Onset of Inflation amid Backreaction from Inhomogeneities

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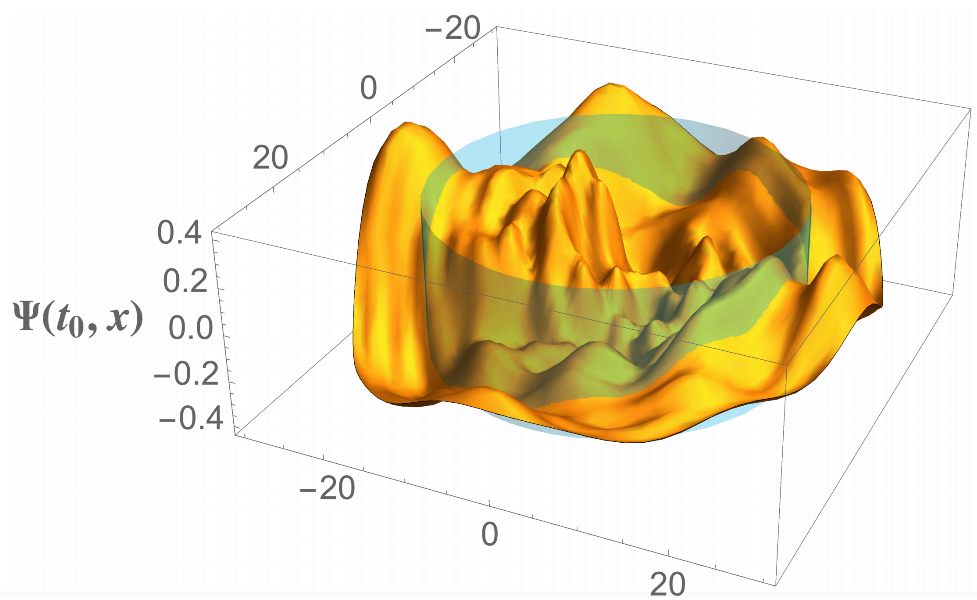


Can a really lumpy spacetime with inhomogeneities on length scales around and well within the Hubble radius, when we include the effects of nonlinear back-reaction, nonetheless flow into inflation?

with David Kaiser
Jolyon Bloomfield
Kiriakos Hilbert

arXiv: 1906.08651

Initial Conditions Problem

- Inflation explains high degree of spatial flatness and homogeneity observed today in our universe at horizon scales
- criticism: in order to begin inflation may require homogeneity over many Hubble volumes
- if inflationary expansion fails to begin under sufficiently inhomogeneous initial conditions then its naturalness is challenged.
- In this work: we study this problem incorporating a well defined set of nonlinear interactions using the Hartree approximation
- complements recent simulations in full (3+1) numerical relativity (e.g. East et al. 2016, Clough et al. 2017)…
- our numerical approach can be applied more efficiently to a wider range of models, across broad regions of phase and parameter space, tracking the evolution of perturbations across a wide range of scales.

Equations of Motion

- Scalar metric perturbations around FLRW background, in longitudinal gauge

$$ds^2 = -\left(1+2\Psi\right)dt^2 + a^2\left(t\right)\left(1-2\Psi\right)h^{ij}\left(\mathbf{x}\right)dx^i dx^j$$

$$h^{ij}\left(\mathbf{x}\right)dx^i dx^j = \frac{dr^2}{\left(1-Kr^2\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- single-field models with minimal couplings to gravity and canonical kinetic term

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V\left(\phi\right)\right]$$

- Coupled equations of motion

$$\Box\phi - \frac{\partial V}{\partial\phi} = 0$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_{\rm pl}^2}T_{\mu\nu} \qquad T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm M}}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V\left(\phi\right)\right]$$

- Quantized field fluctuations and metric perturbations

$$\phi\left(x^\mu\right) \to \hat{\phi}\left(x^\mu\right) = \varphi\left(t\right) + \delta\hat{\phi}\left(x^\mu\right) \qquad \Psi\left(x^\mu\right) \to \hat{\Psi}\left(x^\mu\right)$$

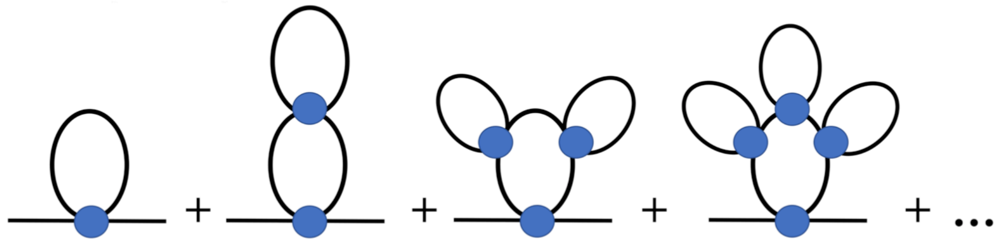
- Procedure: expand the EOM to linear order in $\hat{\Psi}$ and arbitrarily higher order in $\left(\delta\hat{\phi}\right)^n$.
  Implement Hartree approximation to incorporate nonlinear interactions among $\left(\delta\hat{\phi}\right)^n$ —> Capture nonlinear back reaction effects

Hartree Approximation

- Hartree approximation incorporates certain nonlinear structure and captures effects of back-reaction for the self-coupled system.

