

Lattice hadron spectroscopy at zero and finite temperature

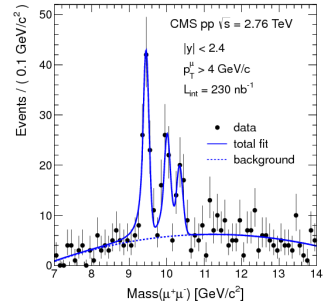
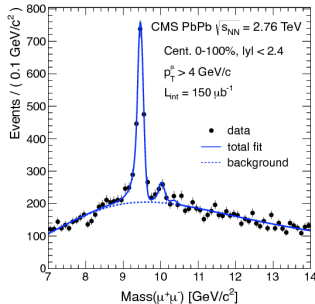
Sinéad M. Ryan
Trinity College Dublin



From Euclidean spectral densities to real time physics, CERN, March 2019

QUARKONIA AND THE QGP - A SHORT HISTORY FROM EXPERIMENTS

- Charmonium suppression observed at SPS (2000) - QGP discovered!
- RHIC ('06,'07) ... Not really ... previous theoretical analysis too crude. Charmonium suppression not enough - competing effects.
- RHIC+LHC: High precision charmonium AND bottomonium - CMS Υ spectrum (2012) compares PbPb collisions (left) with pp collisions (right).



- $\Upsilon(1S)$ survives, $\Upsilon(2S, 3S)$ suppressed. Sequential melting. Υ “cleaner” than J/ψ ... look there! A rich playground for theory.

QUARKONIA AND THE QGP - THEORY

How can we probe the response of quarkonia to the QGP?

- Via potential models
 - Model the heavy quark potential, solve Schrödinger equation
 - Determine the potential via lattice QCD, solve Schrödinger equation
- Via lattice QCD
 - **Calculate quarkonium spectrum directly in finite T lattice QCD**

The situation for both approaches at $T > 0$:

- (an embarrassing) Plethora of potential models with seemingly conflicting results.
- (relativistic) Lattice simulations hindered by thermal (periodic) boundary conditions.

New ideas involve effective field theories (EFTs) and exploiting the separation of scales.

Motivates a lattice EFT approach to Bottomonium at finite T: NRQCD

Lattice preliminaries

**Making measurements at $T=0$:
operators and propagators**

EXCITED STATES

To go beyond ground state spectroscopy the method of choice is the **variational idea**: find operator ϕ to maximise $C(t)/C(t_0)$ from sum of basis operators $\phi = \sum_a v_a \phi_a$

[C. Michael & I. Teasdale. NPB215 (1983) 433]; [M. Lüscher & U. Wolff. NPB339 (1990) 222]

Variational method

If we can measure $C_{ab}(t) = \langle 0 | \phi_a(t) \phi_b^\dagger(0) | 0 \rangle$ for all a, b and solve the generalised eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)},$$

then

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t} [1 + O(e^{-\Delta E t})]$, $t \geq t_0/2$ - gives principal correlator.
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_i^{(n)\dagger} C_{ji}(t_0)$.

For this to be practical, we need

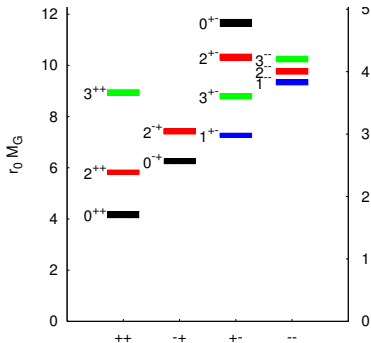
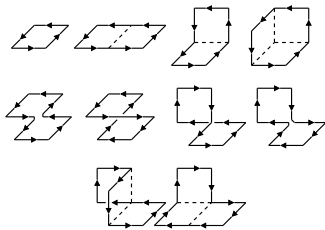
- a “good” basis set that resembles the states of interest
- all elements of this correlation matrix measured

[Blossier et al JHEP 0904 (2009) 094]

GLUEBALLS - AN EARLY SUCCESS

- QCD nonAbelian \Rightarrow allows bound states of glue
- Candidates observed experimentally: $f_0(1370)$, $f_0(1500)$, $f_0(2220)$
- Glueballs can be calculated in lattice QCD
- The interpolating fields are purely gluonic, built from Wilson loops

[Morningstar and Peardon]



DESIGNING GOOD LATTICE OPERATORS - A TALE OF TWO SYMMETRIES



- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
- Classify states by irreps of O_h and relate by subduction to J values of $O(3)$.
- 5 irreps of $O(3)$ and an infinite number for J^P so values are distributed across lattice irreps.

