Towards understanding of charmonia and charmed pentaquarks P_c

Sasa Prelovsek

University of Ljubljana

Jozef Stefan Institute, Ljubljana

University of Regensburg

From Eucledian spectral densities to real-time physics
CERN, 11 march 2019

Outline

Lattice QCD at zero temperature and chemical potential ...

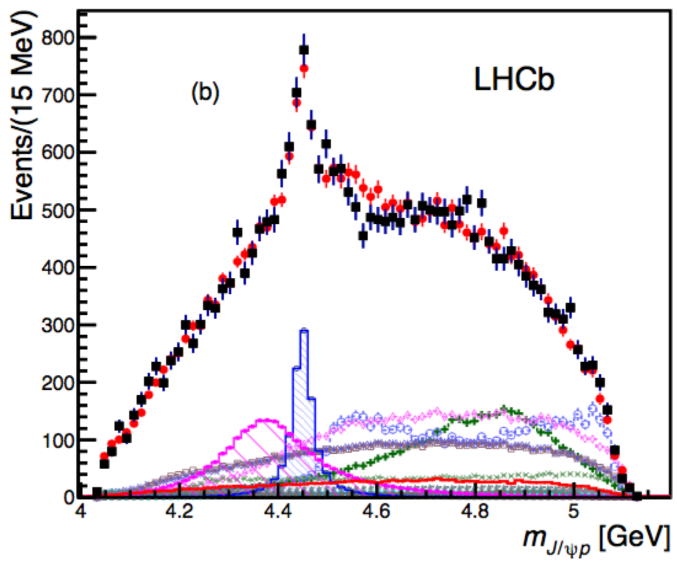
Towards (better) understanding:

- ground and excited charmonia with J<=3

  identify spin and parities of charmonia at rest and in flight

  - lessons are relevant for any other systems you may be interested in

LHCb 2015



- first lattice study of charmed pentquark P_c channel

$$N \;\; J/\psi \rightarrow P_c \rightarrow N \;\; J/\psi$$

  More general lessons

  - scattering / interactions of particles with spin, beyond s-wave
  - construction of two-hadron interpolators
  - why several nearly-degenerate eigenstates appear (even in continuum)

- if there is time: brief discussion on some of ongoing studies

charmonia at rest and in flight: identifying their spin and parities

arxiv:1811.04116, PRD 2019

M. Padmanath, S. Collins, D. Mohler, S. Piemonte, S.P.,

A. Schäfer, S. Weishaeupl

(Regensburg, Ljubljana, Mainz)

$\bar{c}c$ charmonium

Aim

Most of unconventional or exotic hadrons were experimentally discovered in charmonium-like systems.

Important to understand “conventional” charmonia besides addressing non-conventional ones.

Good quantum number of charmonium in its rest frame and in continuum: J^{PC} J=spin

The aim is to study ground and excited charmonia with all J<=3; determine using LQCD:

- for charmonia with p=0: **m**

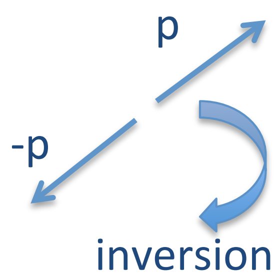
  J^{PC} (next slide)

- for charmonia with p ≠0: **E** $E=\sqrt{m^2+p^2}$ in continuum for strongly stable state

  J^{PC} in particle’s rest frame

I’ll tell later why study

of systems with p ≠0 is essential



note:

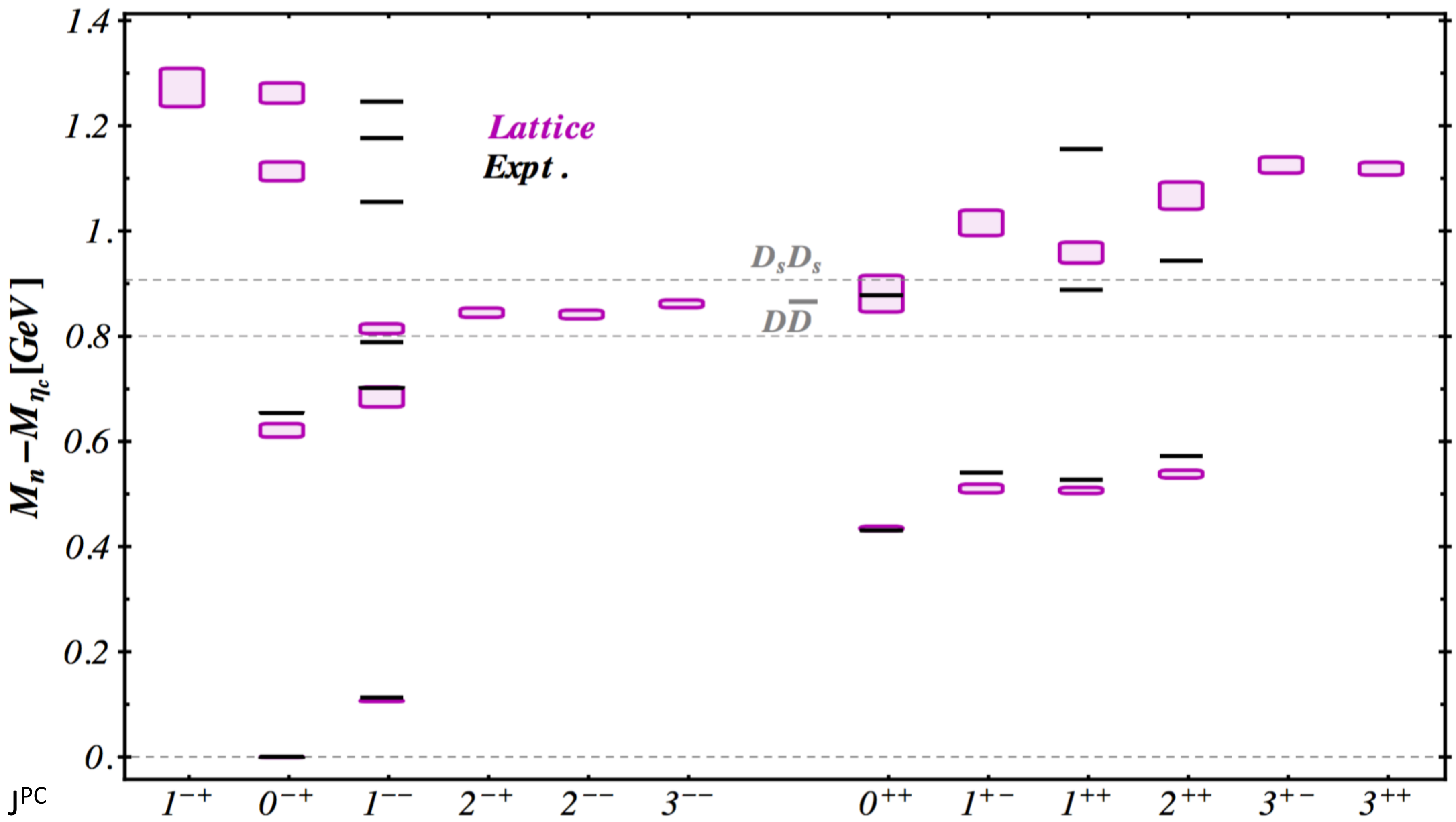
- P is not good q.n. for p ≠0
- particle spin J is not good q.n. for p ≠0

  only total angular momentum (spin+orbital) is good q.n.

What is the aim for p=0 ?

more extensive spectrum at p=0 has been previously obtained by HSC, JHEP 2016

Result: m and J^{PC} of charmonia at p=0



Analogous aim for p≠0.

Simplification of the present work

- Charmonium resonances above strong decay threshold (D$\underline{D}$) treated as strongly stable

only lattice study that took into account strong decays of charmonia
(decay of vector and scalar charmonium to D$\underline{D}$):
Lang, Mohler, Leskovec, S.P., JHEP 2015

We currently purse improved study of charmonia that goes beyond this simplification
(with Regensburg group on the same ensembles) and requires study of scattering.

This simplified study of charmonia at p =0 and p ≠0 and determination of J^{PC} is
important for the ongoing more rigorous study. The study of systems p ≠0 is essential to get
more info on scattering matrix.

- For charmonia slightly below threshold : effects of thresholds ignored

only lattice study that took into account threshold effect
(effect of D$\underline{D}$* on X(3872)):
S.P., Leskovec PRL 2013 ; Padmanath, Lang, S.P., PRD 2015

- multi-hadron interpolators are not considered in this work

How are J^{PC} determined experimentally ?

charmonium -> H_1 H_2

- Lorentz transformation of decay products to charmonium's rest frame
- observed partial wave and q.n. of H_1 and H_2 render J^{PC} of charmonium in its rest frame

Same strategy can not be followed on the lattice:

- Lorentz symmetry is broken
- no decay products for strongly stable states or states treated as strongly stable (in present project)

Info on "stable" charmonia from lattice : E_n , Z^n_i

Considering specific channel : take large number of interpolators with the quantum numbers of this channel

Specific channel : in continuum: J^{PC}

on the lattice: transforms according to given lattice irreducible representation Λ

$$\mathcal{O}_i = \bar{c}\Gamma c, \quad \bar{c}\Gamma' c, \quad \bar{c}\Gamma' D c, \ldots$$

typically 10-30 operators for one channel

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \; \mathcal{O}_j^+(0) | 0 \rangle = \left\langle 0 \right| e^{Ht} \mathcal{O}_i(0) e^{-Ht} \sum_n |n\rangle\langle n| \; \mathcal{O}_j^+(0) \left| 0 \right\rangle$$

$$= \sum_n \langle 0 | \mathcal{O}_i | n \rangle \; e^{-E_n t} \langle n | \mathcal{O}_j^+ | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

$$Z_i^n \equiv \langle 0 | \mathcal{O}_i | n \rangle$$

Carefully constructed O crucial to get info on J^{PC} !

start with $\mathcal{O}^{J^{PC}}$, form from it $\mathcal{O}_\Lambda^{[J^{PC}]}$ and use it in simulation

Lattice details

CLS (= Coordinated Lattice Consortium) N_f =2+1 ensemble (U101) , around 250 configurations

$m_\pi \approx 280$ MeV, $a \approx 0.086$ fm, $N_L=24$, $N_T=128$

relativistic treatment of all quarks (u/d,s,c)

full distillation: Nv=90

m_c slightly smaller than physical

(ongoing scattering simulations aim to explore dependence on m_c and threshold locations;

simulations at two m_c : slightly smaller and slightly larger than physical.

For ongoing study of resonances and threshold effects: changing mc changes position of thresholds;

the aim is to explore how various states depend on the position of thresholds)

Charmonia at rest
p=0

Symmetries : p=0

continuum

C: good

P: good

Rotations:

SO(3)

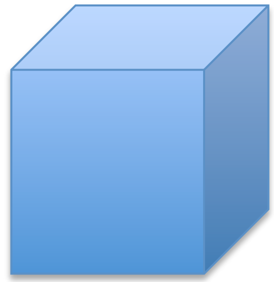
infinite number of elements $R = e^{i\vec{\varepsilon}\vec{J}}$

irreducible representations: J

cubic lattice

C: good

P: good



Rotations: cubic box periodic BC in x,y,z

Octahedral group O

24 elements

E	$8C_3$	$6C'_2$	$6C_4$	$3C_2 = (C_4)^2$

$\mathbf{p} = (0,0,0)$, O_h, $P = \pm$	
Λ (dim)	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3

Construction of interpolators

following earlier work by HSC
(for example PRD 2010)

$$\bar{c}(x)\Gamma c(x), \quad \bar{c}(x)\Gamma \overleftrightarrow{D}_j c(x), \quad \bar{c}(x)\Gamma \overleftrightarrow{D}_j \overleftrightarrow{D}_k c(x)$$

$$O_i^{J^{PC},M}(\mathbf{p}) = \sum_{m_1,m_2,m_3} C_i^{CG}(m_1,m_2,m_3;M)\times \sum_{\mathbf{x}} \bar{c}(x)\Gamma_{m_1}\overleftrightarrow{D}_{m_2}\overleftrightarrow{D}_{m_3}c(x)e^{i\mathbf{p}\cdot\mathbf{x}}.$$

for p=0 these interpolators have good J^{PC}:

$$RO^{J,M}R^{-1} = \sum_{M'} D^J_{MM'}(R^{-1})\ O^{J,M'}$$

O that will transform irreducibly under irrep Λ and row μ ("subduction")

$$O_{\Lambda,\mu} = \sum_{R\in Octah.} T^{\Lambda}_{\mu\mu}(R)^*\ RO$$

$$O^{[J^{PC}]}_{i,\Lambda^C,\mu}(\mathbf{p}=\mathbf{0}) = \sum_M S^{J,M}_{\Lambda,\mu}\ O_i^{J^{PC},M}(\mathbf{p}=\mathbf{0}),$$

typically ~ 10 interpolators in each irrep (for each row)

Subduction coeff: HSC, PRD 2010

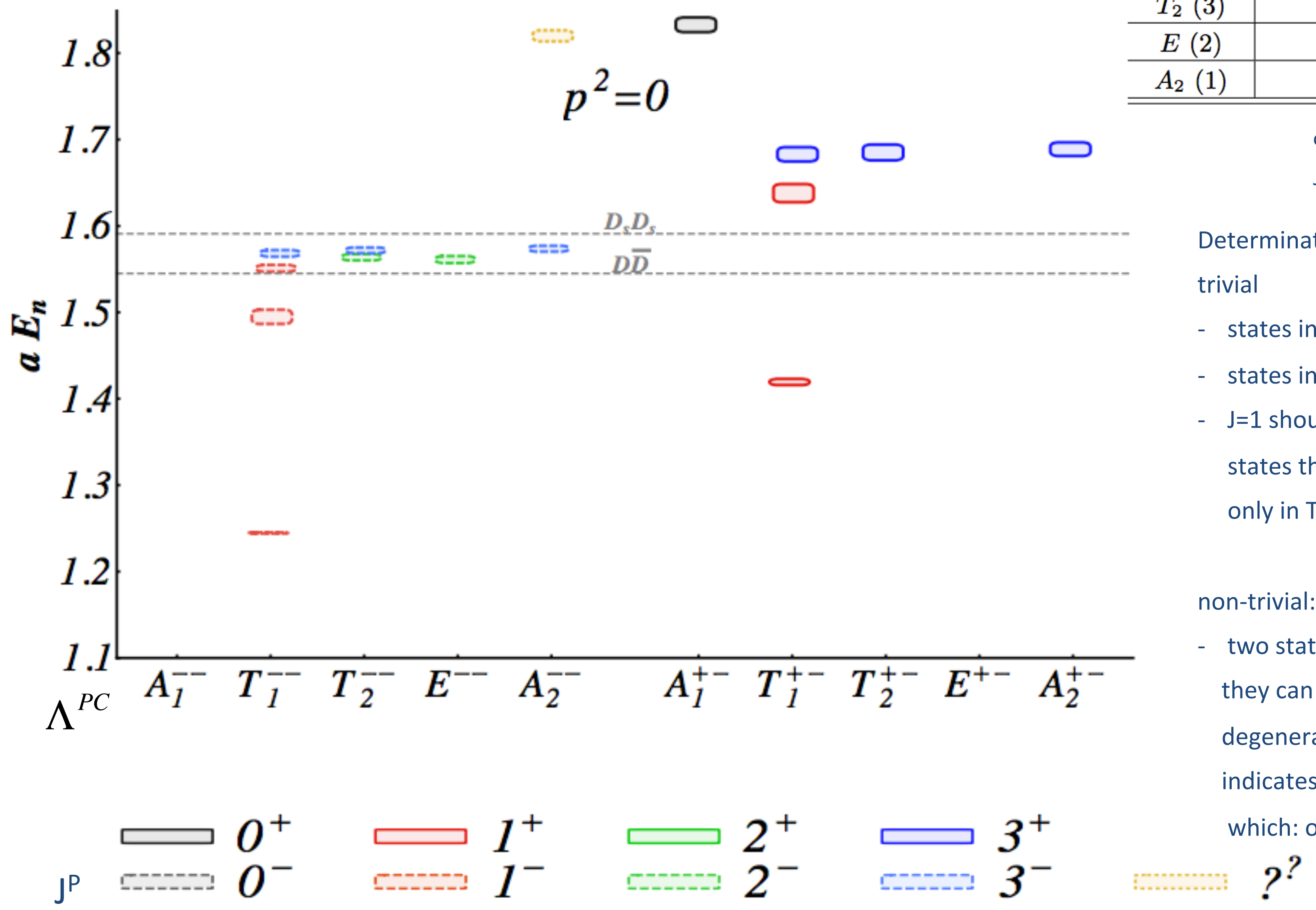
Given irrep contains different J

$\mathbf{p}=(0,0,0),\ O_h,\ P=\pm$	
$\Lambda\ (dim)$	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3

