From quarkonium spectral functions to PDFs: tackling inverse problems on the lattice

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References:

S.Kim, P. Petreczky, A.R., JHEP 1811 (2018) 088
J. Karpie, K. Orginos, A.R., S. Zafeiropoulos arXiv:1901.05408 (accepted at JHEP)
A.R. arXiv:1903.02293

- The real-time challenge
- T>0 quarkonium in-medium spectral functions on the lattice
- Towards parton distribution functions on the lattice
- Conclusion

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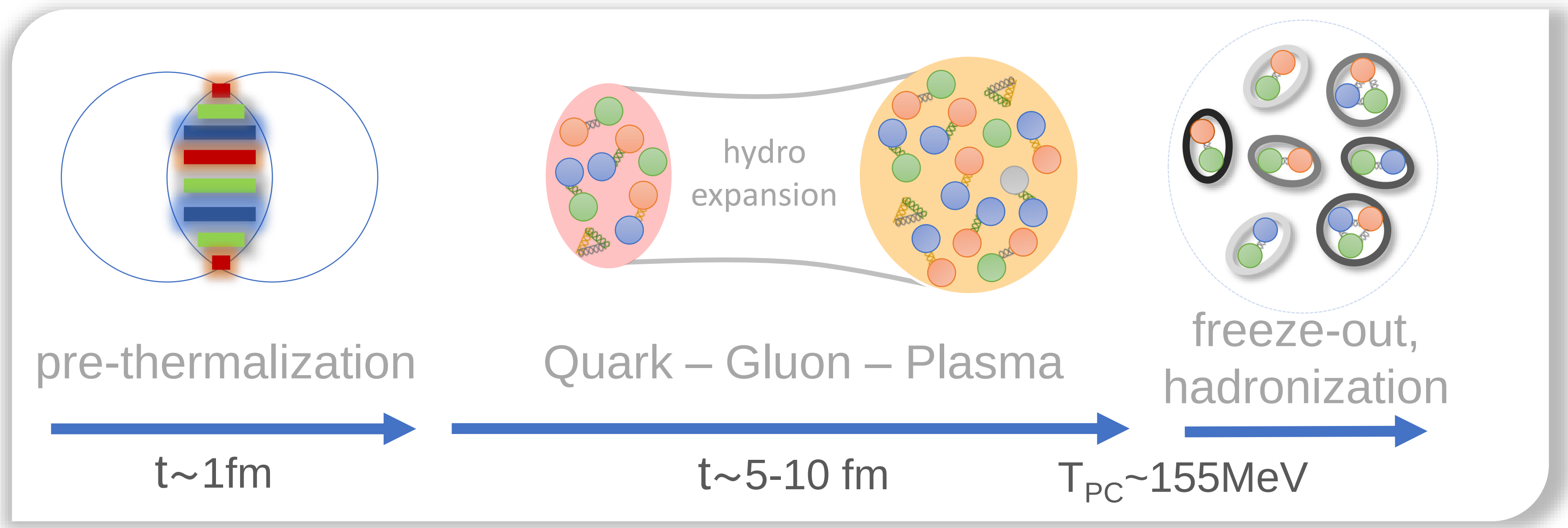
The rich non-perturbative physics of HIC



The rich non-perturbative physics of HIC

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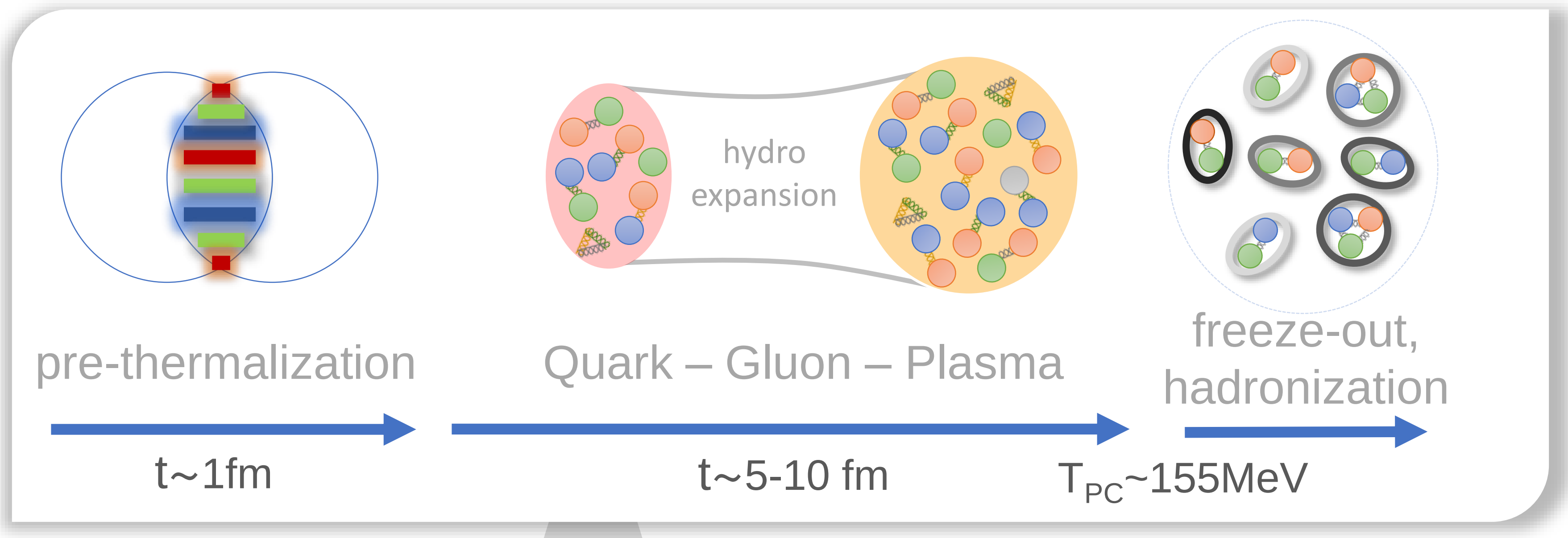

Transport properties of QCD: viscosities, conductivities, …

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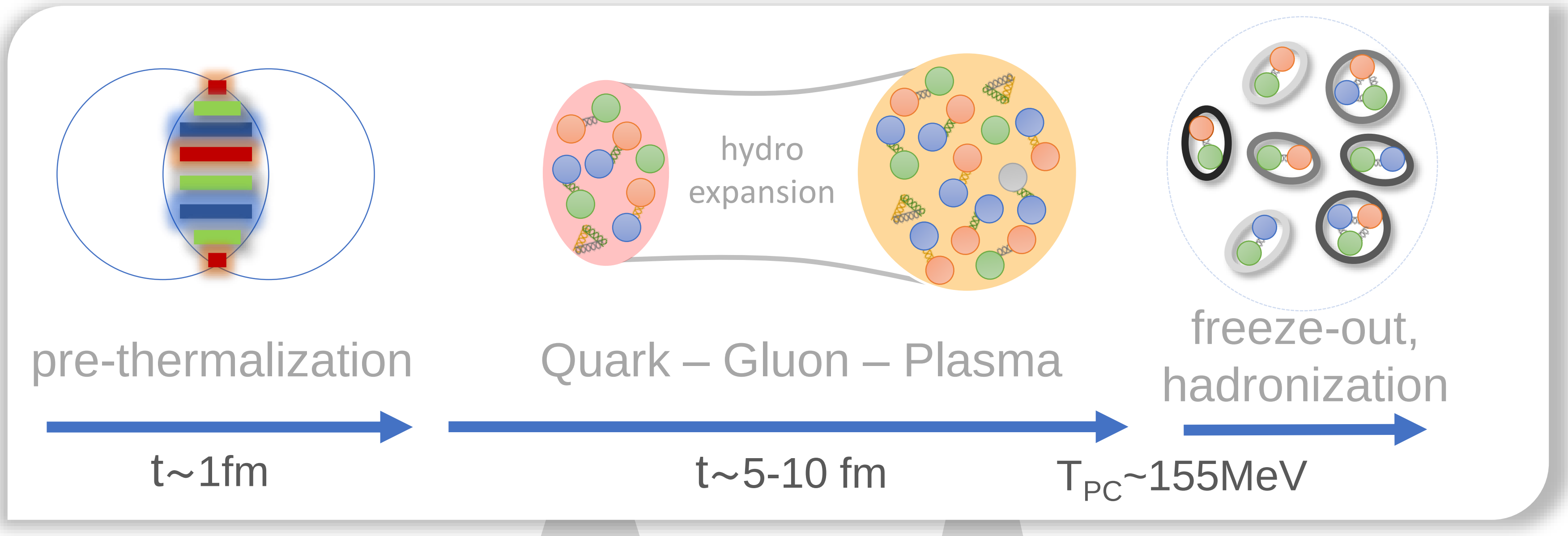

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In-medium properties of heavy quarkonium: equilibration dynamics, melting, etc.

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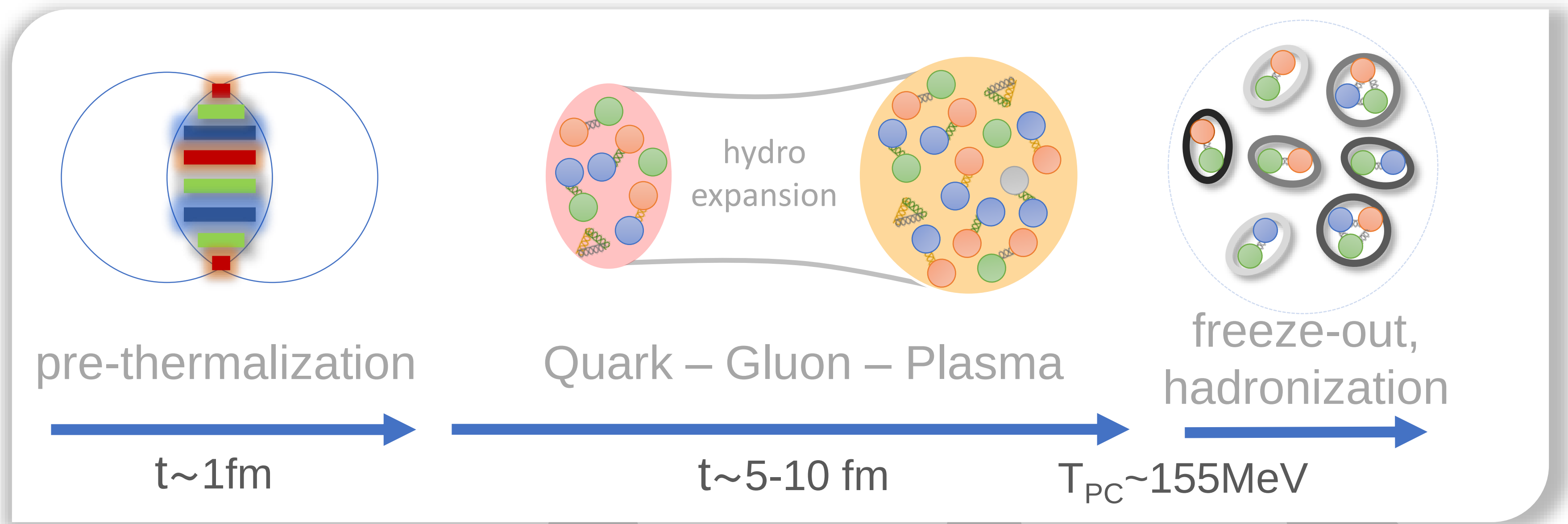

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Hadronization: change of d.o.f. and symmetries

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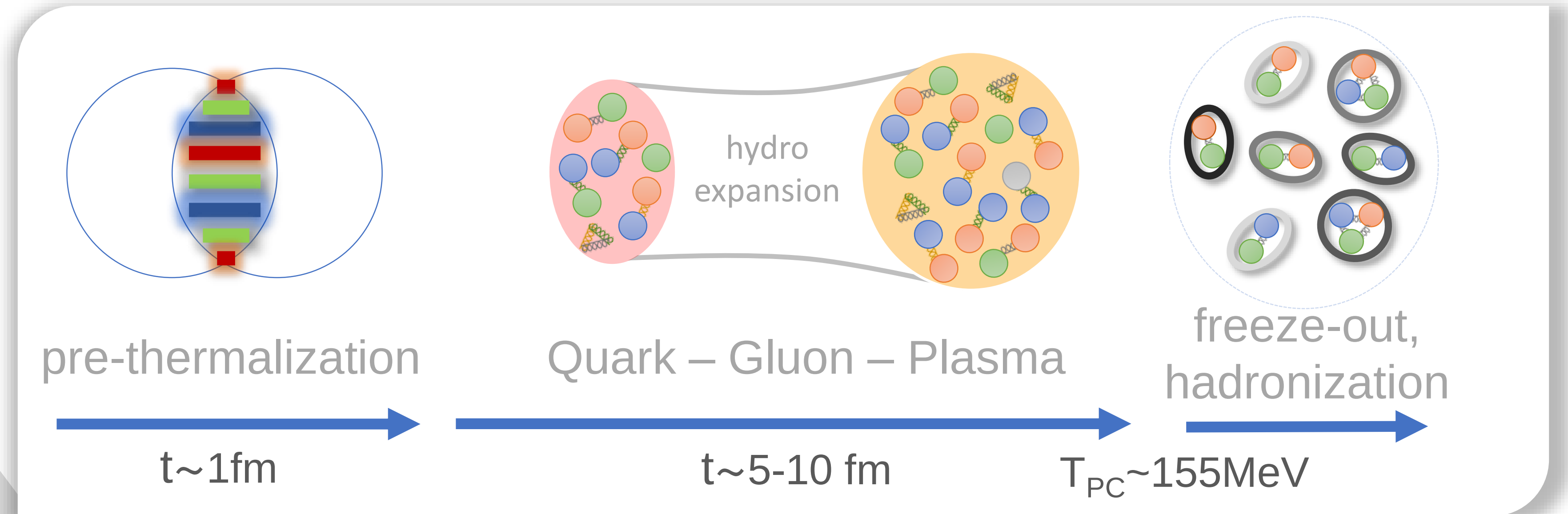

Parton distribution functions of nucleons (and nuclei)

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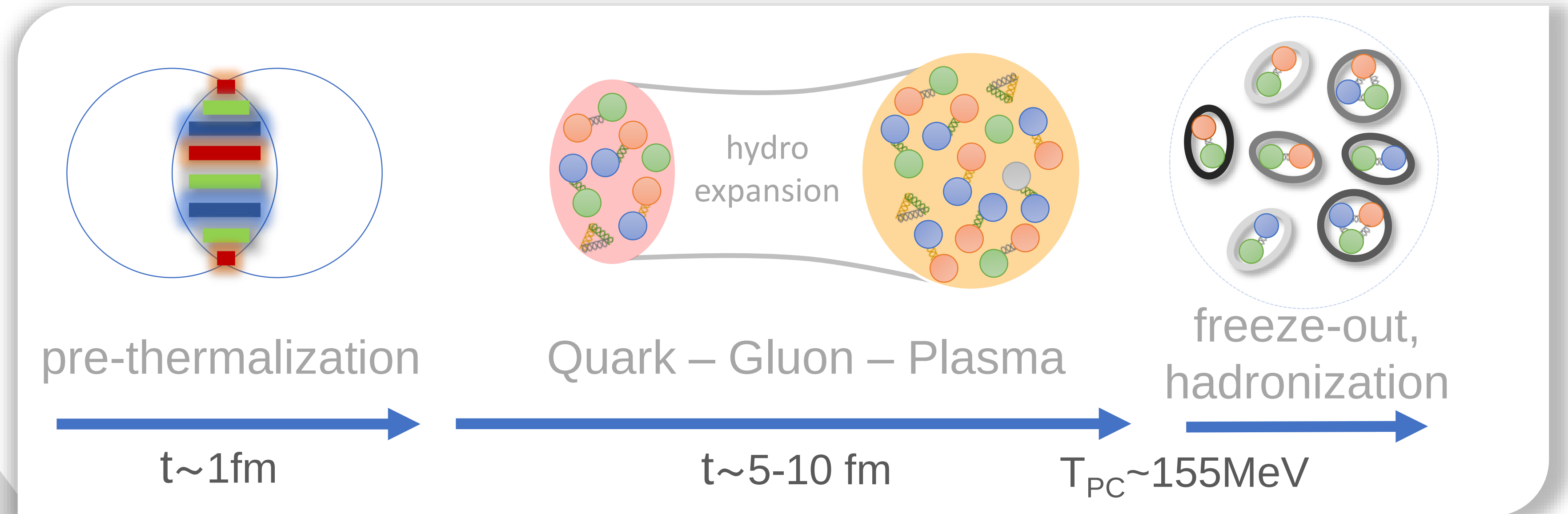

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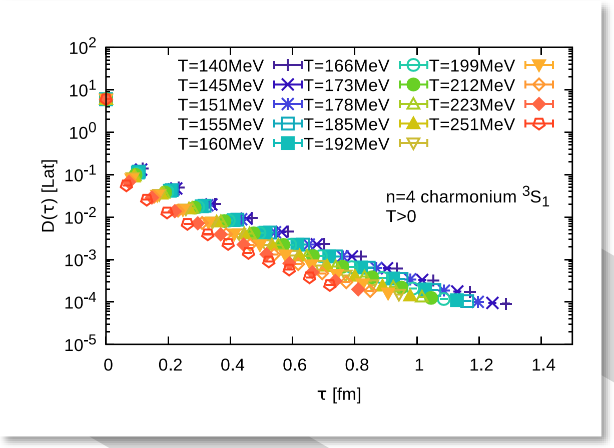
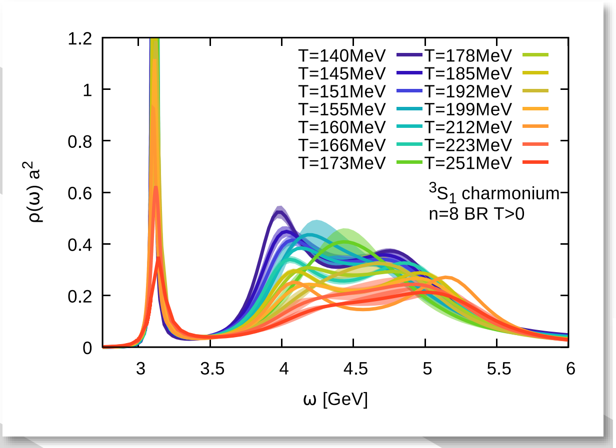
- How can lattice QCD shed light on these dynamical processes?