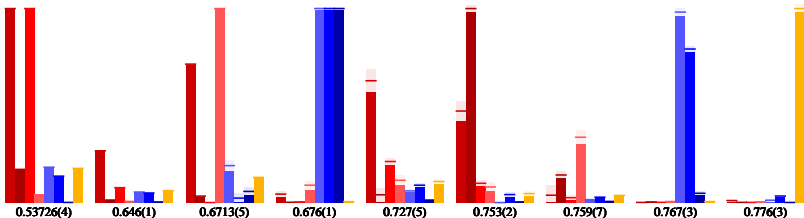
- start with continuum operators, built from n derivatives:

$$\phi = \bar{\psi} \Gamma (D_{i_1} D_{i_2} D_{i_3} \dots D_{i_n}) \psi$$
- Construct irreps of $SO(3)$, then subduce these representations to O_h
- Replace derivative with lattice differences:
- A subduced irrep carries a “memory” of continuum spin J from which it was subduced - it **overlaps** predominantly with states of this J .

J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

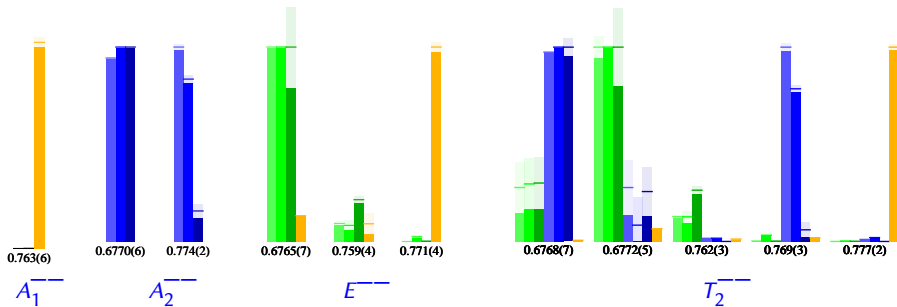
USING GEVP EIGENVALUES - OPERATOR OVERLAPS

- Using $Z = \langle 0 | \Phi | k \rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^- irrep, colour-coding is **Spin 1**, **Spin 2**, **Spin 3** and **Spin 4**.



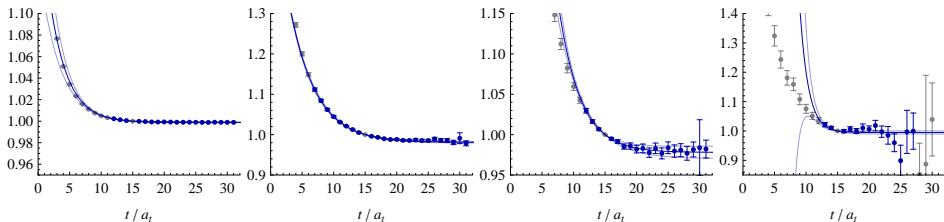
and the rest of the spin 4 state ...

- All polarisations of the spin-4 state are seen
- Spin labelling:



FITTING PRINCIPAL CORRELATORS

- Typical fits for a set of excited states in the T_1^- irrep in charmonium (26 operators!) are



- plotting $\lambda_n(t) \cdot e^{m_n(t_1-t_0)}$ with $t_0 = 15$.
- Expect a plateau at 1.0 if single-exp dominates.
- Anisotropy helps the variational method: improving resolution in the temporal direction.
- Smearing crucial.

Distillation

DISTILLATION DISTILLED...

- A smeared quark field is $\tilde{\psi} = \square\psi$.
- **Define** smearing to be explicitly a very low-rank operator. Rank is $N_D (\ll N_S \times N_C)$.