Strategy to determine J of a state:

$$\langle O^{[J^{PC}]}_{i,\Lambda^C}|\mathbf{0},J^{PC}\rangle \gg \langle O^{[J^{PC}]}_{i,\Lambda^C}|\mathbf{0},J'^{PC}\rangle$$

E=m and J^{PC} for C=-



$\mathbf{p}=(0,0,0),\ O_h,\ P=\pm$	
Λ (*dim*)	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3

only J<4 written

J>=4 expected to be heavier

Determination of J:

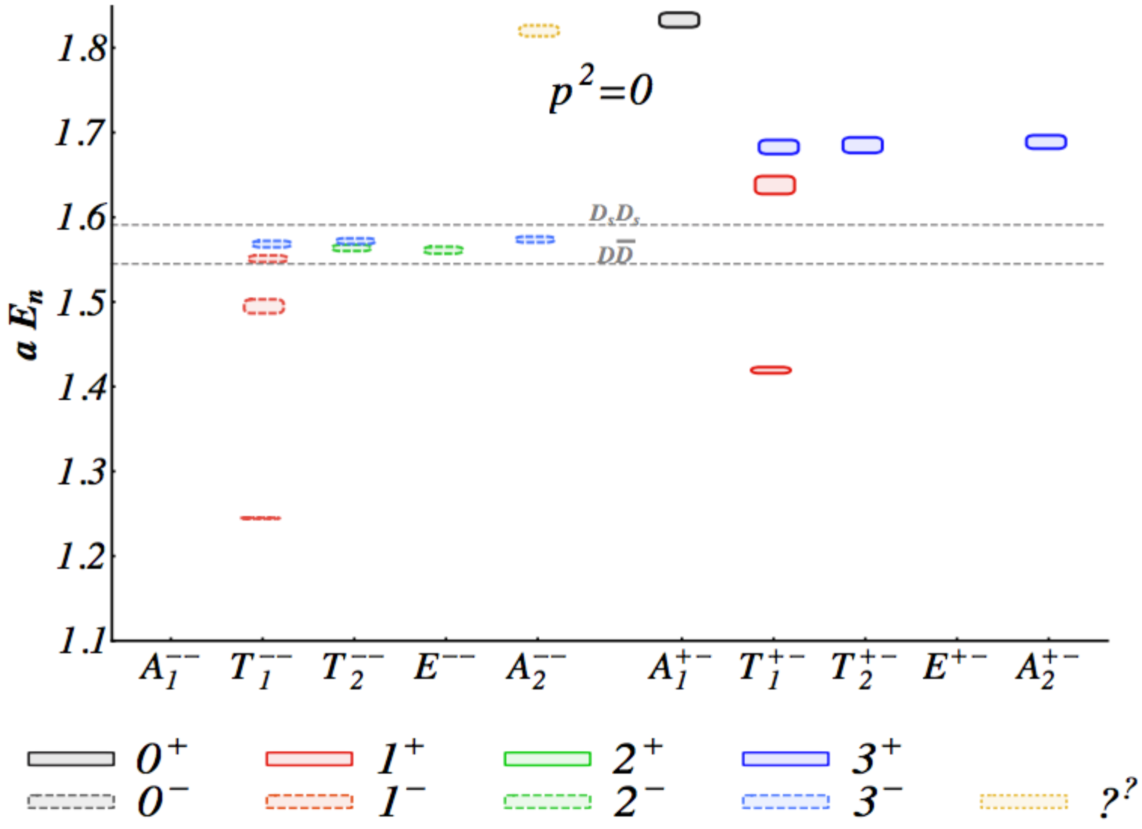
trivial

- states in A2 have J=3
- states in E have J=2
- J=1 should appear only in T1. states that appear only in T1 have J=1

non-trivial:

- two states in T_1^{--} near E~1.55 they can have J=1 or J=3: degeneracy between T1, T2, A2 indicates that one of them is J=3 which: only Z-overlaps can tell

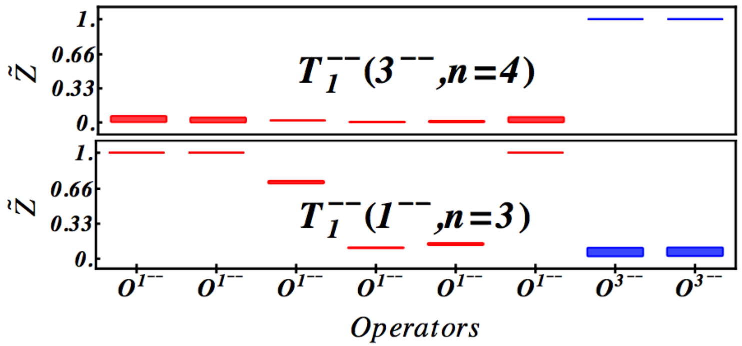
Energies and overlaps, C=-



$\mathbf{p}=(0,0,0),\ O_h,\ P=\pm$	
Λ (*dim*)	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3

$$Z_i^n \equiv \langle 0|\mathcal{O}_i|n\rangle$$

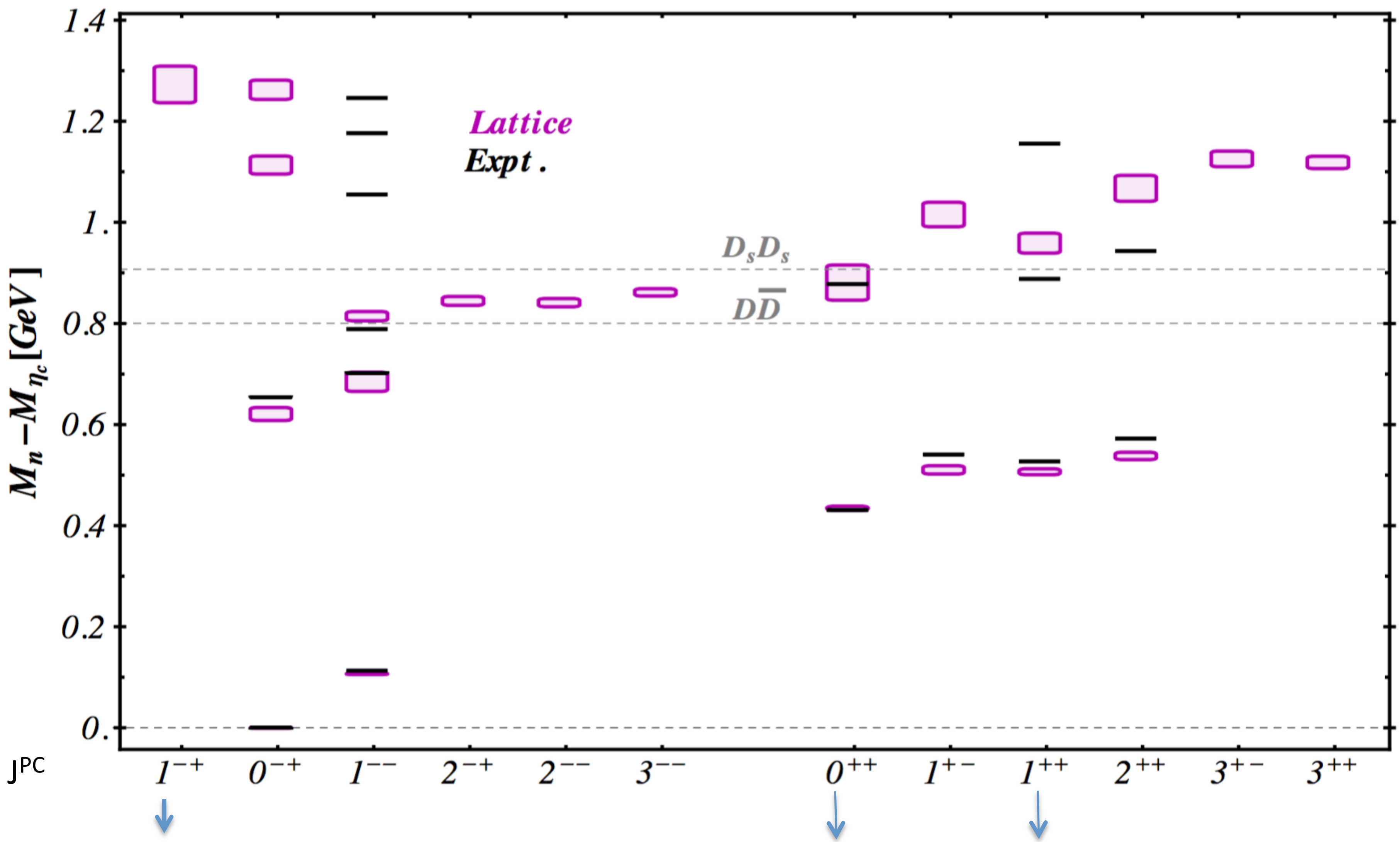
$$\tilde{Z}_i^n = \frac{Z_i^n}{max_n(Z_i^n)} \leq 1$$



$$\langle O_{i,\Lambda^C}^{[J^{PC}]}|\mathbf{0},J^{PC}\rangle \gg \langle O_{i,\Lambda^C}^{[J^{PC}]}|\mathbf{0},J'^{PC}\rangle$$

more extensive spectrum at p=0 has been previously obtained by HSC , JHEP 2016

Result: m and J^{PC} of charmonia at p=0



exotic J^{PC}

hybrid candidate

agrees with new Belle candidate X(3860)

X(3872) to high without DD* threshold effect

Charmonia in flight
$p \neq 0$

$p=(0,0,1)\ 2\pi/N_L$

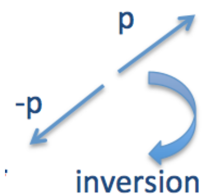
$p=(1,1,0)\ 2\pi/N_L$

Symmetries

continuum

<u>C</u>: good

<u>P</u>: NOT good



<u>Rotations/reflections</u>:

transformations that leave p invariant

rotations around p; little group U(1)

spin J: not good

helicity : good $\lambda = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|}$

$\tilde{\eta}$: good (only for λ=0 states)

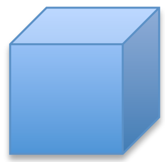
$$\Pi \, |p, J^P, \lambda\rangle = \tilde{\eta} \, |p, J^P, -\lambda\rangle$$

$$\tilde{\eta} \equiv P(-1)^J$$

cubic lattice

<u>C</u>: good

<u>P</u>: NOT good



<u>Rotations/reflections</u>:

transformations that leave box and p invariant: Not much symmetry left !!

p=(0,0,1): group Dic_4 , 8 elements

TABLE VII. Choice of representation matrices for the Dic_4 little group. I denotes the identify transformation, $R(\phi)$ denotes a rotation around the z-axis by ϕ and Π denotes a reflection in the yz plane ($x \to -x$).

Irrep	I	$R(\pi)$	$R(3\pi/2)$	$R(\pi/2)$	Π	$R(\pi)\Pi$	$R(\pi/2)\Pi$	$R(3\pi/2)\Pi$

p=(1,1,0); group Dic_2 , 4 elements

Irrep	I	$R(\pi)$	Π	$R(\pi)\Pi$

irreps: good quantum numbers

helicity: not good

$\tilde{\eta}$: good (only for λ=0 states)

Π is reflection in a plane that contains p; it preserves p

One or two λ contribute to a given irrep : not difficult to determine λ

Many different J^P contribute to a given irrep : difficult to determine J^P !!

refers to J^{PC} in charmonium's rest frame

$\mathbf{p}=(0,0,1)$, Dic_4		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+,\ 1^-,\ 2^+,\ 3^-$
A_2 (1)	0^-	$0^-,\ 1^+,\ 2^-,\ 3^+$
E (2)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$
B_1 (1)	2	$2^\pm,\ 3^\pm$
B_2 (1)	2	$2^\pm,\ 3^\pm$

state with J has |λ|<=J

state with |λ| can have |λ|<=J

$\mathbf{p}=(1,1,0)$, Dic_2		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+,\ 1^-,\ 2^+,\ 3^-$
	2	$2^\pm,\ 3^\pm$
A_2 (1)	0^-	$0^-,\ 1^+,\ 2^-,\ 3^+$
	2	$2^\pm,\ 3^\pm$
B_1 (1)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$
B_2 (1)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$

since helicity is not good, λ=0 and 2 can contribute to A2

state with λ=0 can have J=0,1,2,3,.. ; state with λ=0 has good $\tilde{\eta}\equiv P(-1)^J$

state with with λ=2 can have J=2,3,.. ; state with λ=2 does not have good $\tilde{\eta}$

