Constraining Temporal Oscillations of Cosmological Parameters Using SNe Ia

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Presentation Outline

• Background:
  • Type Ia Supernovae (SNe Ia) and dark energy.
  • The Pantheon set of SNe Ia.

• Searching for oscillatory deviations from $\Lambda$CDM in Fourier Space – why and how?

• Results A: Is the Fourier structure of the Pantheon SNe Ia consistent with $\Lambda$CDM? – Yes.

• Results B: Constraining Alternate Cosmological Models:
  • General methodology.
  • Example.

• Conclusion and future analyses.
Background-type Ia supernovae and DE

- Type Ia supernovae (SNe Ia):
  - Normalizable, luminous standard candles.
  - Distance modulus, $\mu$, and redshift, $z$, describe expansion between occurrence and detection.

\[ \mu(z) = 25 + 5 \log_{10} \left( \frac{d_L}{1\text{Mpc}} \right) = 25 + 5 \log_{10} \left( \frac{c(1 + z)}{1\text{Mpc}} \int_0^z dz' \frac{1}{H(z')} \right) \]
Background-supernovae and DE

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• Dark energy:
  • Measurements of SNe Ia determined that the expansion of the universe is accelerating.
  • The source – dark energy (DE).

Perlmutter 1999.
Background-supernovae and DE

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• The Canonical Cosmological Model: $\Lambda$CDM
  • Cosmologically constant DE, $\Lambda$.
  • Cold dark matter, CDM.

Wollack 2014.
Background-the Pantheon set of SNe Ia

• Consists of 1048 supernovae
• How can this data best be utilized?

The data of Scolnic 2018.
Background-the Pantheon set of SNe Ia

• Consists of 1048 supernovae.
• How can this data best be utilized?
• Some DE theories predict short-lived or oscillatory deviations in expansion.
• With the right statistical technique, could we better constrain these models with the Pantheon data?

The data of Scolnic 2018.
Fourier Transforming SNe Ia Data
Fourier Transforming SNe Ia Data - Why?

• Natural method for detecting oscillatory behaviors in cosmological parameters.
• Well-defined basis for decomposing any model.
Fourier Transforming SNe Ia Data - How?

- Analyze distance modulus residuals.
- Cosmic oscillations in time would be observed as oscillations in conformal time.
- The frequency range is bounded from below by the conformal time period.
- From above by the light crossing time of a galaxy cluster.
- A periodogram contains the Fourier structure of a data set that is discretely sampled in time.
Fourier Transforming SNe Ia Data - How?

• Analyze distance modulus residuals, $\Delta \mu$.

$\Delta \mu_l = \mu_l - \mu_{\Lambda CDM}$
Fourier Transforming SNe Ia Data - How?

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$$\Delta \mu_i = \mu_i - \mu_{\Lambda CDM}$$

$$\tau h_{100} = \int_0^z dz' \frac{1}{H(z')/H_0 \text{100kms}^{-1}\text{Mpc}^{-1}} \frac{1}{100\text{kms}^{-1}\text{Mpc}^{-1}}$$

$$h_{100} = \frac{H_0}{100\text{kms}^{-1}\text{Mpc}^{-1}}$$
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$$f_{\text{min}} = \frac{h_{100}}{h_{100}(\tau_{\text{max}} - \tau_{\text{min}})} \approx 0.08 \text{Gyr}^{-1} h_{100}$$

$$f_{\text{max}} = 1000 f_{\text{min}}$$
Fourier Transforming SNe Ia Data - How?

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$$Q_n = \frac{1}{B} \left| \sum_{j=1}^{N_{\text{SN}}} \Delta \mu_j e^{-2\pi i f_n \tau_j} \right|^2$$
Fourier Transforming SNe Ia Data - How?

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Fourier Transforming SNe Ia Data - Statistics

In a $\Lambda$CDM cosmology, what is the likelihood that at least one periodogram component would be more excursive than the most excursive component of the observed periodogram?

**Method:**

(a) Simulate 10 $g$ sets of SNe observations with Pantheon conformal times in a $\Lambda$CDM universe.

(b) Compute periodogram for this bootstrapped data.

(c) Determine probability distributions for each periodogram component, $Q$.

(d) Account for the "look elsewhere" effect by normalizing by the number of periodogram peaks.
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**Question:** In a $\Lambda$CDM cosmology, what is the likelihood that at least one periodogram component would be more excursive than the most excursive component of the observed periodogram?

**Method:**

(a) Simulate 1000 sets of SNe observations in a $\Lambda$CDM universe with Pantheon conformal times and distance modulus residual scatter.

(b) Compute periodogram for this bootstrapped data.

(c) Determine probability distributions for each periodogram component, $Q_n$.

(d) Account for the “look-elsewhere” effect by normalizing by the number of periodogram peaks.
Results A- Consistency of Pantheon Data with \( \Lambda \)CDM

**Results:**

A roughly 28% chance that at least one component of a \( \Lambda \)CDM periodogram would be more excursive than this most excursive observed peak.