Distillation operator

$$\square(t) = V(t)V^\dagger(t)$$

with $V_{\underline{x},c}^a(t)$ a $N_D \times (N_S \times N_C)$ matrix

- Example (used to date): \square_∇ the **projection operator into \mathcal{D}_∇ , the space spanned by the lowest eigenmodes of the 3-D laplacian**
- Eigenvectors of ∇^2 not the only choice...

- Consider an isovector meson two-point function:

$$C_M(t_1 - t_0) = \langle\langle \bar{u}(t_1) \square_{t_1} \Gamma_{t_1} \square_{t_1} d(t_1) \quad \bar{d}(t_0) \square_{t_0} \Gamma_{t_0} \square_{t_0} u(t_0) \rangle\rangle$$

- Substituting the low-rank distillation operator \square reduces this to a **much smaller** trace:

$$C_M(t_1 - t_0) = \langle \text{Tr}_{\{\sigma, \mathcal{D}\}} [\Phi(t_1) \tau(t_1, t_0) \Phi(t_0) \tau(t_0, t_1)] \rangle$$

- $\Phi_{\beta, b}^{\alpha, a}$ and $\tau_{\beta, b}^{\alpha, a}$ are $(N_\sigma \times N_{\mathcal{D}}) \times (N_\sigma \times N_{\mathcal{D}})$ matrices.

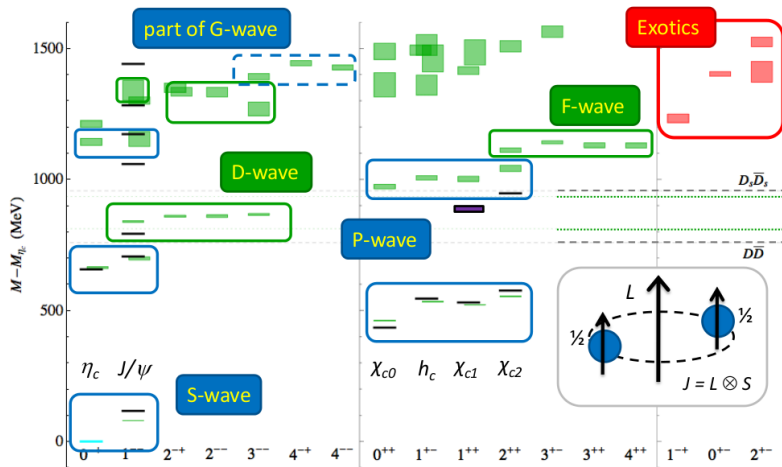
$$\Phi(t) = V^\dagger(t) \Gamma_t V(t)$$

$$\tau(t, t') = V^\dagger(t) M^{-1}(t, t') V(t')$$

The “perambulator”

- Note that propagation (via perambulators) and operator construction are separated. Perambulators can be stored and reused for any later operators of interest.

CHARMONIUM WITH DISTILLATION

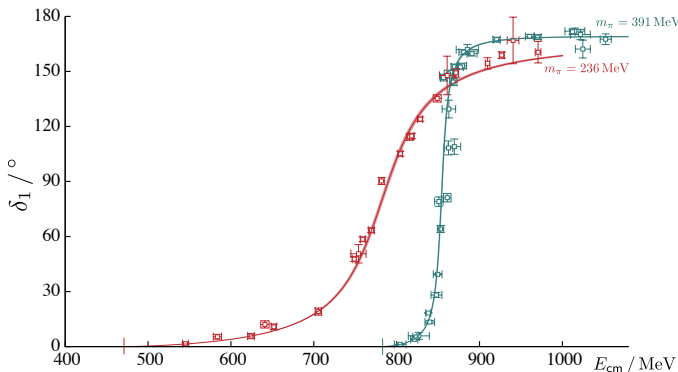


$N_f = 2 + 1$, $m_\pi \sim 400\text{MeV}$. Single hadron operators ie no scattering analysis. Lattice irreps have $\sim 10\text{-}20$ operators in basis.