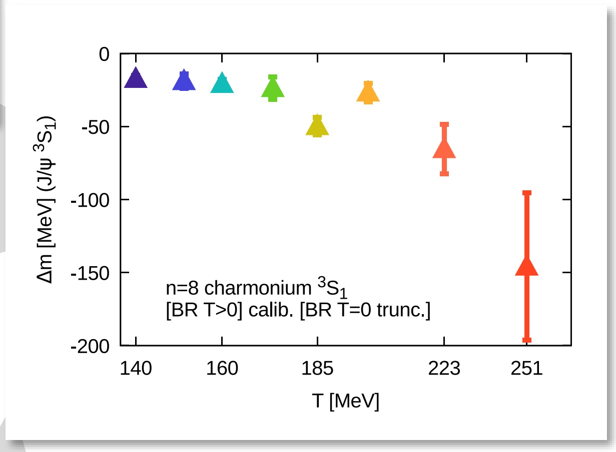
A common challenge



LQCD correlation function
in Euclidean time: **D**



An ill-posed inverse problem



real-time quantity
in momenta: **r**

extracted piece of
dynamical information

Beyond exponential fitting

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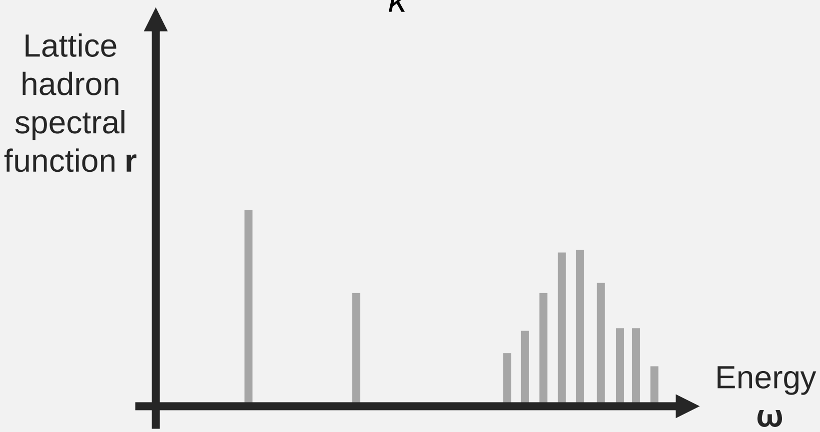
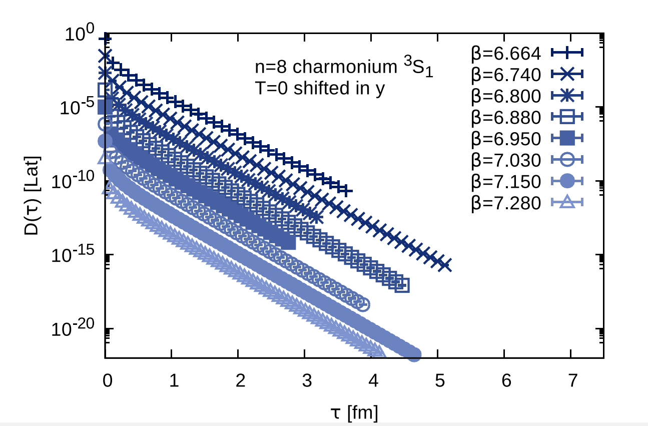
$$\langle MM^\dagger\rangle_{T=0} = \sum_k e^{-E_k\tau}|\langle 0|M|k\rangle|^2$$





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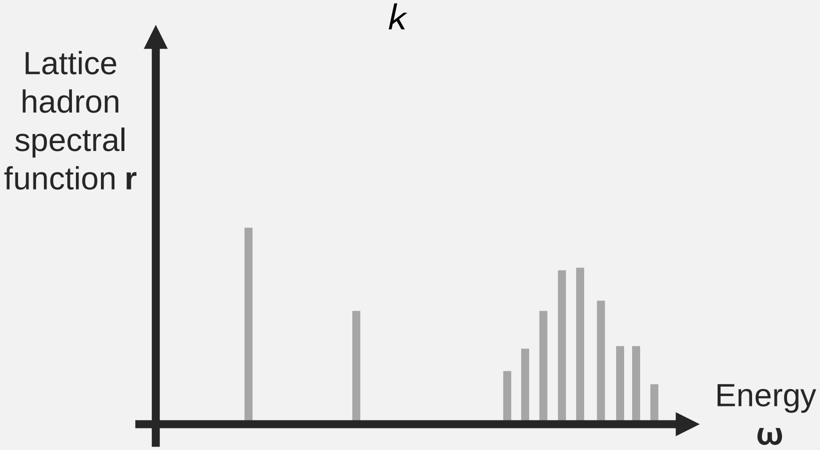
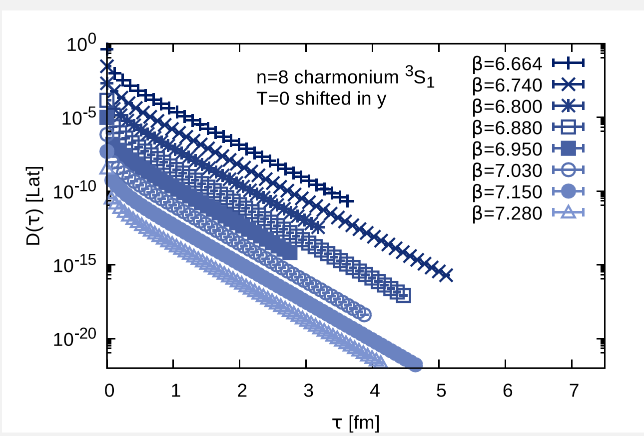
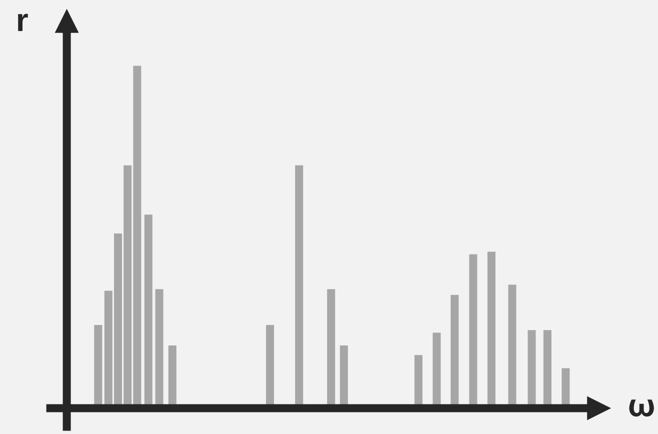
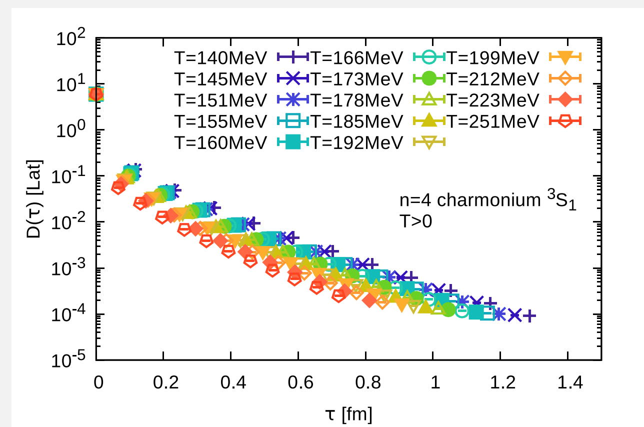




$$\langle MM^\dagger\rangle_{T>0} = \sum_{kl} e^{-\beta E_k} e^{-(E_k-E_l)\tau}|\langle k|M|l\rangle|^2$$





- Intricate structure prevents naive use of T=0 strategies (e.g. GEVP)

The two main challenges

- Invert the spectral representation of the correlator **D**: $\mathbf{D} = \hat{K}\mathbf{r}$

$$K^{T=0}_{\omega\tau} = e^{-\omega\tau} \qquad K^{T>0}_{\omega\tau} = \frac{cosh(\omega(\tau-\beta/2))}{sinh(\omega\beta/2)} \qquad K^{PDF}_{x\nu} = cos(x\nu)$$

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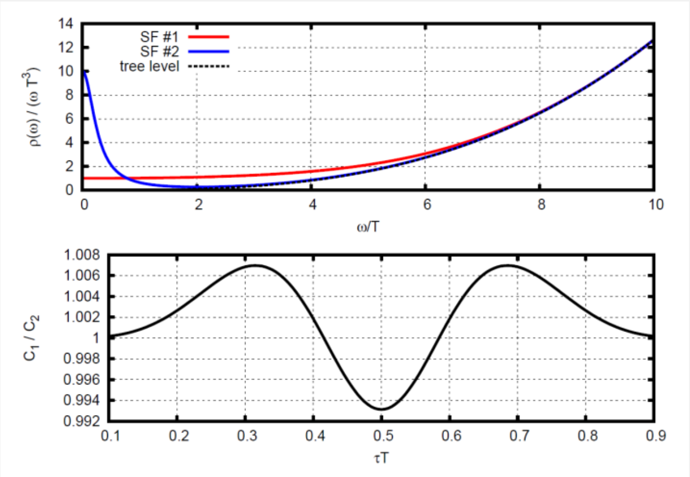

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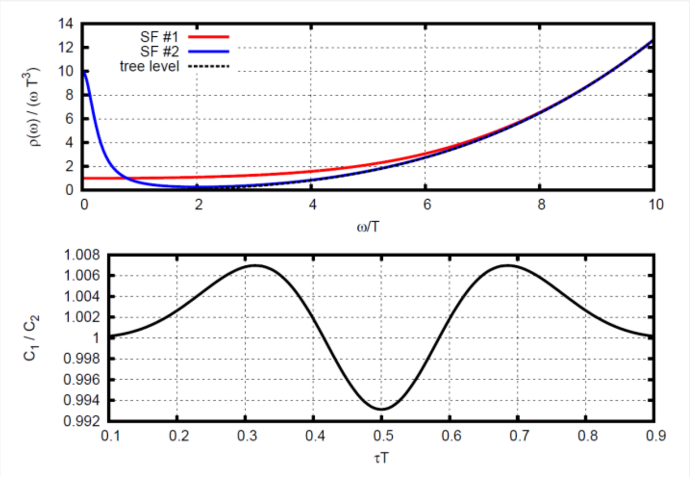

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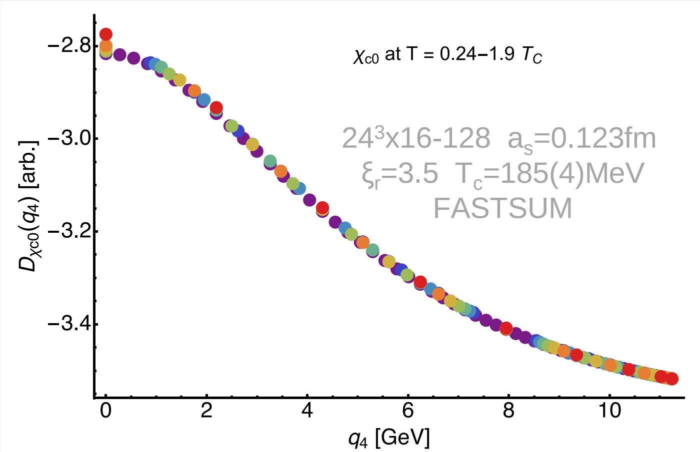

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- Cont. limit: resolve only large w_n behavior

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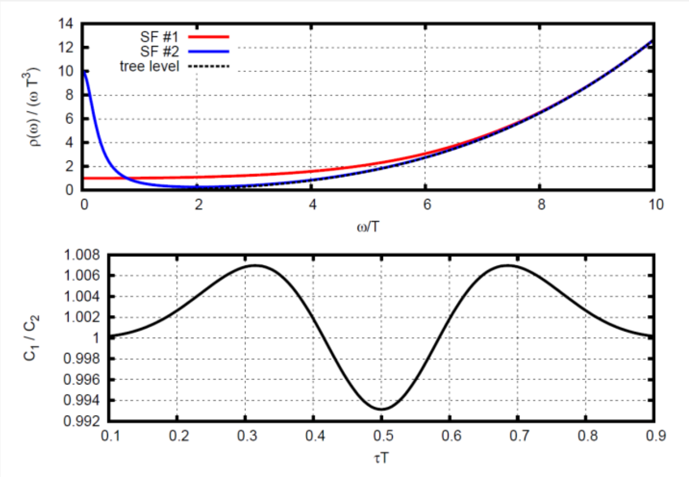

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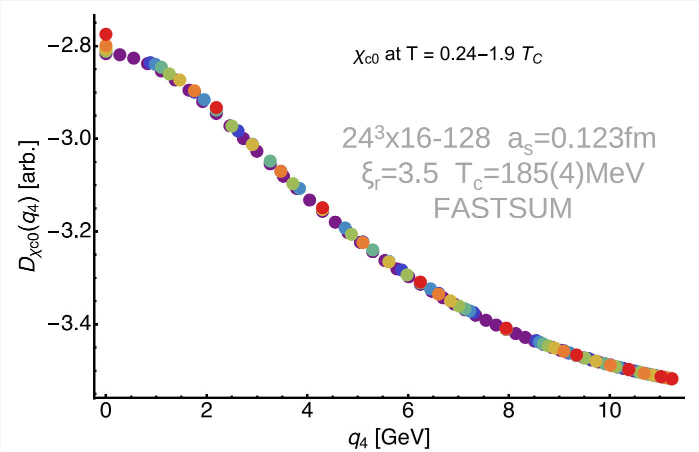

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- How to extract most accurately the information inside lattice data?

The Bayesian strategy

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- Extraction of real-time quantities: inversion of ill-conditioned linear transformation

$$D(\tau) = \int_{-2m_Q}^{\infty} d\omega e^{-\omega\tau} \rho(\omega)$$

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$$D_i = \sum_{l=1}^{N_\omega} \Delta\omega_l \, e^{-\omega_l \tau_i} \rho_l$$

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M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

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- Prior information in both choice of **prior functional S** and **default model m**

Implementing the Bayesian strategy

- Differences in prior information that is incorporated & how to find extremum.

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  - Tikhonov: $$S_T[m,\alpha] = \alpha \int d\omega (\rho - m)^2$$

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    Y.Burnier, A.R. PRL 111 (2013) 18, 182003

    Designed to minimize influence of default model on final answer, weakest curvature of S among priors

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- “Bayesian continuum limit” $N_T \to\infty$ and $\Delta D\to 0$: all version converge to same result

  Choice of S influences how efficiently one converges to this limit
- MEM and BR incorporate positivity but no further info on e.g. analytic structure

  Both methods are susceptible to favor unphysical oscillatory solutions: **Need to develop better priors**

A few words on Backus-Gilbert

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- Simplify the problem as far as possible by linear combinations of input data:

$$\tilde{D}(\tau') = \int d\tau \Sigma(\tau',\tau) D(\tau)$$
$$= \int d\omega \int d\tau \Sigma(\tau',\tau) K(\omega,\tau) \rho(\omega)$$

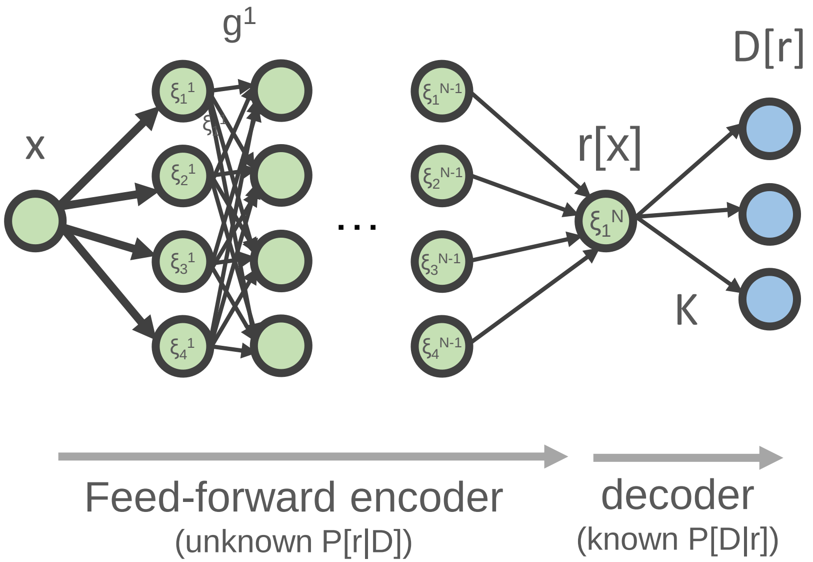
- Task is to construct a resolution function that is as peaked as possible: remains a linear problem

$$\int d\tau \Sigma(\tau',\tau) K(\omega,\tau) \approx \delta(\tau' - \omega)$$

- No default model dependence, but also requires a choice of regularization at intermediate steps

First steps with machine learning

- NN used here as a flexible way to parametrize a function r(x):



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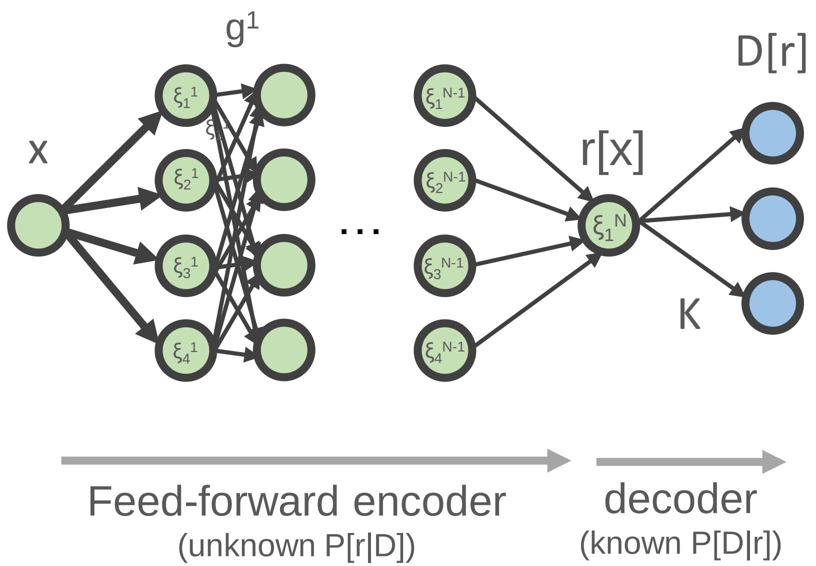

