

Magnetic Field Measurement and Analysis for the Muon $g-2$ Experiment

Ran Hong

(Muon $g-2$ Collaboration)

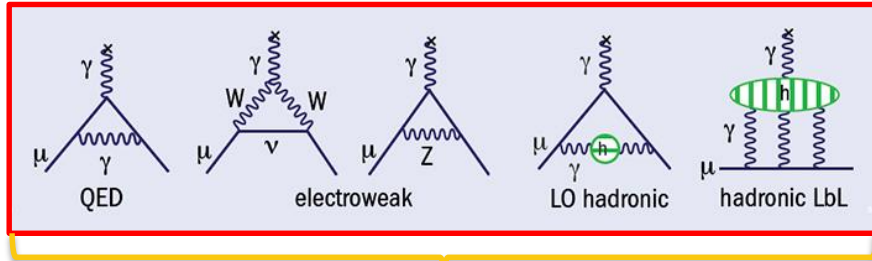
Argonne National Lab



Outline

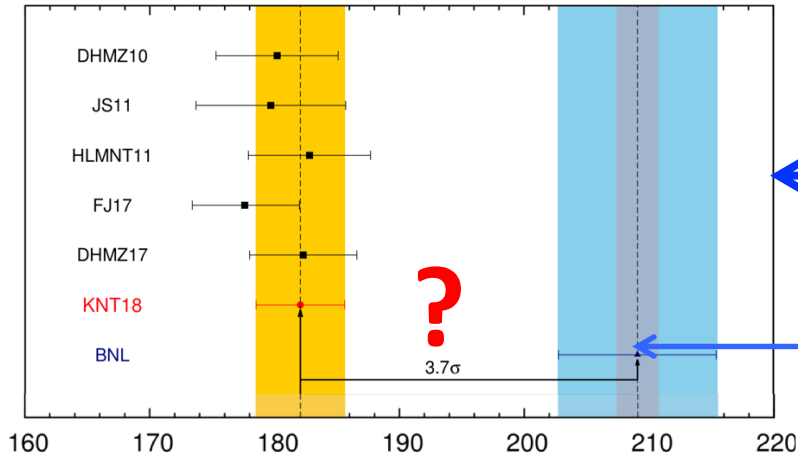
- **Motivation and Experiment Overview**
- **Magnetic Field Measurement System**
 - Storage ring magnet construction
 - Field Measurement devices
 - Analysis Methods
- **Run-1 Summary and Run-2 Upgrades**

Motivation: Muon g-2 and BSM Physics



Three generations of matter (fermions)

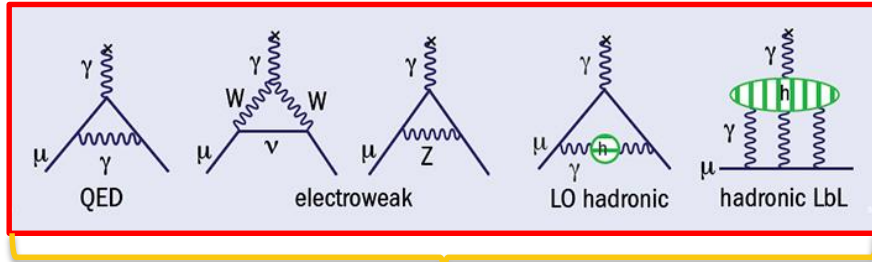
	I	II	III		
mass	2.4 MeV/c ²	1.27 GeV/c ²	171.2 GeV/c ²	0	? GeV/c ²
charge	2/3	2/3	2/3	0	0
spin	1/2	1/2	1/2	1	0
name	u up	c charm	t top	γ photon	H Higgs boson
	4.8 MeV/c ²	104 MeV/c ²	4.2 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
Quarks	d down	s strange	b bottom	g gluon	
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	91.2 GeV/c ²	
	0	0	0	0	
	1/2	1/2	1/2	1	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z⁰ Z boson	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	80.4 GeV/c ²	
	-1	-1	-1	±1	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	W[±] W boson	Gauge bosons



Muon anomalous magnetic moment

We (E989) will do better and reduce the uncertainty by a factor of 4

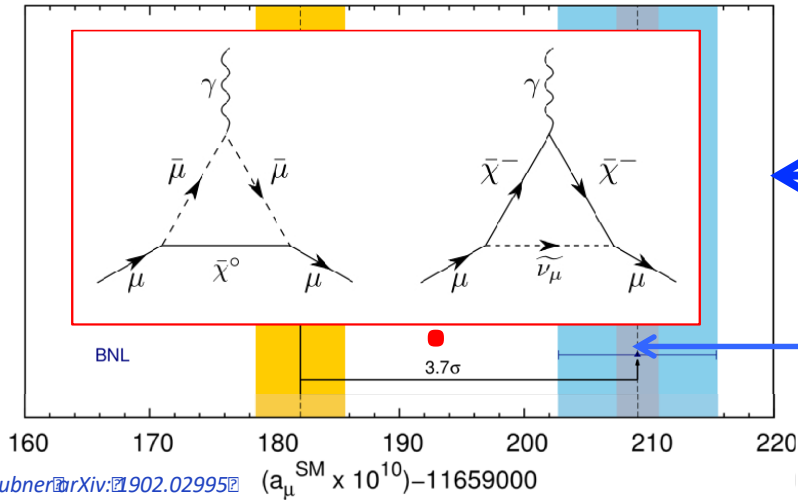
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Gauge bosons



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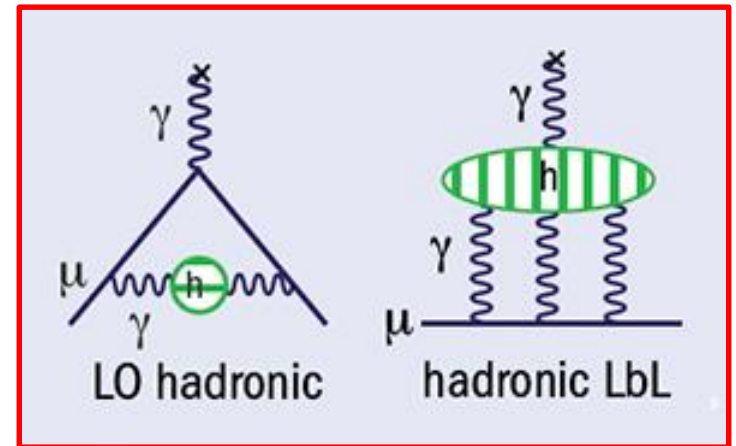
Improve the error bar

Reducing the experimental error bar

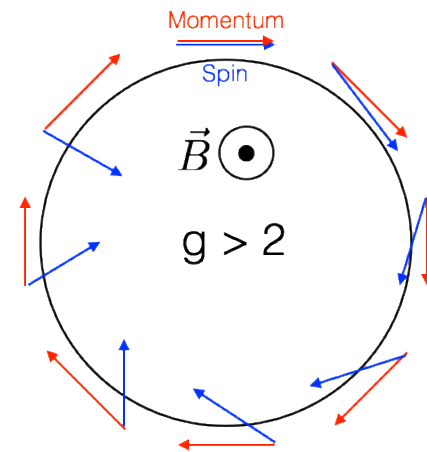
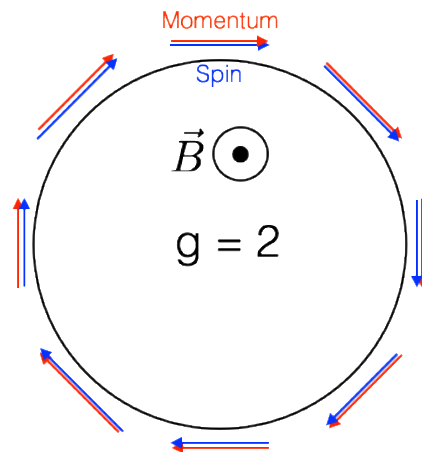
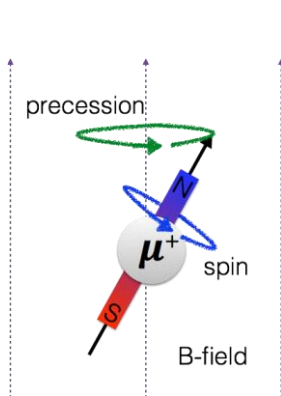
- More statistics: 21 times more data
- Better control on systematic uncertainties

Reducing the theoretical error bar

- Reduce the hadronic vacuum polarization (HVP) calculations by analyzing more $e+e^-$ data
- Improve Hadronic light-by-light calculation by lattice QCD
- Cross-check HVP calculations using lattice QCD



How to measure the muon g-2 value?