- incorporate nonlinear self-interactions of $\delta\hat{\phi}$ into an effective mass for the fluctuations by performing an infinite resummation of the “cactus” diagrams to all orders (Dolan and Jackiw, 1974)

- cactus diagrams dominate at any given order in the calculation of the self-energy:



- higher-order interaction terms $\left(\delta\hat{\phi}\right)^n$ for $n \geq 2$ are replaced by combinations of the dressed two-point function

$$\left(\delta\hat{\phi}\right)^2 \to \langle\left(\delta\hat{\phi}\right)^2\rangle$$
$$\left(\delta\hat{\phi}\right)^3 \to 3\langle\left(\delta\hat{\phi}\right)^2\rangle\delta\hat{\phi}$$
$$\cdots$$

- Nonperturbative in field fluctuations: no assumption made about relative size of $\langle\left(\delta\hat{\phi}\right)^2\rangle$ and ϕ^2
- Linear in gravitational degrees of freedom: does not include terms of the form $\langle\hat{\Psi}\delta\hat{\phi}\rangle$

Nonlinear equations of motion

- Including the nonlinear interactions among field fluctuations in the Hartree approximation:

$$\ddot{\varphi} + 3H\dot{\varphi} + V^{(1)}(\varphi) + \frac{1}{2}V^{(3)}(\varphi)\langle\left(\delta\hat{\phi}\right)^2\rangle = 0$$

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left[\bar{\rho} + \delta\rho_{(2)}\right] - \frac{K}{a^2}$$

- with:

$$\bar{\rho} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$\delta\rho_{(2)} = \frac{1}{2}\left\langle\left(\delta\dot{\hat{\phi}}\right)^2\right\rangle + \frac{1}{2a^2}h^{ij}\left\langle\partial_i\delta\hat{\phi}\partial_j\delta\hat{\phi}\right\rangle + \frac{1}{2}V^{(2)}(\phi)\left\langle\left(\delta\hat{\phi}\right)^2\right\rangle + \frac{1}{8}V^{(4)}(\phi)\left\langle\left(\delta\hat{\phi}\right)^2\right\rangle^2$$

$$\delta\ddot{\phi}_{klm} + 3H\delta\dot{\phi}_{klm} + \left[\frac{k^2}{a^2} + V^{(2)}(\varphi) + \frac{1}{2}V^{(4)}(\varphi)\left\langle\left(\delta\hat{\phi}\right)^2\right\rangle\right]\delta\phi_{klm} = 2\left(\ddot{\varphi} + 3H\dot{\varphi}\right)\Psi_{klm} + 4\dot{\varphi}\dot{\Psi}_{klm}$$

$$\dot{\Psi}_{klm} + H\Psi_{klm} = \frac{1}{2M_{\rm pl}^2}\dot{\varphi}\delta\phi_{klm}$$

- along with the constraint relating the modes Ψ_{klm} $\delta\phi_{klm}$

$$\left[\dot{H} + \frac{2}{3M_{\rm pl}^2a^2}h^{ij}\left\langle\partial_i\delta\hat{\phi}\partial_j\delta\hat{\phi}\right\rangle + \frac{1}{a^2}\left(k^2 - 3K\right)\right]\Psi_{klm} = \frac{1}{2M_{\rm pl}^2}\left[\ddot{\varphi}\delta\phi_{klm} - \dot{\varphi}\delta\dot{\phi}_{klm}\right]$$