The challenge to determine J^P

$\mathbf{p} = (1,1,0)$, Dic_2		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	0^+, 1^-, 2^+, 3^-
	2	$2^\pm$, $3^\pm$
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+
	2	$2^\pm$, $3^\pm$
B_1 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$
	3	$3^\pm$
B_2 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$
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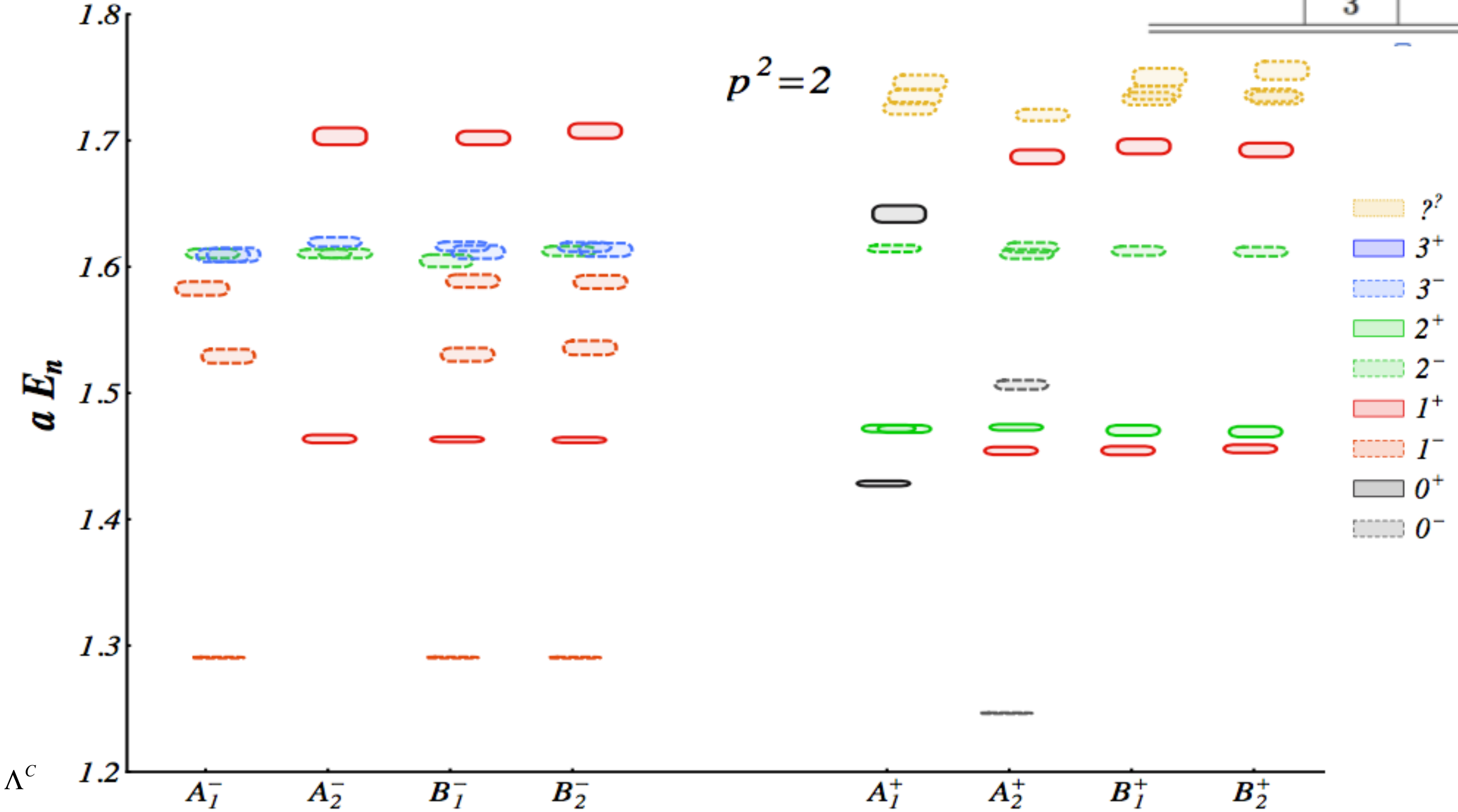


Simple example

$O = \bar{q}\gamma_5\gamma_z q$ transforms according to A2

couples to axial-vector 1^{++} meson (a1 for light quarks) $<\bar{q}\gamma_5\gamma_z q\,|\,1^+> \neq 0$

couples also to pseudoscalar 0^{-+} meson (pion for light quarks) $<\bar{q}\gamma_5\gamma_z q\,|\,0^-(p)> \propto f\ p_z$

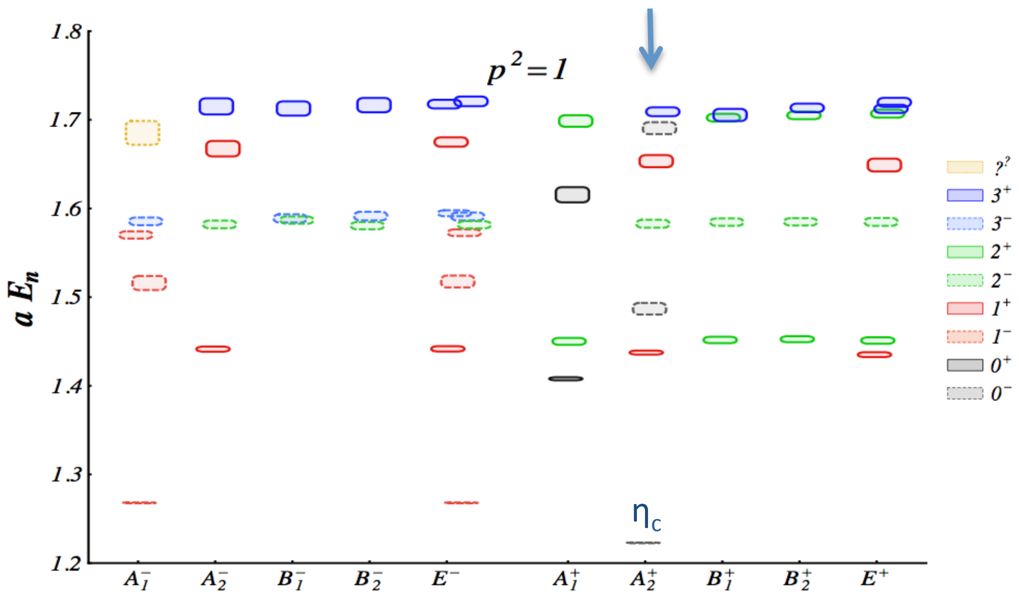


FIG. 11. J^P-identified charmonium spectrum in the moving frame with $\mathbf{p} = (0,0,1)$. Irreps Λ^C of group Dic_4 are presented. The colors indicate J^P of states according to the color-coding (21).

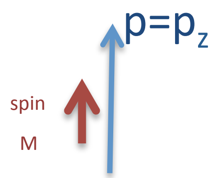
$\mathbf{p} = (0,0,1)$, Dic_4		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+

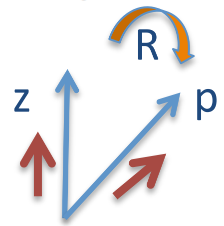
Construction of interpolators

following Hadron Spectrum Coll.
PRD 2012 (Thomas, Edwards, Dudek)
where determination of J^{PC} was applied to
light iso-vector mesons
(it has not been applied to charmonium or other systems yet)

$$\bar{c}(x)\Gamma c(x), \quad \bar{c}(x)\Gamma \overleftrightarrow{D}_j c(x), \quad \bar{c}(x)\Gamma \overleftrightarrow{D}_j \overleftrightarrow{D}_k c(x)$$

$$O_i^{J^{PC},M}(\mathbf{p}) = \sum_{m_1,m_2,m_3} C_i^{CG}(m_1,m_2,m_3;M)\times \sum_{\mathbf{x}} \bar{c}(x)\Gamma_{m_1}\overleftrightarrow{D}_{m_2}\overleftrightarrow{D}_{m_3}c(x)e^{i\mathbf{p}\cdot\mathbf{x}}.$$





$$O_i^{J^{PC},\lambda}(\mathbf{p}) = \sum_M \mathcal{D}^{(J)*}_{M,\lambda}(R)\, O_i^{J^{PC},M}(\mathbf{p})$$

operator with good helicity in continuum

The above operators are REDUCIBLE under discrete groups $Dic_{2,4}$

O that will transform irreducibly under irrep Λ and row μ ("subduction")

$$O_{\Lambda,\mu} = \sum_{R\in Dic_{2,4}} T^{\Lambda}_{\mu\mu}(R)^* \, RO \qquad O_{i,\Lambda^C,\mu}^{[J^{PC},|\lambda|]}(\mathbf{p}) = \sum_{\hat{\lambda}=\pm|\lambda|} S_{\Lambda,\mu}^{\tilde{\eta},\hat{\lambda}}\, O_i^{J^{PC},\hat{\lambda}}(\mathbf{p})$$

Strategy to determine J^P,λ of a |n>:

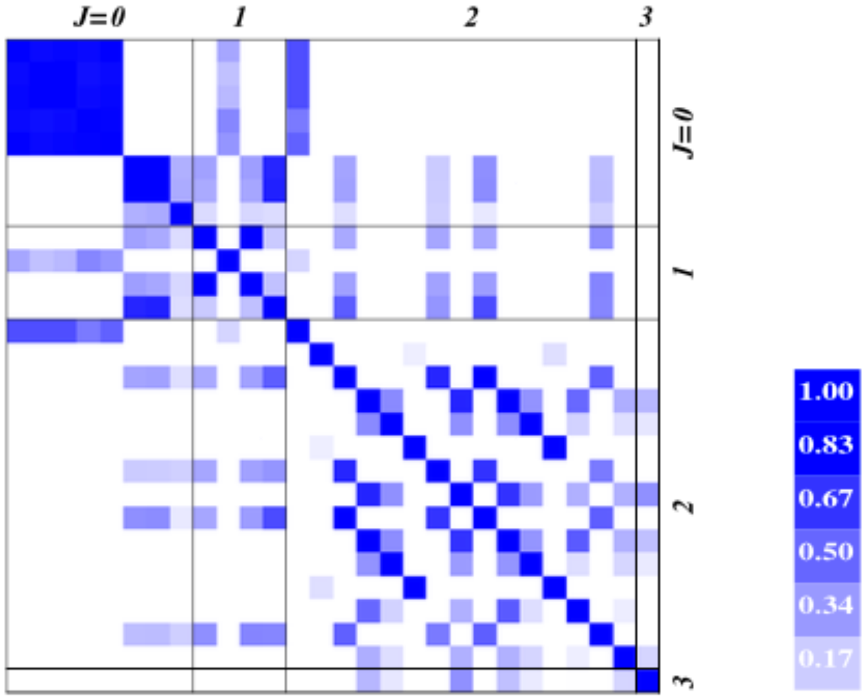
- calculate overlaps Z= <O|n>
- state $|J^P,\lambda\rangle$ couples better to $O^{[J,P,\lambda]}$ than to $O^{[J',P',\lambda']}$

typically 10-30 interpolators in each irrep (for each row)

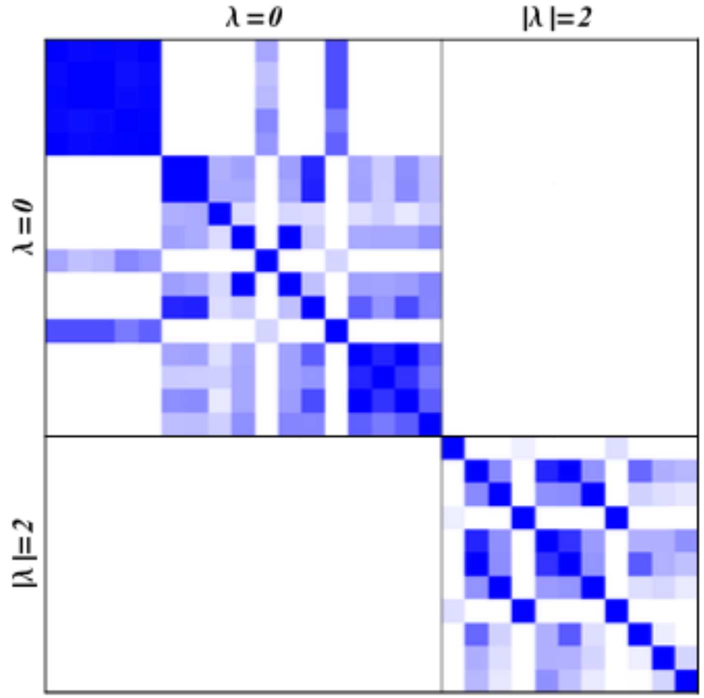
Cross-correlation between different J : non-negligible

Cross-correlation between different λ : small

$$(C_{ij}/\sqrt{C_{ii}C_{jj}})$$



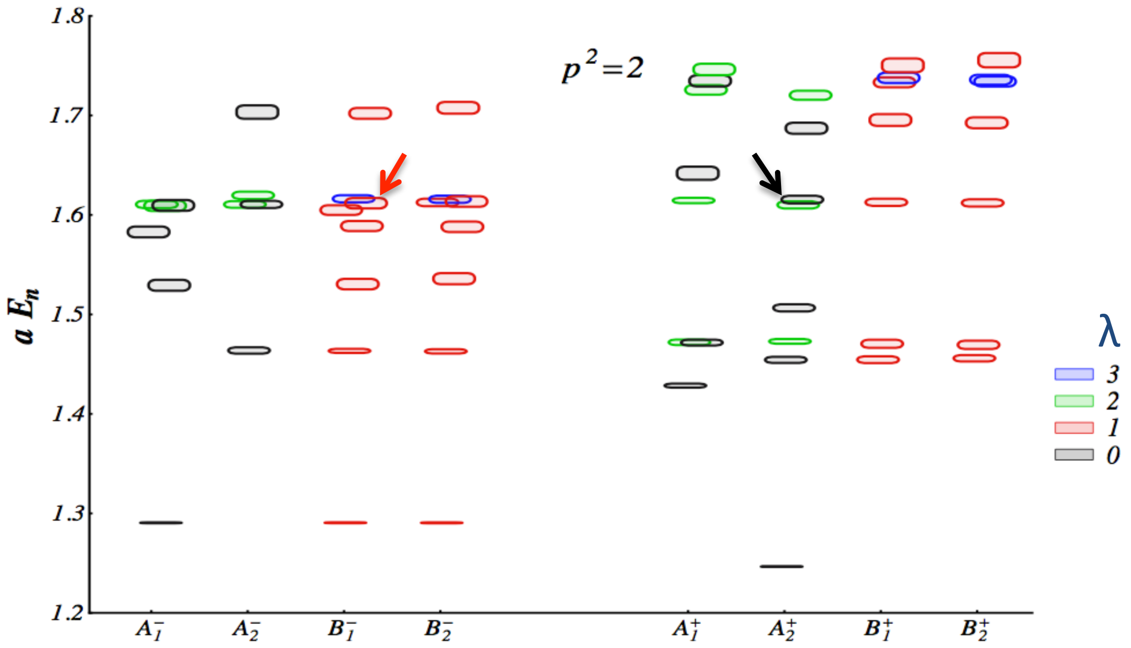
interpolators ordered by J



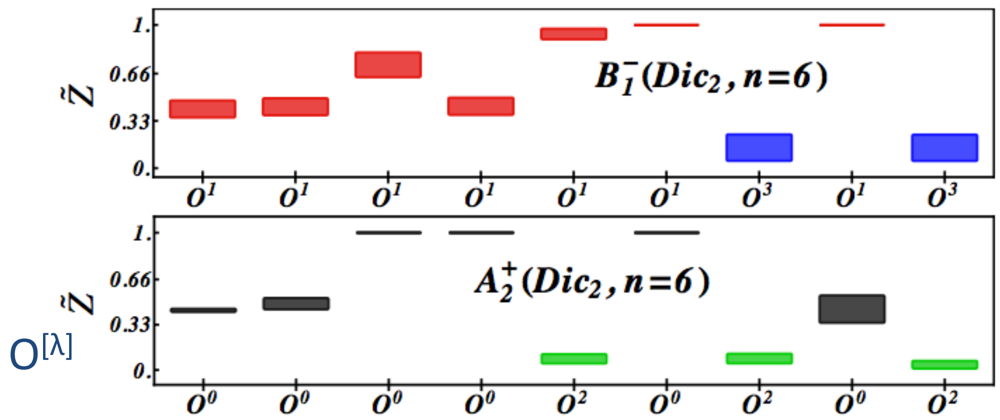
interpolators ordered by λ

p=(1,1,0), irrep A_1 , C=+: 25 interpolators

First step: Determining helicity λ



$\mathbf{p}=(1,1,0),\ Dic_2$		
$\Lambda\ (dim)$	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
$A_1\ (1)$	0^+	$0^+,\ 1^-,\ 2^+,\ 3^-$
	2	$2^\pm,\ 3^\pm$
$A_2\ (1)$	0^-	$0^-,\ 1^+,\ 2^-,\ 3^+$
	2	$2^\pm,\ 3^\pm$
$B_1\ (1)$	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$
$B_2\ (1)$	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$



Helicity is good in continuum

and breaking of continuum symmetry is small

helicity is not difficult to determine

Second step : Determining J^P from known $|\lambda|$, η

Criteria

- $J >= |\lambda|$
- degeneracies of E (J^{PC}) across different irreps that contain this J^{PC}
- $$\langle O_{i,\Lambda^C}^{[J^{PC},|\lambda|]} | \mathbf{p}, J^{PC}, \lambda\rangle > \langle O_{i,\Lambda^C}^{[J^{PC},|\lambda|]} | \mathbf{p}, J'^{P'C}, \lambda\rangle$$
- $$\langle O_{i,\Lambda_1^C}^{[J^{PC},|\lambda|]} | \mathbf{p}, J^{PC}, \lambda\rangle \simeq \langle O_{i,\Lambda_2^C}^{[J^{PC},|\lambda|]} | \mathbf{p}, J^{PC}, \lambda\rangle$$

Challenge: number of charmonia with different J^{PC} in a narrow energy region

Mission impossible if one a single irrep is considered.