Muon precession in a magnetic field

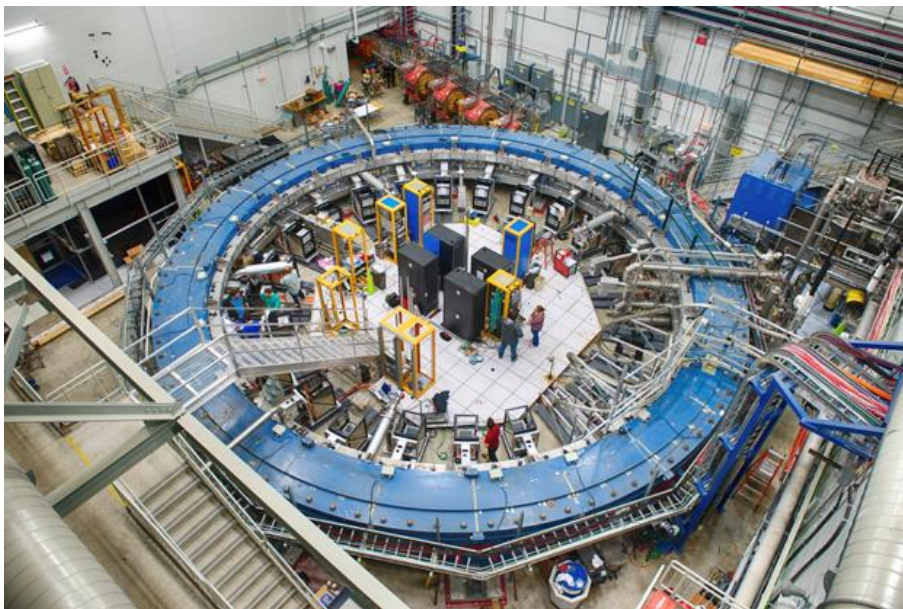
$$\vec{\mu} = g \frac{q}{2m} \vec{s}$$

$$a_m = \frac{g_m - 2}{2}$$

$$a_m (\text{Exp}) = - \frac{m \omega_a}{eB} \quad \text{We measure them}$$

$$\omega_a = \omega_s - \omega_c$$

Experiment overview



Challenge:

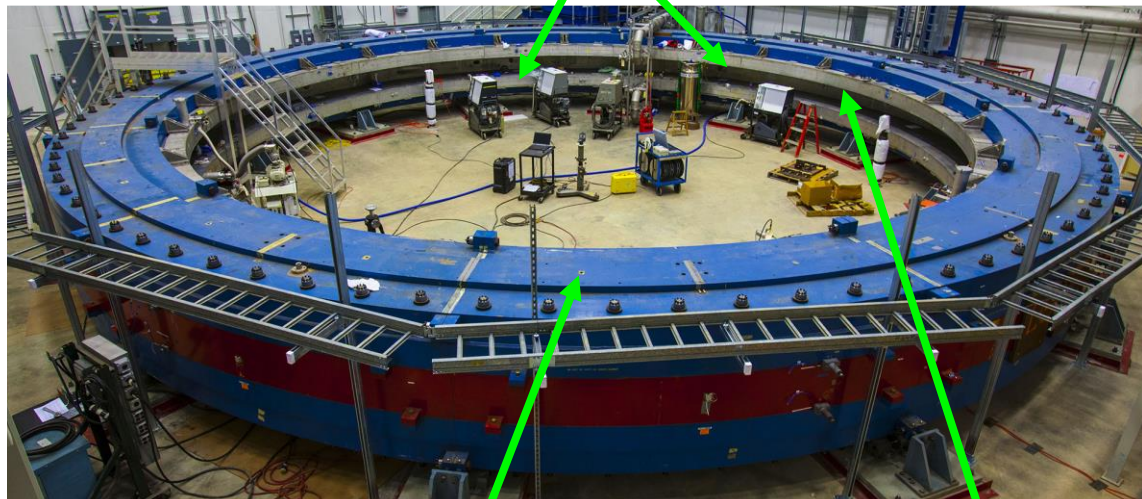
Measuring ω_a and B with systematic uncertainties down to **70 ppb** each

$$a_m(\text{Exp}) = -\frac{mW_a}{eB}$$

	E821	E989
ω_a	180 ppb	70 ppb
B	170 ppb	70 ppb

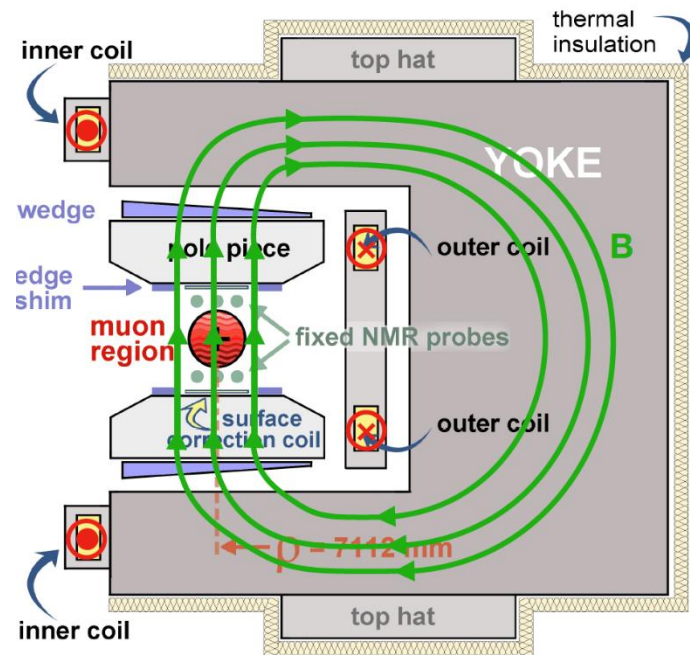
Magnet construction

Super conducting coils



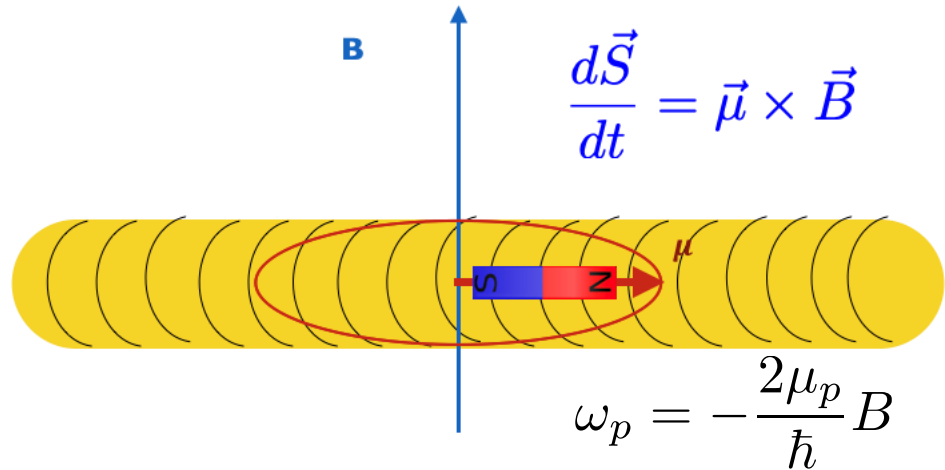
Iron yoke

Uniform field in the gap



Magnetic field measurement device: NMR Probe

- Measure magnetic field using proton Nuclear Magnetic Resonance (NMR)
- Measure NMR frequency ω_p
- Introducing new parameters \hbar , μ_p
- Rewrite a_μ



$$a_m (\text{Exp}) = -\frac{m\omega_a}{eB} \longrightarrow a_\mu (\text{Exp}) = \frac{g_e \omega_a m_\mu \mu_p}{2 \tilde{\omega}_p m_e \mu_e}$$

Annotations for the fraction in $a_\mu (\text{Exp})$:

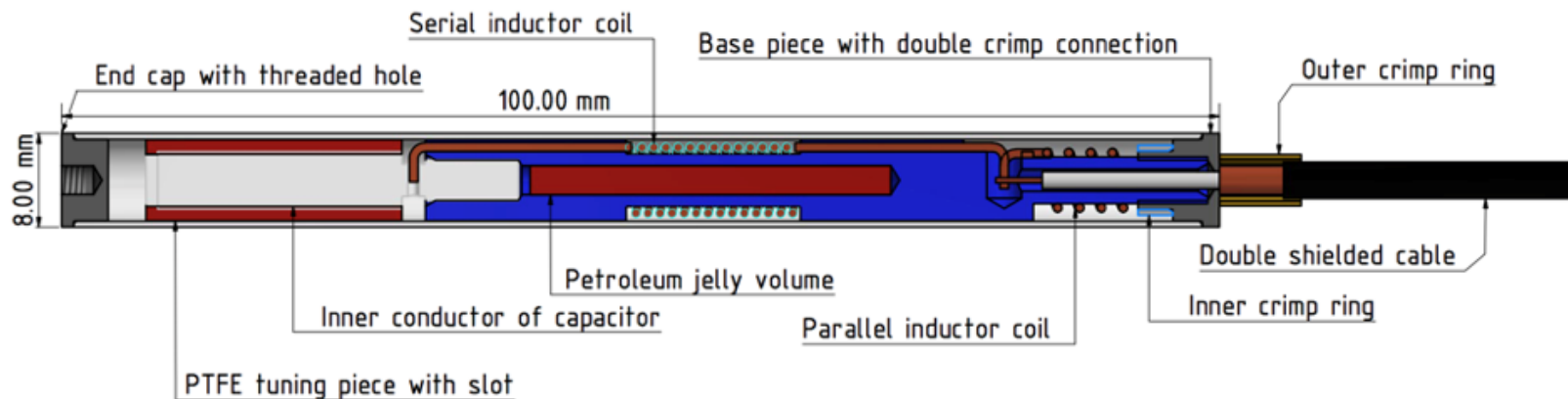
- g_e is circled in red.
- ω_a is circled in red.
- m_μ is circled in purple.
- μ_p is circled in purple.
- 2 is circled in blue.
- $\tilde{\omega}_p$ is circled in blue.
- m_e is circled in purple.
- μ_e is circled in purple.

Scale factors:

- 8 ppb (parts per billion) is indicated by a grey arrow pointing to the μ_p term.
- 22 ppb is indicated by a purple arrow pointing to the μ_e term.
- 0.26 ppt (parts per trillion) is indicated by a black arrow pointing to the $\tilde{\omega}_p$ term.