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g(y): a non-linear "activation function",
last layer linear with zero threshold (ReLU)

- Can a NN provide an appropriate function basis for regularizing the inverse problem?
- When we think of g as Taylor series: each ξ is a convoluted sum of monomials

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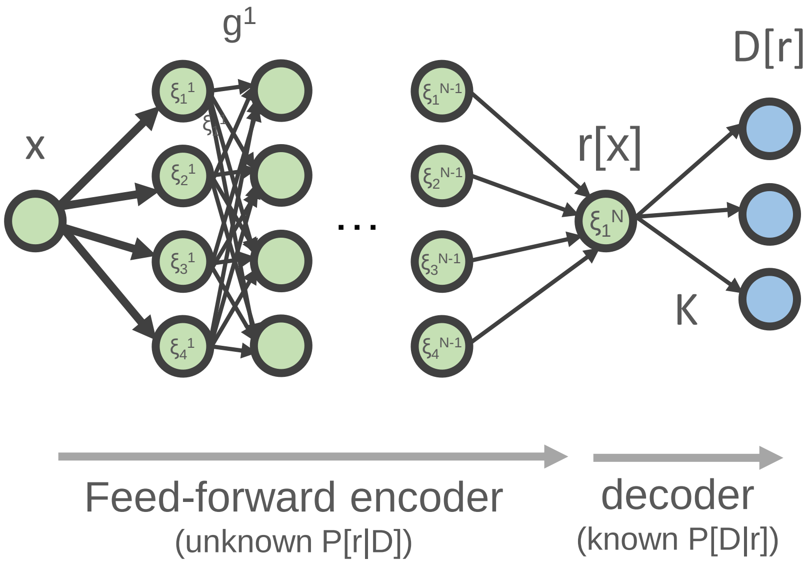

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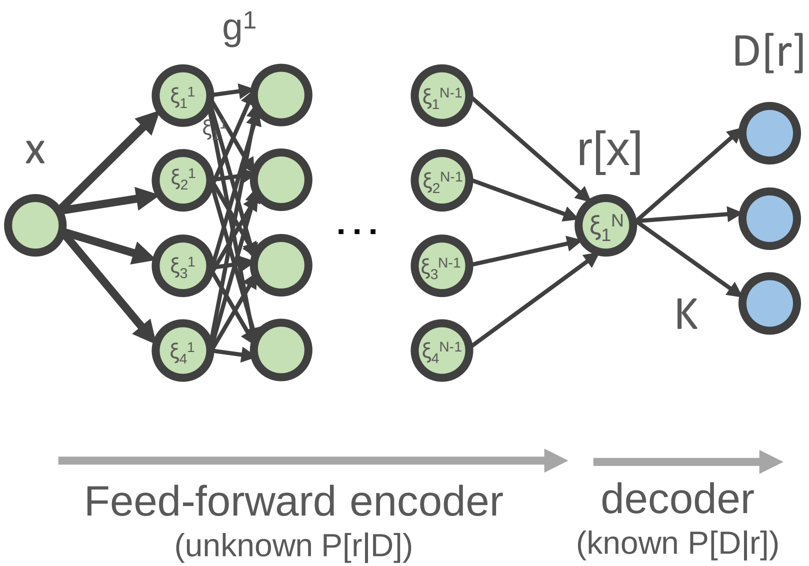

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- Sampling the χ^2 functional space with a particular set of positive definite functions

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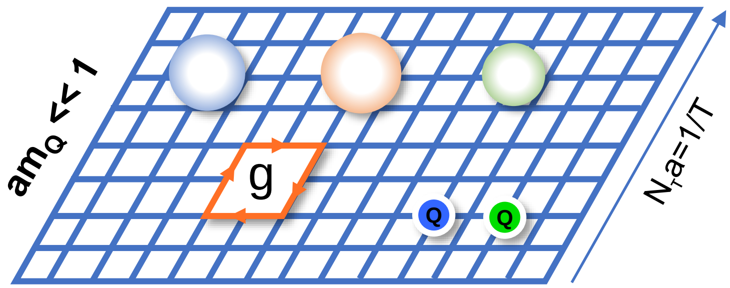
Heavy quarks on the lattice

- Exploit separation of scales to treat heavy quarks non-relativistically



Full Lattice QCD simulation incl. $Q\bar{Q}$
(still too costly)

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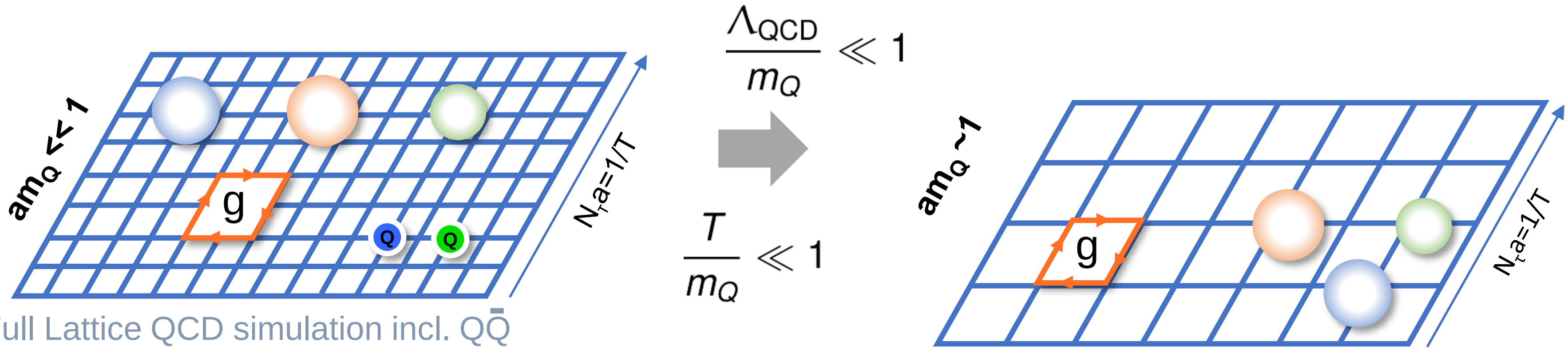

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Lattice QCD simulation without $Q\bar{Q}$

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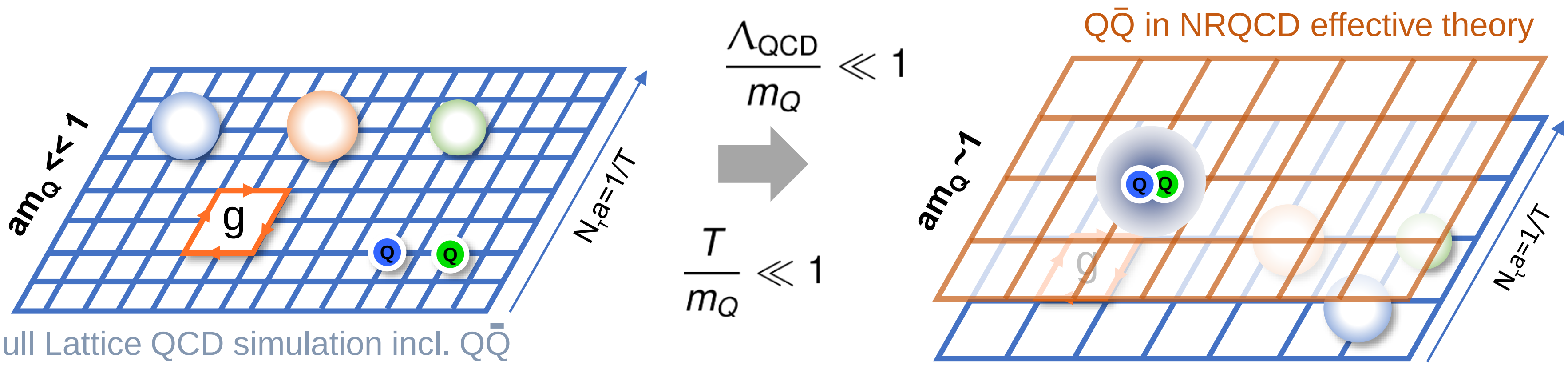

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Lattice QCD simulation without $Q\bar{Q}$

- **Lattice Non-Relativistic QCD** (NRQCD) well established at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in $1/m_Qa$, includes $v\neq0$ contributions
    Thacker, Lepage Phys.Rev. D43 (1991) 196-208
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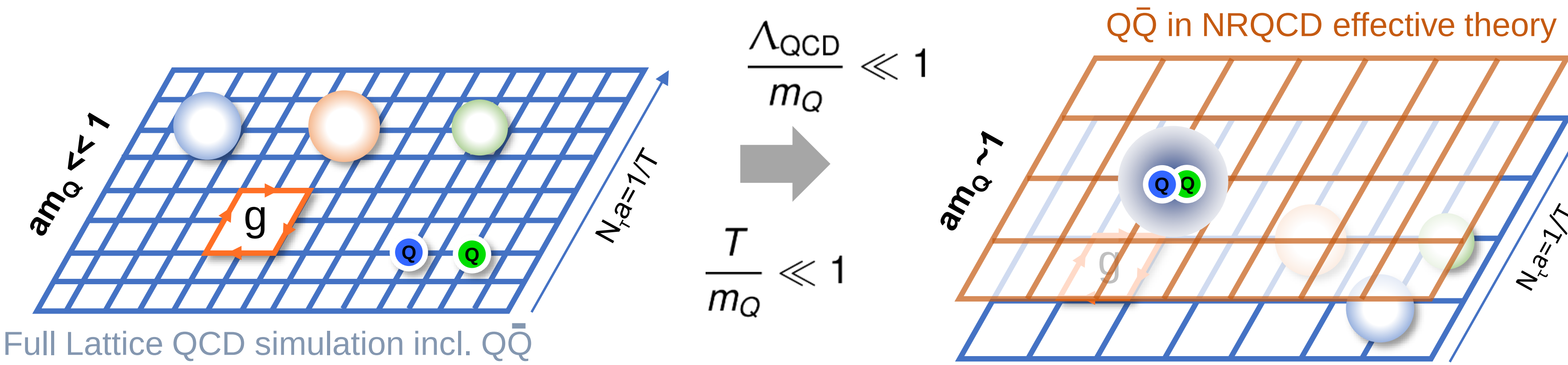

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- **Realistic** & **high statistics** simulations of the QCD medium by HotQCD
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - **m_π=161MeV** T= [**140 – 407**] MeV m_ba= [2.759 – 1.559] Lepage n_b=4
    T=0 N_τ=32-64 T= [140 – 251] MeV **m_ca**= [0.757 – 0.427] Lepage n_c=8
    T>0 N_τ =12
  - Use adaptive time discretization for NRQCD stability via Lepage parameter

NRQCD Euclidean correlators

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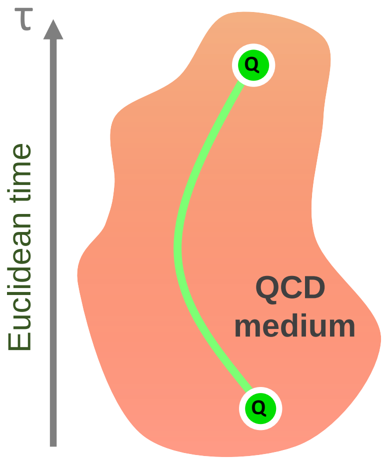

Non-rel. propagator of
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Davies, Thacker Phys.Rev. D45 (1992)

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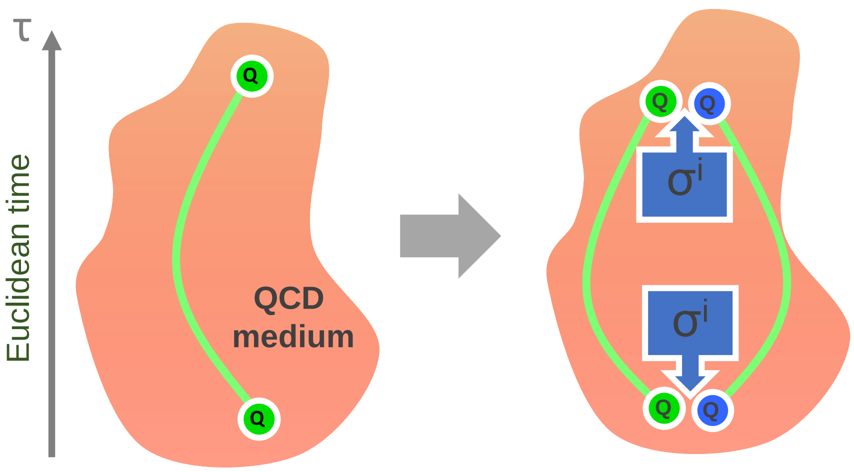

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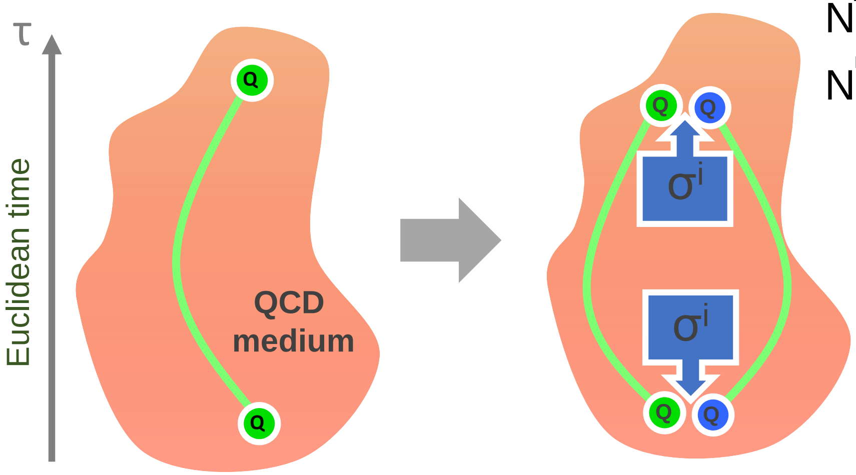
QQ propagator projected to a certain channel

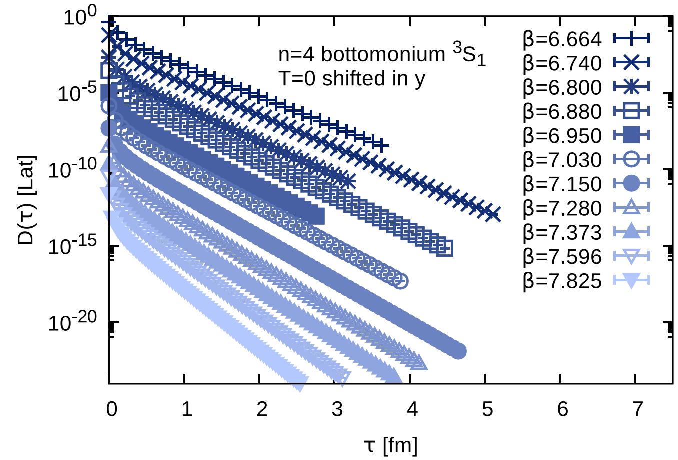
„correlator of QQ wavefct.
$D_{J/\psi}(\tau) \triangleq <\psi_{J/\psi}(\tau)\psi^{\dagger}_{J/\psi}(0)>$ “

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

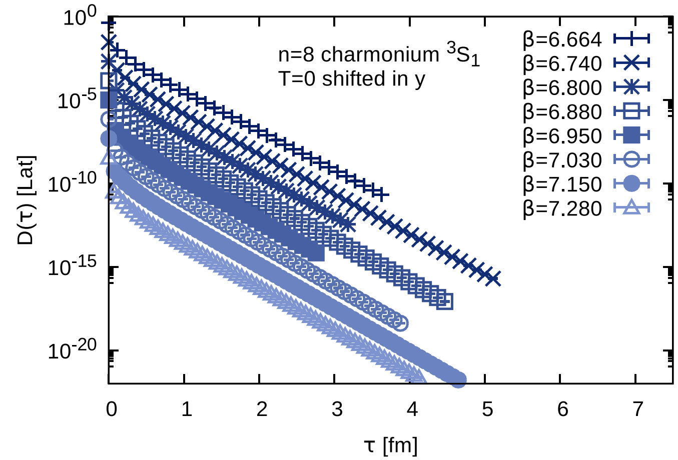
NRQCD Euclidean correlators



$N^{meas}_{bb}(T=0)=400$
$N^{meas}_{bb}(T>0)=1\text{-}4\mathrm{x}10^3$



$N^{meas}_{cc}(T=0)=200$
$N^{meas}_{cc}(T>0)=400$



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Bayesian spectral reconstruciton

- In NRQCD: data D and spectral function ρ are related via **Laplace transform**

$$D(\tau) = \int_{-2m_Q}^{\infty} d\omega e^{-\tau\omega}\rho(\omega) \quad \xleftrightarrow{\text{Fourier}} \quad D(\mu) = \int_{-2m_Q}^{\infty} d\omega \frac{2\omega}{\omega^2+\mu^2}\rho(\omega)$$

  - **Improvement:** incorporate both Euclidean and imaginary frequency data in unfolding

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University of Stavanger

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  - **Improvement:** incorporate both Euclidean and imaginary frequency data in unfolding

- Introduce modified BR method to better estimate systematic uncertainty

Standard BR method (BR)

$$S_{\rm BR} = \alpha \int d\omega \Big(1 - \frac{\rho}{m} + \log\Big[\frac{\rho}{m}\Big]\Big)$$

- Resolves narrow peaked structures with high accuracy
- Ringing in broad structures if recon-structed from small # of datapoints

Y. Burnier, A.R. PRL 111 (2013) 182003

„high gain – high noise“

Low ringing BR method (BR_l)

$$S_{\rm BR_\ell} = \alpha \int d\omega \Big(\kappa\Big(\frac{\partial\rho}{\partial\omega}\Big)^2 + 1 - \frac{\rho}{m} + \log\Big[\frac{\rho}{m}\Big]\Big)$$