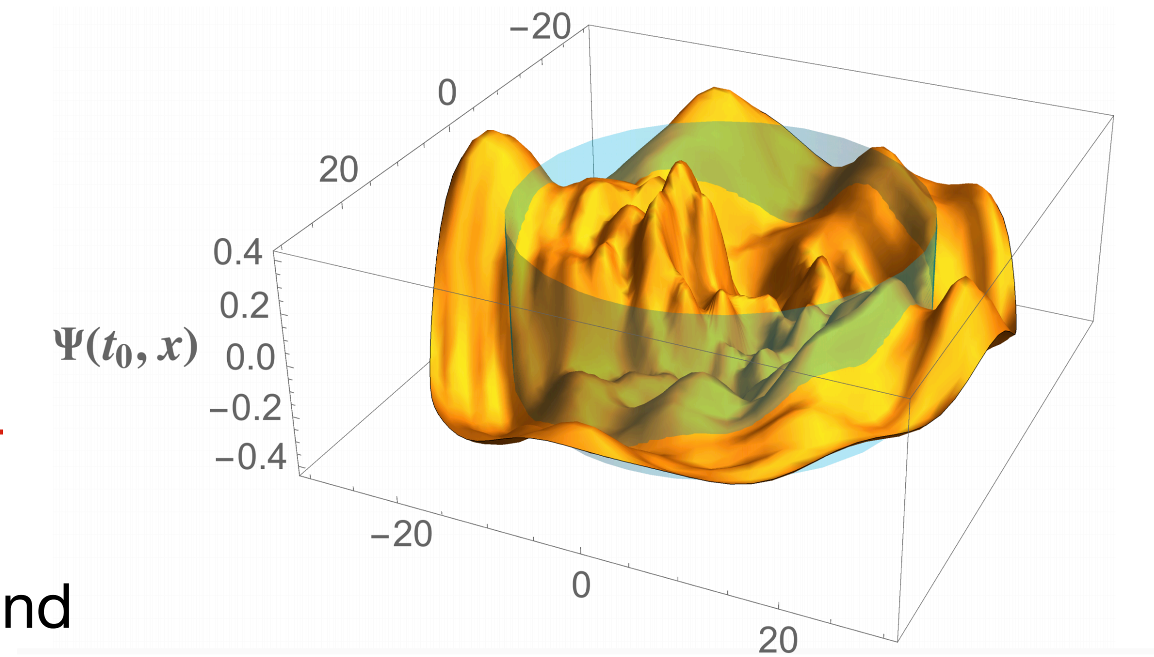
- Nonperturbative in interaction among field fluctuations $\delta\hat{\phi}$
- To linear order in metric perturbations $\hat{\Psi}$
- Fully coupled system incorporates effects of backreaction.

Numerical Results $V(\phi) = \lambda \phi^4 \quad \lambda = 10^{-10} \quad K = 0$

- Particular set of initial conditions $\varphi_0 = 25 M_{\rm pl}$, $\dot{\varphi}_0 = -0.25 M_{\rm pl}^2$ far from inflationary attractor
- Numerically track the evolution of coupled field and metric perturbations with finite comoving spatial volume
- Discrete spectrum of 120 coupled modes k_{nl}

$$\lambda_{min} \lesssim \frac{\pi}{10 H_0} \sim \frac{\pi}{M_{\rm pl}} \qquad \lambda_{\rm max} \sim 3\pi H_0^{-1}$$

