Shapes of Self-Interacting Dark Matter

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Motivations to Study SIDM

- Observationally: small scale problems with cold dark matter
  - Core vs. Cusp, Missing Satellites, Too Big to Fail, Diversity of Rotation Curves
  - Potentially resolved with baryonic feedback processes
- Theoretically: is dark matter collisional or not?
  - We have a very well posed problem with a clear prescription for how to solve it.

What is the goal?

- Compute the viscous cross section

$$\sigma_V = \int d\Omega \, \sin^2\theta \, \frac{d\sigma}{d\Omega}$$

- Include non-perturbative effects - Sommerfeld enhancement
- Have a recipe to do this for arbitrary interactions
  - Compute $2 \to 2$ scattering amplitude
  - Calculate a potential
  - Compute cross section summing over angular momentum modes

Viscous Cross Section

- The viscous cross section is defined as

$$\sigma_V = \int d\Omega \, \sin^2\theta \, \frac{d\sigma}{d\Omega}$$

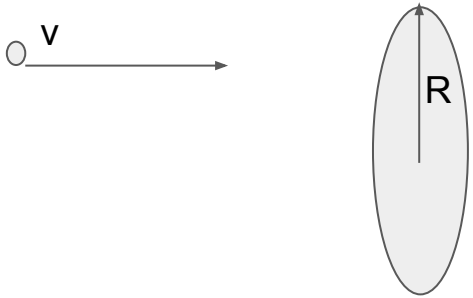
- This cross section regulates forward and backward scattering poles
- Galaxy DM distributions aren't affected by forward and backward scattering so this is a good proxy for what we observe!

Sommerfeld Enhancement

- A Classical Analogy

w/o gravity $\sigma_0 = \pi R^2$

w/ gravity $\sigma = \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$



- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential
  - Solve the Schrodinger Equation

$$S = \frac{|\Psi(0)|^2}{|\Psi^0(0)|^2}$$

Arkani-Hamed, Finkbeiner, Slatyer, Weiner [0810.0713]

Coulomb Potential

$$V(r) = \frac{-\alpha}{2r}$$

- This potential admits an analytic solution for the Sommerfeld enhancement factor

$$S = \left| \frac{\frac{\pi}{\epsilon_v}}{1 - exp[-\frac{\pi}{\epsilon_v}]} \right| \qquad \epsilon_v \equiv \frac{v}{\alpha}$$

- As v becomes large, S starts to approach 1.
- As v approaches 0, S behaves like 1/v and starts to diverge.
- Important in the nonrelativistic limit!

EFT Approach

- Process we consider is DM scattering
- Classify all EFTs with a light mediator and fermionic dark matter
  - Study scalar, vector, pseudoscalar and axial vector interactions
  - **Dirac** and Majorana fermions and **Symmetric** vs. Asymmetric

Type	Process	Channels
Majorana	$\chi\chi \to \chi\chi$	s, t, u
Dirac, Asymmetric	$\chi\chi \to \chi\chi$	t, u
Dirac, Symmetric	$\chi\overline{\chi} \to \chi\overline{\chi}$	s, t

Various Non-Relativistic Potentials

$$V_{scalar}(r) = -\frac{\lambda^2}{4\pi r} e^{-m_\phi r}$$

$$V_{pseudoscalar}(r) = \frac{\lambda^2}{4m_\chi^2 - m_\phi^2} \frac{\delta'(r)}{4\pi r}\left(2S_1 \cdot S_2 - \frac{1}{2}\right) + \frac{\lambda^2}{4\pi} \frac{e^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r} S_1 \cdot S_2 + \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\right]$$

$$+ \frac{\lambda^2}{12\pi m_\chi^2} \frac{e^{-m_\phi r}\delta'(r)}{r} S_1 \cdot S_2$$

$$V_{vector}(r) = -\frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r}\left(\frac{3}{2} + 4S_1 \cdot S_2\right) - \frac{\lambda^2}{4\pi r} e^{-m_A r}$$