- Introduces penalty on arc length of reconstruction $(dL/dw)^2=1+(d\rho/dw)^2$
- Efficiently removes ringing but may lead to overestimated peak widths

c.f. C. Fsicher et.al. PRD98 (2018) 014009

„low gain – low noise“

Calibrating the smooth BR method

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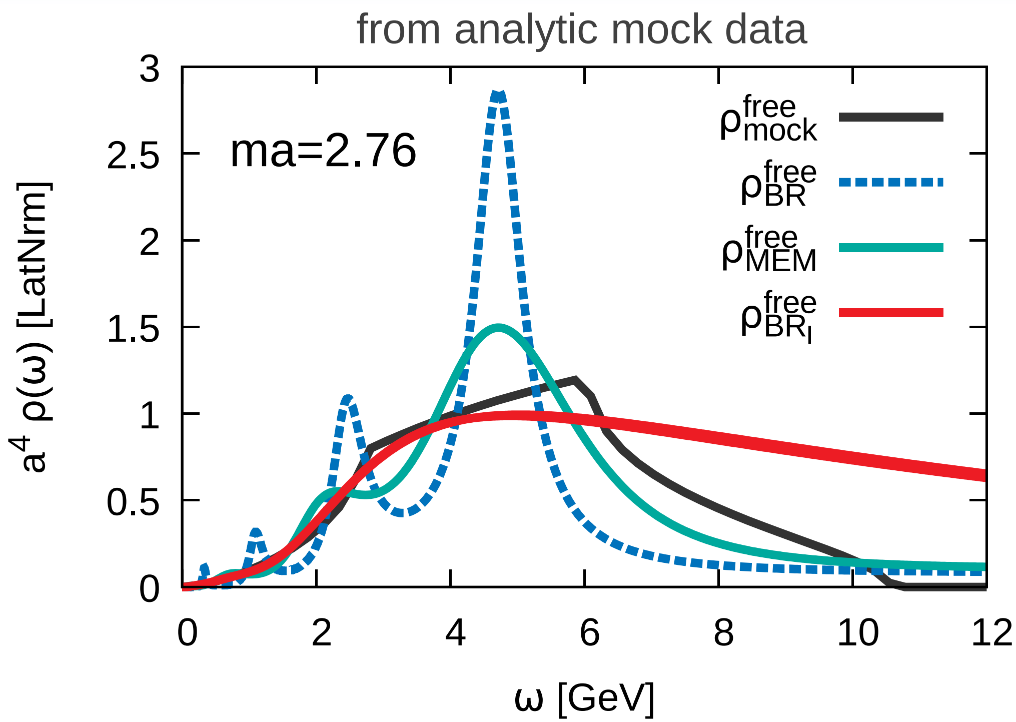
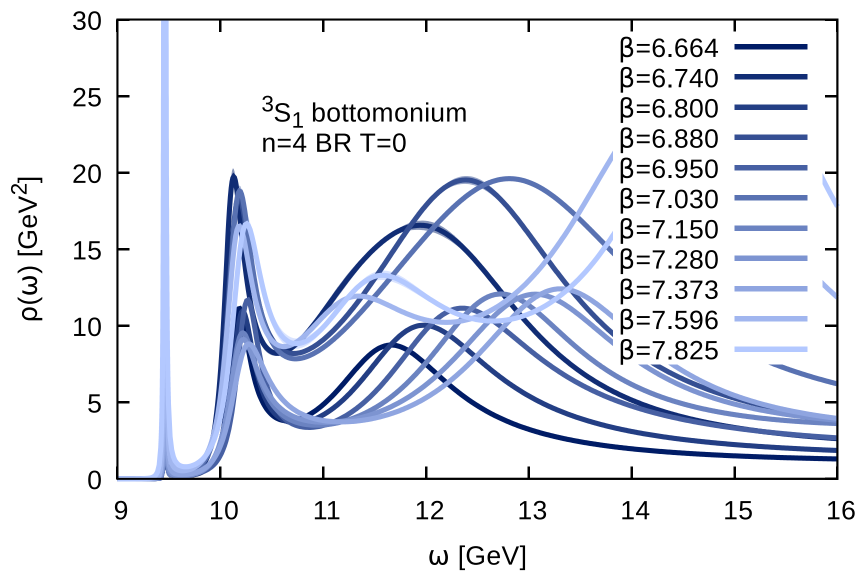


- Hyperparameter κ can be self consistently selected (MEM smoothing implicit N_τ, w_{min})
- Use prior knowledge: free spectral functions known analytically
- In reconstruction from N_τ=12 data: κ=1 successfully suppresses ringing
- Use smooth BR to verify peak existence, standard BR for peak position etc.

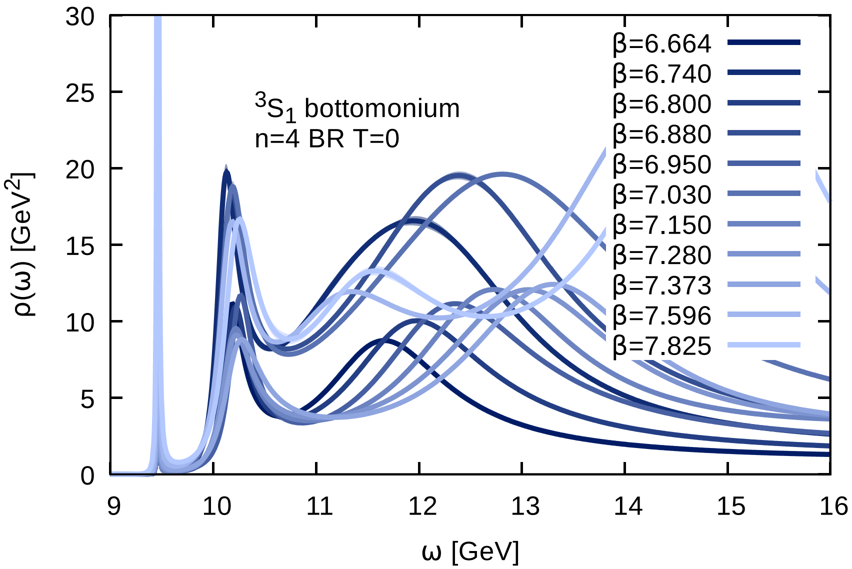
Information content at T=0

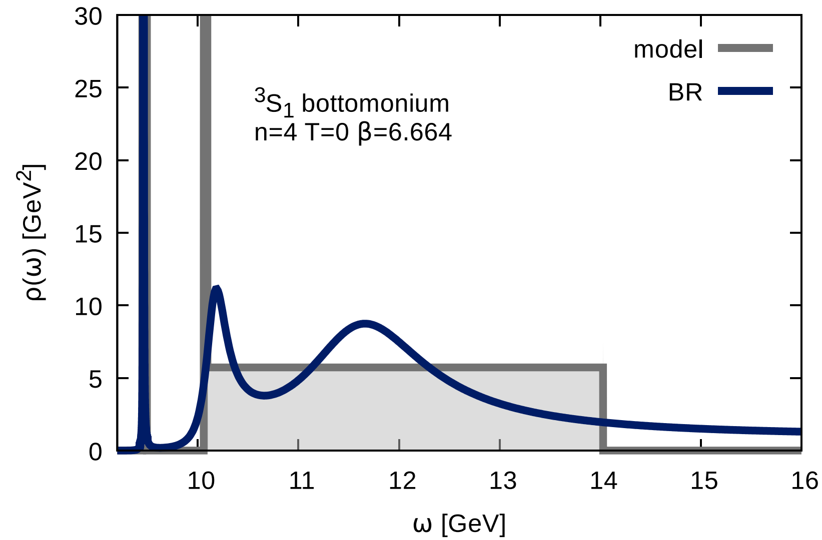


- With a hypothetically perfect extraction, how much can we learn from the data?

Information content at T=0

University of Stavanger





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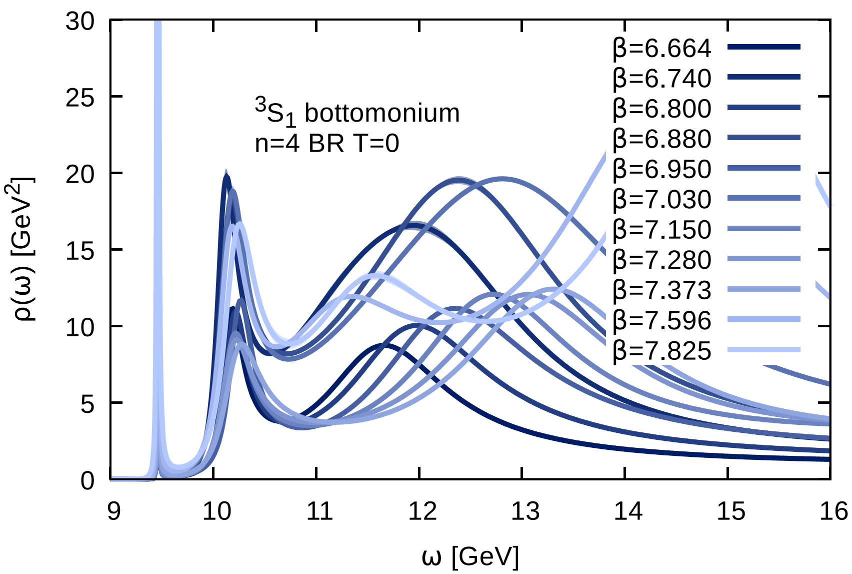
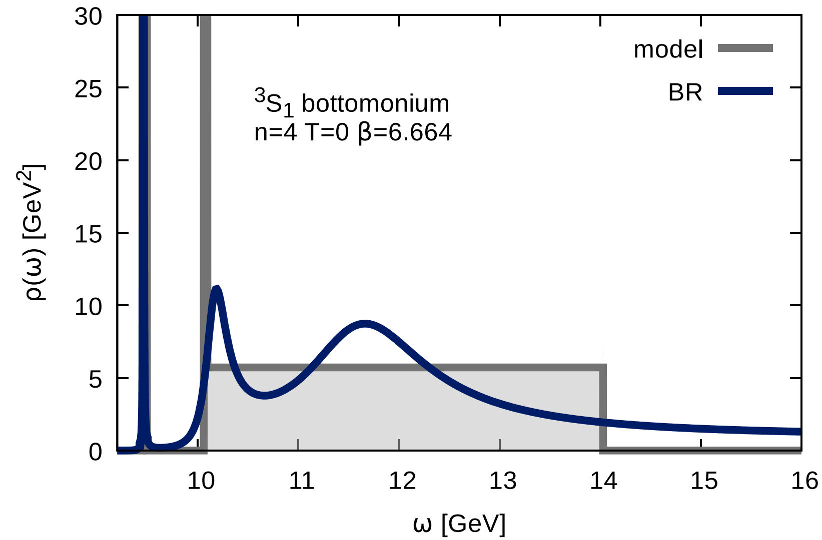
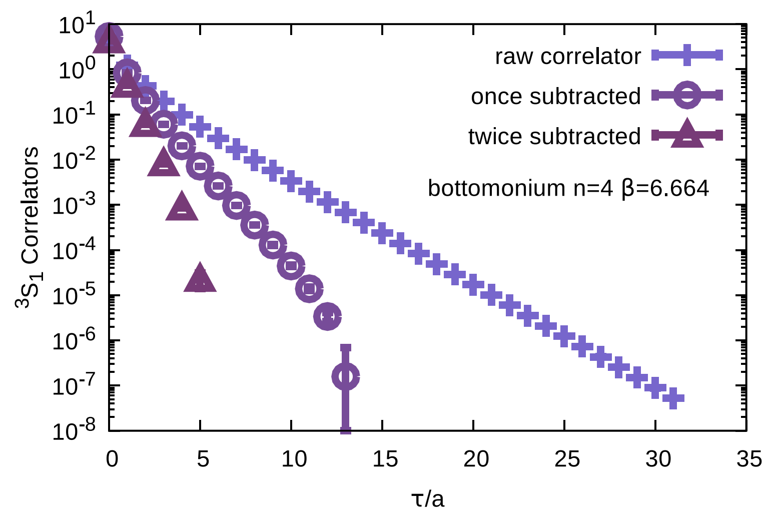


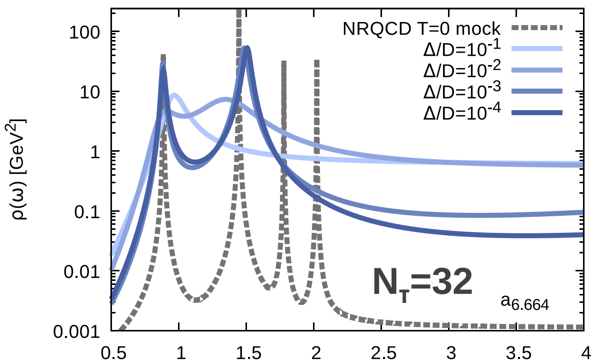


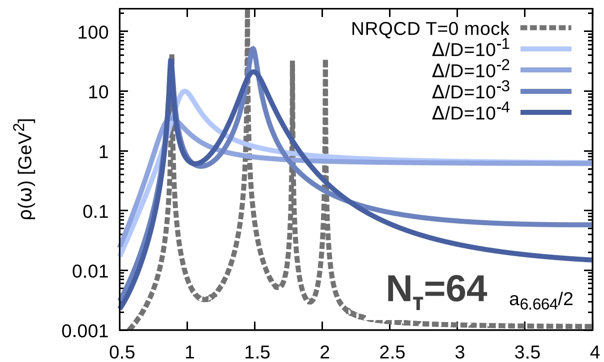
- With a hypothetically perfect extraction, how much can we learn from the data?
- Offer alternative interpretation of correlator: two peaks and a box. Fits correlator $\chi^2/N_\tau \approx 1$
- After subtracting two peaks, only small # of relevant points remain.



How to improve reconstructions?

Mock Test





- Towards the continuum: no significant improvement of bound state reconstruction

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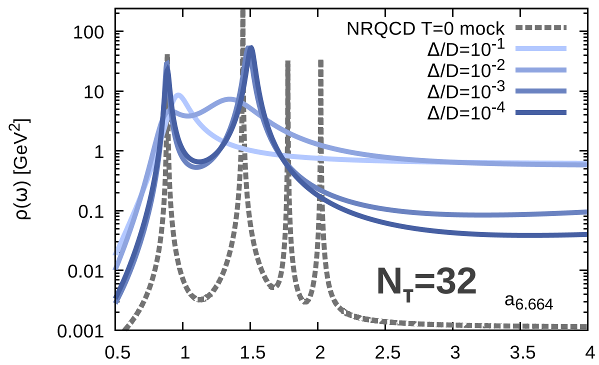

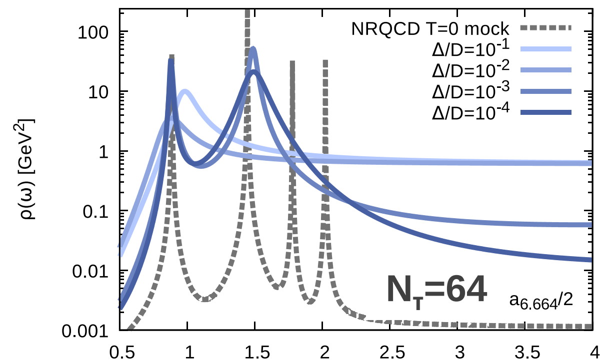

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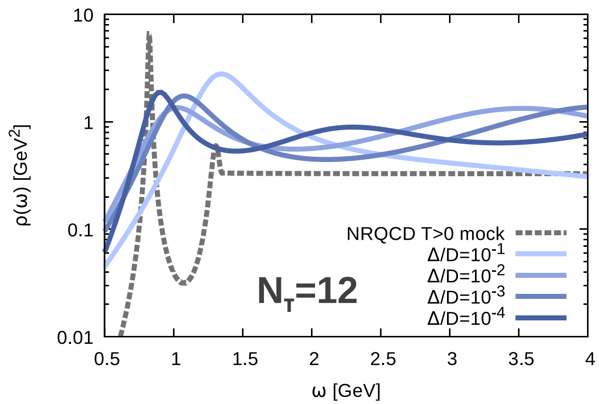

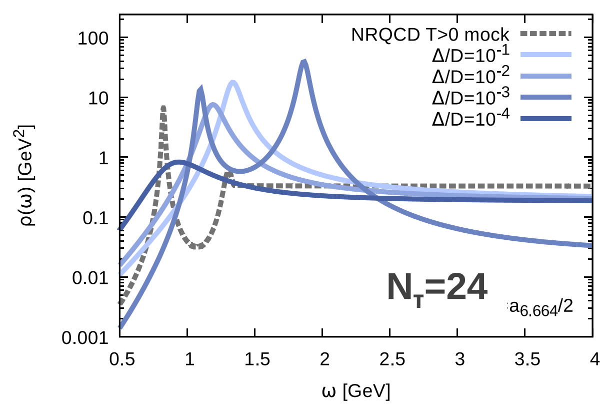

- With e.g. anisotropic lattices, the continuum will be better under control

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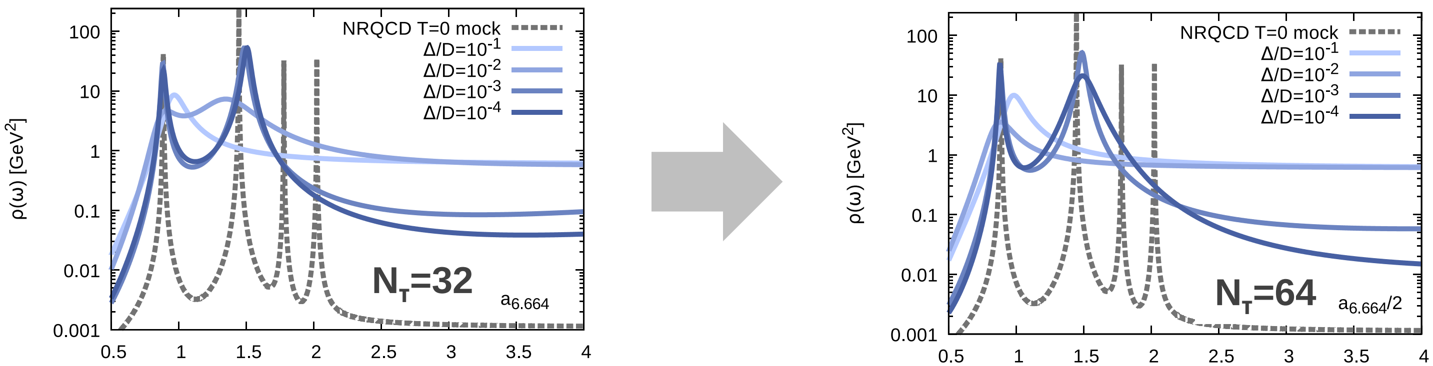

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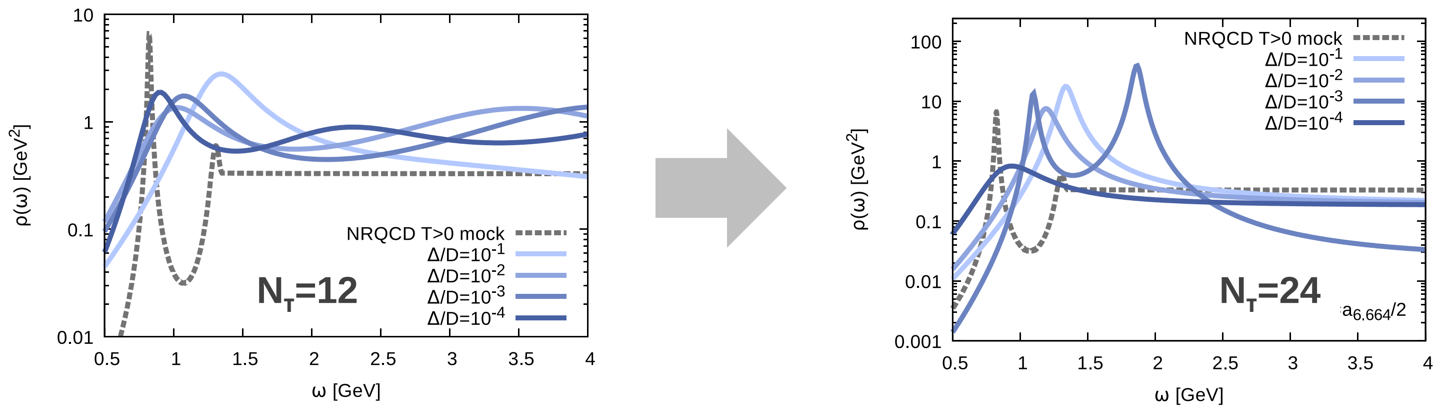

- With e.g. anisotropic lattices, the continuum will be better under control
- Progress needs new ideas: e.g. full QCD multilevel algorithm.