NMR Probes Construction

NMR Probe for field scan and monitor

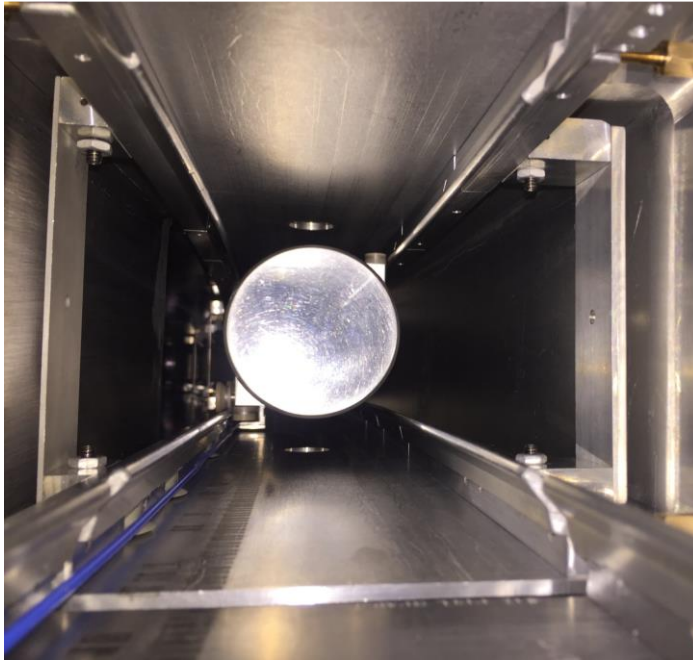


Magnetic Field Measurement Overview

- Spatial distribution: field scan
- Field drift over time: field monitor
- Achieve the proposed accuracy: calibration

In-Vacuum Field Scanner

Trolley in the beam storage region



Field Scanner (Trolley)



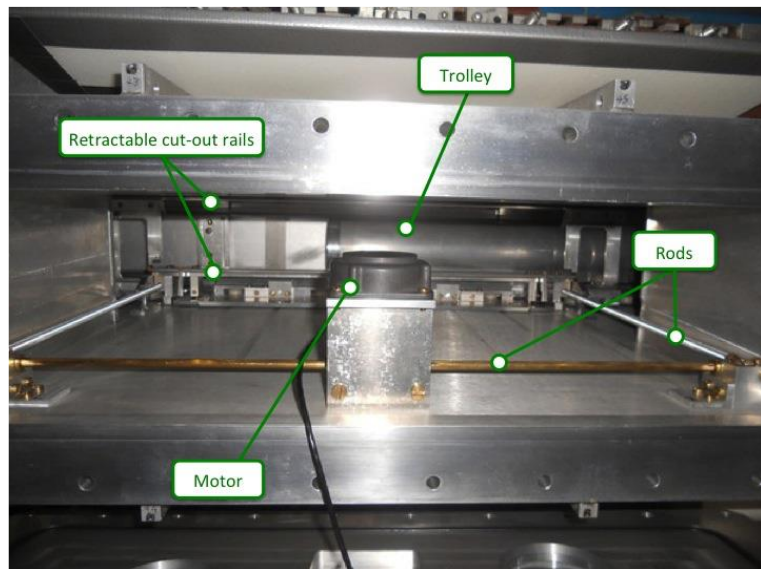
Electronics



Probe holder

In-Vacuum Field Scanner

“Garage” for trolley storage during the beam run



Field Scanner (Trolley)



Electronics



Probe holder

In-Vacuum Field Scanner

- Features
 - Low magnetic foot-print electronics
 - Fully digitized FID waveforms are stored
 - On-board waveform digitization, low noise pick-up
 - Low bit-error rate and phase noise
 - Motions are fully automated and remotely controlled
 - Position determination with high repeatability

Field Scanner (Trolley)

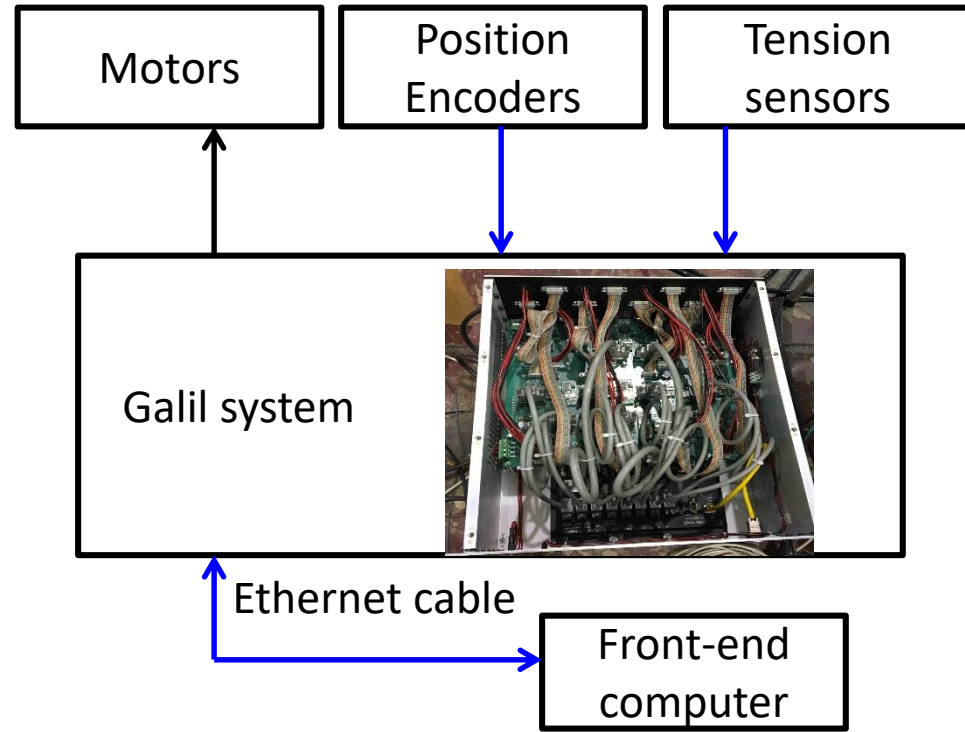
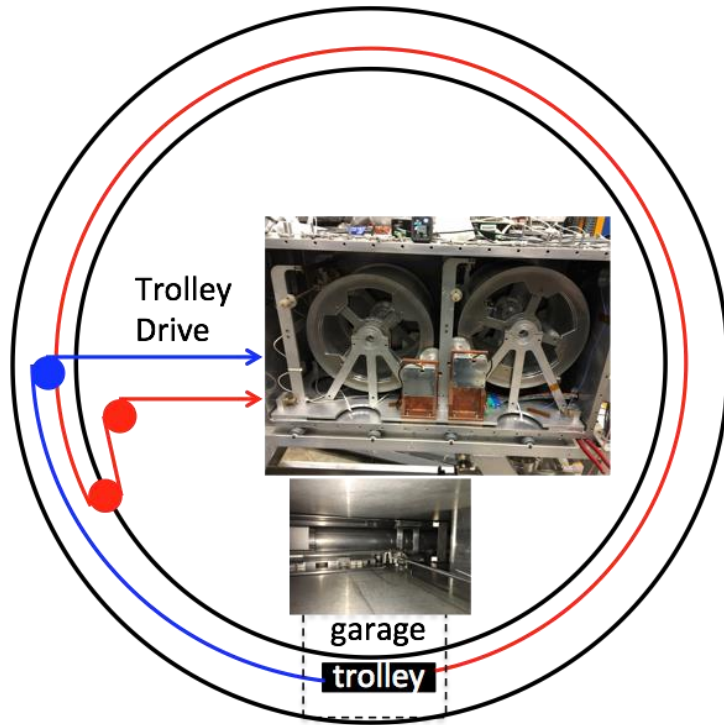