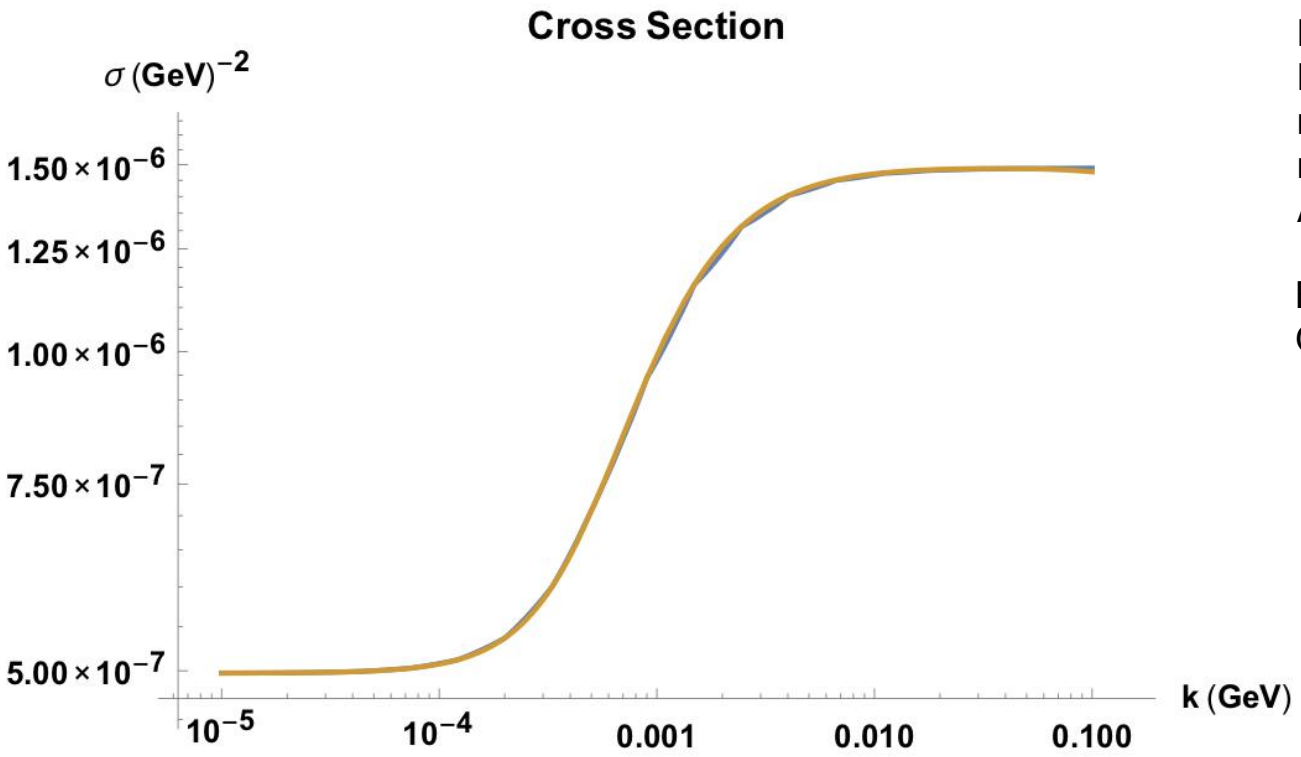
$$V_{axial\,vector}(r) = \frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r}\left(\frac{1}{2} - 2S_1 \cdot S_2\right) - \frac{\lambda^2}{\pi r} e^{-m_A r} S_1 \cdot S_2 + \frac{4m_\chi^2}{m_A^2} V_{pseudoscalar}$$

Computing σ_V

- Use the Lippmann-Schwinger Equations to set the initial conditions
- Solve the Schrodinger Equation
- Match onto the asymptotic form
- **Extract the phase shift (S-matrix)**
- Compute the viscous cross section

$$\sigma_V = \sum_{l=0}^{\infty} \frac{(l+1)(l+2)}{(2l+3)} sin^2(\delta_{l+2} - \delta_l)$$