All (or several relevant) irreps need to be considered and compared

see also Hadron Spectrum Coll. : PRD 2012 (Thomas, Edwards, Dudek)

where determination of J^{PC} was applied to light iso-vector mesons

Charmonia in flight: energies and J^P in their rest-frame

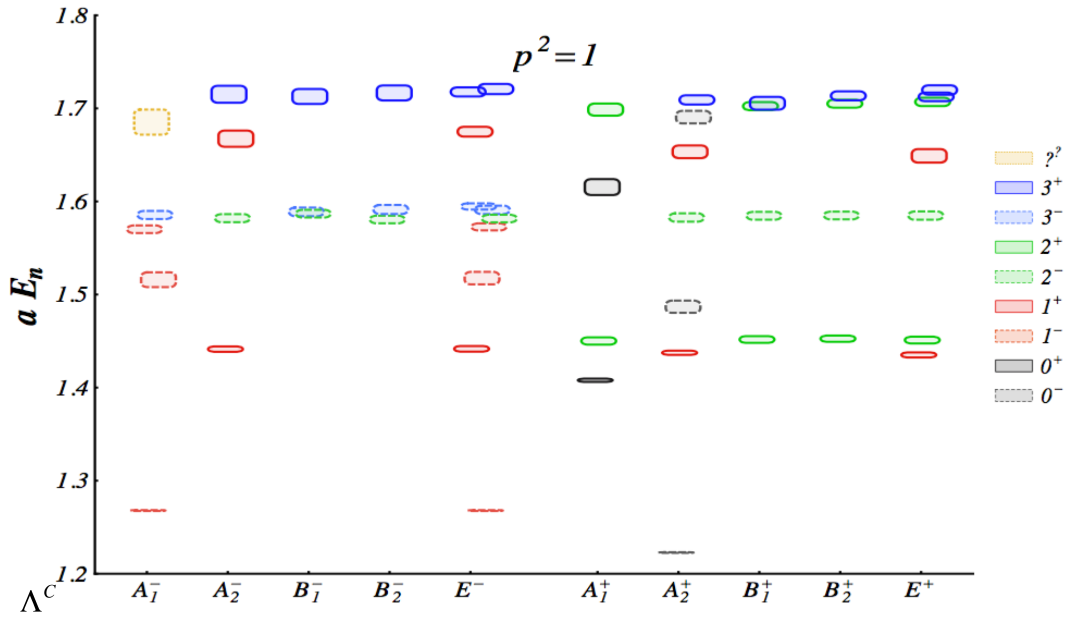
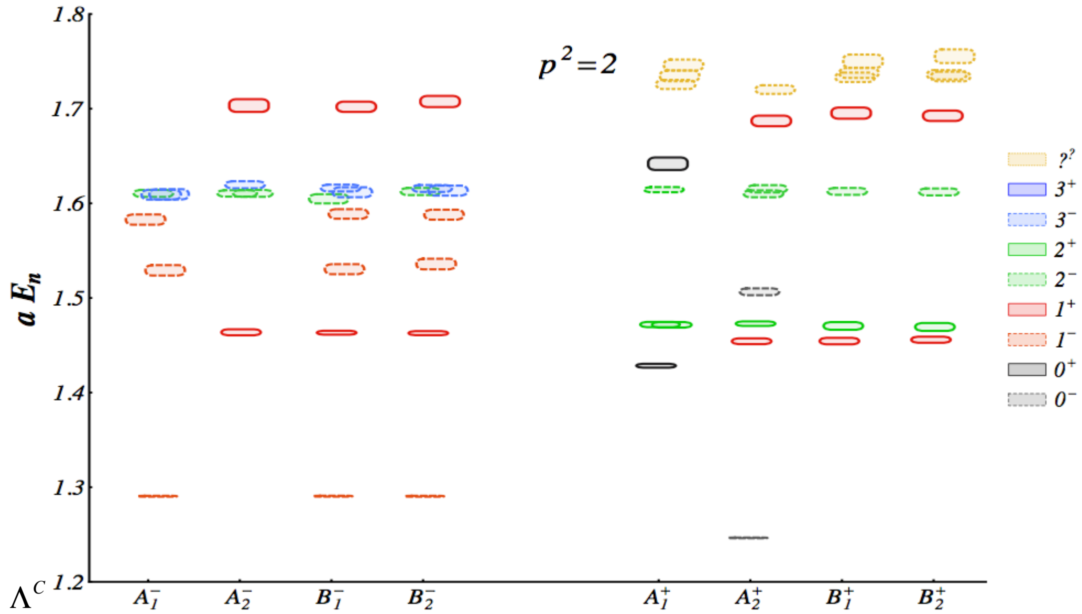


FIG. 11. J^P-identified charmonium spectrum in the moving frame with $\mathbf{p} = (0, 0, 1)$. Irreps Λ^C of group Dic_4 are presented. The colors indicate J^P of states according to the color-coding (21).



$\mathbf{p} = (0,0,1)$, Dic_4		
Λ (*dim*)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	0^+, 1^-, 2^+, 3^-
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+
E (2)	1	$1^\pm$, $2^\pm$, $3^\pm$
	3	$3^\pm$
B_1 (1)	2	$2^\pm$, $3^\pm$
B_2 (1)	2	$2^\pm$, $3^\pm$

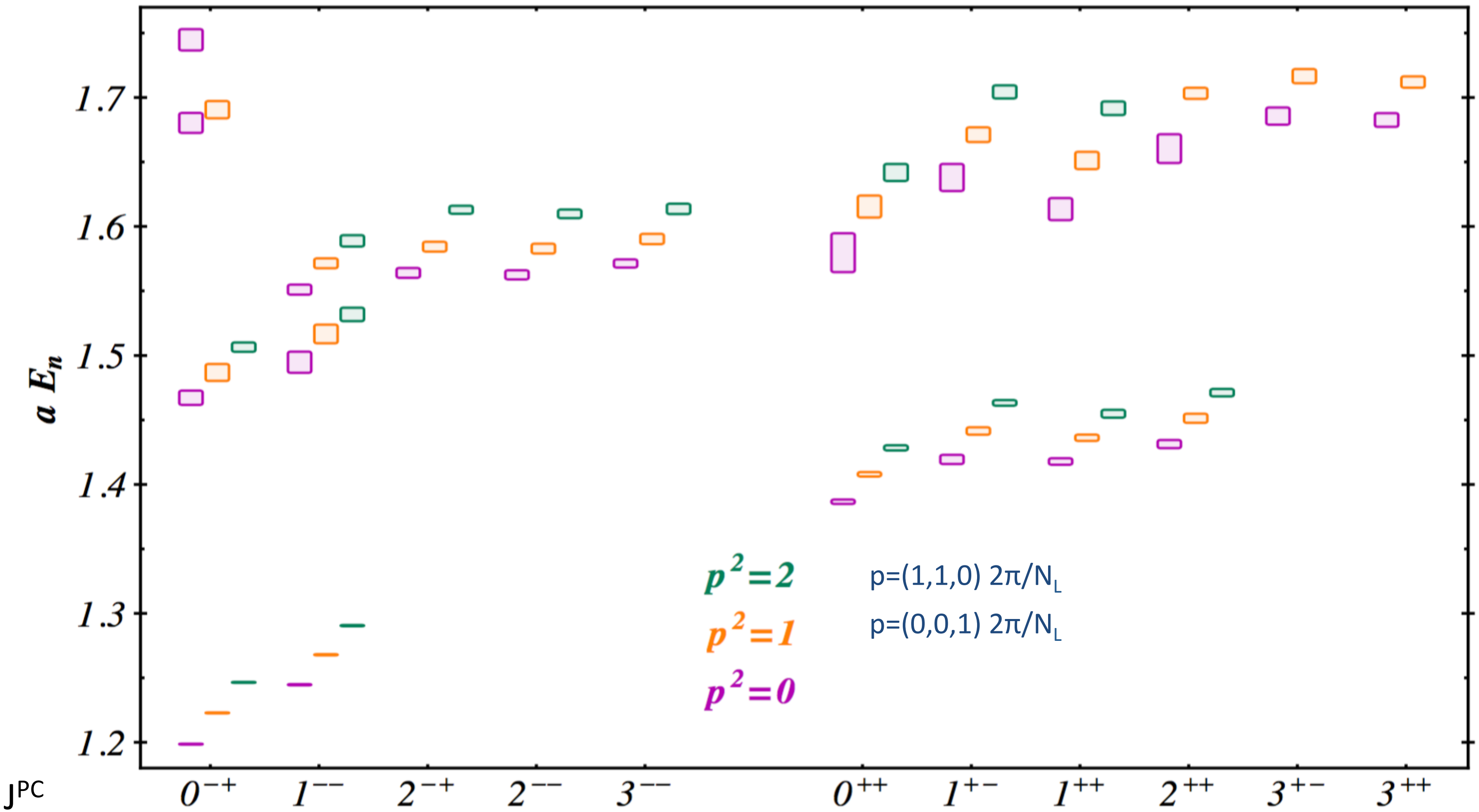
$\mathbf{p} = (1,1,0)$, Dic_2		
Λ (*dim*)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	0^+, 1^-, 2^+, 3^-
	2	$2^\pm$, $3^\pm$
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+
	2	$2^\pm$, $3^\pm$
B_1 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$
	3	$3^\pm$
B_2 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$
	3	$3^\pm$

Result: E and J^{PC} of charmonia at $p \neq 0$

$$E \approx \sqrt{m^2 + p^2}$$

in continuum

J^{PC} denote quantum numbers in particle's rest frame



How are these results useful when considering charmonium resonances with scattering

- scalar resonances appear in DD scattering in s-wave:

  p=(1,1,0) A_1 C=+

- extract 2^- level from scattering analysis

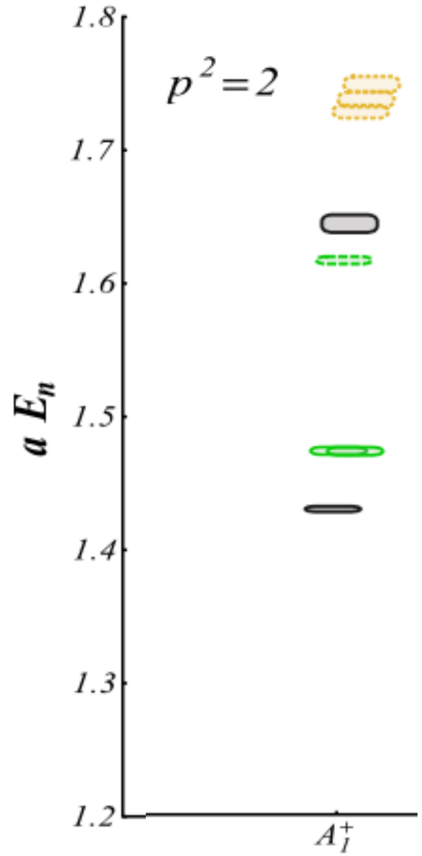
- vector resonances appear in DD scattering in p-wave:

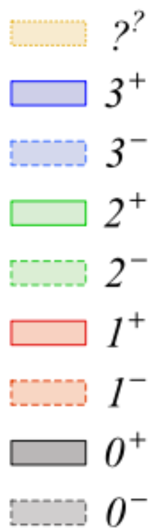
  p=(0,0,1) A_1 C=-

a) assuming 3^- resonance is narrow

extract 3^- level from scattering analysis and determine $\delta_1(p)$

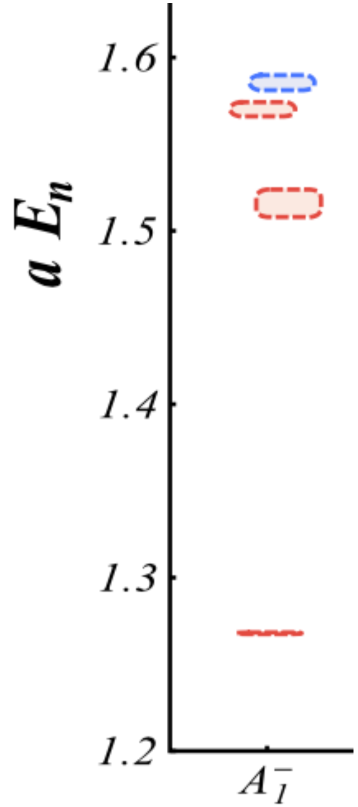
b) keep 3- level and determine $\delta_1(p)$, $\delta_3(p)$





qq levels: above

meson-meson levels: come in addition



Nucleon - J/ψ scattering in pentaquark Pc channel

1811.02285

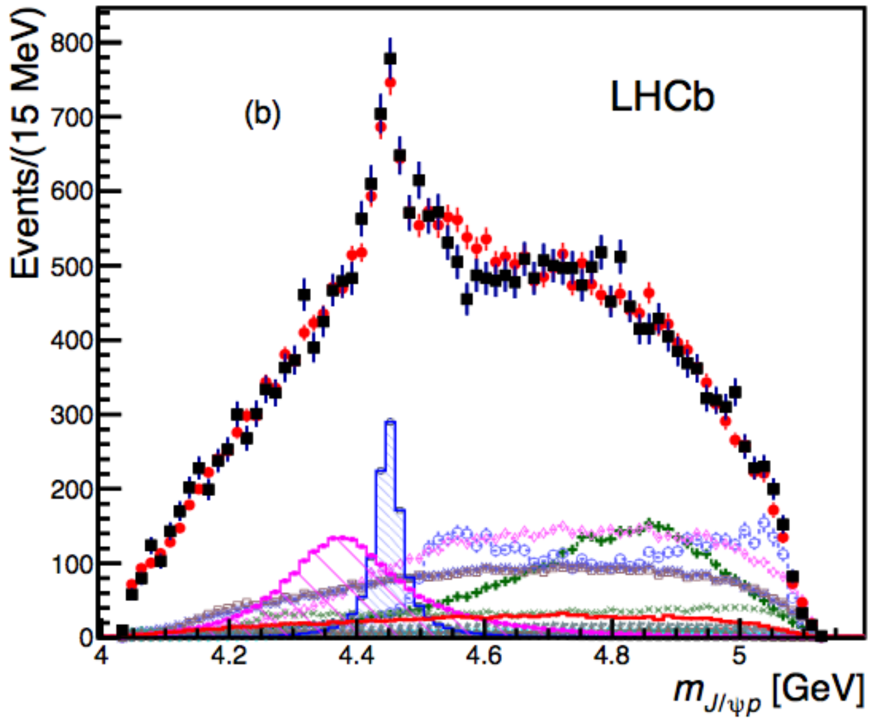
U. Skerbis, S. P. (Ljubljana)