EXTRACTING SCATTERING INFORMATION

- Use the finite volume as a tool
- Related **lattice energy levels in a finite volume** to a decomposition of the scattering amplitude in **partial waves in infinite volume**

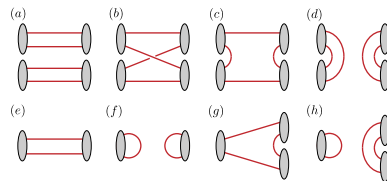
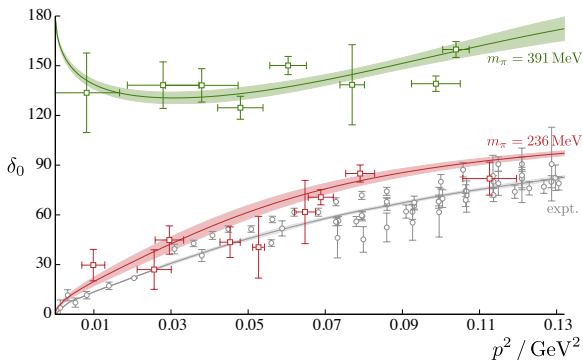
$$\det \left[\cot \delta(E_n^*) + \cot \phi(E_n, \vec{P}, L) \right] = 0$$



- Includes coupled channels, all disconnected diagrams
- See “HadSpec” for more eg a_0 , σ meson etc

SIGMA MESON

Isoscalar $\pi\pi$ scattering including all diagrams.

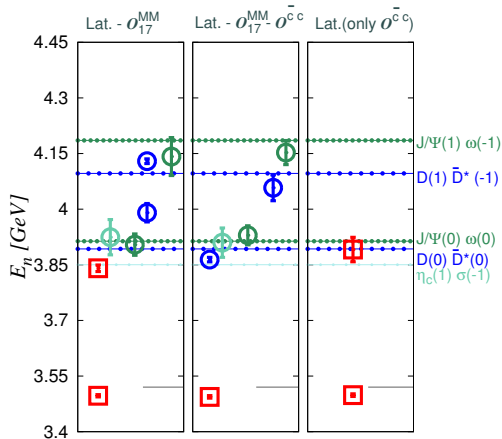
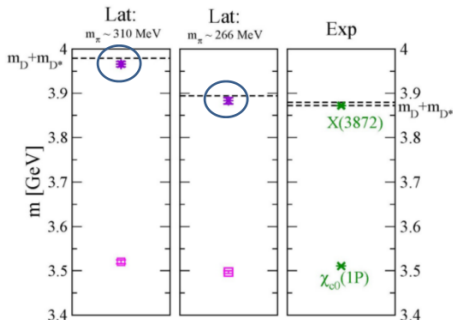


From $m_\pi = 236, 391$ MeV resolve a σ that evolves from a bound-state below $\pi\pi$ threshold at heavier mass to a broad resonance at lighter mass.
 Note that lattice can also be a tool to study mass-dependence!

THE XYZs: X(3872) - A FIRST LOOK (NO COUPLED CHANNELS)

Prelovsek & Leskovec 1307.5172

Padmanath, Lang, Prelovsek 1503.03257



Ground state: $\chi_{c1}(1P)$
 $D\bar{D}^*$ scattering mx: pole just below thr.
 Threshold location? Finite vol effects?

X(3872) not found if $c\bar{c}$ not in basis.

Within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of D^+D^* thresholds: isospin breaking effects important?

**Making measurements at $T=0$:
operators and propagators**

SPECTROSCOPY AT FINITE T: MAXIMUM ENTROPY METHOD

Recall, we calculate correlation functions. These are directly related to spectral functions which contain all information on thermal modifications of the spectrum in that channel.

$$C_i(\tau) = \int d\omega K(\tau, \omega) \rho_i(\omega) \quad \text{and} \quad K(\tau, \omega) = \frac{\cosh\left[\omega\left(\tau - \frac{1}{2T}\right)\right]}{\sinh(\omega/2T)}$$

Eg vector spectral functions yields direct info on thermal dilepton rates:

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\rho_V(\omega, \mathbf{p}, T)}{\omega^2(e^{\omega/T} - 1)}$$

To recover ρ (a function of the continuous energy) from (discrete) C is an ill-posed problem. Use Bayesian methods: Maximum Entropy Method

- Construct ρ that maximises conditional probability $P[\rho|DH]$ of having ρ , given data D and prior H .
- Choice of prior (eg positivity of ρ) defines the method - MEM based on Shannon Jaynes entropy, S :

$$P[\rho|DH] = \exp\left(-\frac{1}{2}\chi^2 + \alpha S\right), \quad \text{with} \quad S = \int d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \log\left(\frac{\rho(\omega)}{m(\omega)}\right) \right]$$

What to look for

- Stable mesons contribute δ function-like peaks to ρ .
- At increasing T - broadening width.