Scattering Theory by Harald Friedrich



Pseudoscalar Potential
Parameter Choices
m_χ = 1 GeV
m_ϕ = 10^{-3} GeV
λ = 10^{-1}

Blue - Numerical Cross Section
Orange - Born Cross Section

Conclusions

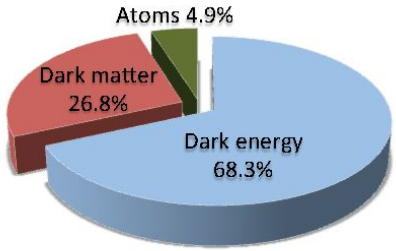
- Understanding the space of theories of SIDM is an interesting theoretical problem to solve
- Sommerfeld enhancement can significantly increase the cross section in the non-relativistic regime
- The viscous cross section is the relevant quantity of interest and we have a well-defined procedure for computing it

Thank You!

Backup

The Universal Energy Budget

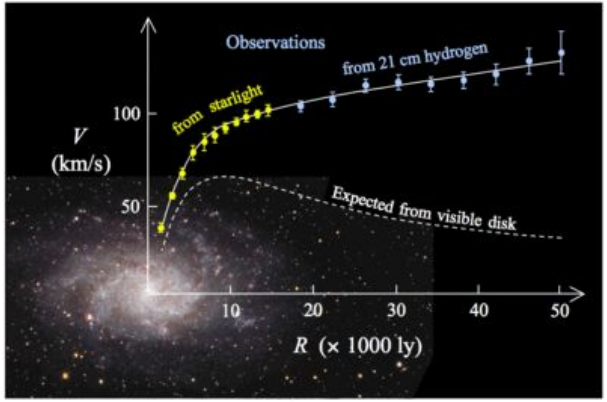
- What makes up our universe?
  - Radiation
  - Ordinary Matter
  - Dark Matter (CDM)
  - Cosmological Constant (Λ)
- ΛCDM is successful on the largest scales



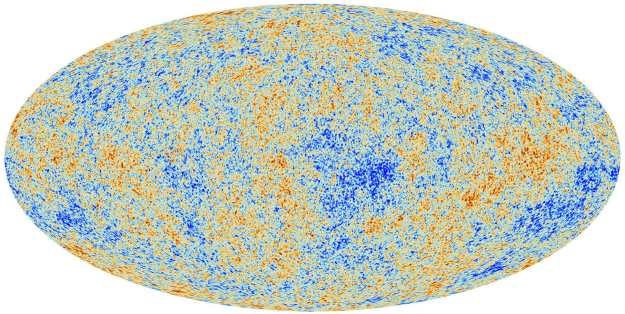
Mitsou [1310.1072]

CDM Signatures & Evidences

- Rotation Curves
- Gravitational Lensing - Bullet Cluster
- Cosmic Microwave Background