$$k_{\rm max} \sim M_{\rm pl}$$

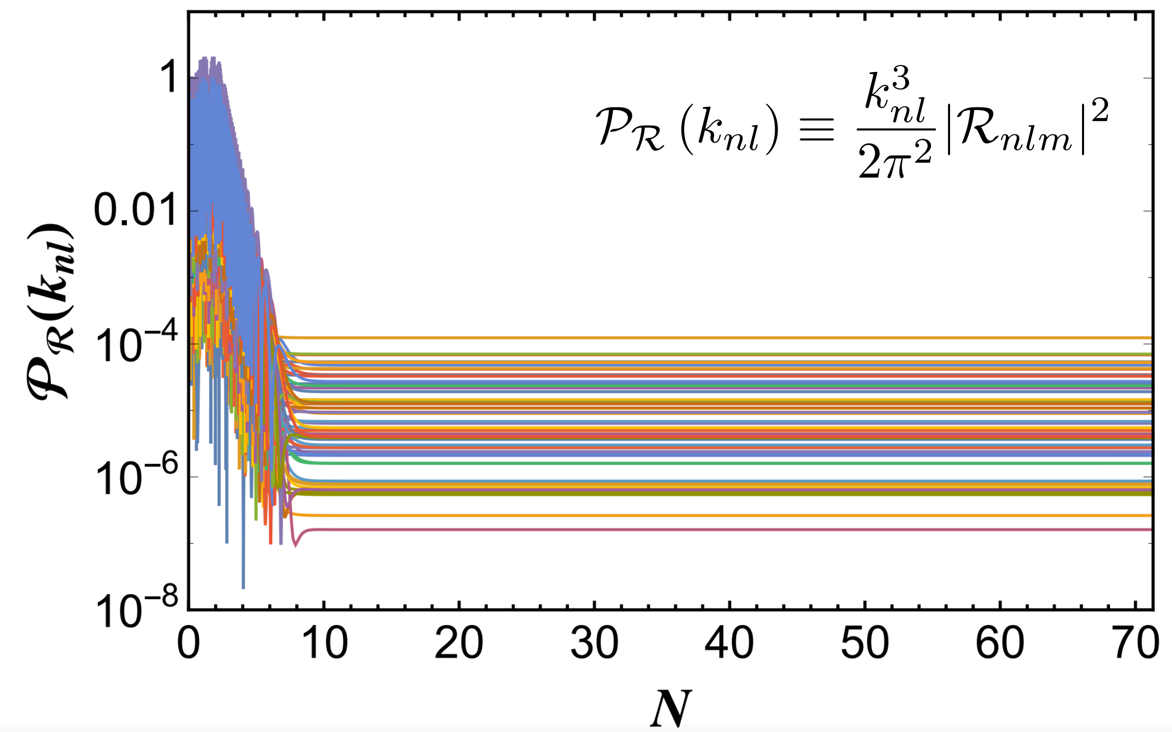


- $\hat{\Psi}(t_0, \mathbf{x})$ begins with substantial structure on sub-Hubble scales
- $\delta\phi_k(t_0)$ and $\delta\dot{\phi}(t_0)$ parameterized randomly around Bunch-Davies

$$\delta\rho_{(2)}(t_0) \simeq \bar{\rho}(t_0)$$

$$H_0 = 0.14 M_{\rm pl} \gg H_{\rm infl}$$

- Initially inhomogeneous system produces a smooth patch on Hubble radius scales before fluctuation modes cross outside the Hubble radius