Pc pentaquark discovery: LHCb 2015

LHCb PRL
1507.03414

$$P_c \to p \;\; J/\psi$$

uud $\bar{c}$c



knowledge of J^P from exp:

- they have opposite parities
- J= 3/2 or 5/2
- favoured J^P in Table

No previous study of Pc on the lattice

Only previous lattice study of N J/ψ scattering:

HALQCD method, only energy region below Pc

T. Sugiura et. all Proccedings of Lattice 2017 conference,

EPJ Web of Conferences 175, 05011 (2018)

irrep Γ^P	J^P
$G_1^{\pm}$	$\frac{1}{2}^{\pm}, \frac{7}{2}^{\pm}$
$G_2^{\pm}$	$\frac{5}{2}^{\pm}, \frac{7}{2}^{\pm}$
$H^{\pm}$	$\frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}, \frac{7}{2}^{\pm}$

LHCb 2015	m (MeV)	Γ (MeV)	favoured J^{PC}
Pc(4380)	4380 ± 40	205 ± 100	$3/2^-$ (L=0)
Pc(4449)	4450 ± 5	40 ± 25	$5/2^+$ (L=1)

O_h

H^- irrep

H^+, G_2^+ irreps

Two-hadron states in Pc channel

Pc(4380)

Pc(4449)

$P_c = \text{uud}\bar{c}c \rightarrow (\text{uud})\,(\bar{c}c)$

light-baryon charmonium

$\rightarrow (\text{uuc})\,(\bar{c}d)$

charmed-baryon charmed-meson

Threshold locations

J^P	L	$m_m + m_b$ [MeV]	meson	barion
$\frac{3}{2}^-$	2^+	3921	$\eta_c(1s)$	p
	0^+	4034	J/ψ	p
	0^+	4293	$D^{*0}(\bar{2}007)$	Λ_c^+
	0^+	4387	D^-	$\Sigma_c^{++}(2520)$
	1^-	4352	χ_{c0}	p
	1^-	4448	χ_{c1}	p
$\frac{3}{2}^+$	1^-	3921	$\eta_c(1s)$	p
	1^-	4034	J/ψ	p
	1^-	4151	$\bar{D}^0$	Λ_c^+
	1^-	4293	$D^{*0}(\bar{2}007)$	Λ_c^+
	1^-	4324	D^-	$\Sigma_c^{++}(2455)$
	1^-	4387	D^-	$\Sigma_c^{++}(2520)$
	0^+	4448	χ_{c1}	p
$\frac{5}{2}^-$	2^+	3921	$\eta_c(1s)$	p
	2^+	4034	J/ψ	p
	1^-	4448	χ_{c1}	p
$\frac{5}{2}^+$	3^-	3921	$\eta_c(1s)$	p
	1^-	4034	J/ψ	p
	1^-	4293	$D^{*0}(\bar{2}007)$	Λ_c^+
	1^-	4387	D^-	$\Sigma_c^{++}(2520)$

Question we address: Do Pc resonances appear in one-channel N J/ψ scattering on the lattice (in approximation where this channel is decoupled from other channels)

$$N\ J/\psi \rightarrow P_c \rightarrow N\ J/\psi$$

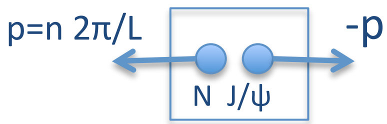
Note: we make only a first step towards Pc ; lots remains to be done

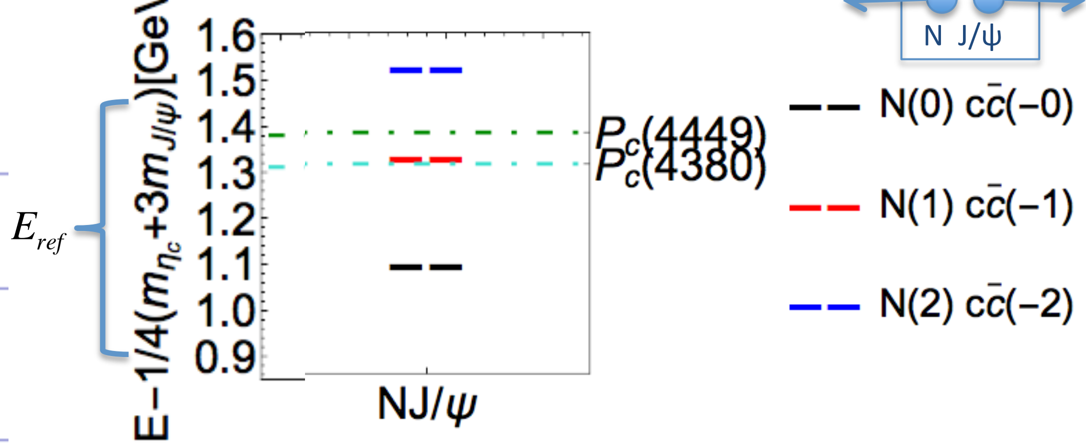
We consider $P_{tot}=0$ since parity is good quantum number in this case

Aim: extract all eigen-states up to **N(p) J/ψ(-p)** for all irreps: p<=2

$$E_{ref} = \tfrac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$

$N_L=16$, a=0.12 fm





Lattice setup

In order to not to get to many N(p) J/Ψ(-p) states below Pc :

small L is welcome for exploratory simulations

16^3 x 32 , a=0.124 fm, L≈2 fm

N_f=2, m_π=266 MeV

Wilson clover, charm quarks: Fermilab approach: $E-E_{ref}$

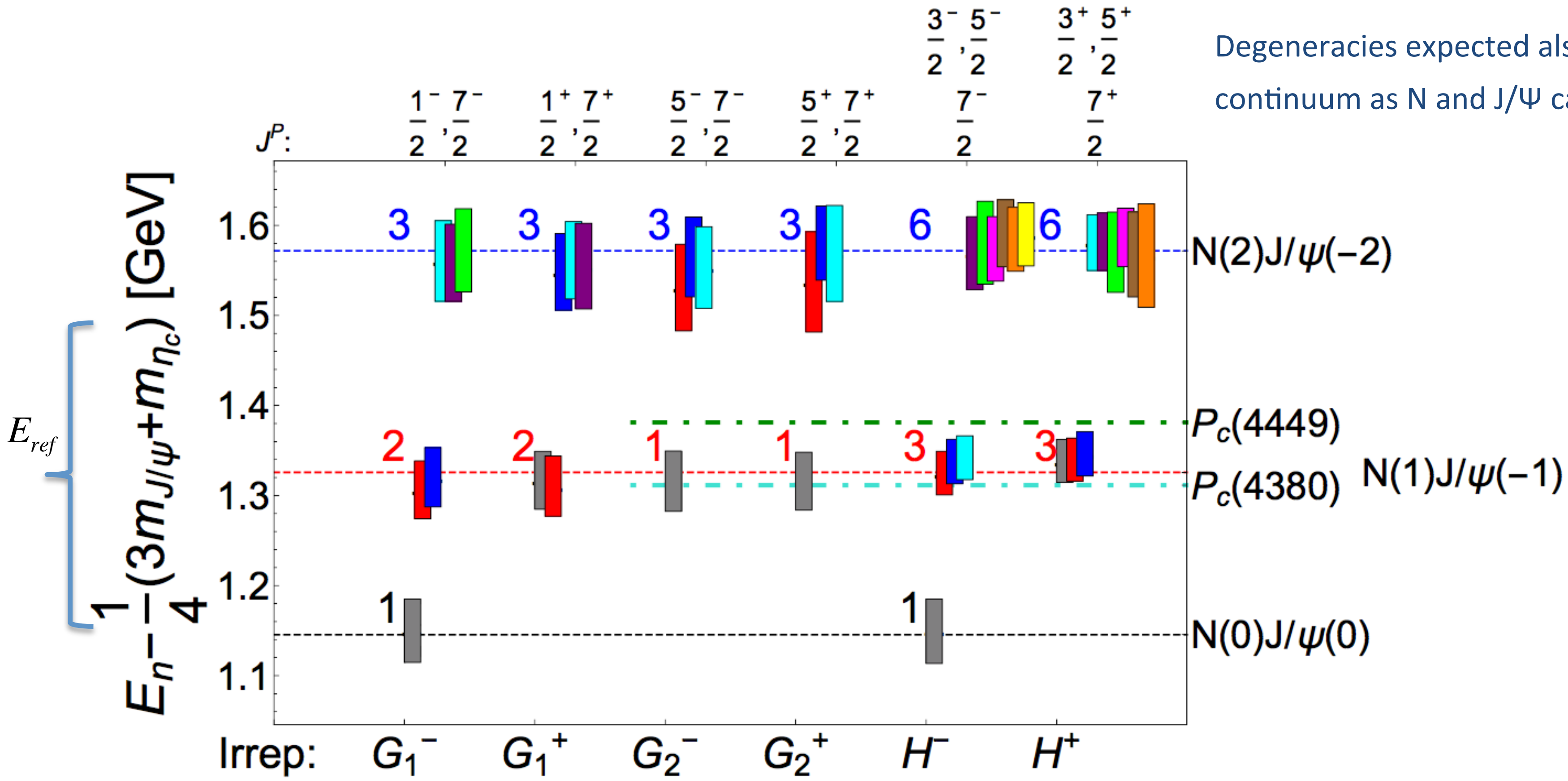
$$E_{ref} = \tfrac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$

Full distillation: N_v=96 for charm quarks

N_v=48 for light quarks

Glimpse at results: N J/Ψ eigen-energies for all irreps
Why there are so many almost-degenerate states?

Degeneracies expected also in continuum as N and J/Ψ carry spin



linearly independent N J/Ψ (non-interacting)

favourite for narrow Pc

example G_2^+ : $N(1/2^+)$ $J/\psi(1^-)$

S=3/2 L=1
S=1/2 L=3
$J^P=5/2^+$ two degenerate eigenstates expected even in continuum

S=1/2 L=3
$J^P=7/2^+$

three degenerate eigenstates expected in G2-
G_2^+

Analogous degeneracies observed by HSC in J/Ψπ and ρπ scattering

operators for N J/Ψ scattering

O="N_{ms} (p) V_i(-p)"

$$V_i(p) = \sum_x \bar{q}\,\Gamma\,\gamma_i\,q\,e^{ipx} \qquad i = x, y, z \qquad \Gamma : (I, \gamma_4)$$

$$N_{\pm 1/2}(p) = \sum_x \epsilon_{abc}\; P_+ \Gamma q_{\frac{1}{2}}\; [q^{bT} \tilde{\Gamma} q^c]\; e^{ipx}$$

$$(\Gamma, \tilde{\Gamma}) : (\mathbb{1}, C\gamma_5),\; (\gamma_5, C),\; (\mathbb{1}, \imath\gamma_4 C\gamma_5)$$

How to combine them (i,ms) to make correct quantum numbers ?

irrep Γ^P	J^P
$G_1^\pm$	$\frac{1}{2}^\pm, \frac{7}{2}^\pm$
$G_2^\pm$	$\frac{5}{2}^\pm, \frac{7}{2}^\pm$
$H^\pm$	$\frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm$

Method 1: projection method:

$$O_{\Gamma,r} = \sum_R T^\Gamma_{rr}(R)\;\; R\; N_{m_s}(p)\; V_i(-p)\; R^{-1}$$

Method 1: operators with projection method

$$O_{\Gamma,r} = \sum_R T^{\Gamma}_{rr}(R) \;\; R\, N_{m_s}(p)\; V_i(-p)\; R^{-1}$$

Example: p=1, H^+ S. P., U.S., C.B. Lang ; JHEP 2017(1), 129

$$O_{H^+,r=1,n=1} = N_{-\frac{1}{2}}(e_x)V_x(-e_x) - N_{-\frac{1}{2}}(-e_x)V_x(e_x) - N_{-\frac{1}{2}}(e_y)V_y(-e_y) + N_{-\frac{1}{2}}(-e_y)V_y(e_y)$$

$$\begin{aligned} O_{H^+,r=1,n=2} = & -N_{-\frac{1}{2}}(e_x)V_y(-e_x) + N_{-\frac{1}{2}}(-e_x)V_y(e_x) + iN_{\frac{1}{2}}(e_x)V_z(-e_x) - iN_{\frac{1}{2}}(-e_x)V_z(e_x) \\ & -N_{-\frac{1}{2}}(e_y)V_x(-e_y) + N_{-\frac{1}{2}}(-e_y)V_x(e_y) + N_{\frac{1}{2}}(e_y)V_z(-e_y) - N_{\frac{1}{2}}(-e_y)V_z(e_y) \\ & -2iN_{\frac{1}{2}}(e_z)\left(V_x(-e_z) - iV_y(-e_z)\right) + 2N_{\frac{1}{2}}(-e_z)\left(V_y(e_z) + iV_x(e_z)\right) \end{aligned}$$

$$\begin{aligned} O_{H^+,r=1,n=3} = & -N_{-\frac{1}{2}}(e_x)V_y(-e_x) + N_{-\frac{1}{2}}(-e_x)V_y(e_x) - 2iN_{\frac{1}{2}}(e_x)V_z(-e_x) + 2iN_{\frac{1}{2}}(-e_x)V_z(e_x) \\ & -N_{-\frac{1}{2}}(e_y)V_x(-e_y) + N_{-\frac{1}{2}}(-e_y)V_x(e_y) - 2N_{\frac{1}{2}}(e_y)V_z(-e_y) + 2N_{\frac{1}{2}}(-e_y)V_z(e_y) \\ & +N_{\frac{1}{2}}(e_z)\left(V_y(-e_z) + iV_x(-e_z)\right) - iN_{\frac{1}{2}}(-e_z)\left(V_x(e_z) - iV_y(e_z)\right) \end{aligned}$$

3 linearly independent operators at p=1: three degenerate state expected in non-interacting limit

Drawback: no info on which (L,S) are related to these three operators

Method 2: operators with partial-wave method

$$O^{|p|,J,m_J,L,S} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S}\, C^{Sm_S}_{s_1 m_{s1},s_2 m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp}) N_{m_{s1}}(Rp) V_{m_{s2}}(-Rp)$$

CalLat: Berkowitz, et. all PLB , 2016(12) 024;
proof of transform. properties S. P., U.S., C.B. Lang ; JHEP 2017

$$R_a O^{J,m_J,S,L} R_a^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R_a^{-1}) O^{J,m'_J,S,L}$$

subduction to irrep

S : HSC; PRD 2010(82), 034508

$$O^{[J,L,S]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r}\, O^{|p|,J,m_J,L,S}$$