M33 Rotation Curve

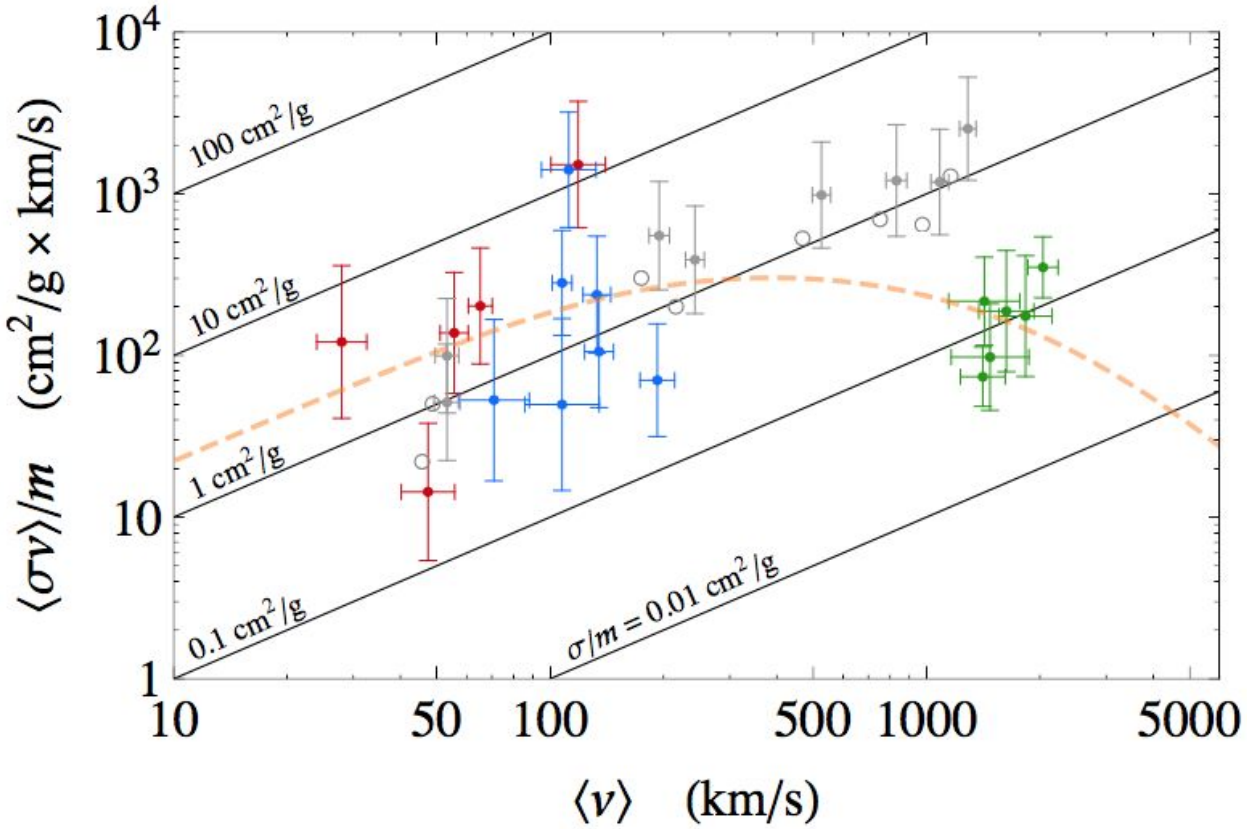


Small Scale Problems with CDM

- Core vs. Cusp Problem
  - Simulations show cuspy profiles whereas rotation curve observations show cored profiles
- Missing Satellites Problem
  - CDM simulations show an overprediction of subhalos and associated dwarf galaxies as compared to observations
- Too Big to Fail Problem
  - Most luminous galaxies predicted to inhabit the most massive subhalos.
  - Massive subhalos are expected to form stars and should host observable galaxies.
  - Low mass galaxies have observed velocities too small to be consistent with the mass of the subhalos they are expected to inhabit.
- Diversity Problem

Baryonic Feedback vs. Self-Interacting Dark Matter

- Supernova driven outflows can help:
  - Flatten the dark matter cusp into a core
  - Deplete baryons and render low mass halos incapable of forming satellites
- SIDM is an interesting alternative
  - Alleviate core vs cusp problem and too big to fail problem by scattering
  - Can give rather interesting signals in experiments depending on how it interacts with the Standard Model
  - Theoretically well motivated question to ask whether dark matter is collisional or not, even if it is just within the dark sector



