/0+**	**++/−+**
$t = -0.18$ GeV^2	250	204	206
$t = -0.24$ GeV^2	367	206	208
$t = -0.30$ GeV^2	415	189	190

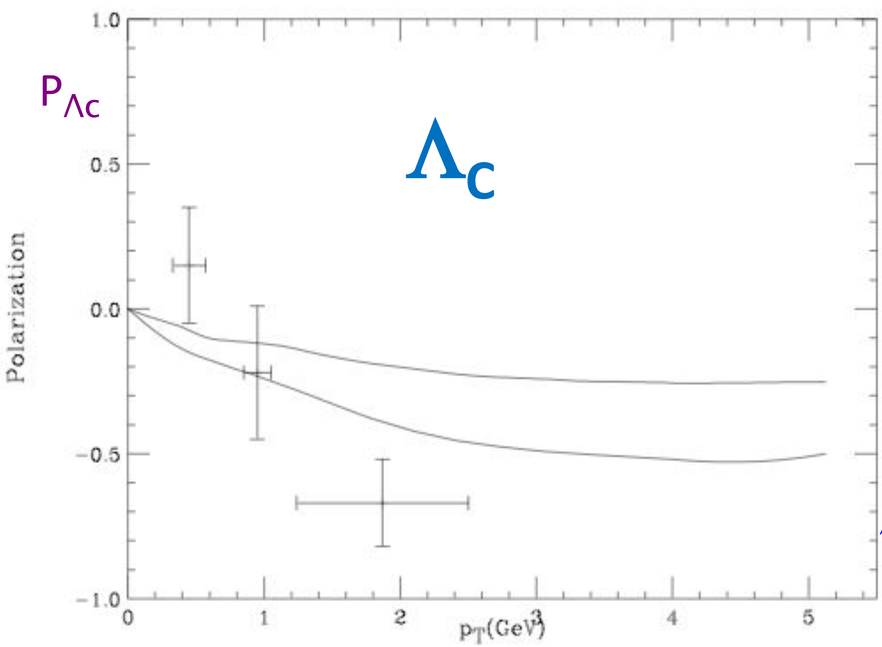
Values of χ^2 (ndf = 208) obtained in the leading-order, leading-twist (++); higher-twist (++/0 +); and next-to-leading-order (++/−+) scenarios. The fit is not performed at the highest value of $-t$ because of the lack of full acceptance in ϕ, resulting in a large statistical uncertainty. The fits include statistical and point-to-point systematic uncertainties