**Conclusion:**

The Fourier spectrum of the Pantheon data set is consistent with \( \Lambda \)CDM.
Results B- Constraining a Cosmological Model
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• Choose alternate cosmological model:
  • Ex: Oscillating DE energy density.

\[ \rho_{DE} = \rho_{DE,can} (1 + A_{\rho} \sin(2\pi f_{\rho} \tau + \phi_{\rho})) \]
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- Compute the distance modulus residuals.

- Select rejection probability ratio, \( R_{uv} \).

- Compute the periodogram.
  - Ex: \( A_\rho = 0.15 \)
    \( \varphi_\rho = 0 \)
    \( f_\rho = 10 f_{\text{min}} \)

- Reject periodograms with components that are \( R_{uv} \) times more extreme than the most extreme Pantheon component.
Results B- Constraining a Cosmological Model

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\]

• Compute the distance modulus residuals.
• Select rejection probability ratio, \( R_{rej} \).

Example:

\( R_{rej} = 0.01 \)

“We reject those cosmologies with at least one periodogram component that is more than 100 times more excursive than the most excursive periodogram component of the Pantheon data set.”
Results B-Constraining a Cosmological Model

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  \[ \rho_{DE} = \rho_{DE,can}(1 + A_\rho \sin(2\pi f_\rho \tau + \phi_\rho)) \]
• Compute the distance modulus residuals.
• Select rejection probability ratio, \( R_{rej} \).
• Compute the periodogram.
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- Reject periodograms with components that are \( R_{rej} \) times more extreme than the most extreme Pantheon component.
Conclusion and Future Work

• The Fourier structure of the Pantheon data set is consistent with $\Lambda$CDM.
• Fourier analysis provides stronger constraints on certain alternate cosmological models than standard $\chi^2$ analysis.
• Strongly rule out any model with a Fourier amplitude of $>35$ mmags.
• Results detailed in Brownsberger 2019.
Conclusion and Future Work

• The Fourier structure of the Pantheon data set is consistent with $\Lambda$CDM.
• Fourier analysis provides stronger constraints on certain alternate cosmological models than standard $\chi^2$ analysis.
• Strongly rule out any model with a Fourier amplitude of $> 35$mmags.
• Results detailed in Brownsberger 2019.
• With one year of LSST data (Carroll 2014), these constraints will fall to $\sim 2$mmags.
• Future analyses could also study 3-dimensional structure of SNe Ia signals.
Full Constraints Placed on Alternate Cosmological Model Periodograms
A universe with variable DE energy density

- Dark energy density that scales in time by a scaling $X(\tau)$:
  \[ \rho_{DE} \rightarrow X(\tau) \rho_{DE} \]
- Evolve differential equations forward in $z$

\[ \frac{d\tau'}{dz} = \frac{1}{H'} \quad \tau'(z = 0) = 0 , \]
\[ \frac{d d_L'}{dz} = \frac{d_L'}{1 + z} + \frac{1 + z}{H'} \quad d_L'(z = 0) = 0 , \]

\[ H' \equiv \frac{H}{H_0} = \sqrt{(1 + z)^3 \Omega_{m,0} G' + (1 + z)^4 \Omega_{r,0} G' + X(\tau, z) \Omega_{\Lambda}} , \]
\[ \tau' \equiv \tau H_0 = \tau h_{100} 100 \text{ km s}^{-1} \text{ Mpc}^{-1} , \]
\[ d_L' \equiv d_L H_0 / c . \]
Fitting the Periodogram Components

- If the distance modulus residuals, \( \Delta \mu \), were evenly spaced in \( \tau \), drawn from a function with no \( \tau \) dependence, and had Gaussian uncertainties, then

\[
\left. r_n(Q) \right|_{\mu(\tau)=0}, \text{Gaussian uncertainties, & evenly spaced } \tau = 1 - e^{-Q/c_Q},
\]

- We generalize this with the lower incomplete gamma function:

\[
r_n(Q) = \frac{\gamma\left(1/b_n, (Q/a_n)^{b_n}\right)}{\gamma\left(1/b_n, \infty\right)}
\]

- We determine the limiting periodogram values by numerically solving

\[
r_n(Q_{\text{max},n}) = (1 - P_{\text{rej}})^{1/N_{\text{peak}}}
\]
More on Our Model Constraining Analysis

- Ideally, we would use bootstrap analysis to recompute the distribution of periodogram values for each considered non-standard cosmology
  - Not computationally feasible for a range of parameter values
- We look at the deviation of the alternate cosmology from $\Lambda$CDM relative to the deviation of the Pantheon data from $\Lambda$CDM.

\[ R_{r\chi^2_\nu} = \frac{\int_{r\chi^2_{\nu,\Lambda CDM}}^\infty d(\chi^2)p_\nu(\chi^2)}{\int_{r\chi^2_{\nu,\Lambda CDM}}^\infty d(\chi^2)p_\nu(\chi^2)} \]

\[ R_{\text{Fourier}} = \frac{1 - (\max(r_n(Qn,\Lambda CDM)))^{N_{\text{peak}}}}{1 - (\max(r_m(Qm,\text{Pantheon})))^{N_{\text{peak}}}} \]

- The idea: a large deviation from $\Lambda$CDM would have been detected in our analysis. No such deviation was detected, and so theories that predict such deviation can be ruled out.