Shapes of Self-Interacting Dark Matter

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Motivations to Study SIDM

- **Observationally: small scale problems with cold dark matter**
  - Core vs. Cusp, Missing Satellites, Too Big to Fail, Diversity of Rotation Curves
  - Potentially resolved with baryonic feedback processes

- **Theoretically: is dark matter collisional or not?**
  - We have a very well posed problem with a clear prescription for how to solve it.
What is the goal?

- Compute the viscous cross section
  \[ \sigma_V = \int d\Omega \sin^2 \theta \frac{d\sigma}{d\Omega} \]
- Include non-perturbative effects - Sommerfeld enhancement
- Have a recipe to do this for arbitrary interactions
  - Compute 2 → 2 scattering amplitude
  - Calculate a potential
  - Compute cross section summing over angular momentum modes
Viscous Cross Section

- The viscous cross section is defined as
  \[ \sigma_V = \int d\Omega \sin^2 \theta \frac{d\sigma}{d\Omega} \]

- This cross section regulates forward and backward scattering poles
- Galaxy DM distributions aren’t affected by forward and backward scattering so this is a good proxy for what we observe!
Sommerfeld Enhancement

- A Classical Analogy
  
  \[
  \begin{align*}
  \text{w/o gravity} \quad \sigma_0 &= \pi R^2 \\
  \text{w/ gravity} \quad \sigma &= \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{\text{esc}}^2}{v^2}\right)
  \end{align*}
  \]

- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential
  - Solve the Schrödinger Equation

\[
S = \frac{\left|\Psi(0)\right|^2}{\left|\Psi^0(0)\right|^2}
\]

Arkani-Hamed, Finkbeiner, Slatyer, Weiner [0810.0713]
Coulomb Potential

- This potential admits an analytic solution for the Sommerfeld enhancement factor

\[
S = \left| \frac{\pi}{\epsilon_v} \frac{\epsilon_v}{1 - \exp[-\frac{\pi}{\epsilon_v}]} \right| \quad \epsilon_v \equiv \frac{v}{\alpha}
\]

- As \( v \) becomes large, \( S \) starts to approach 1.
- As \( v \) approaches 0, \( S \) behaves like \( 1/v \) and starts to diverge.
- Important in the nonrelativistic limit!
EFT Approach

- Process we consider is DM scattering
- Classify all EFTs with a light mediator and fermionic dark matter
  - Study scalar, vector, pseudoscalar and axial vector interactions
  - Dirac and Majorana fermions and Symmetric vs. Asymmetric

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Various Non-Relativistic Potentials

\[ V_{\text{scalar}}(r) = -\frac{\lambda^2}{4\pi r} e^{-m_\phi r} \]

\[ V_{\text{pseudoscalar}}(r) = \frac{\lambda^2}{4m_\chi^2 - m_\phi^2} \frac{\delta'(r)}{4\pi r} \left( 2S_1 \cdot S_2 - \frac{1}{2} \right) + \frac{\lambda^2}{4\pi} \frac{e^{-m_\phi r}}{m_\chi^2} \left[ \frac{m_\phi^2}{3r} S_1 \cdot S_2 + \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3} \left( 1 + m_\phi r + \frac{m_\phi^2 r^2}{3} \right) \right] \]

\[ + \frac{\lambda^2}{12\pi m_\chi^2} \frac{e^{-m_\phi r} \delta'(r)}{r} S_1 \cdot S_2 \]

\[ V_{\text{vector}}(r) = -\frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r} \left( \frac{3}{2} + 4S_1 \cdot S_2 \right) - \frac{\lambda^2}{4\pi r} e^{-m_A r} \]

\[ V_{\text{axial vector}}(r) = \frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r} \left( \frac{1}{2} - 2S_1 \cdot S_2 \right) - \frac{\lambda^2}{\pi r} e^{-m_A r} S_1 \cdot S_2 + \frac{4m_\chi^2}{m_A^2} V_{\text{pseudoscalar}} \]
Computing $\sigma_V$