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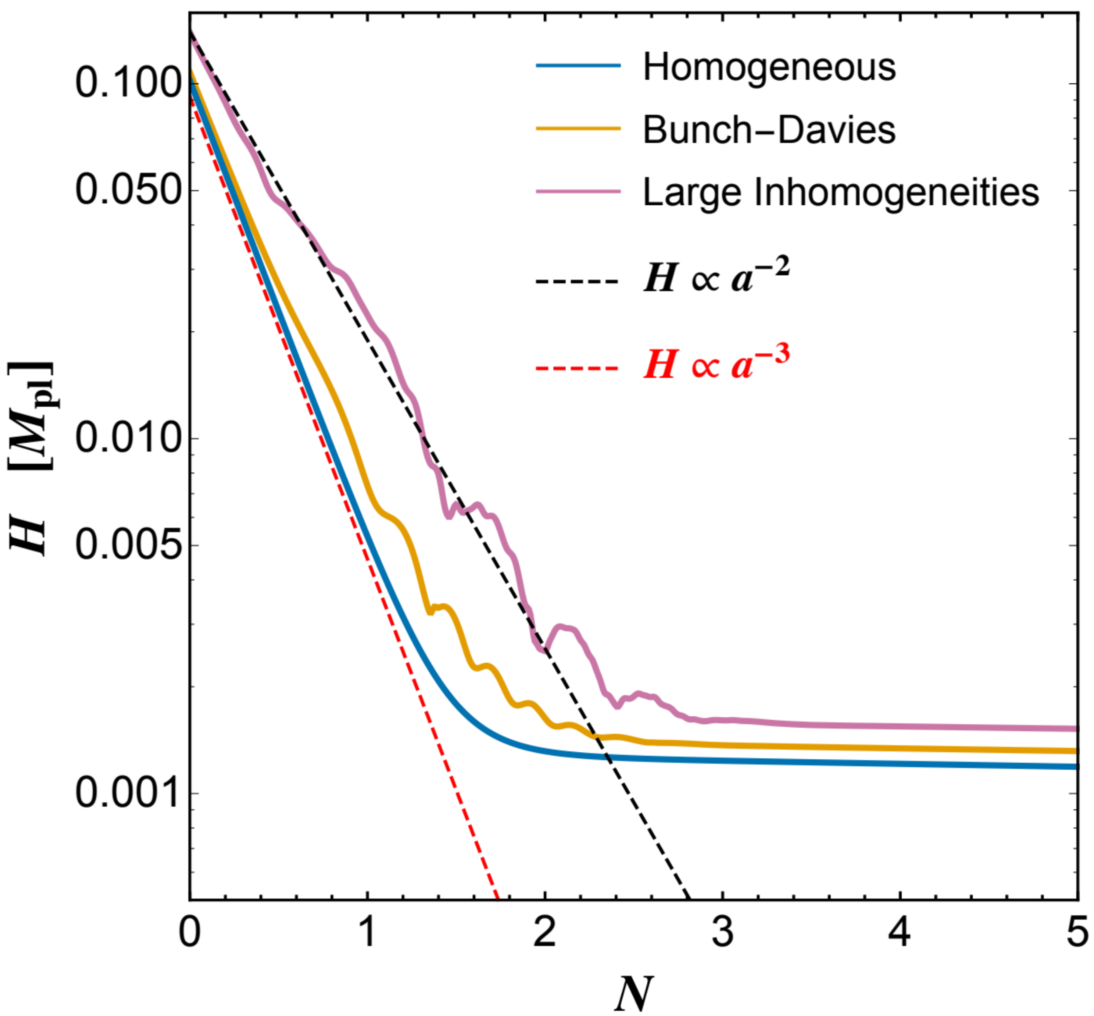