Towards finite temperature

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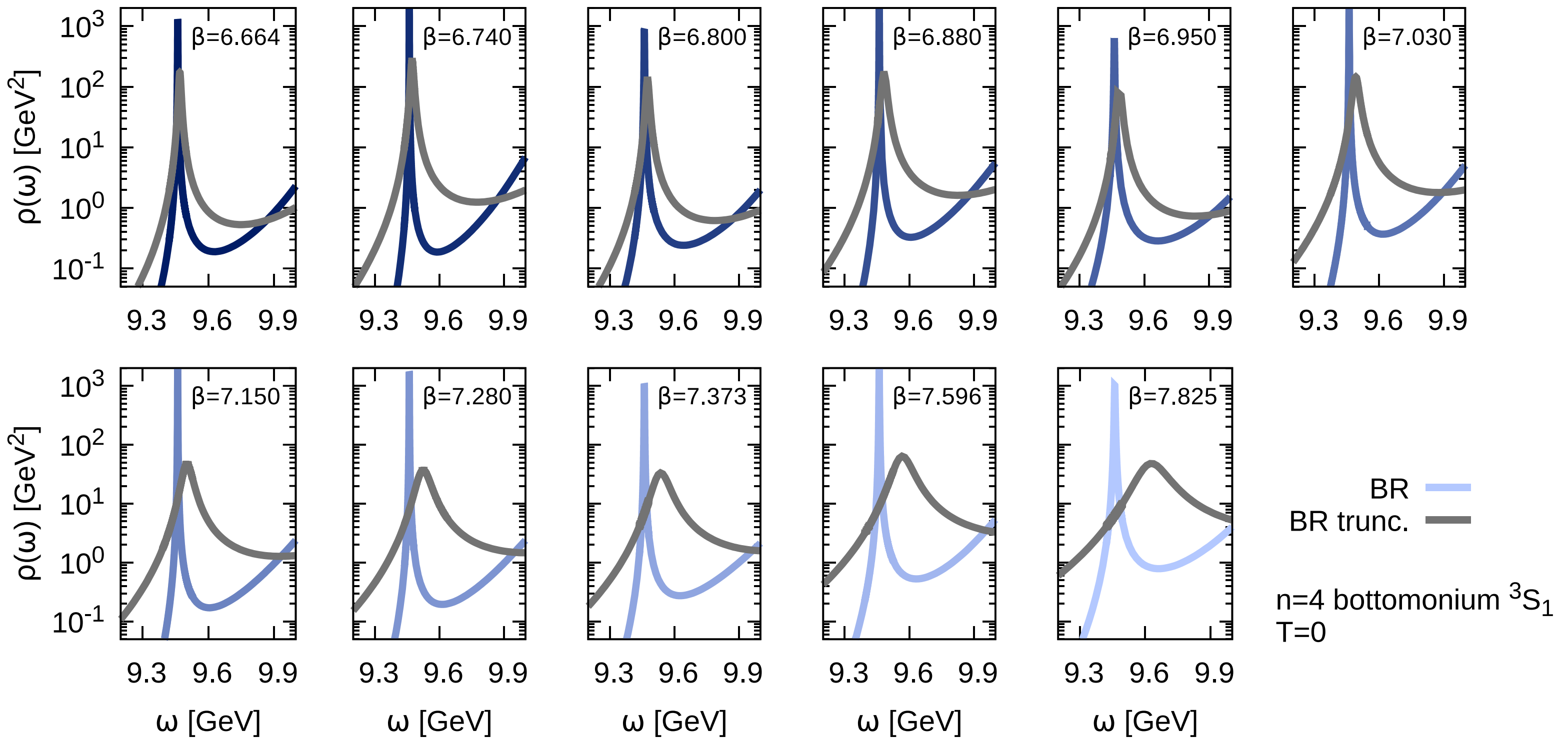

- "High-gain" BR method resolves T=0 ground state very well from N_T=48-64 points
- How does accuracy suffer from limited Euclidean extent at T>0 (N_T=12) ?

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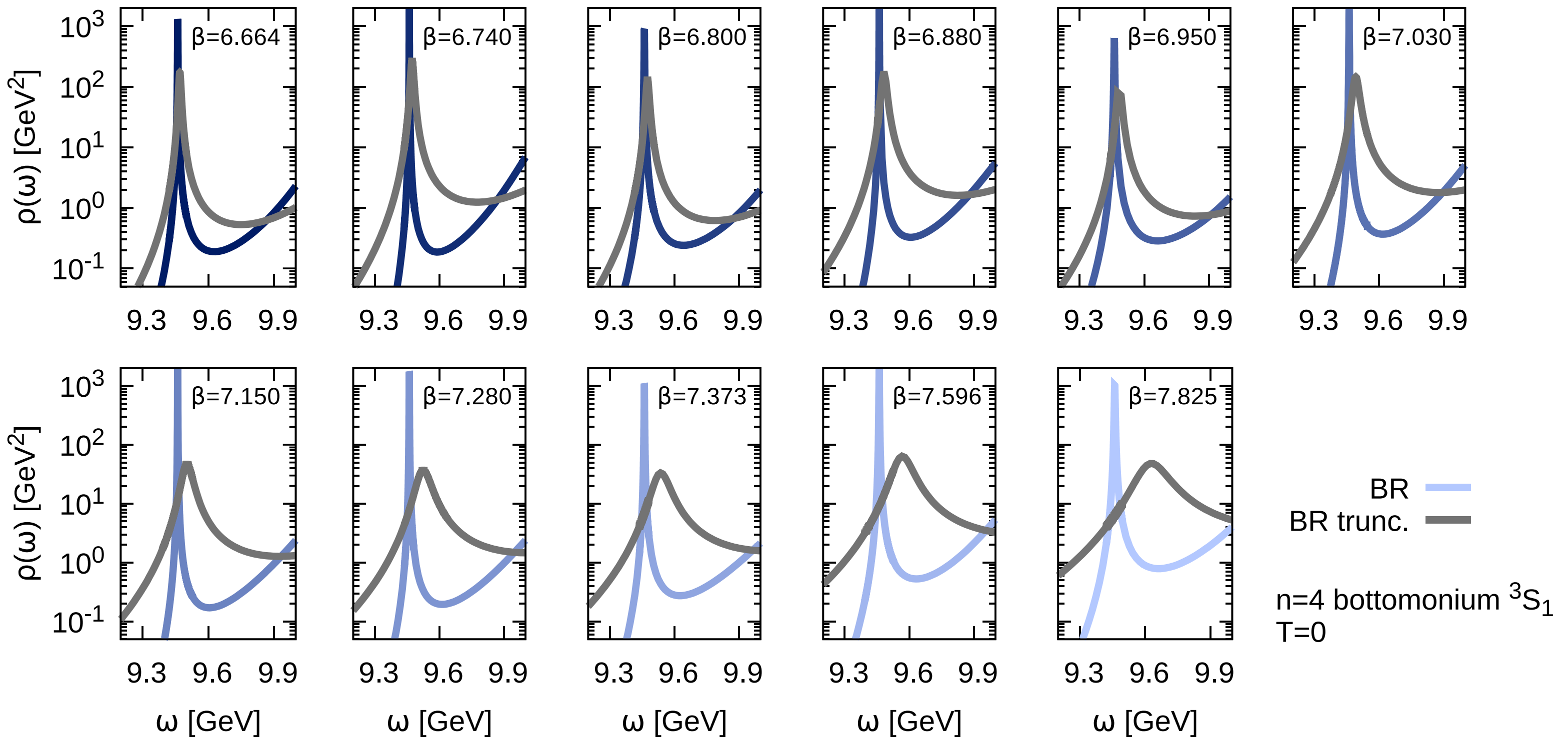

- "High-gain" BR method resolves T=0 ground state very well from N_T=48-64 points
- How does accuracy suffer from limited Euclidean extent at T>0 (N_T=12) ?

→ Systematic shift of peaks to higher frequencies, as well as broadening. needs to be accounted for when analyzing T>0 spectra

Bottomonium S-wave at T>0

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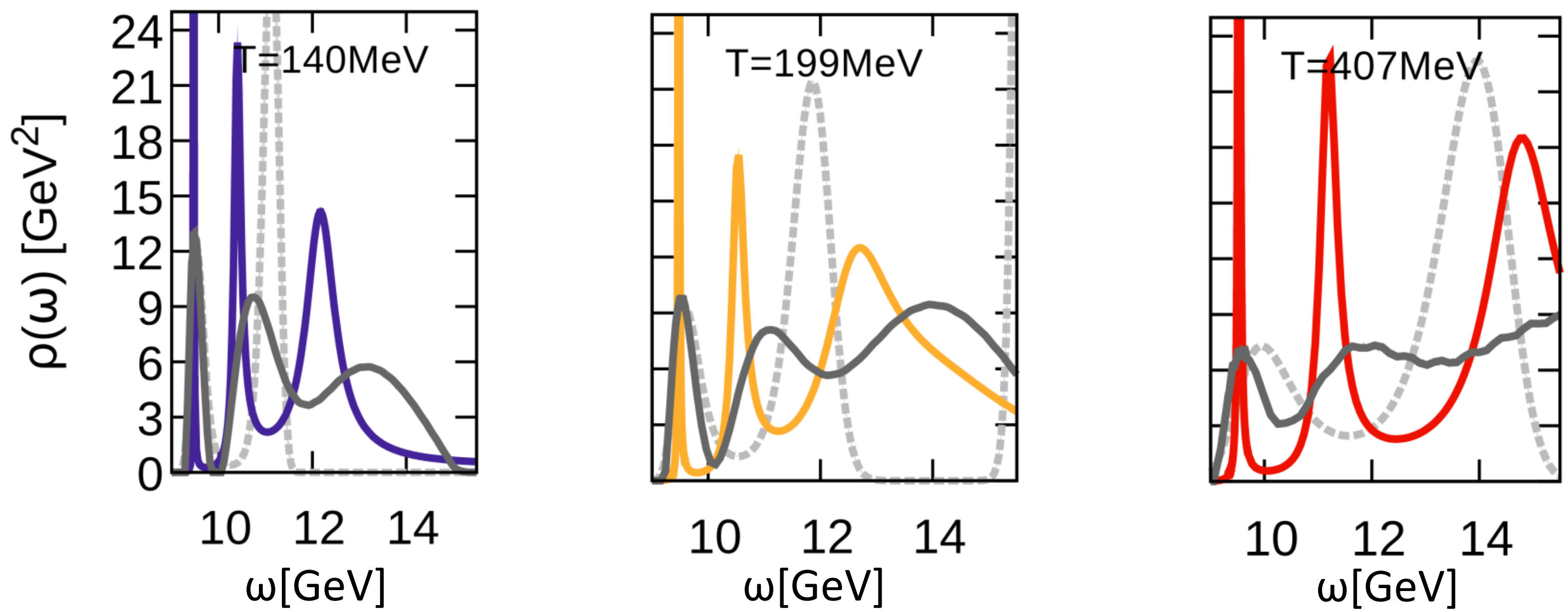
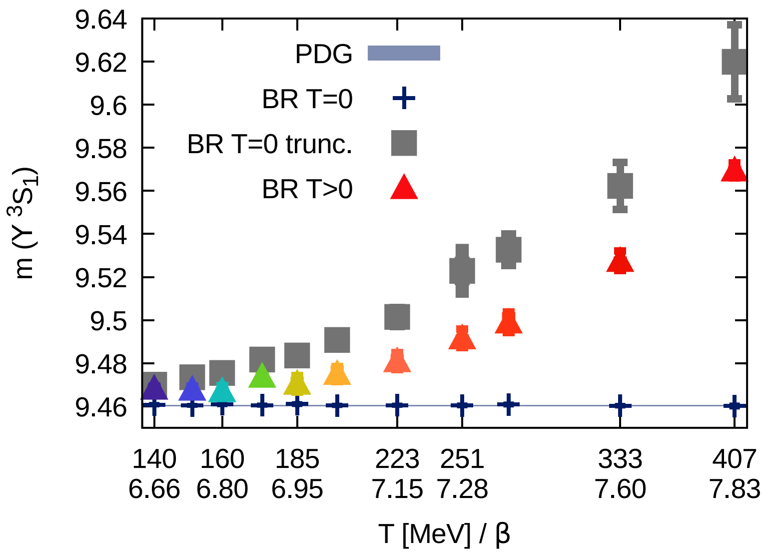
- Inspection of Upsilon (1S) ground state in medium modification

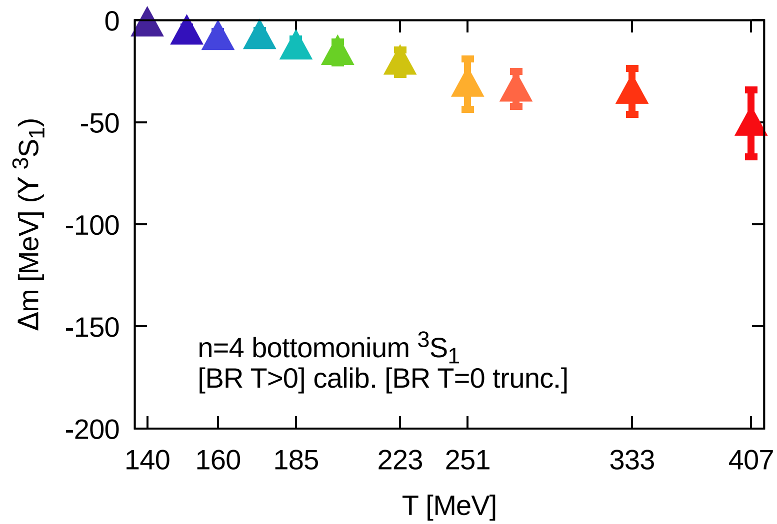


- Smooth BR method and MEM show very similar peak heights
- Peak position of original BR method and smooth BR method consistent

In-medium mass shifts

- Need to establish correct baseline to compare in-medium masses to





- Truncated T=0 reconstruction shows artificial shift to higher frequencies (gray sq.)
- In-medium shifts at T=140MeV very close to truncated results (no in-medium mod.)
- At higher temperatures masses lie clearly below the baseline
  (compatible with non-perturbative potential based computations (pNRQCD))

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- The real-time challenge
- T>0 quarkonium in-medium spectral functions on the lattice
- Towards parton distribution functions on the lattice
- Conclusion

The parton structure of nucleons

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- Parton distribution functions: form factors of hadronic matrix elements

$q_i(x,\mu)$ what fraction of hadron momentum x, does the parton I carry at resolution μ [GeV]

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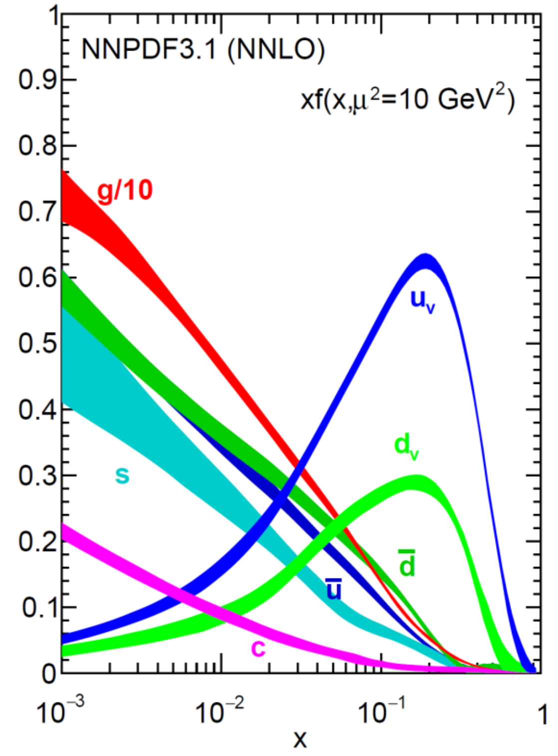
Experimental data → elaborate fitting procedures (templates, NN) →



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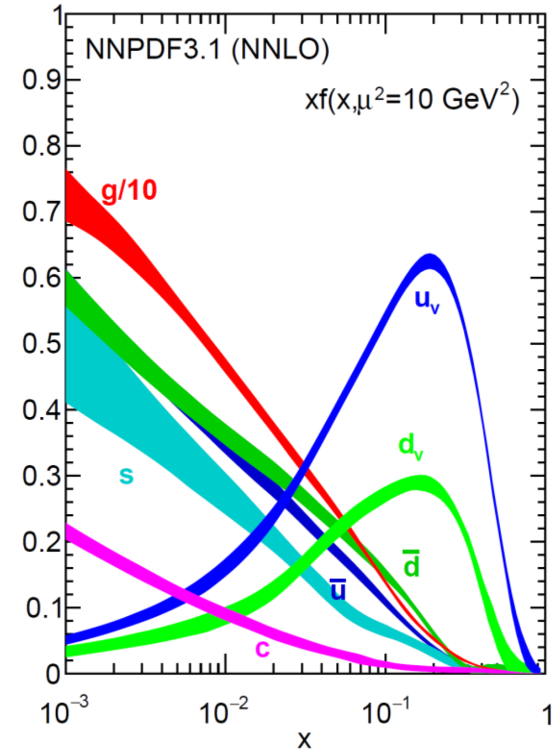