Example: p=1, H^+ : 3 linearly independent operators -> 3 degenerate eigenstates in non-int. limit

$$O^{[J=\frac{3}{2},L=1,S=\frac{3}{2}]}_{H+,r=1} = 3\ O_{H+,r=1,n=1} + i(4\ O_{H+,r=1,n=2} - O_{H+,r=1,n=3})$$

$$O^{[J=\frac{3}{2},L=1,S=\frac{1}{2}]}_{H+,r=1} = O^{[J=\frac{5}{2},L=3,S=\frac{1}{2}]}_{H+,r=1} = 3\ O_{H+,r=1,n=1} + i(O_{H+,r=1,n=2} + 2\ O_{H+,r=1,n=3})$$

$$O^{[J=\frac{3}{2},L=3,S=\frac{3}{2}]}_{H+,r=1} = O^{[J=\frac{5}{2},L=1,S=\frac{3}{2}]}_{H+,r=1} = 3\ O_{H+,r=1,n=1} - i(O_{H+,r=1,n=2} + O_{H+,r=1,n=3})$$

$$O^{[J=\frac{5}{2},L=3,S=\frac{3}{2}]}_{H+,r=1} = 12\ O_{H+,r=1,n=1} + i(O_{H+,r=1,n=2} - 4\ O_{H+,r=1,n=3})$$

Number of degenerate N J/Ψ eigen-states in non-interacting limit
=
Number of linearly independent N J/Ψ operators

explicit expressions for all p=0,1 operators

S. P., U.S., C.B. Lang ; JHEP 2017

irrep	$N(p)J/\psi(-p)$		
	$p^2=0$	$p^2=1$	$p^2=2$
G_1^+	0	2	3
G_1^-	1	2	3
G_2^+	0	1	3
G_2^-	0	1	3
H^+	0	3	6
H^-	1	3	6

for each of
this operator-type
we use two vector
and three nucleon choices:
6 =2*3 times more operators
than number of states expected
in non-interacting limit

→ from previous two pages

General remark on two-hadron operators

Explicit expressions all for $H^{(1)}(p)H^{(2)}(-p)$

- PV, PN, VN, NN

- in three methods (projection, partial-wave, helicity)

- including proofs for all methods

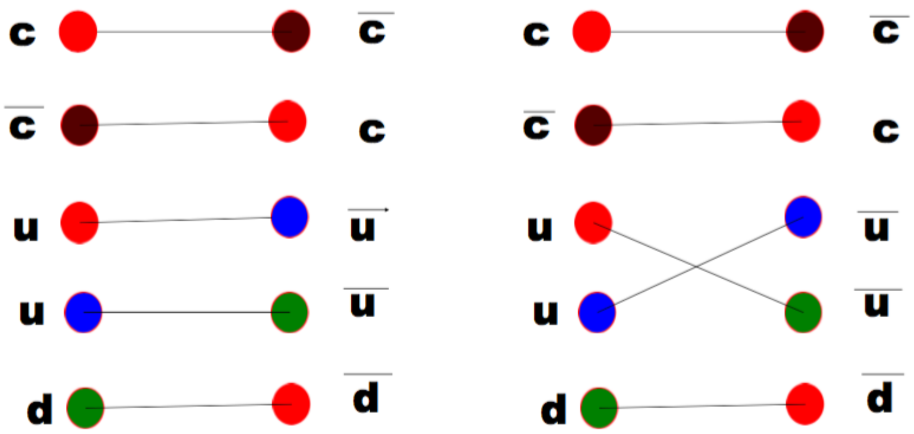
- all irreps, $|p|=0,1$

given in [S. P., U. Skerbis, C.B. Lang, arXiv:1607:06738, JHEP 2016]

operators from three methods are consistent (not equal) with each other

Correlation matrices for N J/ψ system

$O = \Sigma \; N_{ms} (p) \, V_i(-p)$



Wick contractions:

no quark line connects N and/ Jψ

charm annihilation omitted

$$C = \sum \left\langle \langle 0| N_{m'_s}(p') \bar{N}_{m_s}(p) |0\rangle \langle 0| V_{i'}(p') V_i^\dagger(p) |0\rangle \right\rangle$$

separately pre-calculated for all momenta and polarizations

Correlation matrices and eigenstates

$$C_{ij}(t) = \langle 0|O_i(t)\bar{O}_j(0)|0\rangle =$$

$$= \sum_{n=1}^{\mathrm{N}} e^{-E_n t}\langle 0|O_i|n\rangle\langle n|\bar{O}_j|0\rangle, \;\; i,j = 1,\ldots,\mathrm{N}$$

Number of interpolators N in each irrep

irrep	G_1^-	G_1^+	G_2^-	G_2^+	H^-	H^+
NJ/ψ	6×6	5×6	4×6	4×6	10×6	9×6

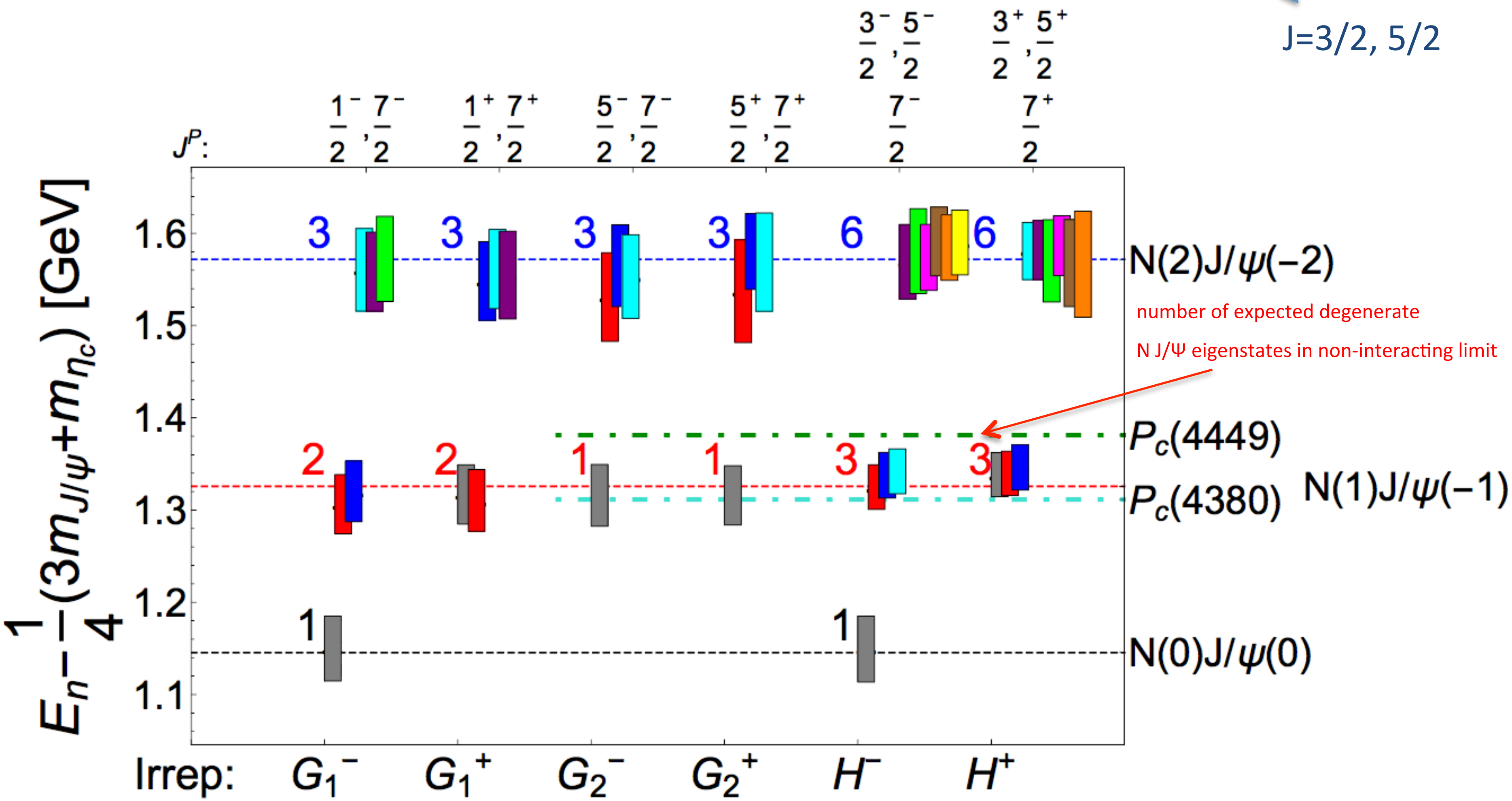
extracting eigen-energies using GEVP

$$C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$$

$$\lambda^{(n)}(t)_{\mathrm{large\ t}} = A_n e^{-E_n t}$$

Result: N J/Ψ eigen-energies for all irreps (including Pc channels)

J=3/2, 5/2



- E consistent with non-interacting energies (dashed lines); no significant energy shifts observed
- number of states consistent with non-interacting case: carefully constructed operators crucial for this
- no additional eigenstate (related to Pc) observed

Analytic prediciton of E_n based on P_c assuming coupling only to N J/Ψ

Relation between E_n and δ for arbitrary spin [Briceno, PRD89, 074507 (2014)]

S= I + i M = $e^{2i\delta}$

$$\det_{\mathrm{oc}}\left[\det_{lSJm_J}\left[\mathcal{M}^{-1}+\delta\mathcal{G}^V\right]\right]=0$$

$c \propto Z_{00}$

$$[\delta\mathcal{G}_j^V]_{Jm_J,lS;J'm_{J'},l'S'}=\frac{ik_j^*\delta_{SS'}}{8\pi E^*}n_j\left[\delta_{JJ'}\delta_{m_Jm_{J'}}\delta_{ll'}+i\sum_{l'',m''}\frac{(4\pi)^{3/2}}{k_j^{*l''+1}}c^{\mathbf{d}}_{l''m''}(k_j^{*2};L)\right.$$
$$\left.\times\sum_{m_l,m_{l'},m_S}\langle lS,Jm_J|lm_l,Sm_S\rangle\langle l'm_{l'},Sm_S|l'S,J'm_{J'}\rangle\int d\Omega\, Y^*_{l,m_l}Y^*_{l'',m''}Y_{l',m_{l'}}\right]$$

assume that P_c resides only in a single partial wave (L, S) and that there is no interaction in the other channels.

$$\cot\delta_{(L,S)}=\frac{2Z_{00}(1;\ p^2(\frac{2\pi}{\mathrm{L}})^2)}{\sqrt{\pi}\ \mathrm{L}\ p}$$

Luscher's relation between E_n and δ

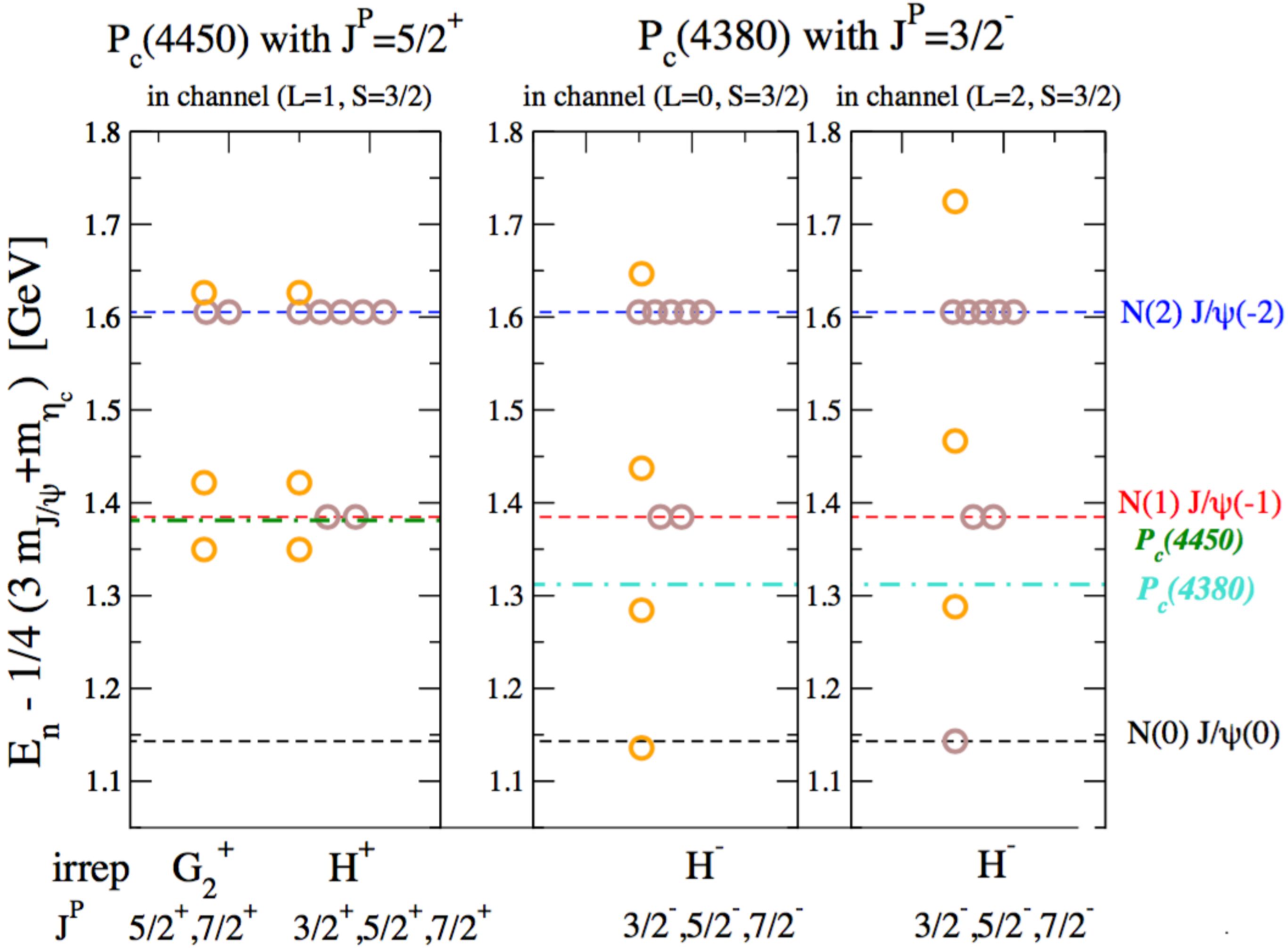
BW form for P_c assumed, M_{Pc} and Γ_{Pc} taken from exp , E_n predicted

$$\cot\delta_{(L,S)}=\frac{M_{P_c}-E^2}{E\ \Gamma(E)}\qquad \Gamma(E)=\Gamma_{P_c}\left(\frac{p(E)}{p(M_{P_c})}\right)^{2L+1}\frac{M_{P_c}^2}{E^2}$$

Analytic prediciton based on $P_c(4449)$ or $P_c(4380)$ assuming coupling only to N J/Ψ

$$\cot\delta_{(L,S)} = \frac{M_{P_c} - E^2}{E\,\Gamma(E)} = \frac{2Z_{00}(1;\ p^2(\frac{2\pi}{L})^2)}{\sqrt{\pi}\ L\ p}$$

$$\Gamma(E) = \Gamma_{P_c}\left(\frac{p(E)}{p(M_{P_c})}\right)^{2L+1}\frac{M_{P_c}^2}{E^2}\ .$$