Electronics



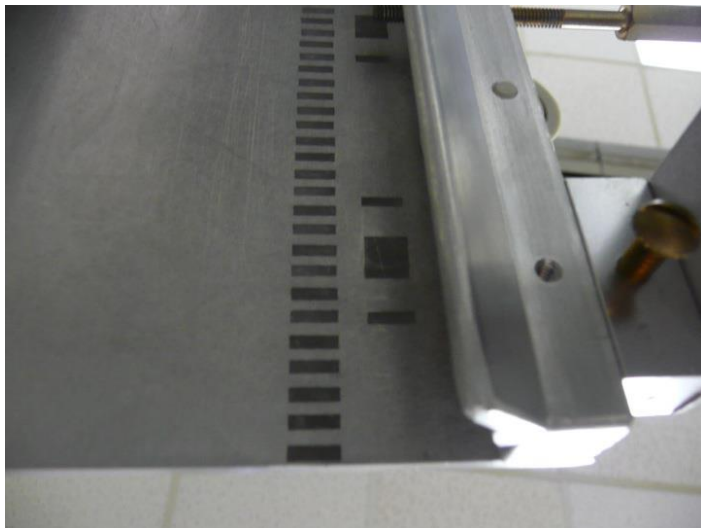
Probe holder

In-Vacuum Field Scanner: Motion Control

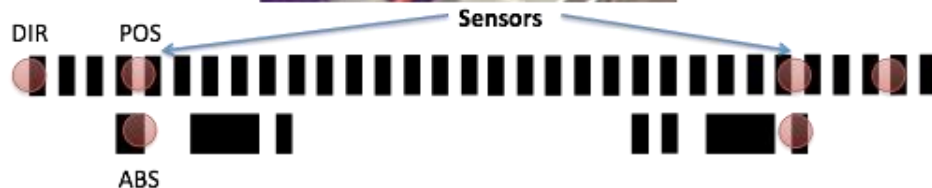


In-Vacuum Field Scanner: Position Determination

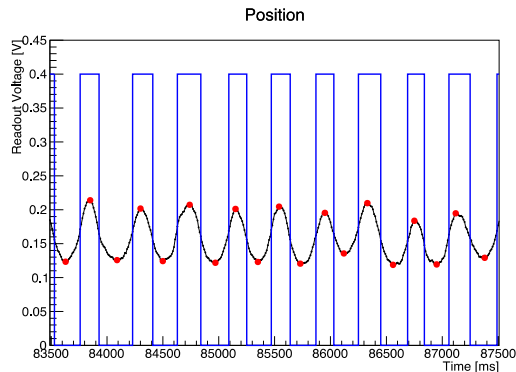
Barcode printed on the floor



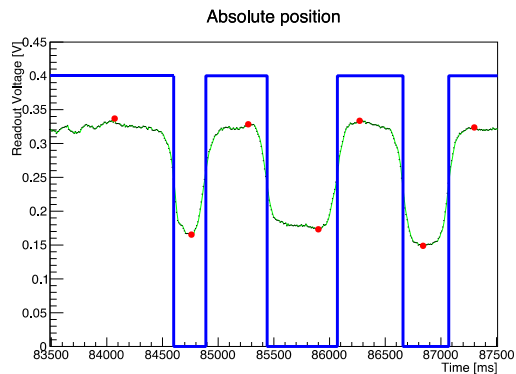
Barcode scanner



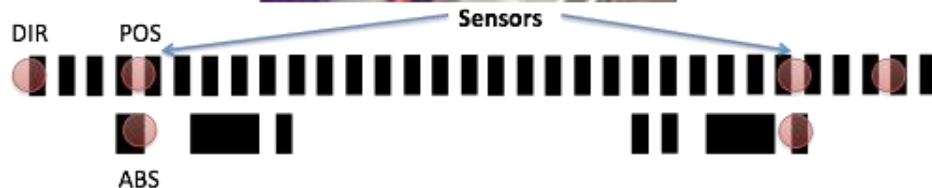
In-Vacuum Field Scanner: Position Determination



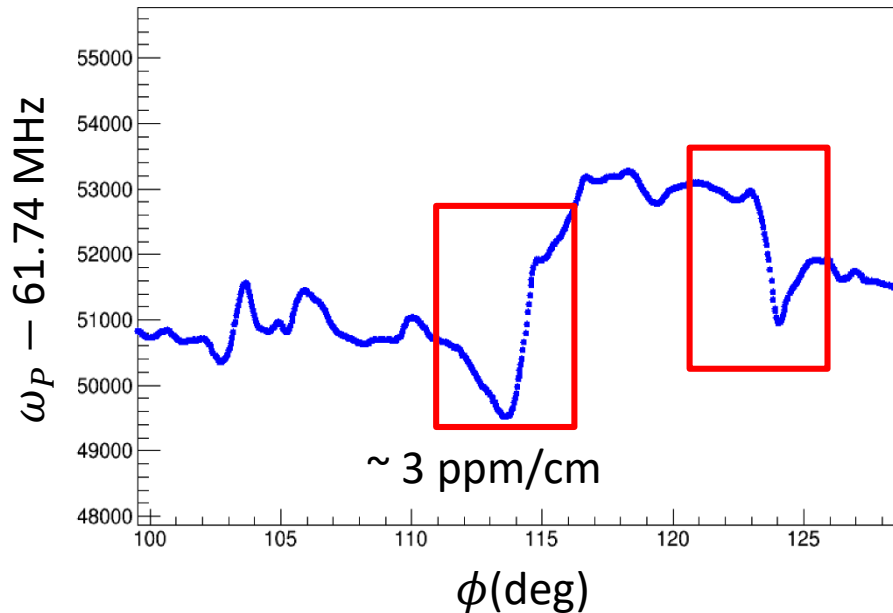
or



Barcode scanner



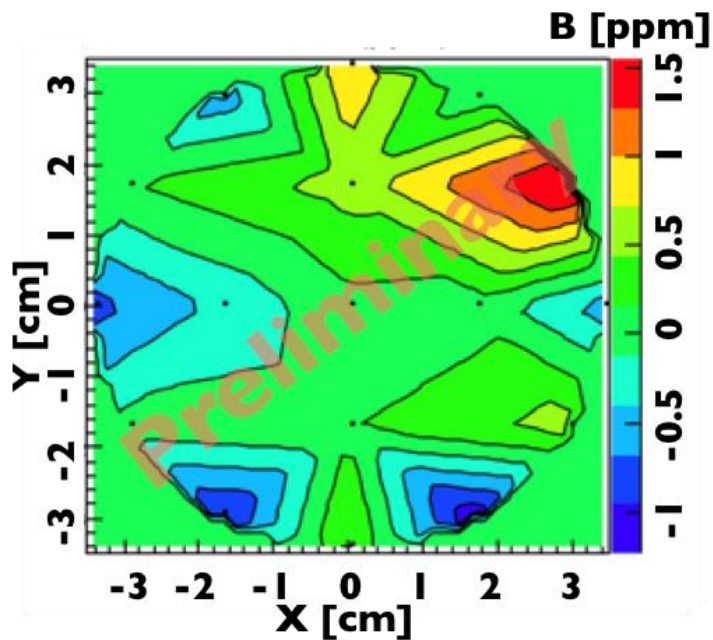
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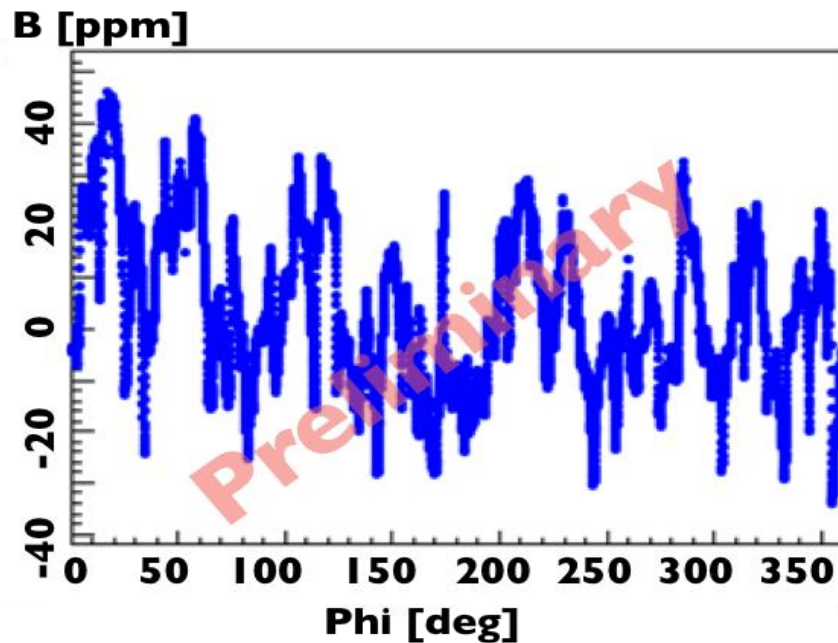
Reasons for barcode

- Because of the cable stretch, position derived from motor encoders is not accurate ($\pm 2 \text{ cm}$)
- Need high repeatability between different scans for field tracking study, particularly at high-gradient region

In-Vacuum Field Scanner: Example Field Map (5/16/2018)