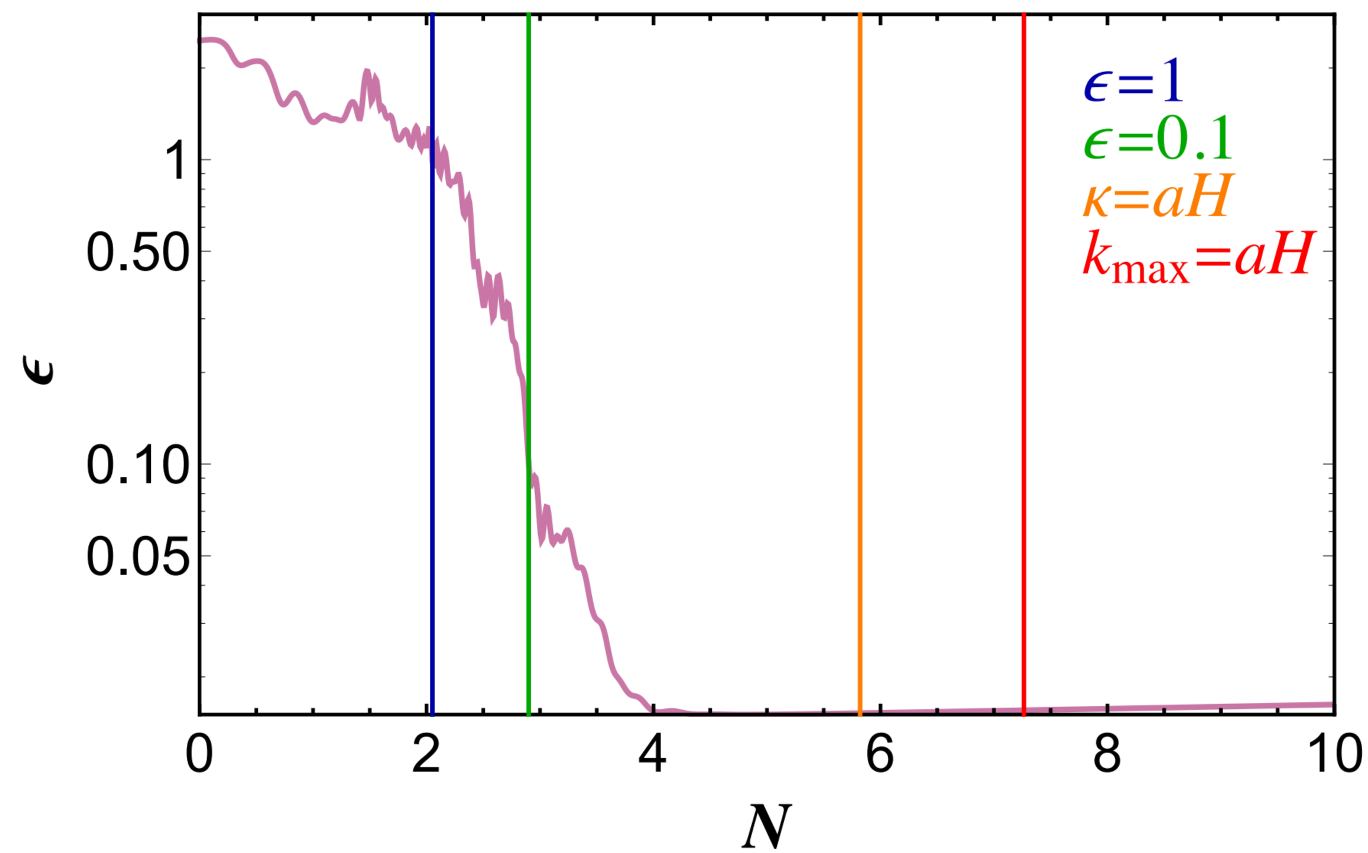
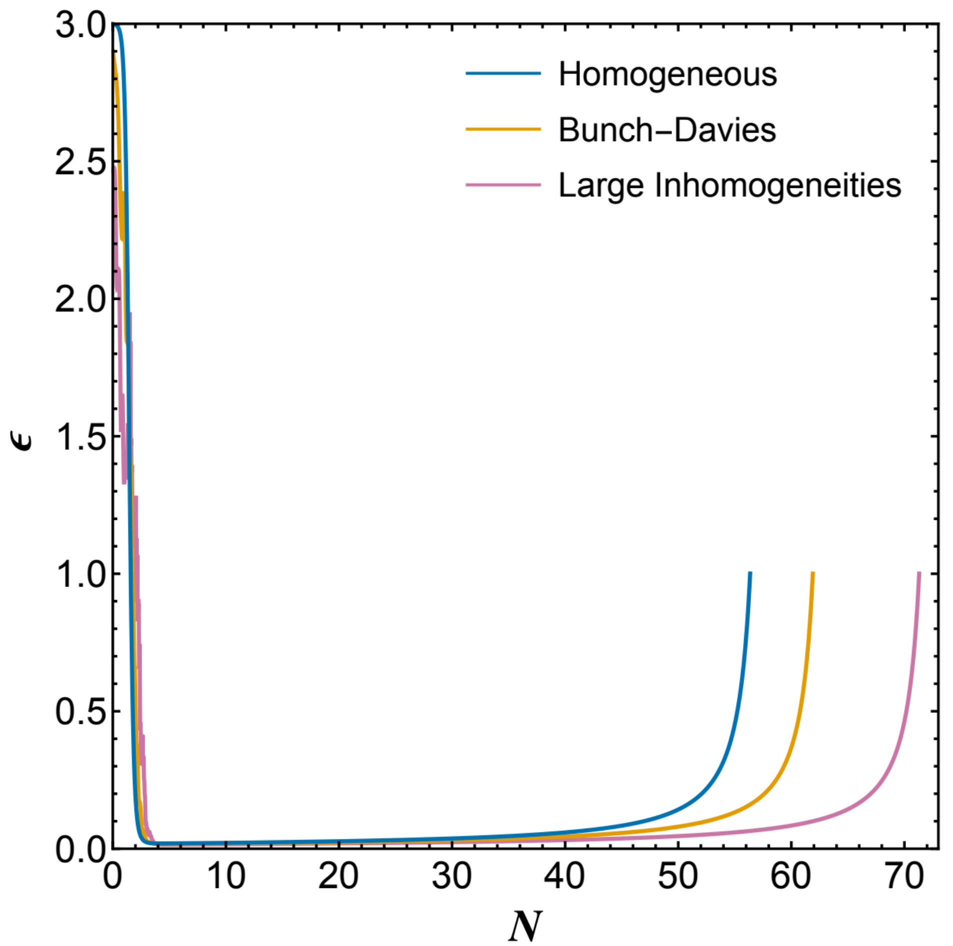