Red: Dwarf galaxy data

Blue: Low Surface Brightness galaxy data

Green: Cluster data

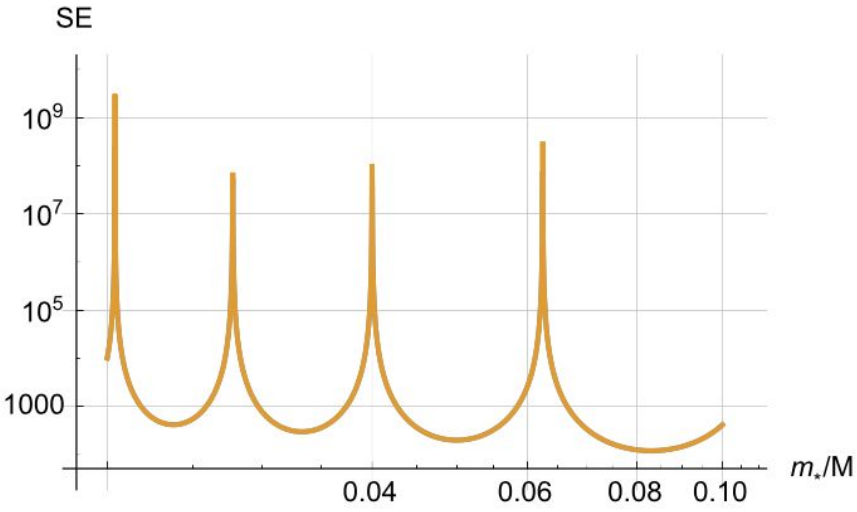
Gray: SIDM N-body simulation halos

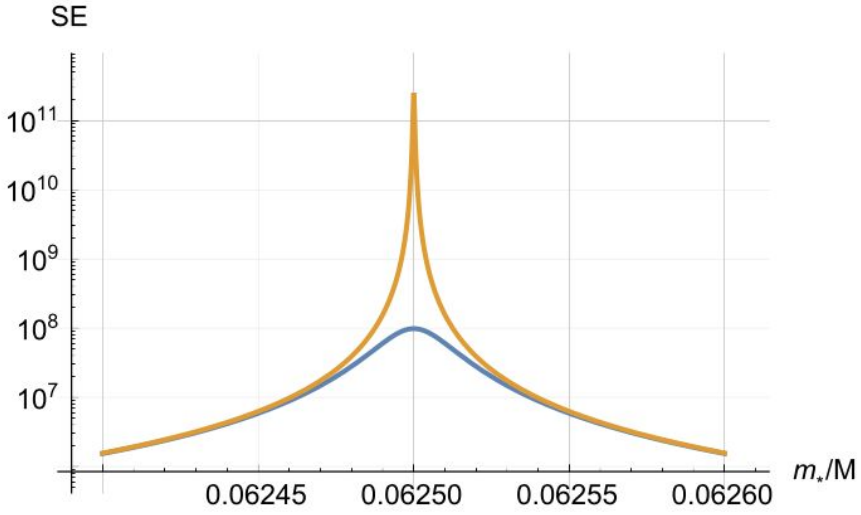
Best fit dark photon model curve shown

Kaplinghat, Tulin, Yu [1508.03339]

Hulthen Potential

$$V(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}.$$





Blum, Sato, Slatyer [1603.01383]

How do we renormalize?

- Introduce a UV cutoff
  - Removes high momentum states
  - Softens the short range behavior
- Add local counterterms
  - Systematically removes cutoff dependence
  - Derivative expansion
- Make sure we have the correct long range behavior

Coulomb Potential Example

- Step 1: Introduce the UV cutoff

$$\frac{1}{r} \to \frac{4\pi}{q^2} \to \frac{4\pi}{q^2} e^{-a^2 q^2/2} \to \frac{erf(r/\sqrt{2}a)}{r}$$

- Step 2: Add a local counterterm

$$V_{eff} = -\frac{\alpha}{r} erf(r/\sqrt{2}a) + 2\pi\alpha c a^2 \delta_a^3(r)$$

- Step 3: Perturbative matching

$$-\frac{4\pi\alpha}{q^2} e^{-a^2 q^2/2}(1 + ca^2 q^2/2) = -\frac{4\pi\alpha}{q^2}(1 + (c-1)a^2 q^2/2 + \mathcal{O}(a^4 q^4))$$