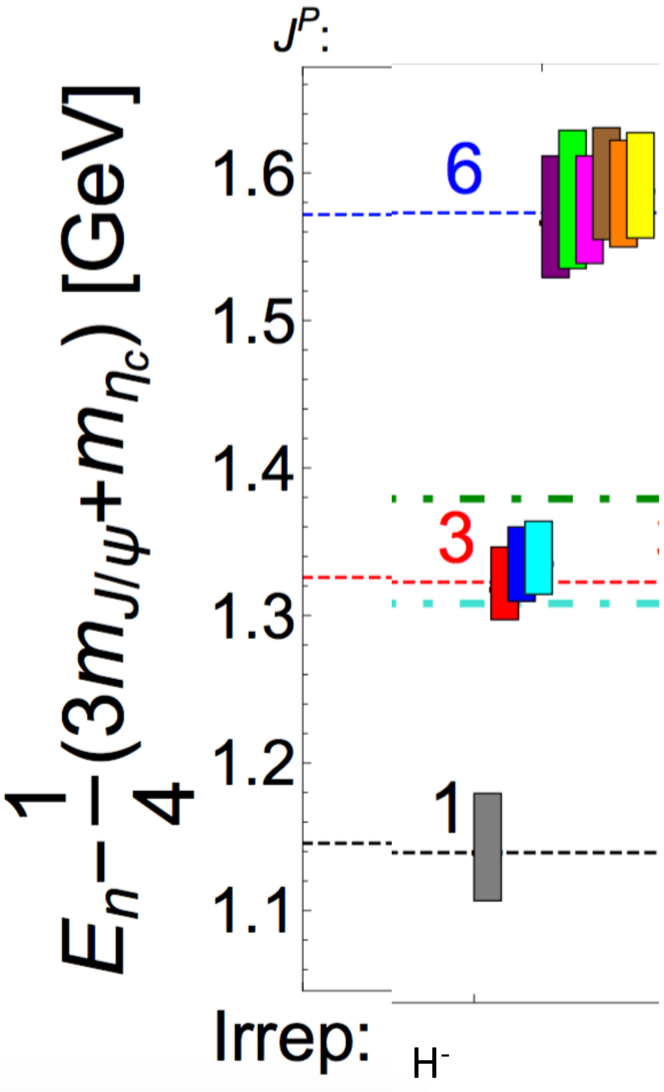


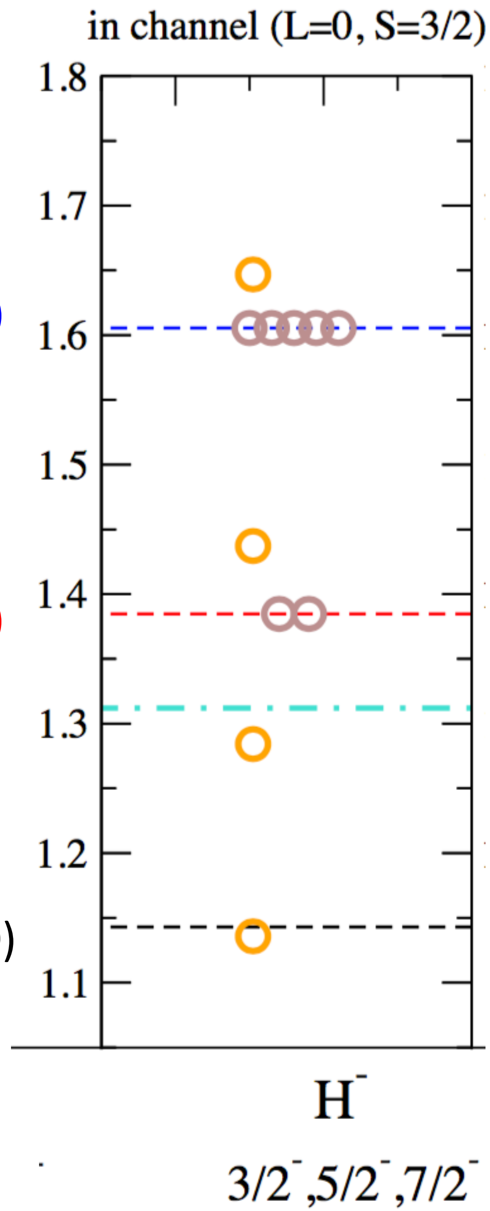
Predicts one extra level with respect to interacting case

Comparing lattice data and analytic prediction for one-channel Pc(4380) with $J^P = 3/2^-$

10=1+3+6 states expected in non-interacting limit

10 states found





11 states expected in case of Pc

Conclusions concerning P_c (so far)

- Lattice spectra do not support the scenario where a P_c resonance couples only to N J/Ψ decay channel and is decoupled from other channels.
- Lattice results indicate that **the existence of Pc resonance within one-channel N J/Ψ scattering is not favored in QCD**.
- This might **suggest that the strong coupling between the N J/Ψ with other channels might be responsible for the existence of the Pc resonances in experiment.**
- Future lattice simulations of the coupled-channel scattering will be needed to confirm or refute this hypotheses.

ongoing analysis of one-channel and coupled-channel scattering

Collins, Padmanath, Piemonte, Mohler, S.P. (preliminary)

Vector resonance: one-channel scattering D^+D^-

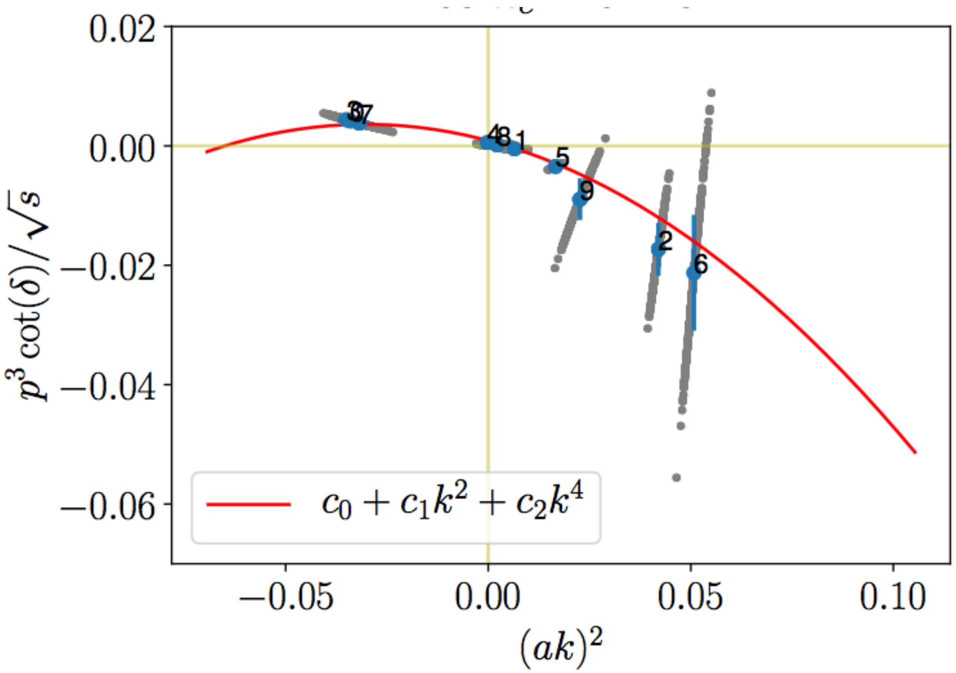
$\psi(3770) \rightarrow D^+ D^-$

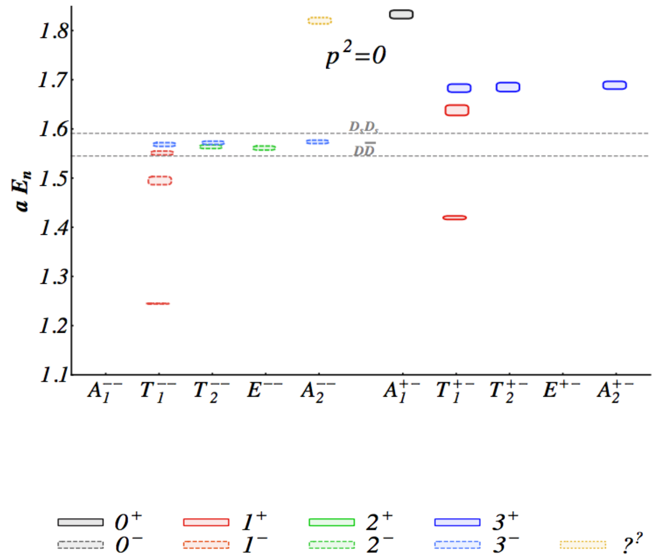
Study of one-channel DD scattering:

p=0 and p≠0 crucial to get that many phase-shift points

scattering matrix $S(k)=e^{2 i \delta(k)}$ δ(k)=p-wave phase shift

preliminary (Lattice 2018)





crucial to know

which of the energy-levels

corresponds to 3^-

and exclude it from analysis

of p-wave phase shift

simplified case when only one partial-wave L contributes

One channel scattering

$$S(E) = e^{2i\delta(E)} = I + 2iT$$

Luscher's equation: $$f[E_n, \delta(E_n)] = 0 : \quad E_n \to \delta(E_n)$$

Two coupled channel scattering

a: O=H_1 H_2

b: O=H'_1 H'_2

↓

E_n

$$S(E) = \begin{vmatrix} \underset{a \to a}{\eta(E)\, e^{2i\delta_a(E)}} & \underset{a \to b}{i\sqrt{1-\eta^2(E)}e^{i(\delta_a(E)+\delta_b(E))}} \\ \underset{b \to a}{i\sqrt{1-\eta^2(E)}e^{i(\delta_a(E)+\delta_b(E))}} & \underset{b \to b}{\eta(E)\, e^{2i\delta_b(E)}} \end{vmatrix} = I + 2iT(E)$$

generalized Luscher's (det) eq.:

1 equation with three unknowns

$$f[E_n, \delta_1(E_n), \delta_2(E_n), \eta(E_n)] = 0 : \quad E_n \to ??$$

Parametrizing T matrix and

determine parameters from the fit to all E_n

$$\mathrm{Re}[T_{ij}^{-1}(E)] = a_{ij} + b_{ij}E^2 + c_{ij}E^4 + \ldots$$

$$\text{fit to all } \mathrm{E}_n : \quad \text{values } \mathrm{a}_{ij}, b_{ij}, c_{ij}$$

Collins, Padmanath, Piemonte, Mohler, S.P. (preliminary)

Scalar resonances: coupled-channel scattering D^+D^- - $D_s^+D_s^-$

- not settled yet which is first excited scalar charmonia
- testing group before attacking more exotic channels
- CLS ensembles, two volumes, three different total momenta

$$O \approx DD = (\bar{c}u)(\bar{u}c) + (\bar{c}d)(\bar{d}c)$$

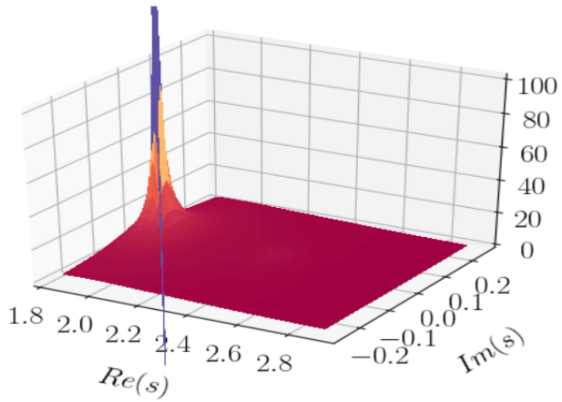
$$D_s D_s = (\bar{c}s)(\bar{s}c)$$

preliminary

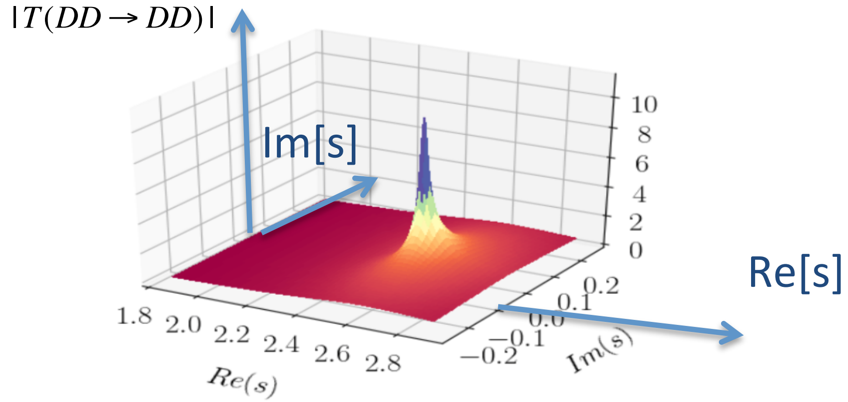
Lattice 2018

|T(DD->DD)|

on four sheets

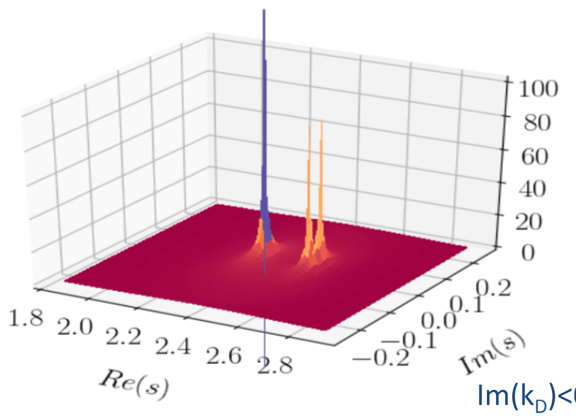


(a) I (+,+) $Im(k_D)>0$, $Im(k_{Ds})>0$

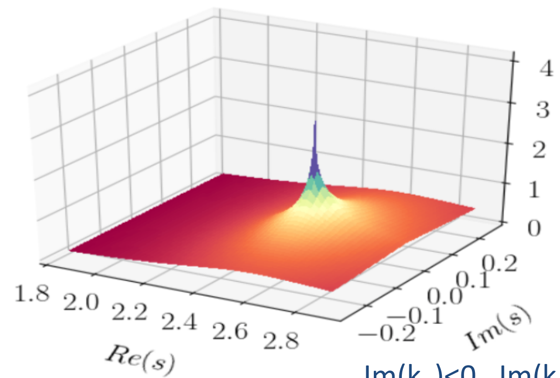


(b) II (+,-) $Im(k_D)>0$, $Im(k_{Ds})<0$

$s=E^2_{cm}$



$Im(k_D)<0$, $Im(k_{Ds})>0$



$Im(k_D)<0$, $Im(k_{Ds})<0$

Conclusions

- “Stable” charmonia at rest and in flight: E and J^{PC} could be determined for J<=3

  tip: consider many irreps, carefully construct O, analyze overlaps

- effects of strong decays and thresholds on charmonia: underway

- Charmed pentaquark: lattice results indicate that the existence of Pc resonance within a one-channel N J/Ψ scattering is not favored in QCD.