- Simple fitting function successful: $p(x) = \dfrac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} x^a (x-1)^b$

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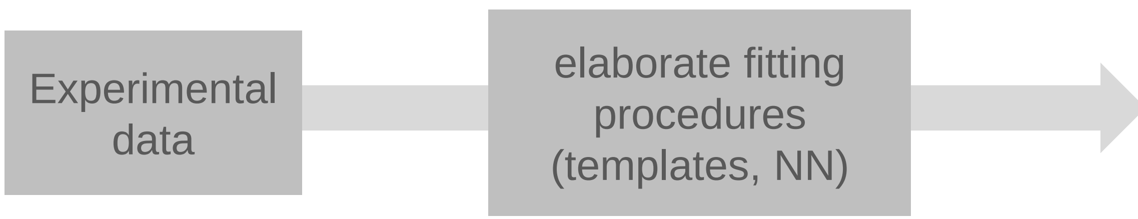

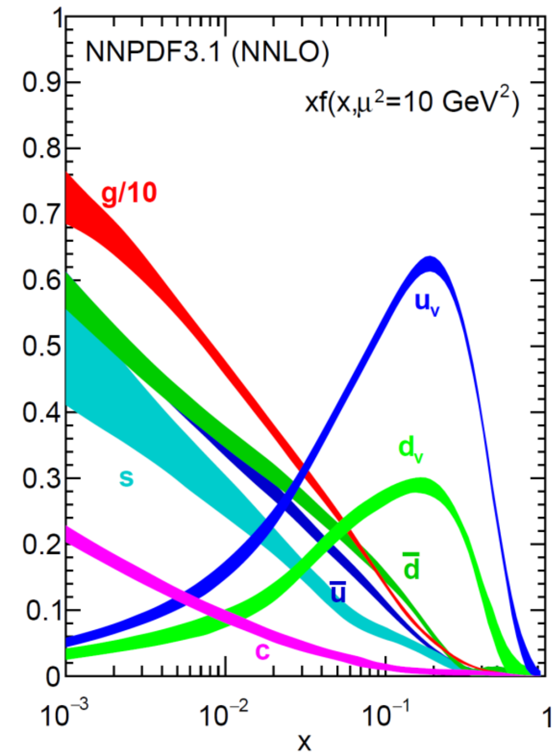

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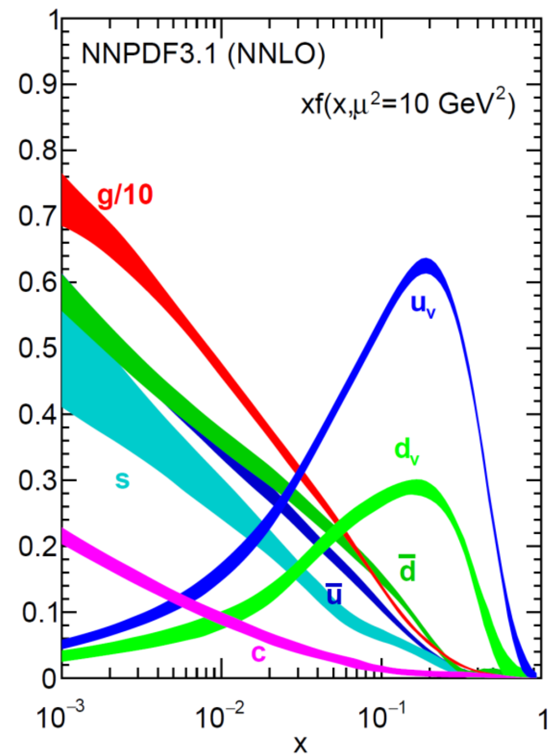

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?

- Theory goal: provide genuine first principles constraints to PDFs via LQCD.

Towards PDF's from the lattice

University of Stavanger

- Three strategies on the market: Quasi PDF's, Pseudo PDFs, Hadronic Tensor

Quasi PDF (Ji, 2013)

1. solve inverse problem for quasi PDF

$$M(\nu, \frac{\nu^2}{p_z^2}) = \int_0^\infty dy\, cos(\nu y)\tilde{q}(y, p_z)$$

2. match result to PDF at large p_z

$$\tilde{q}(y, p_z) \longrightarrow q(x, \mu)$$

Needs 3-point function:

$$\langle H(p)|\bar{\psi}(z)\frac{\tau^3}{2}\gamma^\alpha W(z;0)\psi(0)|H(p)\rangle = p^\alpha M(\nu, z^2) + z^\alpha N(\nu, z^2)$$

+ perturbative matching

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Pseudo PDF (Radyushkin, 2017)

1. match M to Ioffe time PDF Q

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Hadronic Tensor (Liu, 1994)

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iqz}\langle N(p)|[J^\dagger_\mu(z), J_\nu(z)]|N(p)\rangle$$
$$= \Pi^{(1)}_{\mu\nu}F(x, Q^2) + \Pi^{(2)}_{\mu\nu}F_2(x, Q^2)$$

$$F_1(x, Q^2) \rightarrow q(x, Q^2) \qquad F_2(x, Q^2) \rightarrow xq(x, Q^2)$$

1. Compute renormalized lattice $W^E_{\mu\nu}$

2. Analytic continuation to Minkowski

$$W^E_{\mu\nu}(\mathbf{q}, \mathbf{p}, \tau) = \int d\omega\, e^{-\tau\omega}W_{\mu\nu}(\mathbf{q}, \mathbf{p}, \omega)$$

Requires computation of 4-point function

The pseudo PDF inverse problem

- Ill-posed inverse problem due to interplay of finite ν and limited integration range

$$Q(\nu,\mu) = \int_0^1 dx\, cos(\nu x) q(x,\mu) \xrightarrow[\text{datapoint } \nu=[0,\nu_{max}]]{\text{discretize with } N_\nu} \mathbf{Q} = K\mathbf{q}$$

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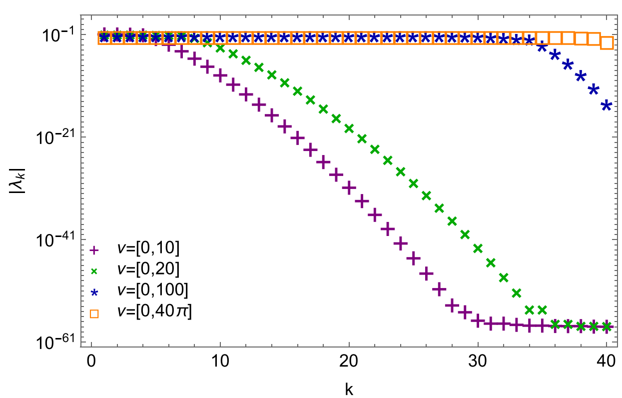
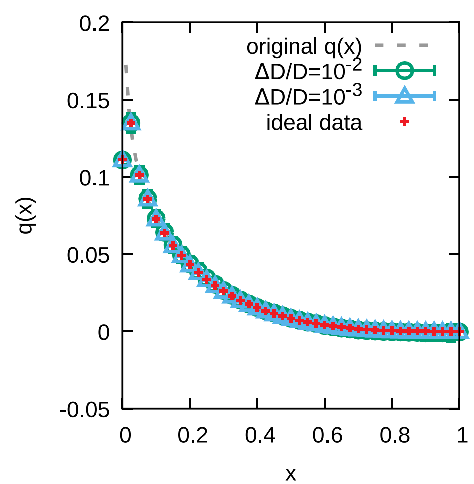
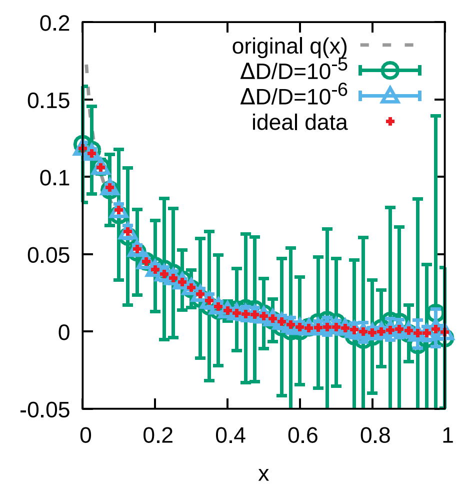
- Direct reconstructions are challenged: (e.g. SVD regularization for K^{-1})





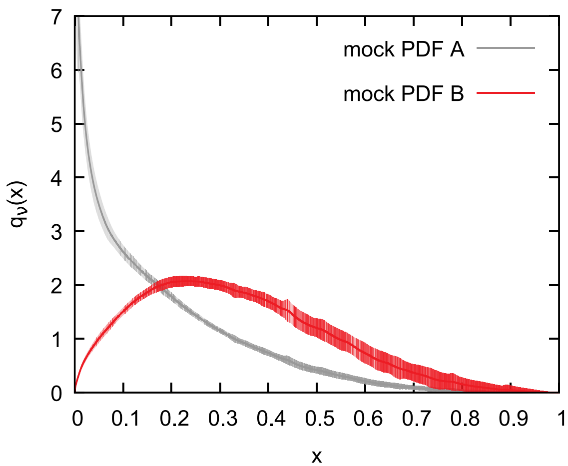


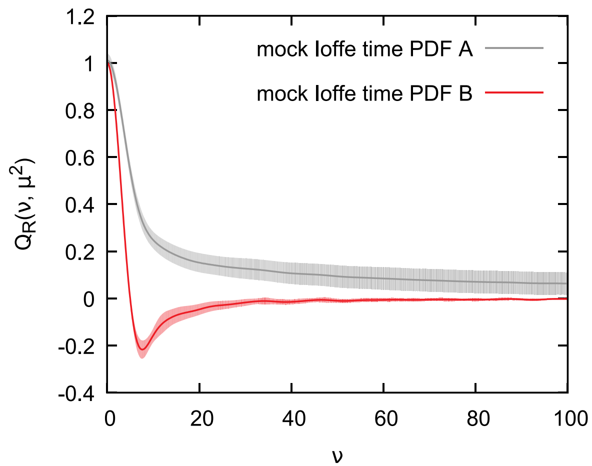
  - For realistic ν_{max} ~ 20 direct inversion even with 10^{-5} relative error not robust

More advanced reconstructions

University of Stavanger

- Our goal: identify most promising reconstruction approach via mock tests





input data: 100 replicas of
- NNPDF pheno PDF **(A)**
- q(x=0)=0 deformed version **(B)**

for realistic conditions discretize via $\nu_{max} \leq 20$, N_ν = 10-20

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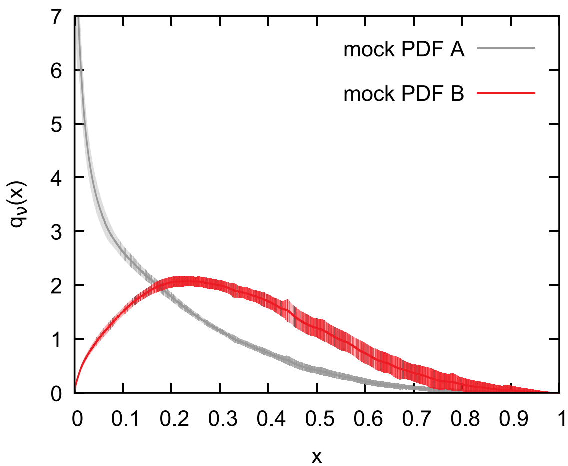

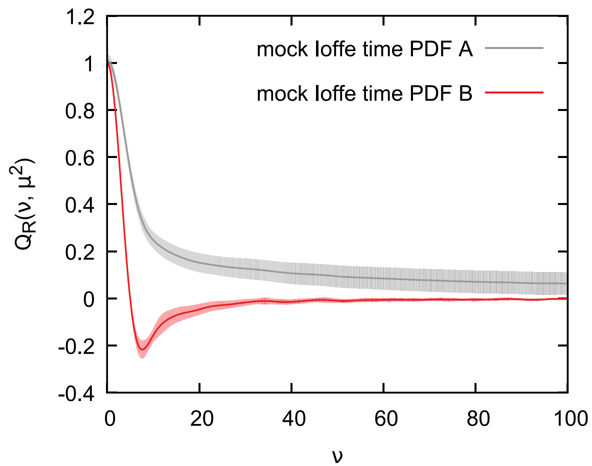

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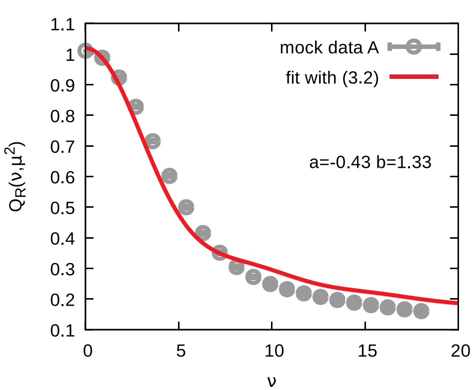
- Prior knowledge on singularities needed for accurate reconstruction: fit Q with pheno ansatz



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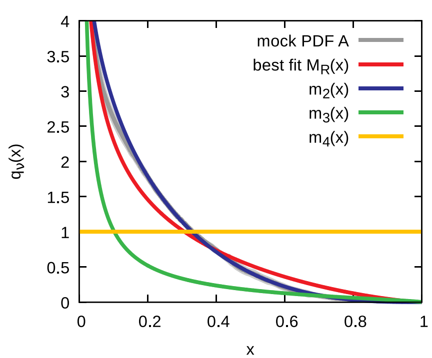

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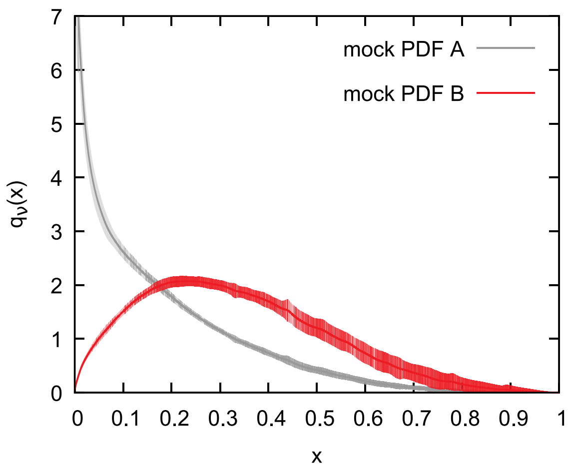
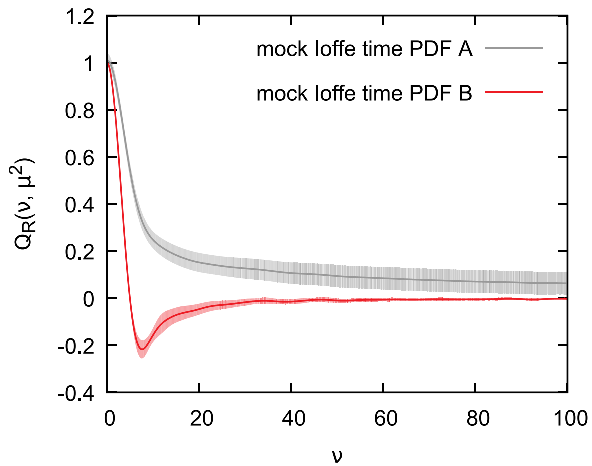
- Our goal: identify most promising reconstruction approach via mock tests

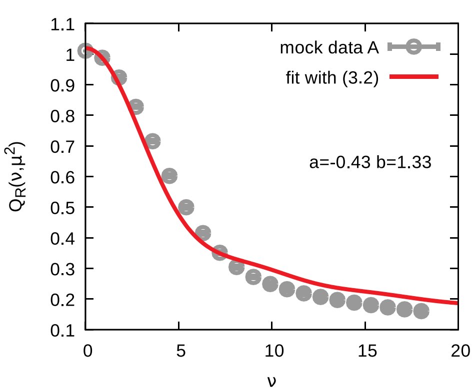




input data: 100 replicas of
- NNPDF pheno PDF **(A)**
- q(x=0)=0 deformed version **(B)**

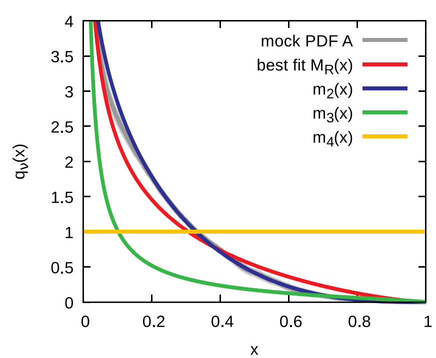
for realistic conditions discretize via $\nu_{max} \leq 20$, N_ν = 10-20

- Prior knowledge on singularities needed for accurate reconstruction: fit Q with pheno ansatz



p(x)=

$x^a(x-1)^b$



for Backus Gilbert precondition with best polynomial fit (incorporate divergence at x=0):

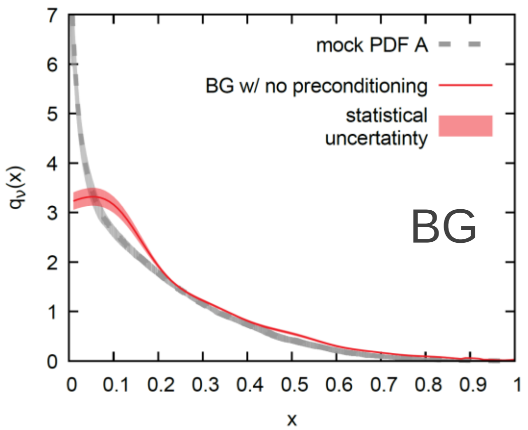
$$Q(\nu,\mu) = \int_0^1 dx \cos(\nu x) p(x) \frac{q(x,z^2)}{p(x)}$$

for Bayesian methods supply best polynomial fit as default model

$$m(x) = p(x)$$

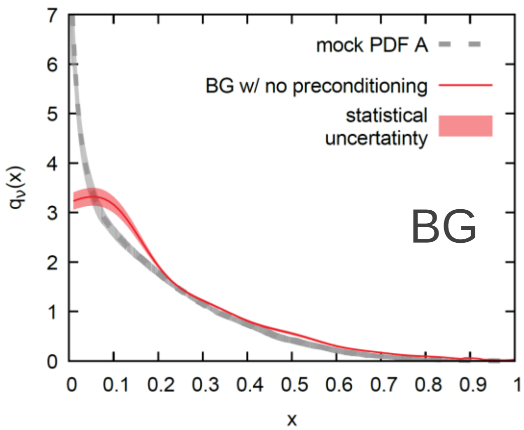
Comparison of approaches (A)