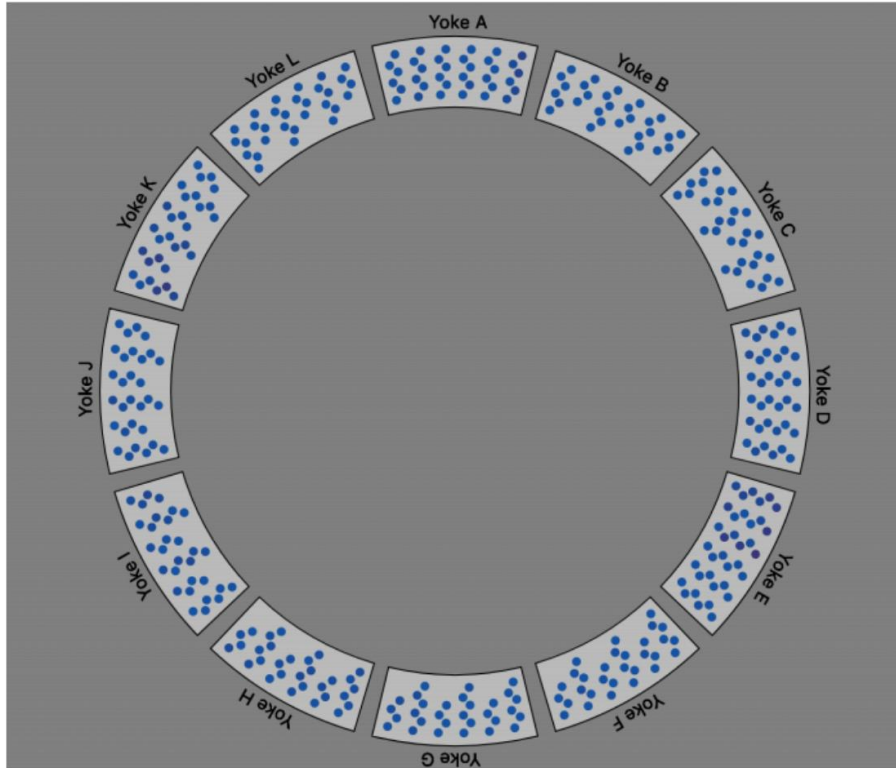


Transverse



Azimuthal

Field Monitor

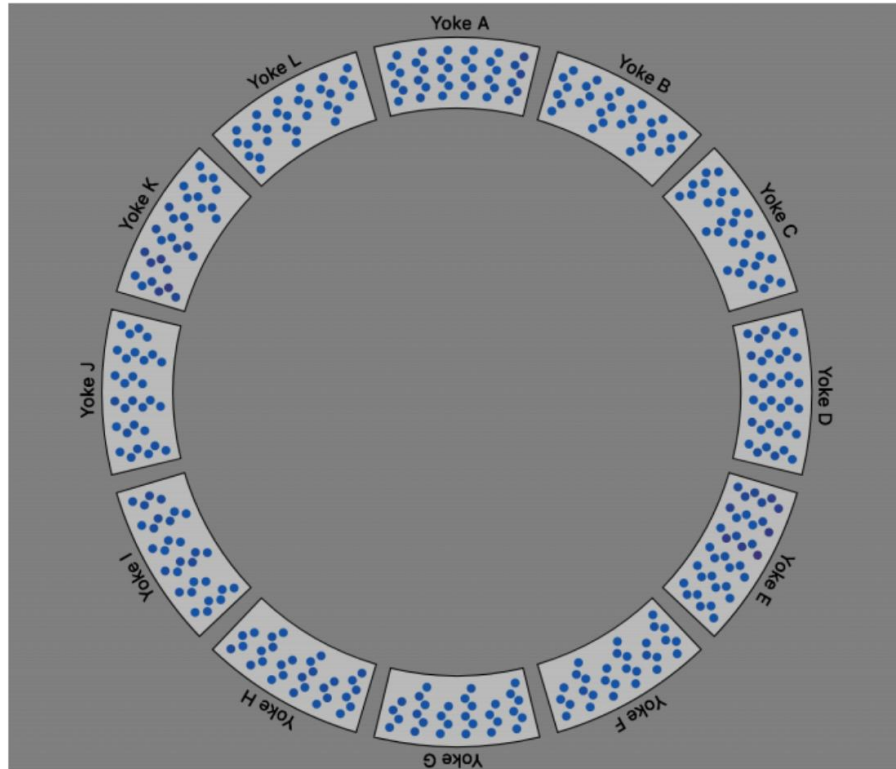


378 Fixed probes

- Outside the vacuum chamber

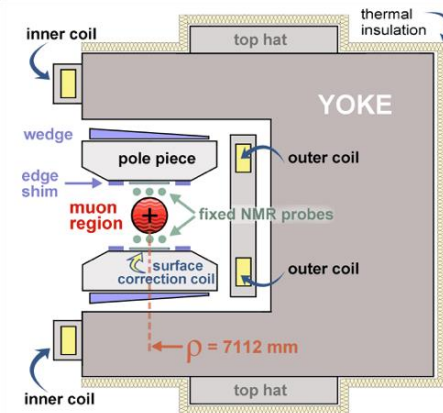


Field Monitor



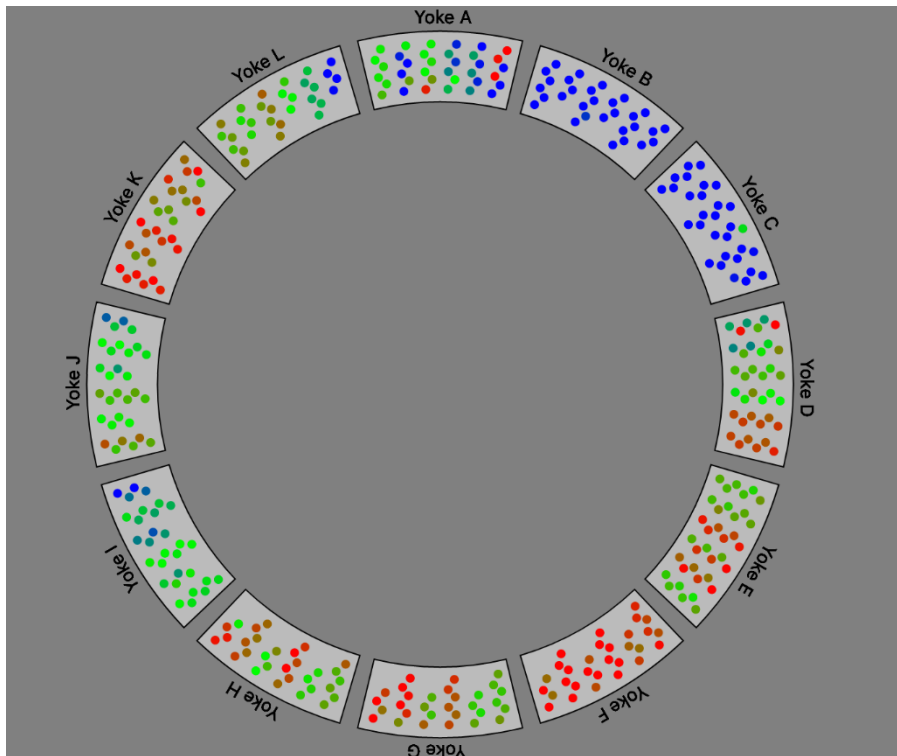
378 Fixed probes

- Outside the vacuum chamber
- Around the muon storage region
- Offline analysis: interpolate field in between the scans



g-2 Magnet in Cross Section

Field Monitor



378 Fixed probes

- Outside the vacuum chamber
- Around the muon storage region
- Offline analysis: interpolate field in between the scans

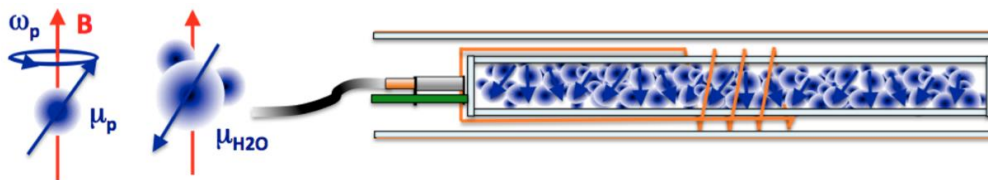
Features

- Fast online analysis with better algorithm (accelerated by GPU)
- Fast repetition rate (1.6s, ~10 times faster than E821)
- Power-supply feedback system to stabilize the field azimuthal average

Probe Calibration

Why calibration is needed?

- Probe and its materials perturb the field
- Protons are inside molecules



$$\omega_p^{\text{meas}} = \left[1 - \sigma(\text{H}_2\text{O}, T) - \left(\epsilon - \frac{4\pi}{3} \right) \chi(\text{H}_2\text{O}, T) - \delta_m \right] \omega_p^{\text{free}}$$

Protons in H₂O molecules, diamagnetism of electrons screens protons => local B changes

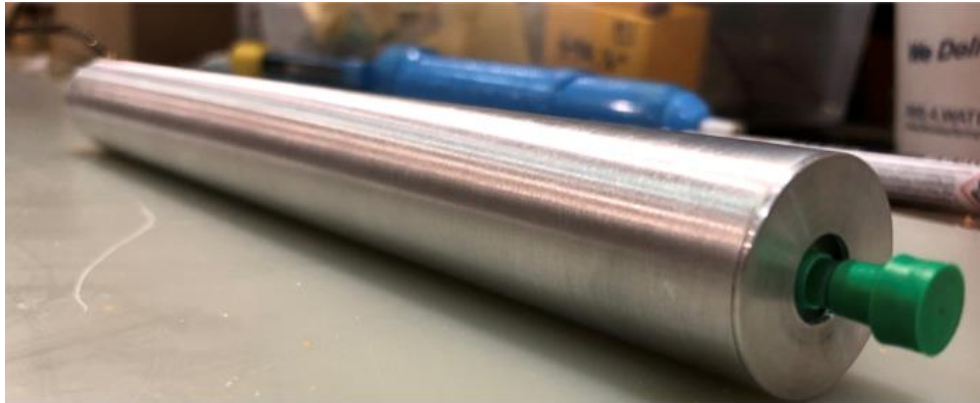
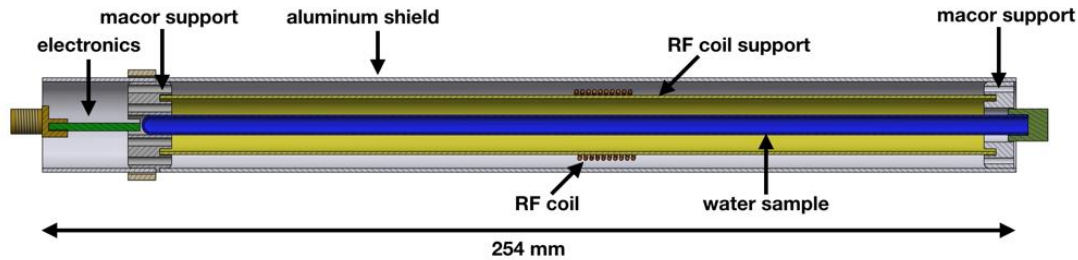
- $\sigma = 25\,680(2.5) \times 10^{-9}$ at 25 deg C [Y. Neronov and N. Seregin, Metrologia **51**, 54 (2014)]

Magnetic susceptibility of water gives shape-dependent perturbation

- $\epsilon = 4\pi/3$ (perfect sphere)
- $\epsilon = 2\pi$ (infinite cylinder) when probe is perpendicular to B
- $\chi_{\text{H}_2\text{O}} = -720(3) \times 10^{-9}$ [B. H. Blott and G. J. Daniell, Meas. Sci. Technol. **4**, 462 (1993)]