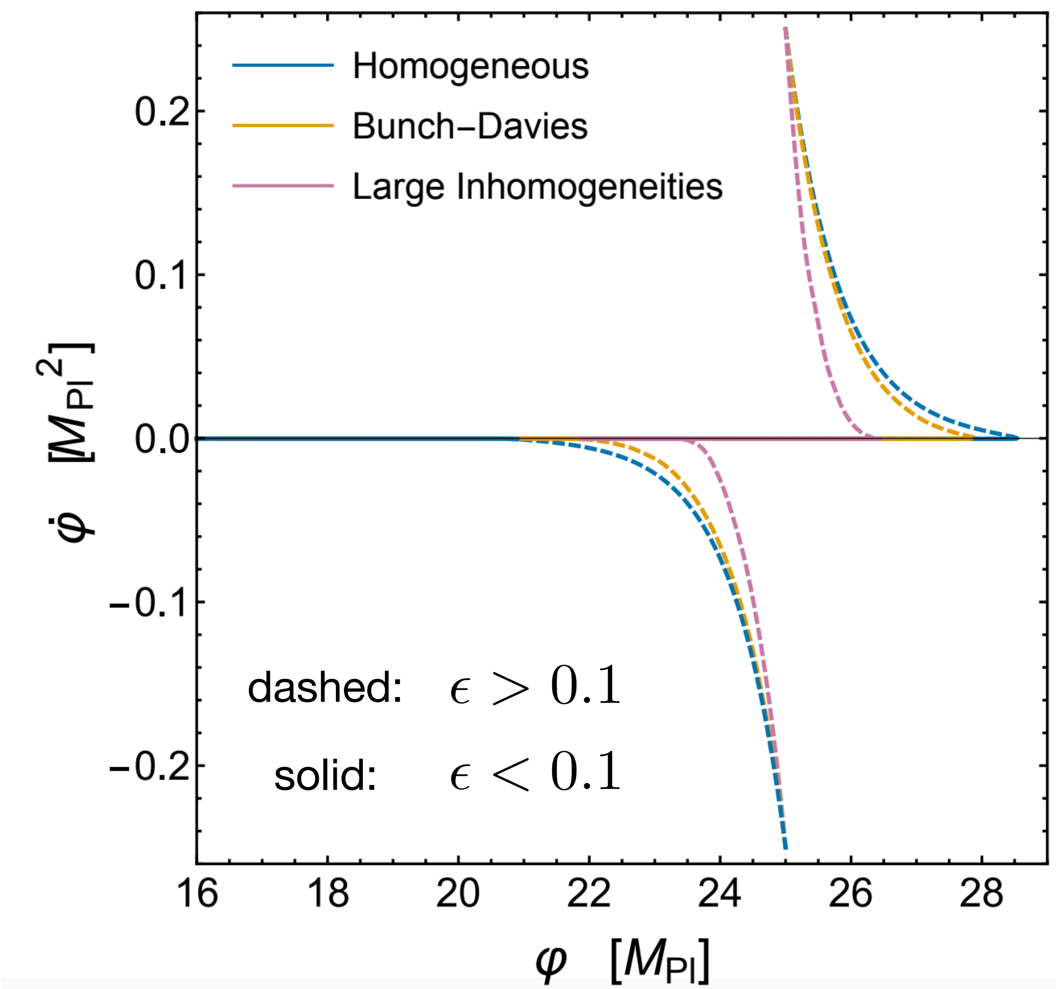


- Initially inhomogeneous system flows along inflationary attractor ($\epsilon < 0.1$) by $\ln a \sim 3$
- Fluctuation mode with $k \sim M_{\rm pl}$, below which there is most power in fluctuations, crosses outside Hubble radius by $\ln a \sim 6$
- System enters a slowroll phase of inflation in the face of substantial inhomogeneity on sub-Hubble scales

Numerical Results $V(\phi) = \lambda \phi^4 \quad \lambda = 10^{-10} \quad K = 0$

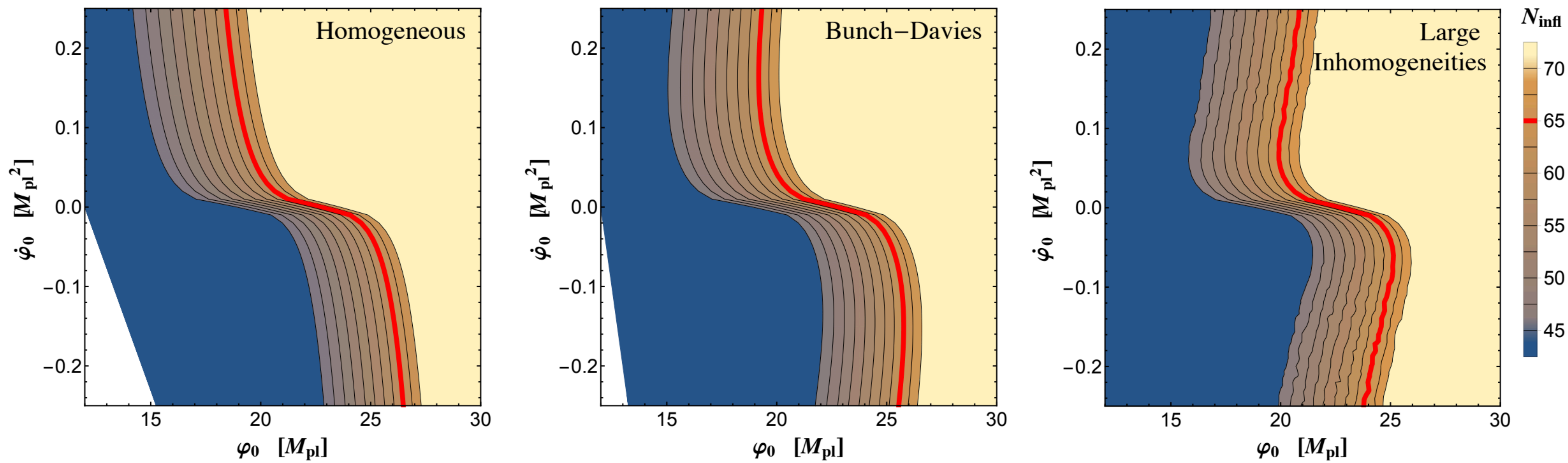
- Nontrivial effects of inhomogeneities on inflation