Single Spin Asymmetry

$$pp \rightarrow \Lambda^{\uparrow} (\Lambda_c^{\uparrow}) X$$

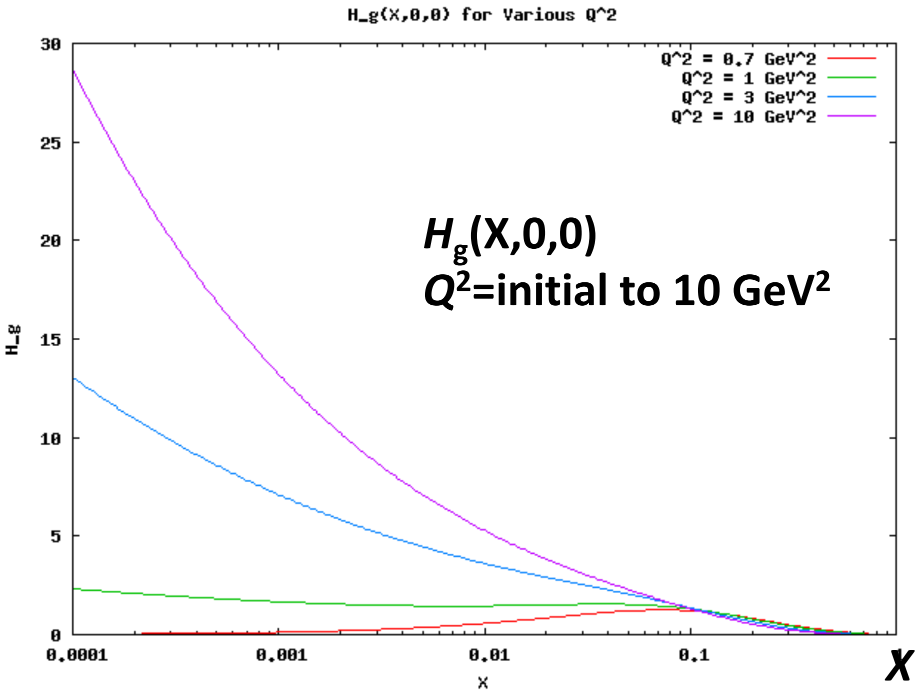


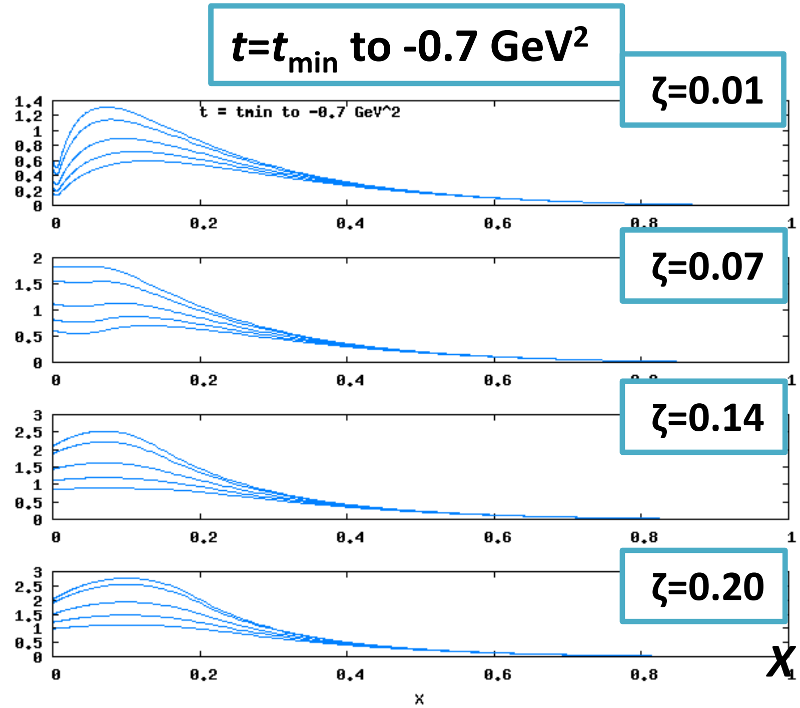
K. Heller, PRD1997
curves from model of
Dharmaratna & GRG PRD '90 & '97



E791, PLB 471, 449 (2000)
$\pi^- + p \rightarrow \Lambda_C + X$
curves from GRG hep-ph/9907573

After pdf's vs. Q^2 → fix x dependence
Regge behavior determines t dependence
Spectator determines ζ dependence





$H_g(X,\zeta,t)$
Q^2=initial

from J. Poage

Gluon & Sea quark distributions
Spectator Model

- N→ ***g*** + "color octet N" spectator ($8 \otimes 8 \supset 1$)
  (could be spin ½ or 3/2)
- (N→ ***anti-u*** + color 3 "tetraquark" uuud)

- How to normalize?

$$H_g(x,\xi,t)_{Q^2} \rightarrow H_g(x,0,0)_{Q^2} = xG(x)_{Q^2}$$

Evolution & small x phenomenology

- Sea quark distributions $H_{anti\text{-}u}(x,0,0)$ . . .

- Use pdf's to fix x dependence

- Small x ~ Pomeron
- Model generalizes to GTMDs > TMDs . . .

Top spin correlations & gluon polarizations

$\rho_{t',\bar{t}';t,\bar{t}}$	UP,UP	LP,LP	UP,LP + LP,UP
$++,++$	$\gamma^{-2}(1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$-4\gamma^{-2}\beta^2 sin^2\theta$
$+-,+-$	$\beta^2 sin^2\theta(2-sin^2\theta))$	$-\beta^2 sin^4\theta$	0
$++,--$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(+1+\beta^2(1+sin^4\theta))$	$+4\gamma^{-2}\beta^2 sin^2\theta$
$+-,-+$	$\beta^2 sin^4\theta$	$-\beta^2 sin^2\theta(2-sin^2\theta))$	0
$++,+-$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-4\gamma^{-1}\beta^2 sin\theta cos\theta$
$++,-+$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$4\gamma^{-1}\beta^2 sin\theta cos\theta$

11 using values of

$$G^{(1)}_{\Lambda_{N1},R,R} + G^{(1)}_{\Lambda_{N1},L,L} = G^{(1)}_{\Lambda_{N1},XX} + G^{(1)}_{\Lambda_{N1},YY} = G^{(1)}_{\Lambda_{N1},UP}$$

$$G^{(1)}_{\Lambda_{N1},R,L} + G^{(1)}_{\Lambda_{N1},L,R} = G^{(1)}_{\Lambda_{N1},YY} - G^{(1)}_{\Lambda_{N1},XX} = G^{(1)}_{\Lambda_{N1},LP}$$

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming g+g→ t + t-bar in single plane CM
γ & β for top & antitop in CM.
θ = top production angle in CM relative to (t+tbar) momentum direction in lab
Taking X-Z plane for p+p→$(t+tbar)_{CM}$+X gives ϕ dependence to
t+tbar plane for opposite helicities: Re($e^{\pm(1or2)i\phi} \cdot e^{\pm(-i(1or2)\phi)}$)
leading to cos2ϕ for UP,LP and LP,UP and cos4ϕ modulations
for LP,LP.