Magnetization of probe materials perturbs the field at site of protons

Probe Calibration



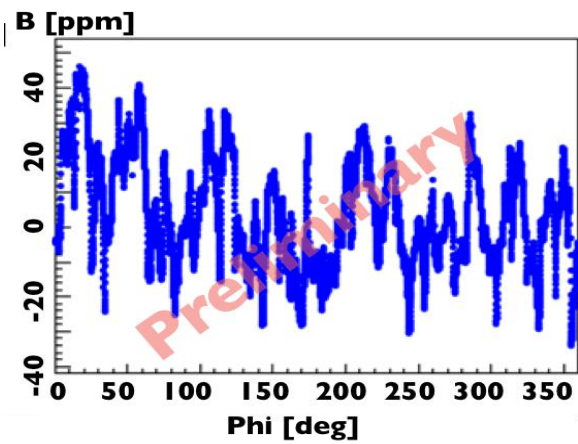
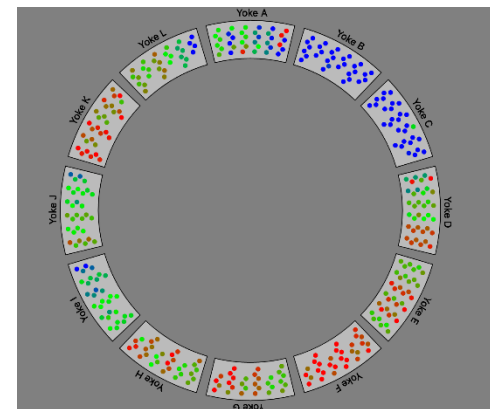
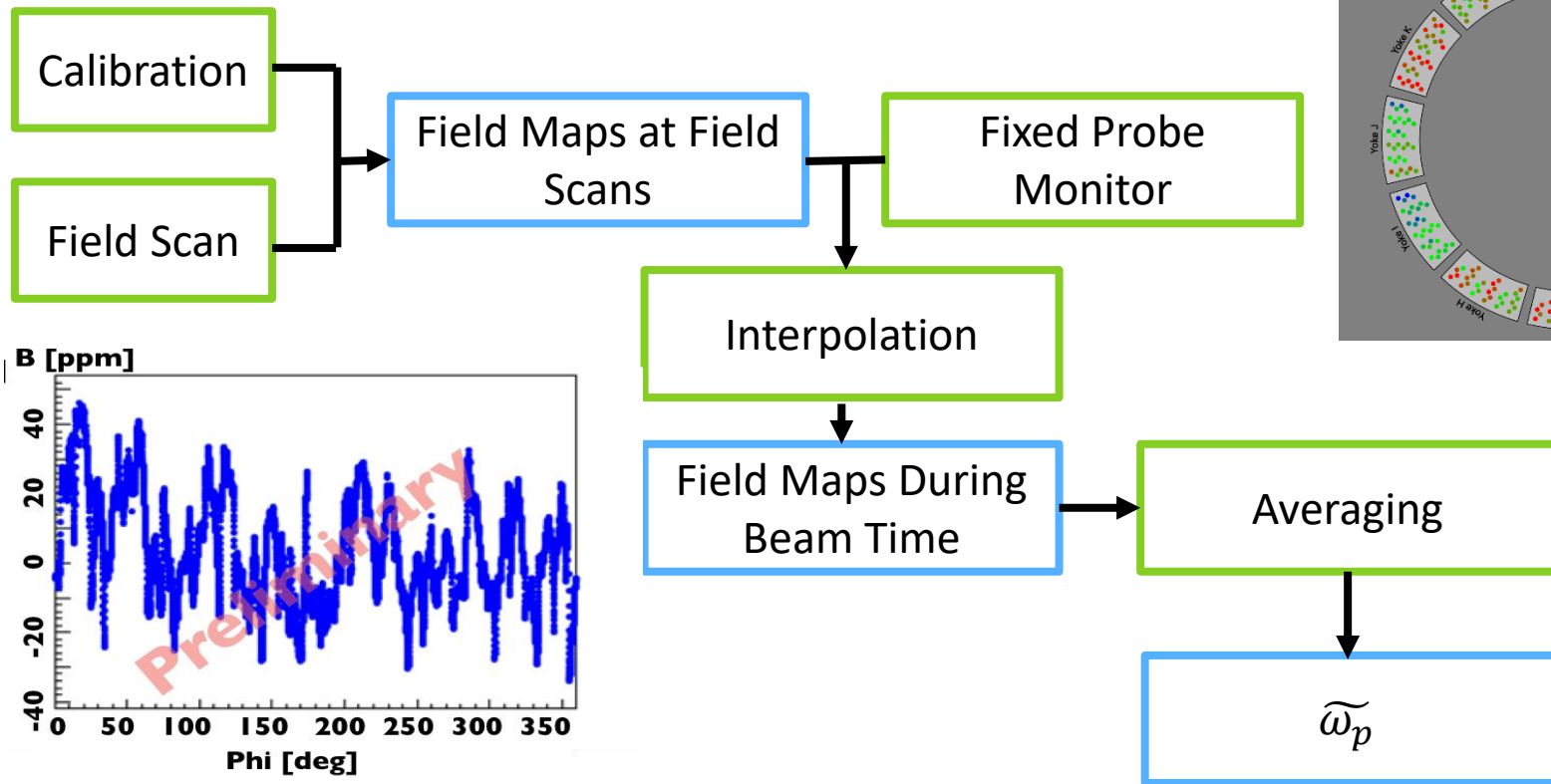
Plunging probe

- Well-defined geometry
- Low-perturbation material
- Vacuum compatible
- Motorized positioning system

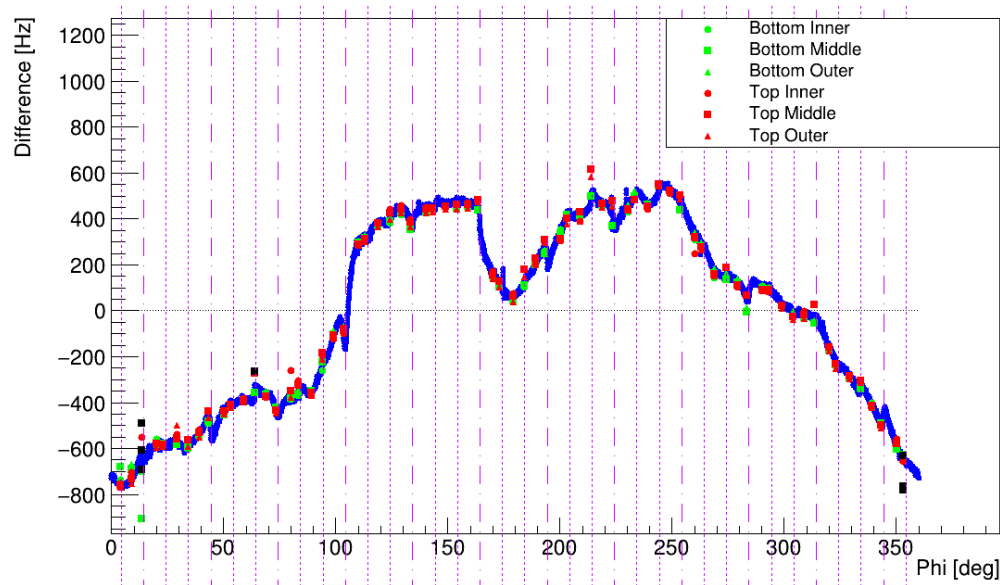
Calibration

- Use the trolley probe and the plunging probe to measure **the field at the same position**
- Shim the field at the calibration position
- Account for misalignment and field drift

Data analysis



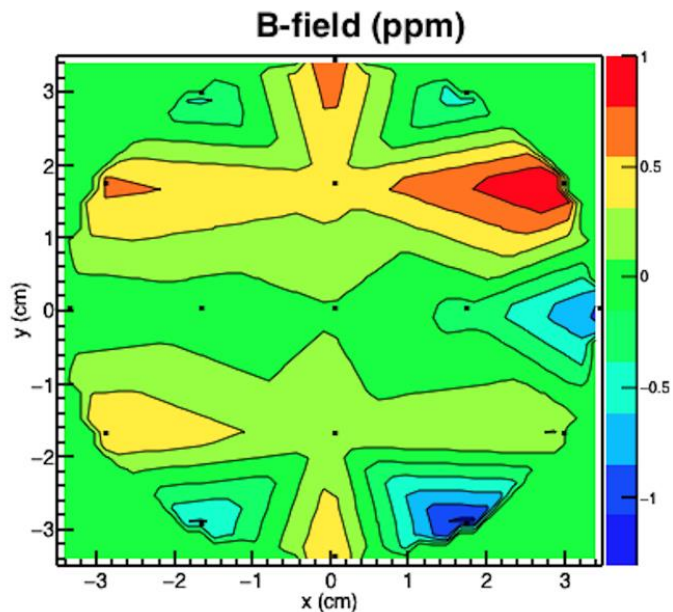
Data Analysis: Field Multipole Tracking



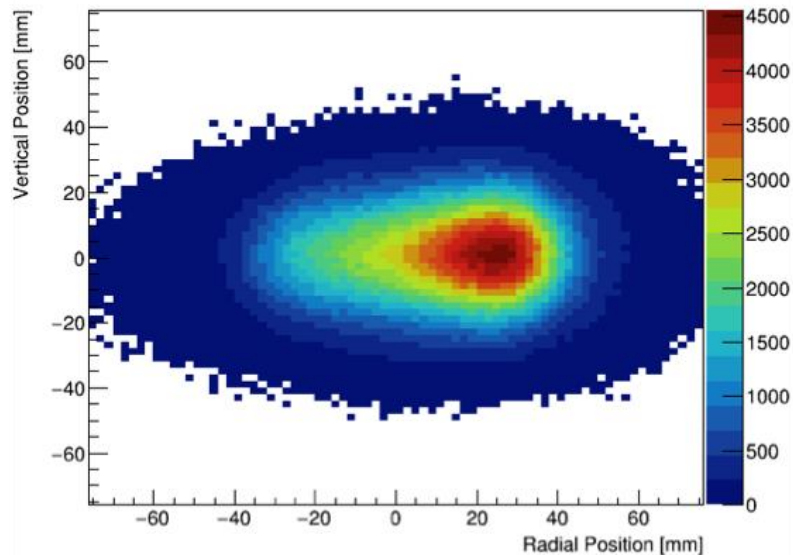
Difference between two field scans

- Blue Line: Dipole field (cross-sectional average) measured by the trolley
- Red/Green Dots: Fixed probe measurements

Data Analysis: Averaging



Azimuthal Average of the field in the field aperture



Transverse beam distribution

Data Analysis Progress

- Data production and quality control are stream-lined
- Analysis in advanced stages:
 - FID frequency extraction
 - Calibration
- On-going efforts
 - Field interpolation
 - Averaging over the muon distribution
 - Motion effect in the field mapping
 - Beam-related transient field

Projected improvement on the field measurement

Category	E821 (ppb)	E989 (ppb)	Methods
Absolute probe calibration	50	35	More uniform field for calibration
Trolley probe calibration	90	30	Better alignment between trolley and the plunging probe
Trolley measurement	50	30	More uniform field, less position uncertainty
Fixed probe interpolation	70	30	More stable temperature
Muon distribution	30	10	More uniform field, better understanding of muon distribution
Time dependent external magnetic field	-	5	Direct measurement of external field, active feedback
Others*	100	30	More uniform field, trolley temperature monitor, etc
total	170	70	

* Higher multipoles, trolley temperature and power supply, kicker eddy currents, etc.