- Use the Lippmann-Schwinger Equations to set the initial conditions
- Solve the Schrodinger Equation
- Match onto the asymptotic form
- **Extract the phase shift (S-matrix)**
- Compute the viscous cross section

\[
\sigma_V = \sum_{l=0}^{\infty} \frac{(l+1)(l+2)}{(2l+3)} \sin^2(\delta_{l+2} - \delta_l)
\]
Pseudoscalar Potential
Parameter Choices
\( m_\chi = 1 \text{ GeV} \)
\( m_\phi = 10^{-3} \text{ GeV} \)
\( \lambda = 10^{-1} \)

Blue - Numerical Cross Section
Orange - Born Cross Section
Conclusions

- Understanding the space of theories of SIDM is an interesting theoretical problem to solve
- Sommerfeld enhancement can significantly increase the cross section in the non-relativistic regime
- The viscous cross section is the relevant quantity of interest and we have a well-defined procedure for computing it
Thank You!
Backup
The Universal Energy Budget

- What makes up our universe?
  - Radiation
  - Ordinary Matter
  - Dark Matter (CDM)
  - Cosmological Constant ($\Lambda$)
- $\Lambda$CDM is successful on the largest scales

Mitsou [1310.1072]
CDM Signatures & Evidences

- Rotation Curves
- Gravitational Lensing - Bullet Cluster
- Cosmic Microwave Background
Small Scale Problems with CDM

● Core vs. Cusp Problem
  ○ Simulations show cuspy profiles whereas rotation curve observations show cored profiles

● Missing Satellites Problem
  ○ CDM simulations show an overprediction of subhalos and associated dwarf galaxies as compared to observations

● Too Big to Fail Problem
  ○ Most luminous galaxies predicted to inhabit the most massive subhalos.
  ○ Massive subhalos are expected to form stars and should host observable galaxies.
  ○ Low mass galaxies have observed velocities too small to be consistent with the mass of the subhalos they are expected to inhabit.

● Diversity Problem
Baryonic Feedback vs. Self-Interacting Dark Matter

- Supernova driven outflows can help:
  - Flatten the dark matter cusp into a core
  - Deplete baryons and render low mass halos incapable of forming satellites
- SIDM is an interesting alternative
  - Alleviate core vs cusp problem and too big to fail problem by scattering
  - Can give rather interesting signals in experiments depending on how it interacts with the Standard Model
  - Theoretically well motivated question to ask whether dark matter is collisional or not, even if it is just within the dark sector
Red: Dwarf galaxy data
Blue: Low Surface Brightness galaxy data
Green: Cluster data
Gray: SIDM N-body simulation halos

Best fit dark photon model curve shown

Kaplinghat, Tulin, Yu [1508.03339]
Hulthen Potential

\[ V(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}. \]

Blum, Sato, Slatyer [1603.01383]
How do we renormalize?

- Introduce a UV cutoff
  - Removes high momentum states
  - Softens the short range behavior
- Add local counterterms
  - Systematically removes cutoff dependence
  - Derivative expansion
- Make sure we have the correct long range behavior
Coulomb Potential Example

- **Step 1: Introduce the UV cutoff**
  \[
  \frac{1}{r} \rightarrow \frac{4\pi}{q^2} \rightarrow \frac{4\pi}{q^2} e^{-a^2 q^2 / 2} \rightarrow \frac{erf(r / \sqrt{2a})}{r}
  \]

- **Step 2: Add a local counterterm**
  \[
  V_{\text{eff}} = -\frac{\alpha}{r} erf(r / \sqrt{2a}) + 2\pi \alpha c a^2 \delta_a^3(r)
  \]

- **Step 3: Perturbative matching**
  \[
  - \frac{4\pi \alpha}{q^2} e^{-a^2 q^2 / 2}(1 + c a^2 q^2 / 2) = - \frac{4\pi \alpha}{q^2} (1 + (c - 1) a^2 q^2 / 2 + \mathcal{O}(a^4 q^4))
  \]