Well known obstacles

- Temperature dependence enters
 - kinematically due to periodic B.C. (periodicity of K)
 - dynamically due to propagation thru' a temperature-dependent mediumand these effects should be disentangled.
- Problematic region for small ω (energy): constant contribution to transport, susceptibilities etc. The melting of quarkonia obscured by a constant contribution

Aarts et al, '05, Umeda '07, Petreczky et al '07-'09

- Stability of the algorithm to construct ρ and the dependence on H are relevant systematics to be dealt with.

QUARKONIA AT FINITE TEMPERATURE: MEM & NRQCD

- EFT of QCD where physics above m_Q integrated out.
- $\mathcal{L}_{\text{nrqcd}}$ ordered in powers of $v = |\mathbf{p}|/m_Q$ and $v^2 \sim 0.1$ for b quarks.
- Use $\mathcal{O}(v^4)$ improved NRQCD with (tadpole-improved) tree-level coefficients.
- Propagators - solving an initial value problem

$$G(\mathbf{x}, \tau = 0) = S(\mathbf{x}) G(\mathbf{x}, \tau = a\tau) = \left(1 - \frac{H_0}{2n}\right)^n U_4^\dagger(\mathbf{x}, 0) \left(1 - \frac{H_0}{2n}\right)^n G(\mathbf{x}, 0)$$

In NRQCD:

- No thermal boundary condition (kinematical temperature dependence) - a simple spectral relation: $G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} e^{-\omega'\tau} \rho(\omega')$ ($\omega = 2M + \omega'$). Heavy quarks not in thermal equilibrium with light-quark-gluon system but appear as probes.
- Troublesome constant (additive) contribution absent - only energies above $2M$ are present.

WHAT TO EXPECT WHEN QUARKS ARE NOT BOUND?

Consider free quarks in continuum NRQCD, with $E_{\mathbf{p}} = \frac{\mathbf{p}}{2M}$

As an example: S and P wave correlators

Burnier, Laine, Vepsäläinen '08

$$G_S(\tau) \sim \int d^3p \exp(-2E_{\mathbf{p}}\tau) \sim \tau^{-3/2}$$

$$\rho_S(\omega) \sim \int d^3p \delta(\omega - 2E_{\mathbf{p}})$$

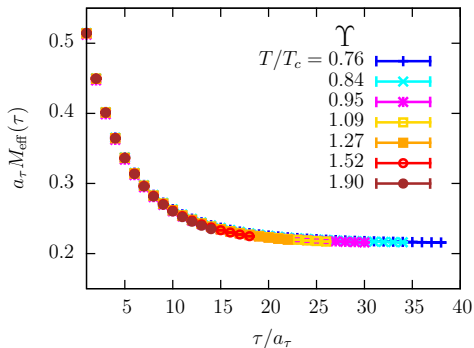
$$G_P(\tau) \sim \int d^3p \mathbf{p}^2 \exp(-2E_{\mathbf{p}}\tau) \sim \tau^{-5/2}$$

$$\rho_P(\omega) \sim \int d^3p \mathbf{p}^2 \delta(\omega - 2E_{\mathbf{p}})$$

- Temperature dependence only enters via the medium!
- In the free continuum case - power law decay for large euclidean time, τ
- Expect modifications from interactions, finite lattice spacing, etc in a realistic case

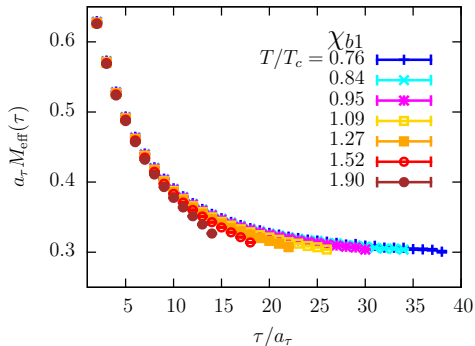
EFFECTIVE MASS: COMPARING S AND P WAVE CORRELATORS

Υ : S wave



Very little T dependence.