- Scattering of hadrons with spins: several almost-degenerate eigenstates appear

- Simulation of several coupled-channels in relation to Pc resonances: open challenge

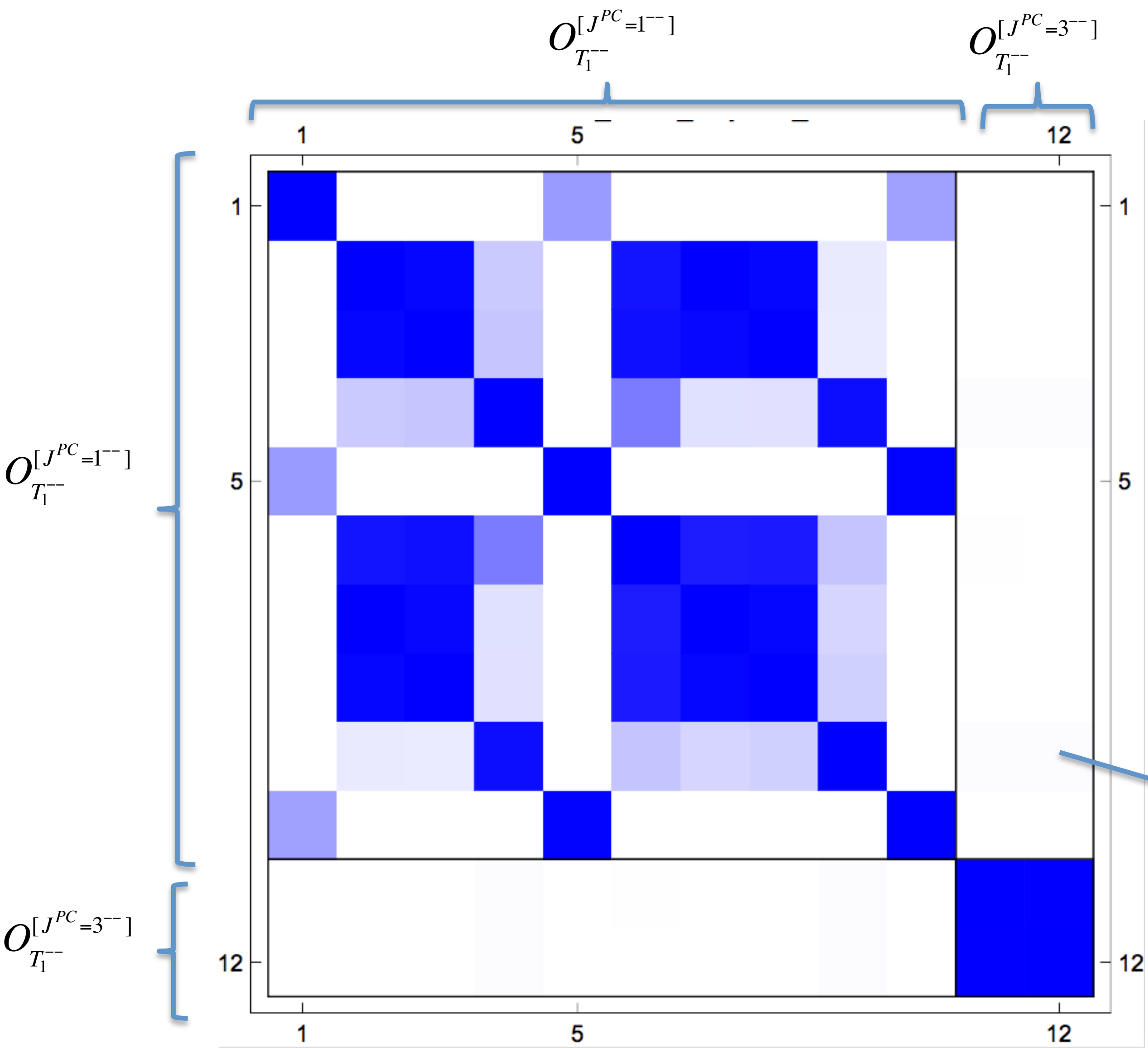
  stastistical noise related o baryons needs to be reduced

  $P_{tot} \neq 0$ would render more info on S, but parity is lost ...

Backup

Breaking of continous rotational symmetry is small

$\mathbf{p}=(0,0,0),\ O_h,\ P=\pm$	
$\Lambda\ (dim)$	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3



$$\left(C_{ij}/\sqrt{C_{ii}C_{jj}}\right)$$

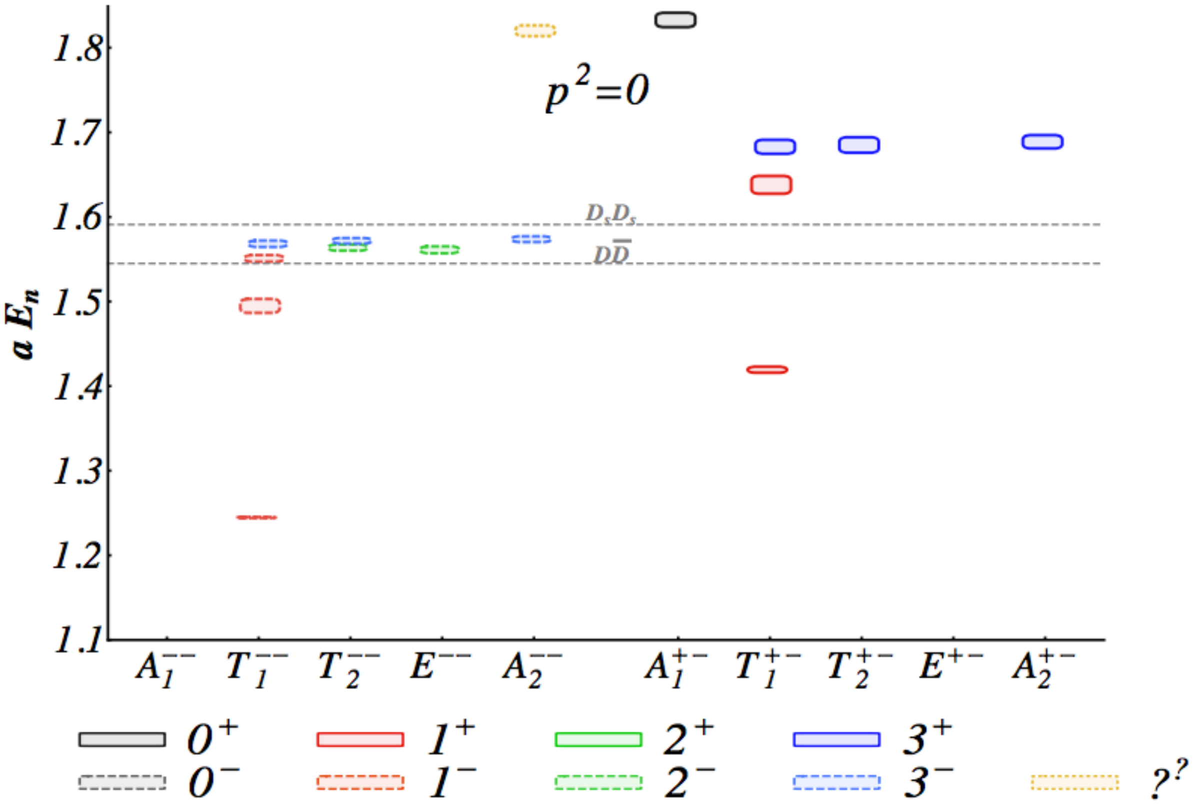
at t=5; i,j=1,..,12

non-zero but small

$\mathbf{p}=0,\ O_h,\ \Lambda^{PC}$							
A_1^{++}	5	A_1^{+-}	3	A_1^{-+}	7	A_1^{--}	2
T_1^{++}	8	T_1^{+-}	8	T_1^{-+}	4	T_1^{--}	12
T_2^{++}	6	T_2^{+-}	4	T_2^{-+}	5	T_2^{--}	5
E^{++}	5	E^{+-}	3	E^{-+}	5	E^{--}	3
A_2^{++}	1	A_2^{+-}	0	A_2^{-+}	1	A_2^{--}	2
$\mathbf{p}=(0,0,1),\ Dic_4,\ \Lambda^C$				$\mathbf{p}=(1,1,0),\ Dic_2,\ \Lambda^C$			
A_1^+	14	A_1^-	18	A_1^+	25	A_1^-	27
A_2^+	20	A_2^-	12	A_2^+	31	A_2^-	21
B_1^+	11	B_1^-	9	B_1^+	23	B_1^-	29
B_2^+	11	B_2^-	9	B_2^+	23	B_2^-	29
E^+	23	A^-	29				

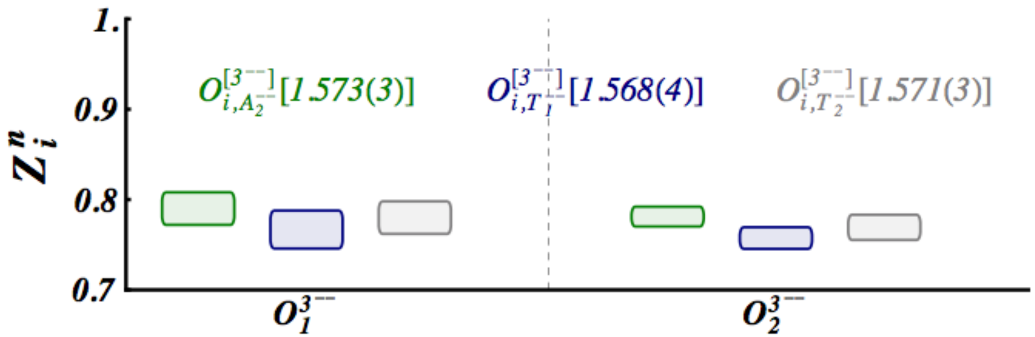
TABLE II. Number of interpolators with up to two derivatives used in computing correlation matrices of each lattice irrep in the rest frame (top) and in the moving frames (bottom) with momentum $\mathbf{p}=(0,0,1)$ on the left and $\mathbf{p}=(1,1,0)$ on the right.

Tests with degeneracies of Z-factors



$\mathbf{p}=(0,0,0)$, O_h, $P=\pm$	
Λ (*dim*)	J
A_1 (1)	0
T_1 (3)	1, 3
T_2 (3)	2, 3
E (2)	2
A_2 (1)	3

C=-1



$$\langle O^{[J^{PC}]}_{i,\Lambda_1^C}|\mathbf{0},J^{PC}\rangle \simeq \langle O^{[J^{PC}]}_{i,\Lambda_2^C}|\mathbf{0},J^{PC}\rangle.$$

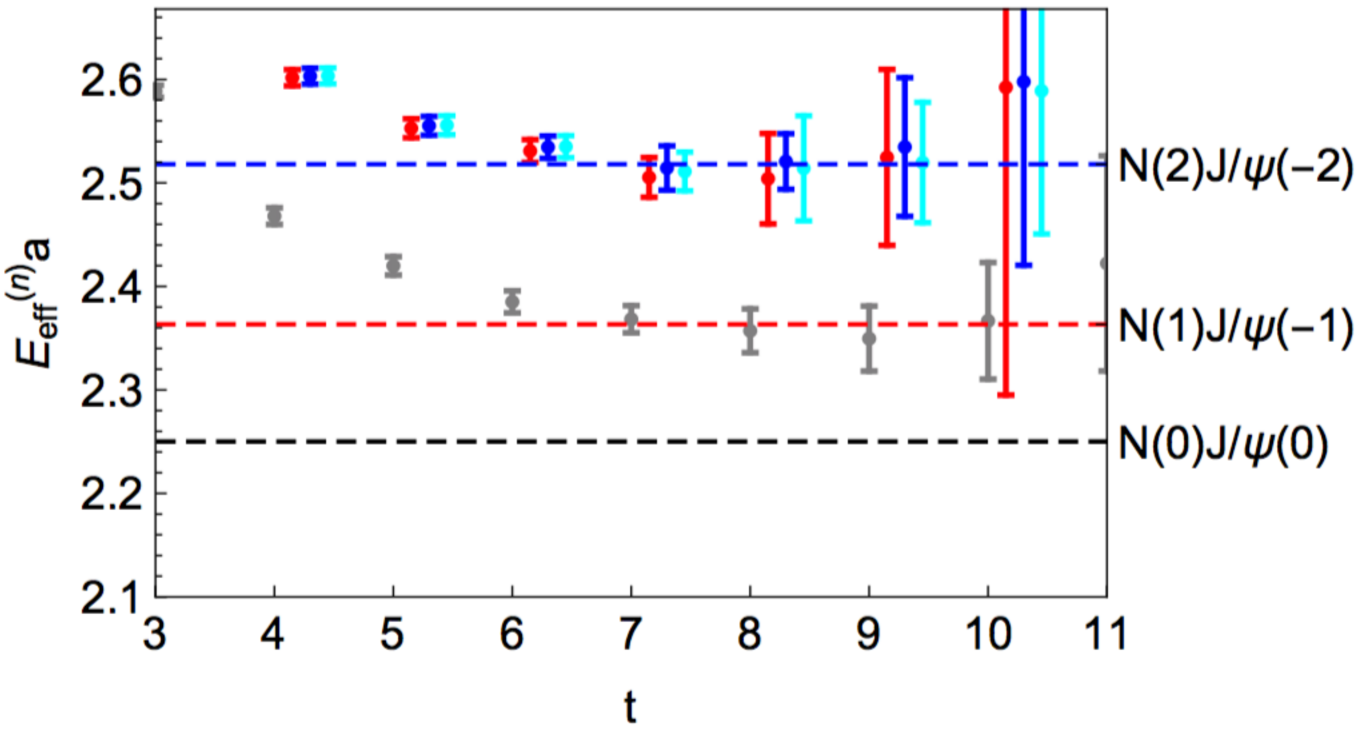


FIG. 3. Effective energies (13) for NJ/ψ system in G_2^+ irrep. This gives the eigen-energy E_n in the plateau region. We observe all $N(p)J/\psi(-p)$ eigenstates, expected in the non-interacting limit: this number is 0, 1 and 3 states for $p^2 =$ 0, 1 and 2, respectively (Table II). No additional eigenstate is found. The non-interacting energies (1) are indicated by the dashed lines.

Charmonium -> $D^+ D^-$

study of DD scattering and determining scattering matrix $S(E_{cm})$

study with p=0 and p≠0 to get more info on S

p=0

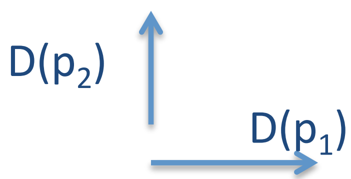
Luscher's relation

E^{lat} -> $S(E^{lat})$

$$E^{lat} \approx \sqrt{m_D^2 + \vec{p}^2} + \sqrt{m_D^2 + (-\vec{p}_1)^2}$$

$$\vec{p}_1 = \vec{n}_1 \, \tfrac{2\pi}{N_L}$$

p=p1+p2≠0



generalized Luscher's relation

E^{lat} -> $S(E^{lat}_{cm})$

$$(E^{lat}_{cm})^2 \approx E^2 - \vec{p}^2 = (\sqrt{m_D^2 + \vec{p}_1^2} + \sqrt{m_D^2 + \vec{p}_2^2})^2 - (\vec{p}_1 + \vec{p}_2)^2$$

$$\vec{p}_1 = \vec{n}_1 \, \tfrac{2\pi}{N_L}, \quad \vec{p}_2 = \vec{n}_2 \, \tfrac{2\pi}{N_L}$$

Remember: no scattering and decay in present work; this is only motivation to study charmonia in flight