APS DPF 2019, Boston, Ran Hong, Argonne National Laboratory



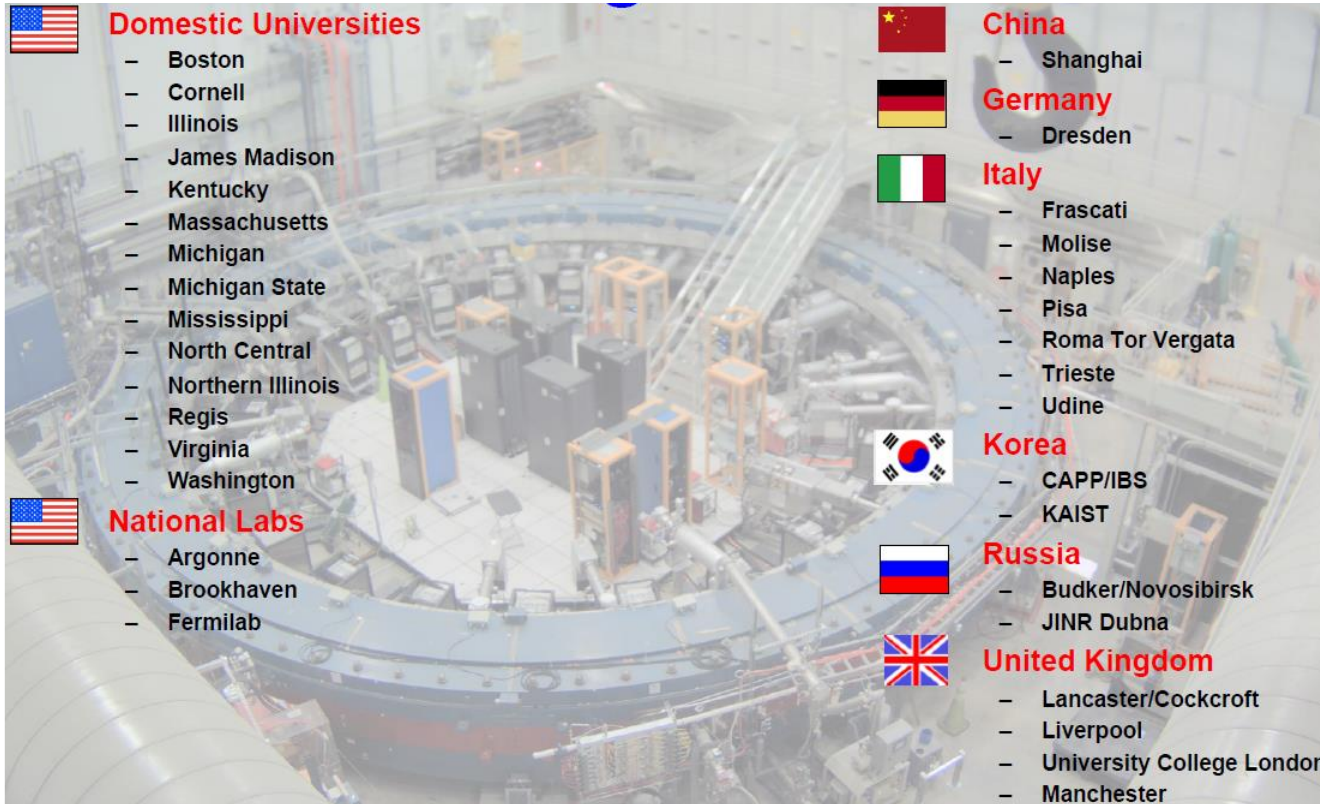
Run-1 Summary

- The high-precision magnetic field measurement system for the Muon $g-2$ experiment (E989) is commissioned.
 - Field scanning trolley
 - Fixed probes for field tracking
 - Calibration probe
- Operation in Run-1 (2018) was successful
 - ~30 Field Scans
 - 100% uptime when the magnet is On
 - Data quality: >95%
- Data analysis framework is developed. Field interpolation between scans and averaging over the muon distribution are the main on-going studies.

Run-2 Upgrades

- More systematic study runs
 - Stationary trolley runs: studying the relationship between the trolley and nearby fixed probes
 - Stepper trolley runs: motion effects, resolution
 - More frequent field scans: studying the interpolation
- Better field scan scheduling
 - Covering different times of a day (different temperatures)
- Synchronize fixed probe trigger with the muon beam
 - Study the beam-related transient field

Thanks to All Collaborators



Domestic Universities

- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- North Central
- Northern Illinois
- Regis
- Virginia
- Washington

National Labs

- Argonne
- Brookhaven
- Fermilab

China

- Shanghai

Germany

- Dresden

Italy

- Frascati
- Molise
- Naples
- Pisa
- Roma Tor Vergata
- Trieste
- Udine

Korea

- CAPP/IBS
- KAIST

Russia

- Budker/Novosibirsk
- JINR Dubna

United Kingdom

- Lancaster/Cockcroft
- Liverpool
- University College London
- Manchester

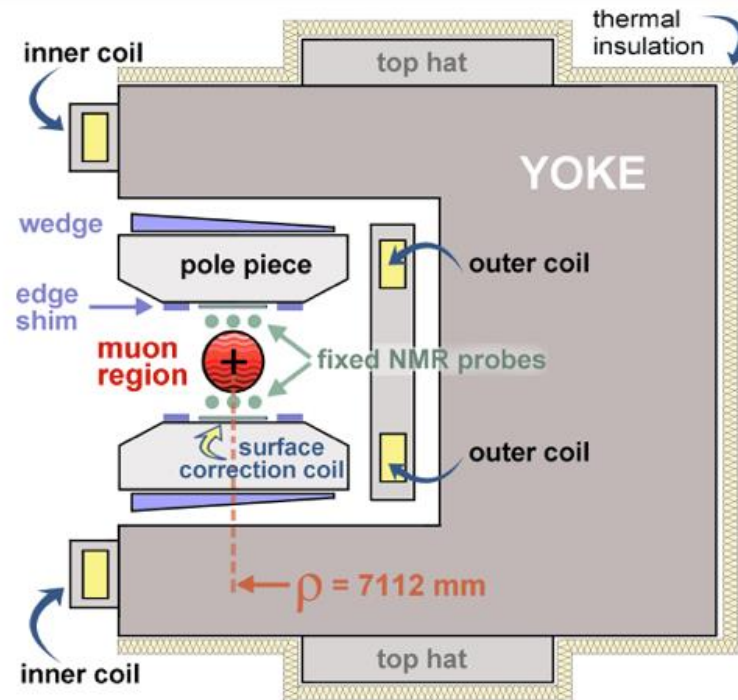
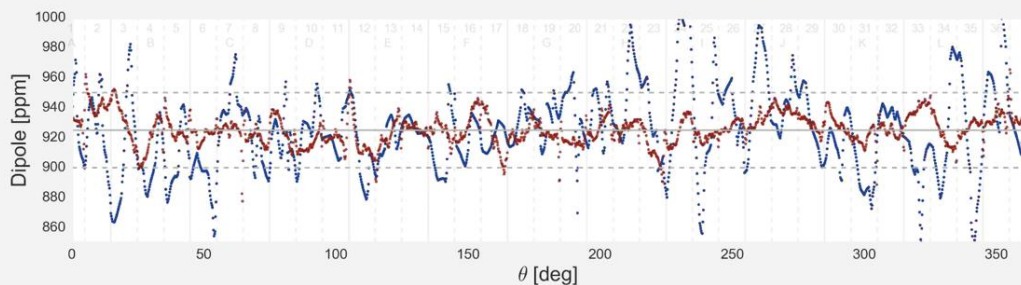
Passive Magnet Shimming

Iron strip laminations



E821 (BNL)

E989 (FNAL)

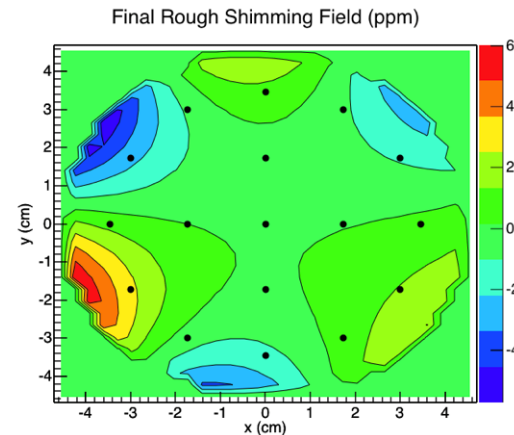


g-2 Magnet in Cross Section

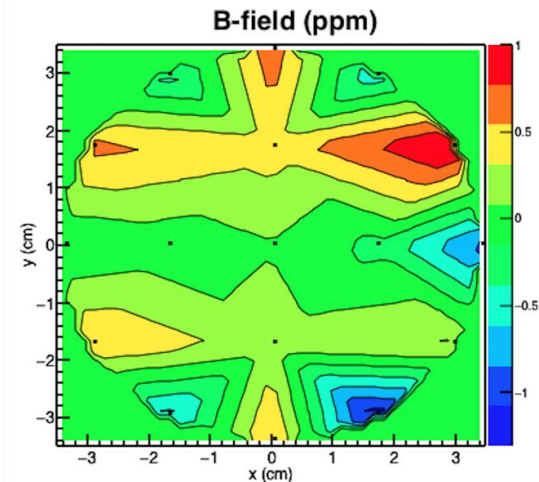
Active Magnet Shimming



Before
active shimming



After
active shimming



200 Concentric surface current coils to
cancel the transverse non-uniformity

NMR Signal Analysis

Realistic case: probe in non-uniform field

$$f(t) = Ae^{-t/\tau_2} e^{i\omega_0 t} \int_{-\infty}^{+\infty} g(\Delta\omega) e^{i\Delta\omega t} d\Delta\omega$$