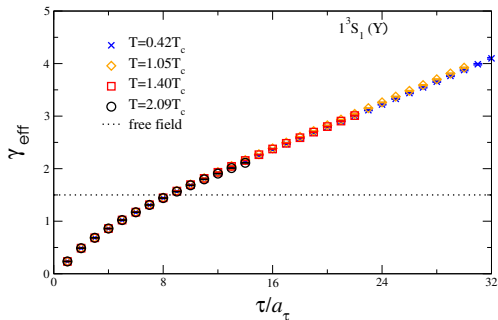
χ_{b1} : P wave



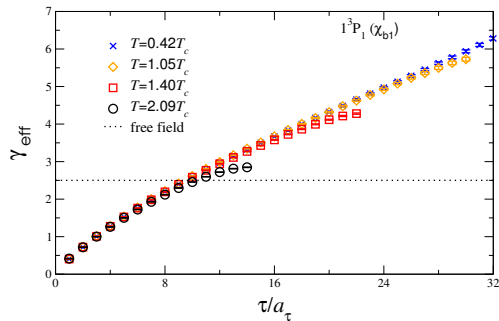
Significant T dependence. Rules out pure exponential decay at $T \sim 2T_c$

EFFECTIVE POWER: COMPARING S AND P WAVE CORRELATORS

$$\gamma_{\text{eff}} = -\tau \frac{G'(\tau)}{G(\tau)} \rightarrow \tau$$

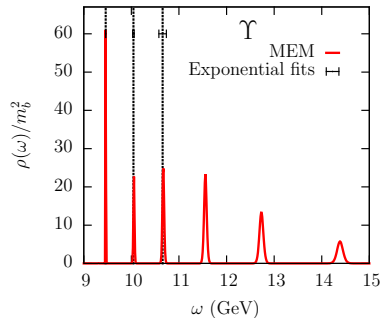
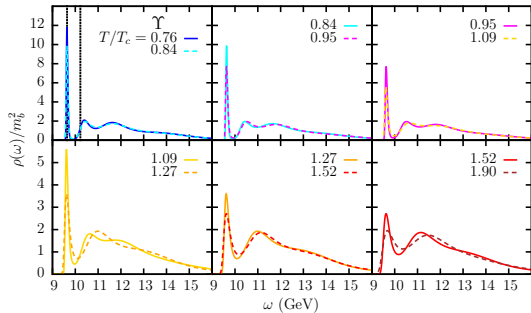


Υ S wave

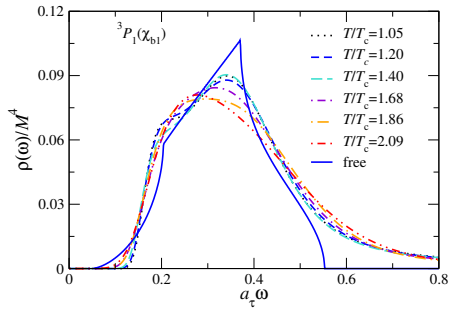
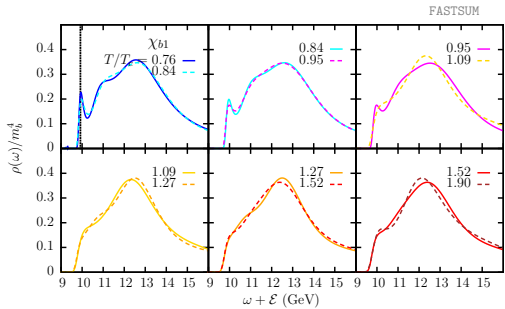