- Test battery BG – MEM – BR – Tickhonov, based on ν_{max} = 10, N_ν = 12



Comparison of approaches (A)

- Test battery BG – MEM – BR – Tickhonov, based on v_{max} = 10, N_v = 12
  - fitted p(x) captures PDF well: methods incorporating m=p strongly in end result at advantage



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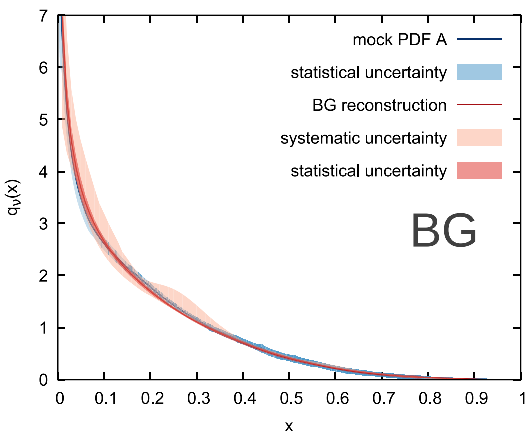

Comparison of approaches (A)

University of Stavanger

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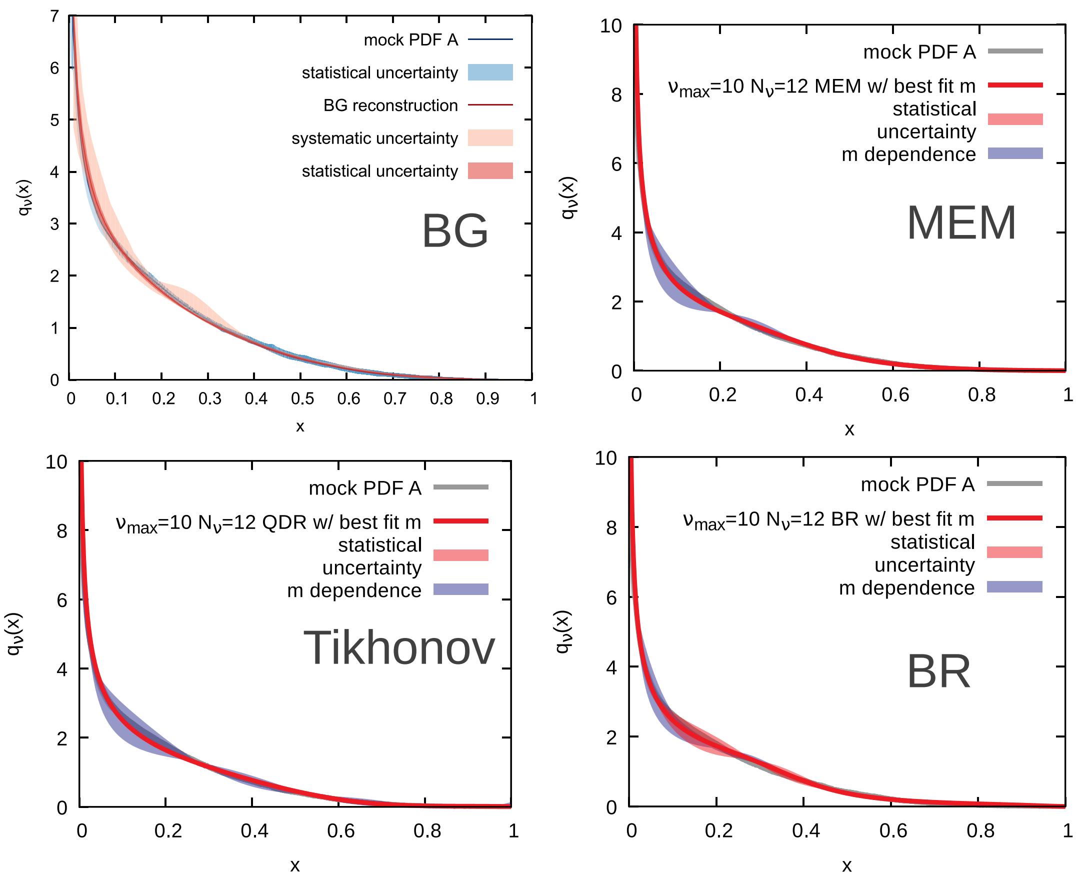

- Similar performance among all methods for scenario **(A)** if prior information in q(x->0) present

- Systematic errors from changing the preconditioning polynomial (fit parameters x2 or x0.5)

Comparison of approaches (B)

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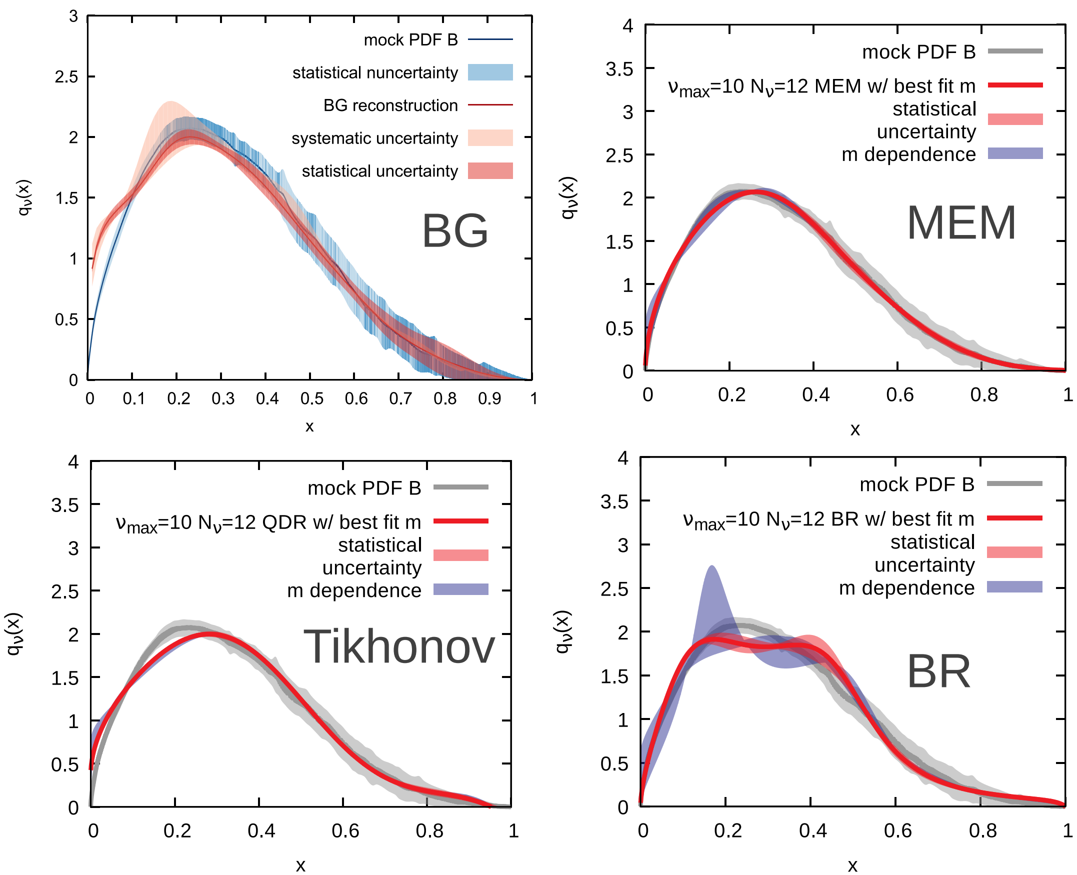
- Test battery BG – MEM – BR – Tickhonov, based on v_{max} = 10, N_v = 12
  - fitted p(x) captures PDF well: methods incorporating m=p strongly in end result at advantage

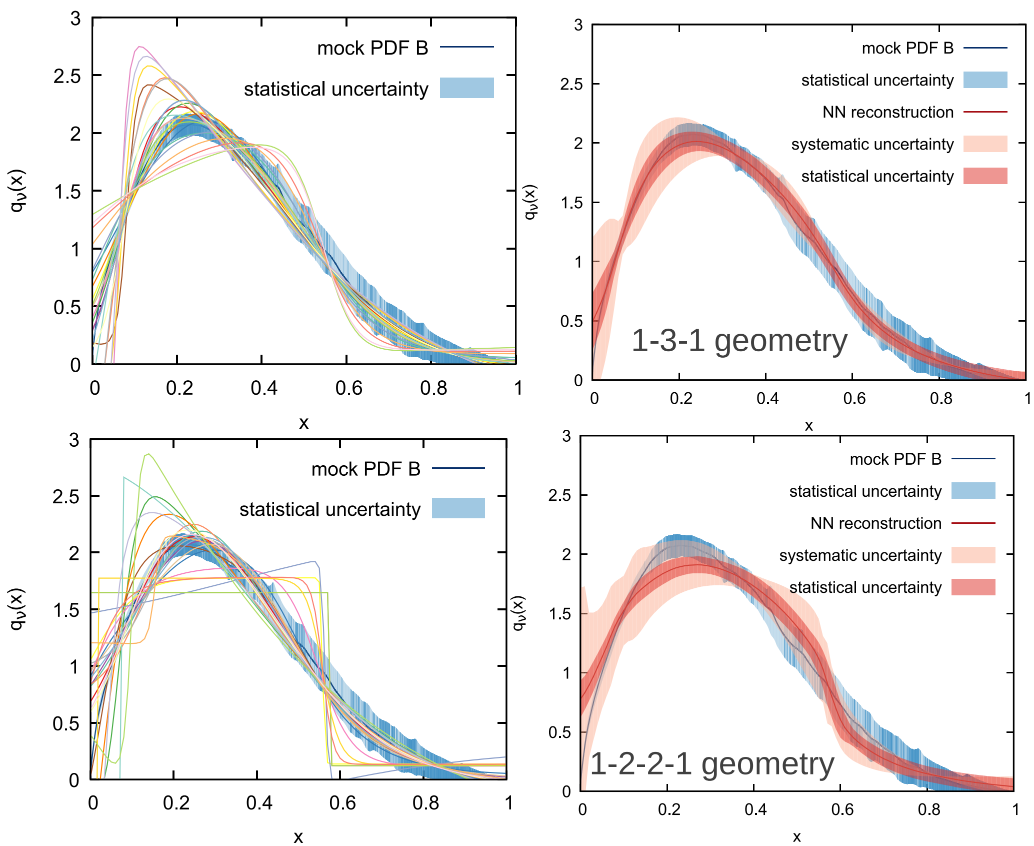


- MEM most accurate and precise for scenario **(B),** BG and Tikhonov struggle with q(0)=0.
- Systematic errors from changing the preconditioning polynomial (fit parameters x2 or x0.5)
- Extending input data to v_{max} = 20, N_v = 24 leads to sign. improved results and all Bayesian methods comparable

Neural network approach

University of Stavanger

- Current interpretation: sampling of degenerate χ^2 functional with restricted basis



- Use genetic algorithm (sim. annealing) on average of 100 replicas to generate a set of 25 optimal NNs: variation as systematic uncertainty
- Generate optimal NN for each of the 100 replicas: variation as statistical uncertainty

Neural network approach

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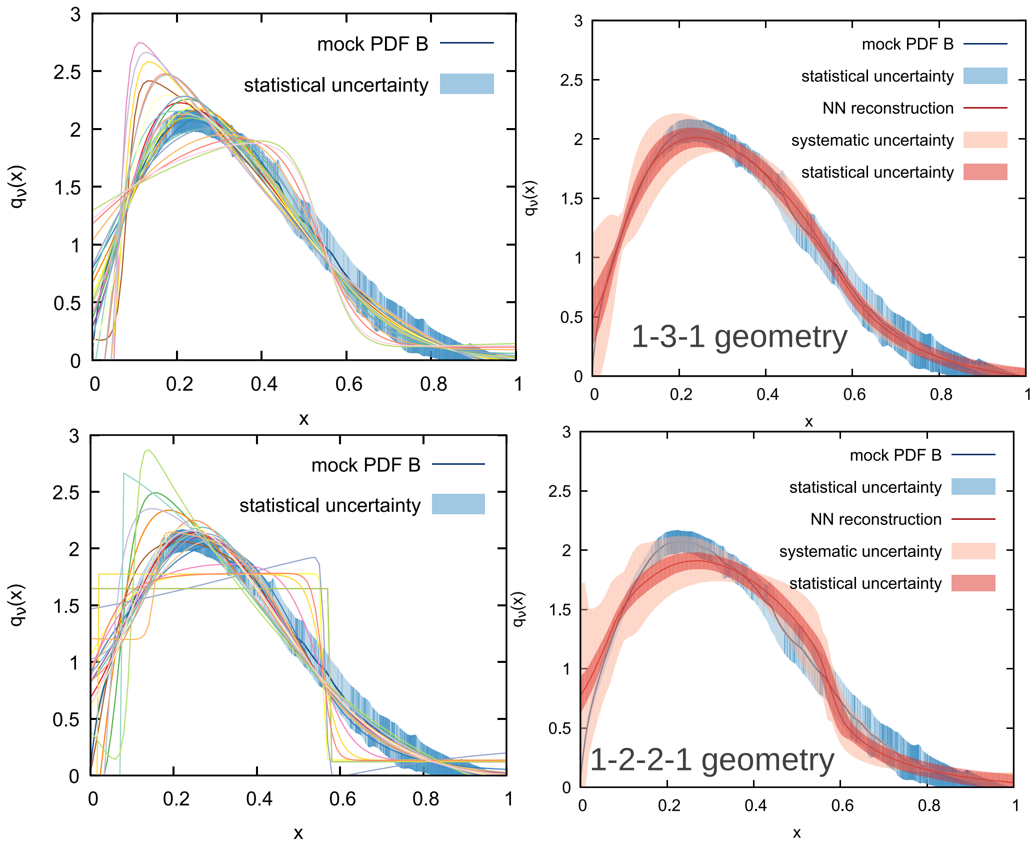

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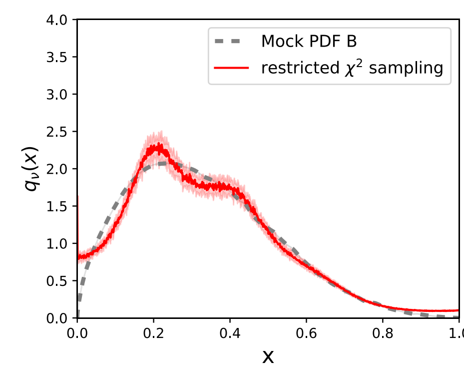

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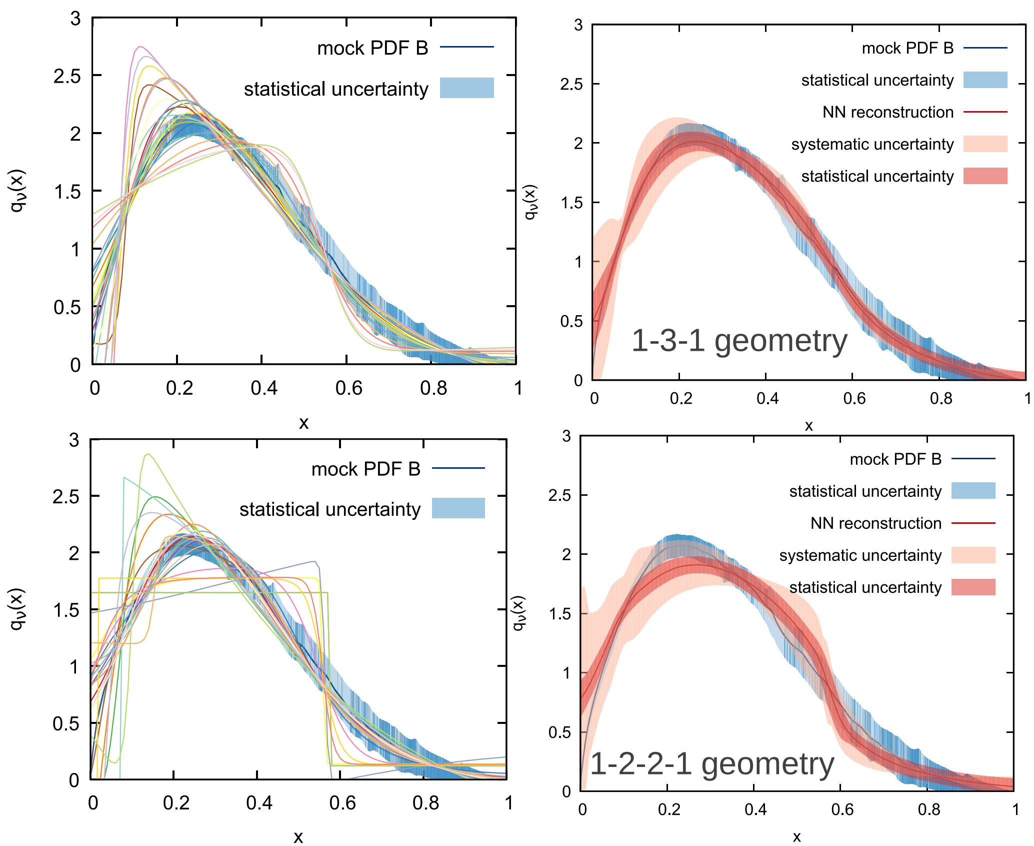

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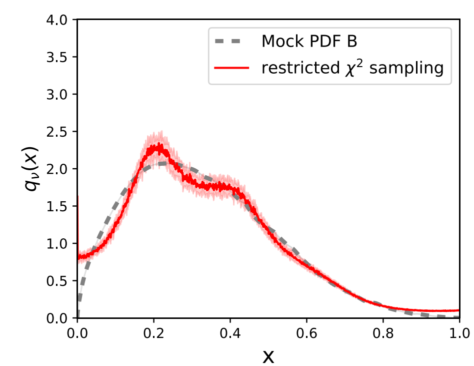

- Need to better understand influence of NN geometry on end result, systematic not quantified

- The real-time challenge
- T>0 quarkonium in-medium spectral functions on the lattice
- Towards parton distribution functions on the lattice
- Conclusion

Conclusion

- Physics of HIC urges non-perturbative understanding of QCD real-time dynamics
- Lattice QCD real-time information hidden behind ill-posed inverse problems
  - Main challenges: **exponential information loss** and **limited Euclidean time extent**
- Two concrete examples of recent progress:
  - **In-medium heavy quarkonium** spectral functions: new estimate for in-medium mass shifts from a better understanding of reconstruction systematics
  - **Parton Distribution Functions:** first step beyond naïve inversions. Promising for accurate extraction of PDFs directly from upcoming lattice QCD simulations

Conclusion

University of Stavanger

- Physics of HIC urges non-perturbative understanding of QCD real-time dynamics
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  - **Parton Distribution Functions:** first step beyond naïve inversions. Promising for accurate extraction of PDFs directly from upcoming lattice QCD simulations
- A lot of work remains to be done:
  - **Improving Bayesian methods:** need to incorporate more specific prior information
  - **Improve understanding of NN reconstruction:** e.g. influence of NN geometry

Backup slides

Non Bayesian approaches

Projection methods: Pade, Cuniberti

Find "optimal" approximation to the Matsubara correlator $D(\omega_n)$

$$C(\omega) = \cfrac{z_0}{1 + \cfrac{z_1(\omega-\omega_1)}{1+\cfrac{z_2(\omega-\omega_2)}{\vdots}}}$$

z_i chosen such that $C(\omega_n)=D(\omega_n)$

Corresponds to a projection onto rational basis functions

Spectral functions from analytic continuation $\mathrm{Im}[C(i\omega)]$

No default model, but requires very small statistical errors. Also reconstruction does not necessarily respect spectral rep.