Inverse Fourier Transform of $g(\Delta\omega)$!

$$\begin{aligned} G(t) &= \int_{-\infty}^{+\infty} g(\Delta\omega) e^{i\Delta\omega t} d\Delta\omega = C(t) + iS(t) \\ &= \sqrt{C^2(t) + S^2(t)} e^{i \tan^{-1} S(t)/C(t)} \end{aligned}$$

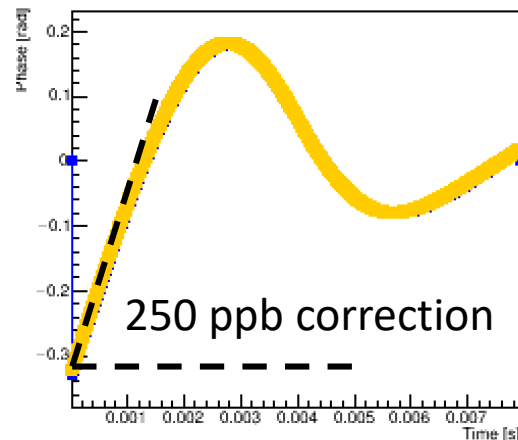
$$f(t) = Ae^{-t/\tau_2} \sqrt{C^2(t) + S^2(t)} e^{i(\omega_0 t + \phi(t))}$$

$$\phi(t) = \tan^{-1} S(t)/C(t)$$

- Symmetric distribution
 $g(\Delta\omega)$: $\phi(t) = 0$, ω_0 is average frequency $\bar{\omega}$
- Asymmetric distribution,

$$\bar{\omega} = \omega_0 + \left. \frac{d\phi}{dt} \right|_{t=0}$$

Linear phase fit residual



Data Analysis: Calibration

Calibration Runs

- Trolley, PP swapping
- Shimmed field

Shimmed Field Scan Runs

- PP scan locally around the calibration position
- Φ direction scan: using PP or the trolley

PP ΔB Runs

- PP ΔB measurements in all 3 directions

$$C = B_{PP} - B_T + \sum_{i=R,Y,\Phi} B_{Si}(\Delta x_i)$$
$$\Delta x_i = \frac{\Delta B_{PPi} - \Delta B_{Ti}}{B'_{Ai}}, i = R, Y, \Phi$$
$$B'_{Ai} = \frac{\partial B_A}{\partial x_i}$$

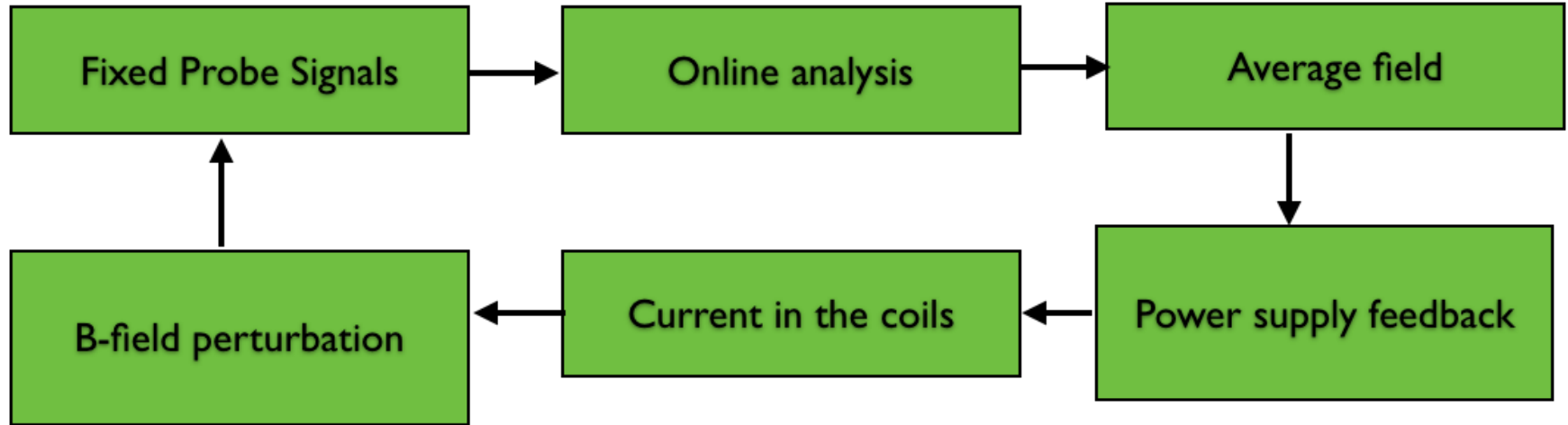
Trolley ΔB Runs

- Trolley ΔB measurements in all 3 directions

Trolley ΔB Runs and Azimuthal Scan Runs

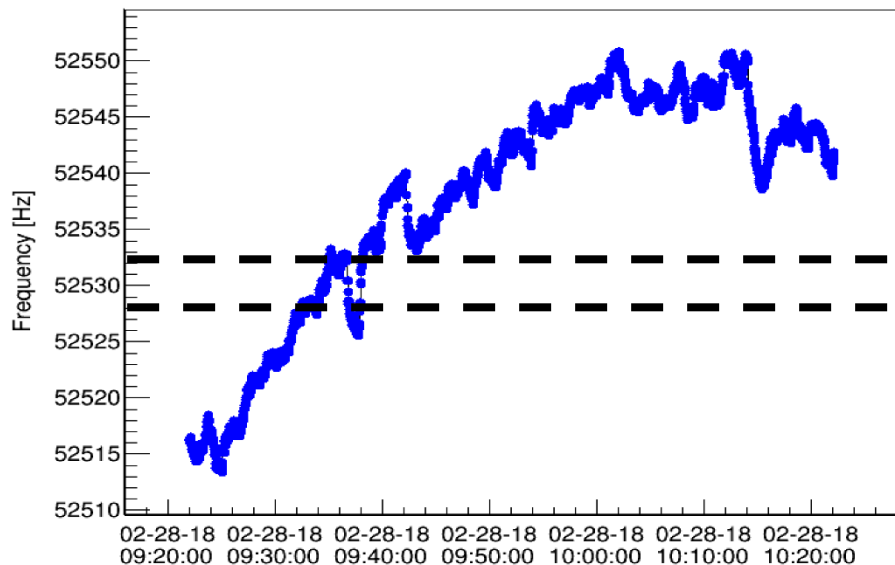
- Applied transvers gradients (R,Y): Trolley ΔB Runs
- Applied longitudinal gradients (Φ): Trolley azimuthal scans with azimuthal coils on/off

Field Monitor and Stabilization

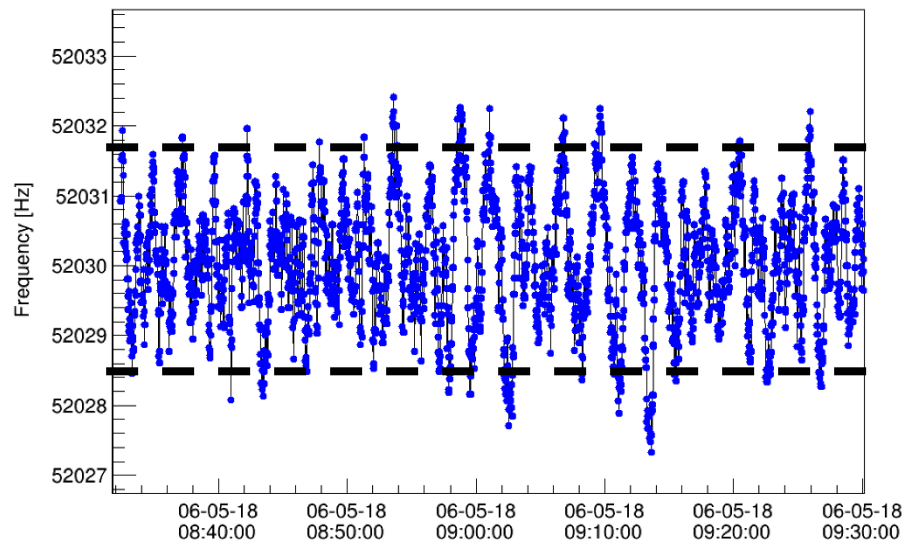


Field Monitor and Stabilization

Feedback OFF



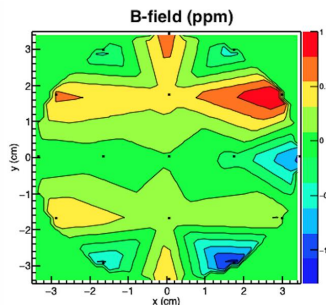
Feedback ON



--- +/- 30 ppb band

Data Analysis: Averaging

Multipole Expansion



$$= B(r, \theta) = B_0 + \sum_{n=0}^4 \left(\frac{r}{r_0} \right)^n [a_n \cos(n\theta) + b_n \sin(n\theta)]$$