χ_{b1} P wave

MEM: IN PRACTICE



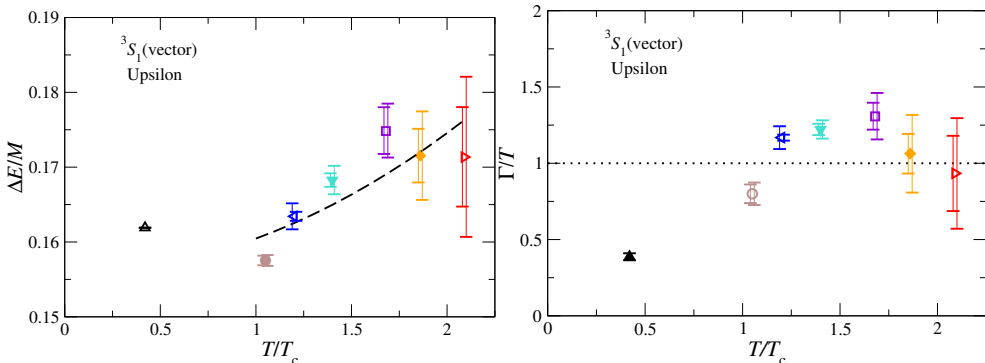
P WAVES IN DETAIL FROM MEM: χ_{b1} IN THE PLASMA



- Other χ_b states similar. Understanding/controlling systematics crucial.
- Structure of spectral functions from NRQCD similar to $\rho_{P\text{wave}}^{\text{free, lat}}(\omega)$.
- Qualitatively different to S waves - Melting immediately above T_c and of unbound, quasi-free b quarks.

MASSES AND WIDTHS: Υ AT FINITE TEMPERATURE

- Extract peak position and width by fitting (left side) peaks to a Gaussian.



Temperature dependence of the mass shift ΔE and width Γ

- Errors include fitting systematics
- Dashed line a comparison with analytic results (Brambilla 2010)

VARIATIONAL IDEAS AT FINITE T

- Expect the temperature dependence of bound states to differ from more loosely bound molecular or other multi-hadron-like states. Studying temperature dependence to elucidate structure?
- Can the temperature dependence of hybrids give some insight to their structure?
- Can ideas for better operators improve precision at finite temperature and help with studies of melting and suppression?
- WHOT-QCD collaboration have explored variational ideas to extract excited states at finite T.

- 0810.1567, Umeda et al: Ground and first excited states of $J/\psi, \chi_c$ from a basis of smeared source and sink operators.
Studied the shape and vol-dependence of BS wavefunctions to distinguish bound and scattering states.
- 1104.33842, Ohno et al: Adapted variational method to calculate locations and heights of spectral functions - and compared to MEM.
Very dependent on “good” operators to resolve a signal in the range of available t at high temperatures.

COMBINING MEM & VARIATIONAL METHODS

Recall, in general

$$C_{\Gamma}(\vec{p}, t) = \int d\omega \rho_{\Gamma}(\omega, \vec{p}) \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

$$C_{\Gamma}(t) = \sum_k \rho_{\Gamma}(m_k) \frac{\cosh[m_k(t - N_t/2)]}{\sinh[m_k N_t/2]} \text{ with } a = 1; 1/T = N_t.$$

And in the variational set up $C_{\Gamma}(t)_{ij} = \sum_{\vec{x}} \langle \phi_{\Gamma}(\vec{x}, t); \phi_{\Gamma}^{\dagger}(\vec{0}, 0) \rangle_j$. with $C_{\Gamma}(t)_{11}$ the unsmearred correlator is defined above.

Then using $C_{\Gamma}(t) = C_{\Gamma}(t_0) V \Lambda V^{-1}$ an *effective spectral function* is

$$\rho_{\Gamma}(m_k^{\text{eff}}(t; t_0)) = (C_{\Gamma}(t_0) V)_{1k} (V^{-1})_{k1} \frac{\sinh[m_k^{\text{eff}}(t, t_0) N_t/2]}{\cosh[m_k^{\text{eff}}(t, t_0)(t_0 - N_t/2)]}$$

- Another handle on excited states.
- While only the (1,1) element is related to $\rho(m_k^{\text{eff}})$ all $k = 1, \dots, n$ contribute.
- For sufficiently large t, t_0 $m_k^{\text{eff}}(\rho(m^{\text{eff}}))$ approaches $m_k(\rho(m_k))$ the mass and spectral function of the k^{th} state.
- Ohno et al (2011) concluded “a judicious choice of operators necessary ... and this is a severe limitation”. *Time to revisit?*