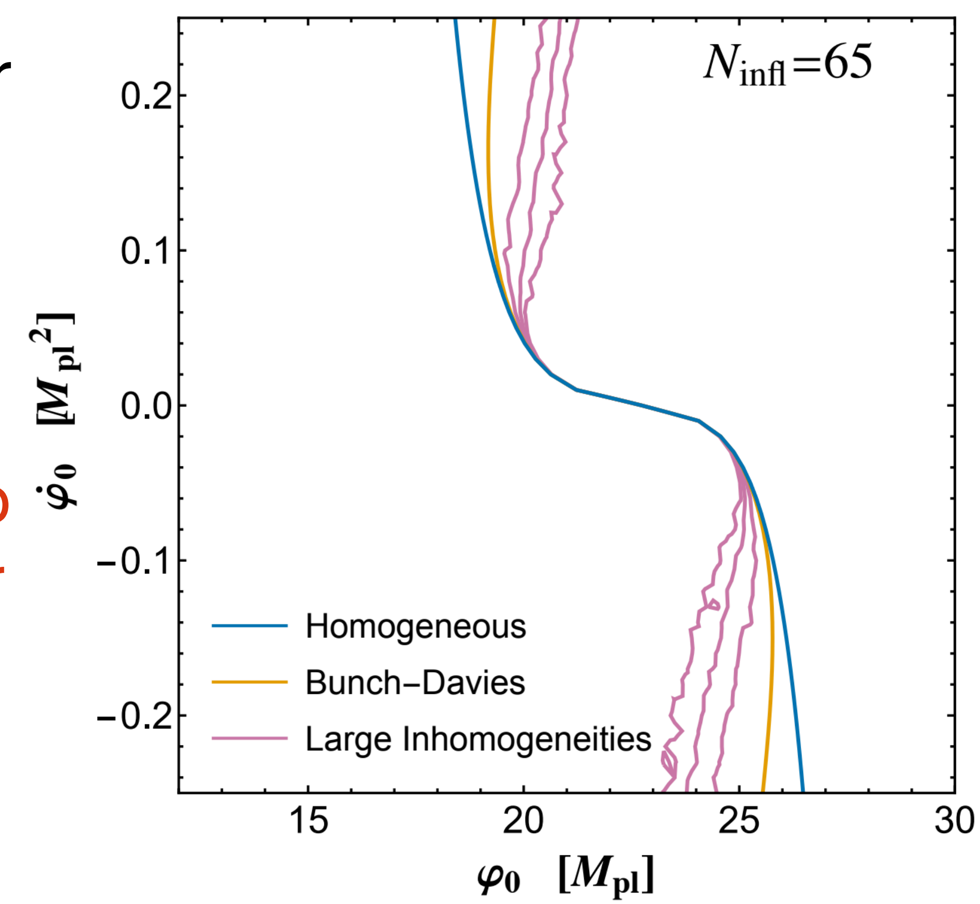


- Initial inhomogeneities extended the duration of inflation
- Large inhomogeneities and nonlinear backreaction increase effect of Hubble drag on $\varphi(t)$ —> φ traverses shorter distance before arriving at slow roll attractor
- For initial $\dot{\varphi}_0 > 0$ ($\dot{\varphi}_0 < 0$) the inhomogeneous system enters the inflationary attractor at smaller (larger) φ and spends less (more) time evolving along the inflationary attractor

Phase Space of Initial Conditions



- Inhomogeneous system reaches slow roll attractor while significant power remains in fluctuations on sub-Hubble scales
- Inhomogeneities backreact and increase Hubble drag —> more quickly damp out field velocity —> regions of phase space which begin ``rolling up the hill” (`down the hill”) of their potential no longer have large advantage (disadvantage) flowing into inflation that they do in the homogeneous case
- Volume of the projected phase space which yields sufficient inflation is conserved



Conclusions

- Amidst large inhomogeneities and including nonlinear effects in the Hartree approximation, system still finds inflationary attractor, even while significant power remains in inhomogeneities on sub-Hubble scales
- Inhomogeneities and nonlinearities have nontrivial effects on the flow into inflation: system with large $-\left|\dot{\phi}_0\right|$ pushed onto slow roll attractor along which it evolves for longer — opposite effect for large $+\left|\dot{\phi}_0\right|$
- In general, large-field inflation in a simple potential like $V(\phi) = \lambda\phi^4$ appears robust in the face of significant inhomogeneities
- Unique quantitative measure of robustness: volume of phase space which yields sufficient inflation remains unchanged under inhomogeneities and nonlinearity
- Future: nontrivial effects of inhomogeneities on inflation are general and our computationally simpler and more efficient approach allows us to explore the initial conditions problem for more models over a wide range of scales