Backus-Gilbert Method

Attempt to simplify the problem as far as possible by linear combinations of input data:

$$\tilde{D}(\tau') = \int d\tau \Sigma(\tau',\tau) D(\tau)$$
$$= \int d\omega \int d\tau \Sigma(\tau',\tau) K(\omega,\tau) \rho(\omega)$$

Goal: construct a resolution function

$$\int d\tau \Sigma(\tau',\tau) K(\omega,\tau) \approx \delta(\tau'-\omega)$$

No default model dependence, but also requires a choice of regularization at intermediate steps

Motivation: Quarkonium in HIC

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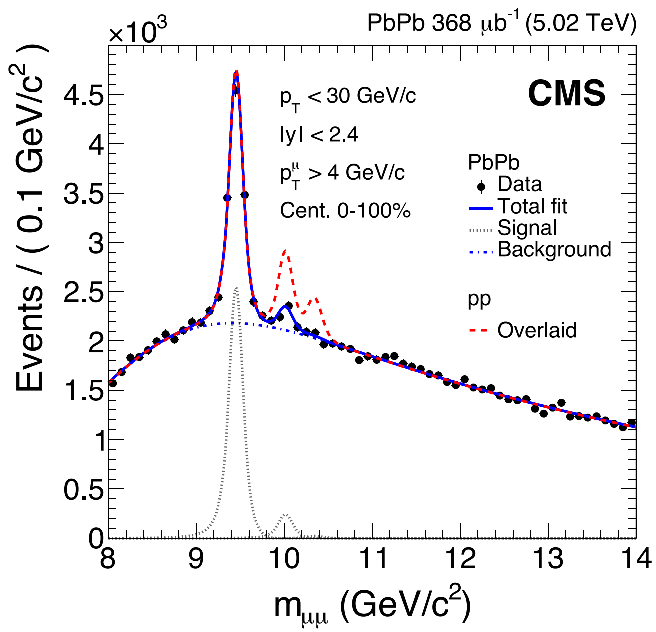
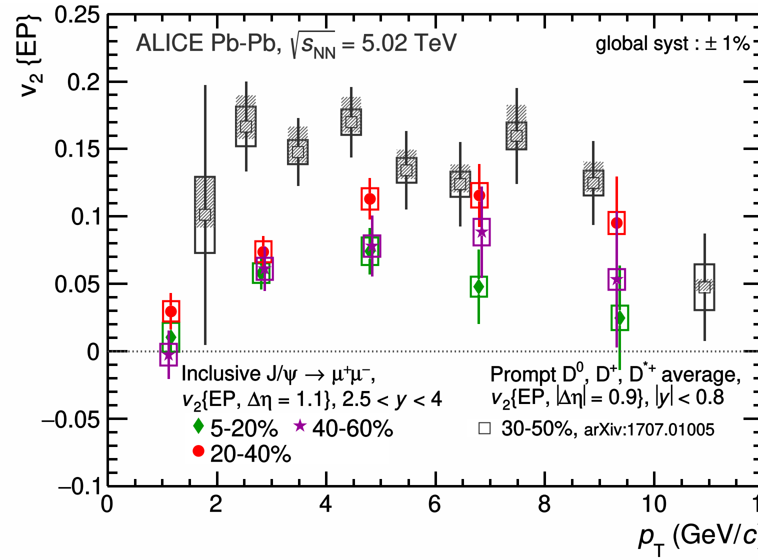
- Heavy quarkonium: **Precision probes** of the QGP



[CMS collaboration] PRL120 (2018) 142301

Bottomonium: a non-equilibrium probe of the full QGP evolution



[ALICE collaboration] PRL119 (2017) 242301

Charmonium: a partially equilibrated probe, sensitive to the late stages

- Idealized setting: properties of quarkonium in a static medium?

Towards a modern Bayesian approach

- Elevate to a full Bayesian analysis: sample the posterior via Monte Carlo
  - Much better control over both statistical and systematic uncertainties
  - Self consistent treatment of e.g. weight α possible, no ad-hoc assumptions needed

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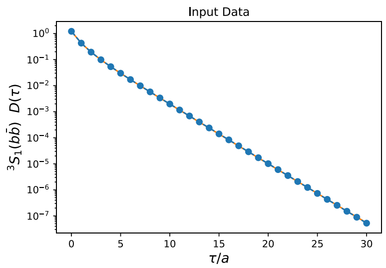

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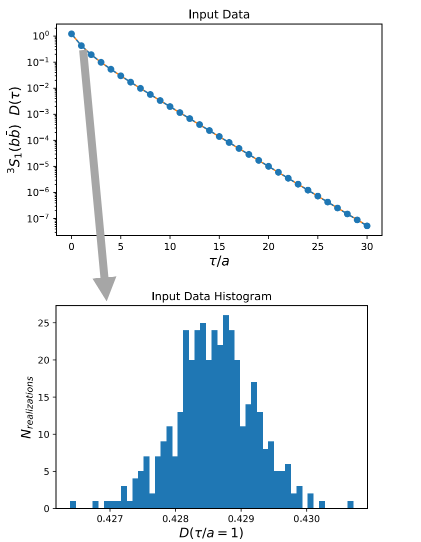

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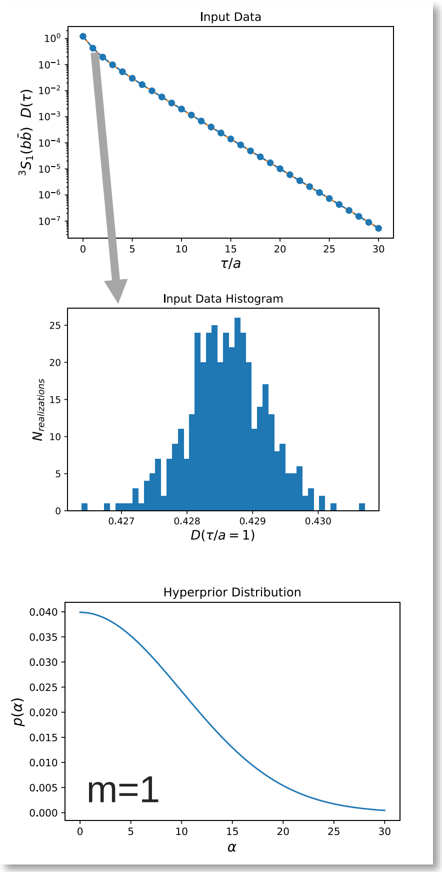

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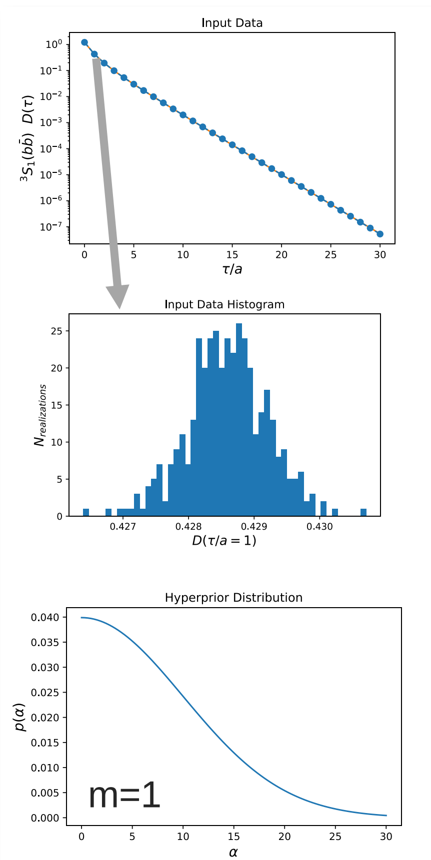

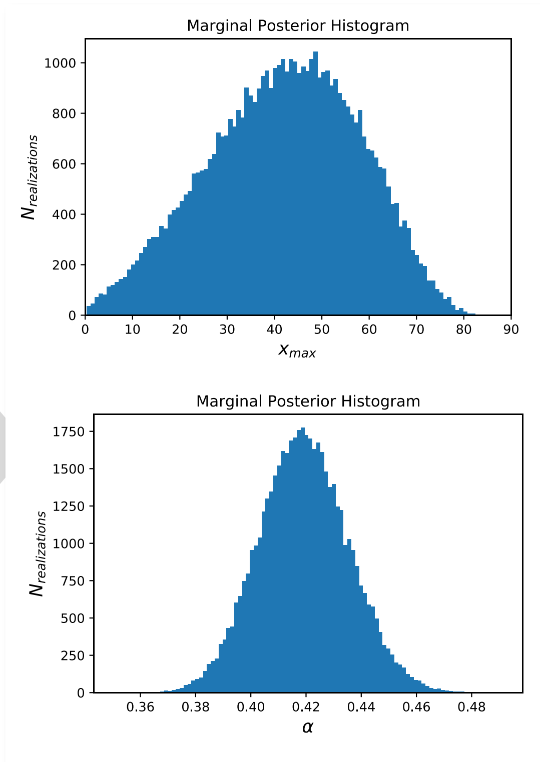

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

Towards a modern Bayesian approach

University of Stavanger

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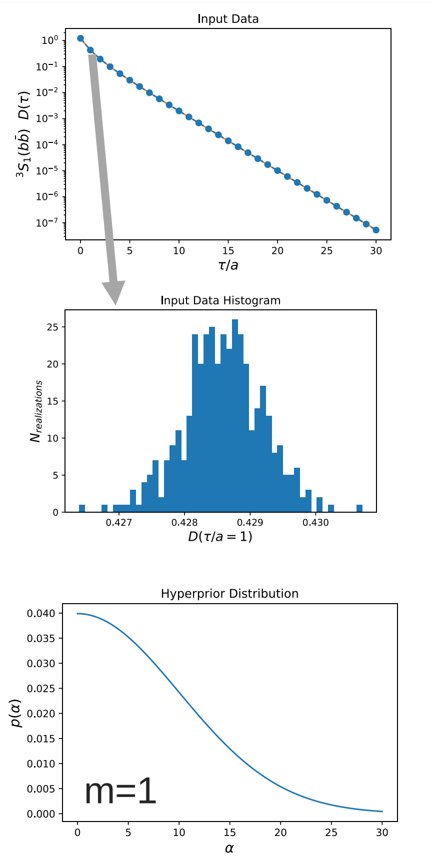

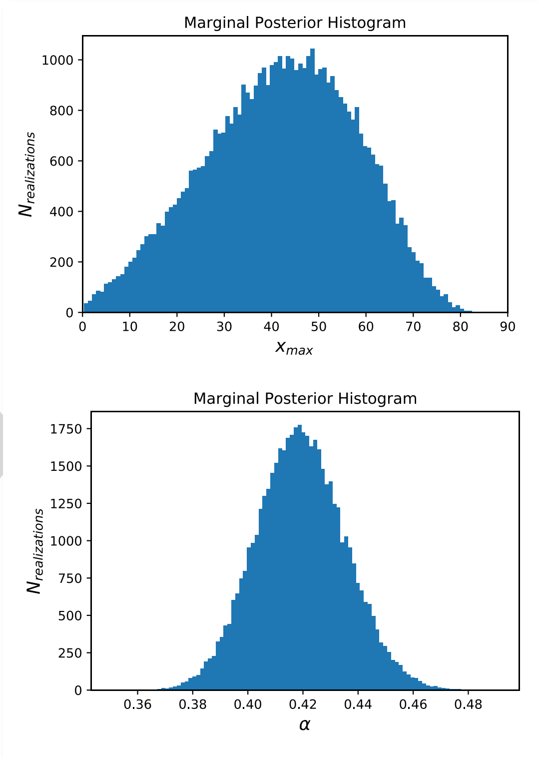

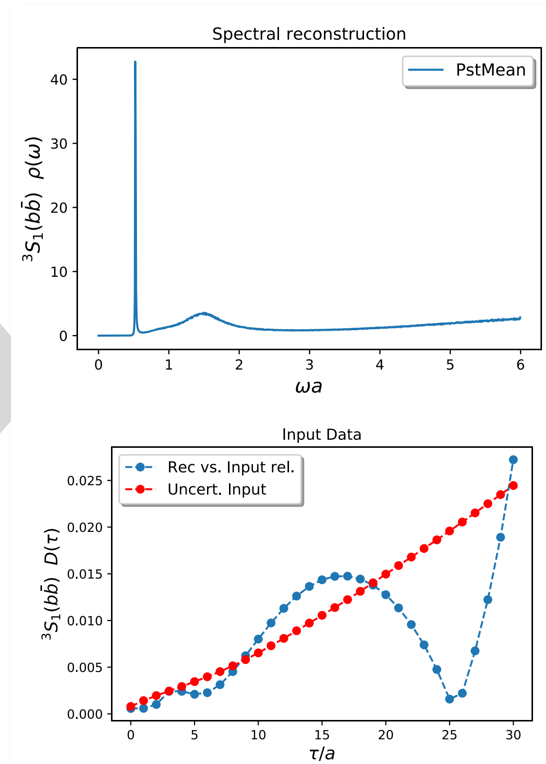

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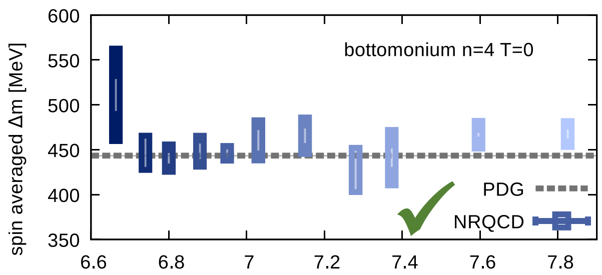
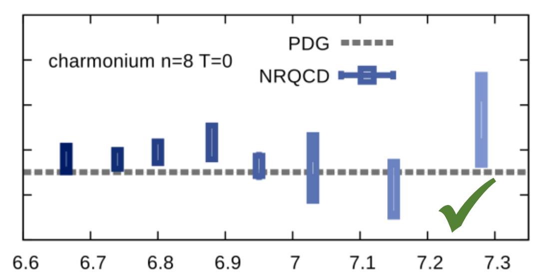
Mass splittings at T=0

- With T>0 spectra goal, no T=0 specific NRQCD improvements: Accuracy?





Spin weighted difference between S- and P-wave (c.f. potential model: dep. only on central pot.)

Mass splittings at T=0

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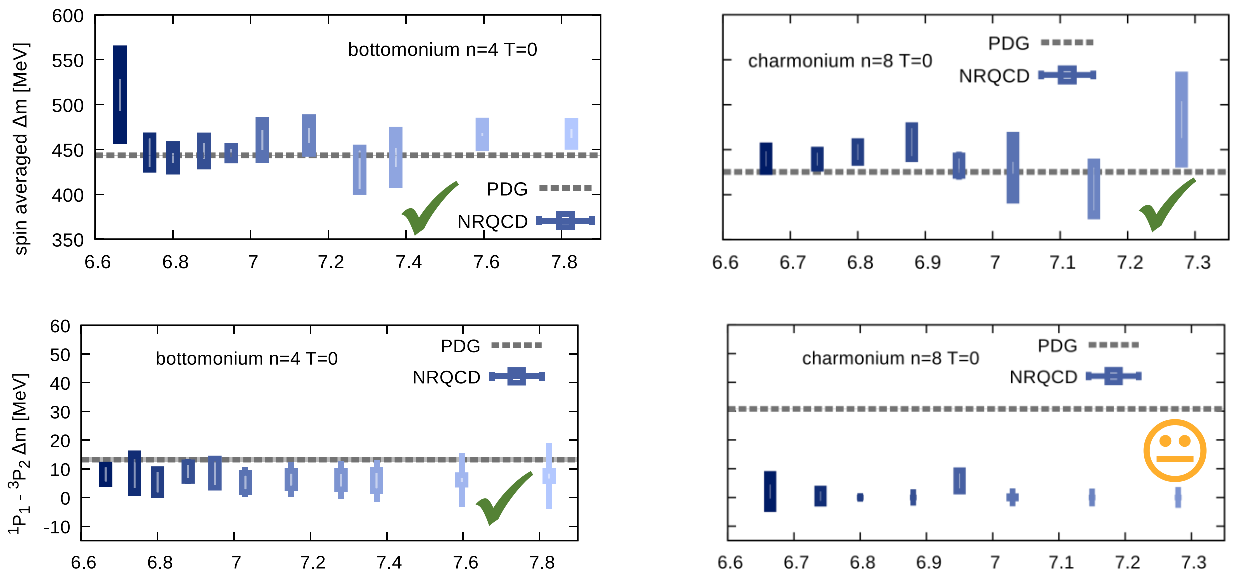

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P-wave $^1P_1$ $^3P_2$ splitting (c.f. potential model: spin-orbit coupling)

Mass splittings at T=0

University of Stavanger

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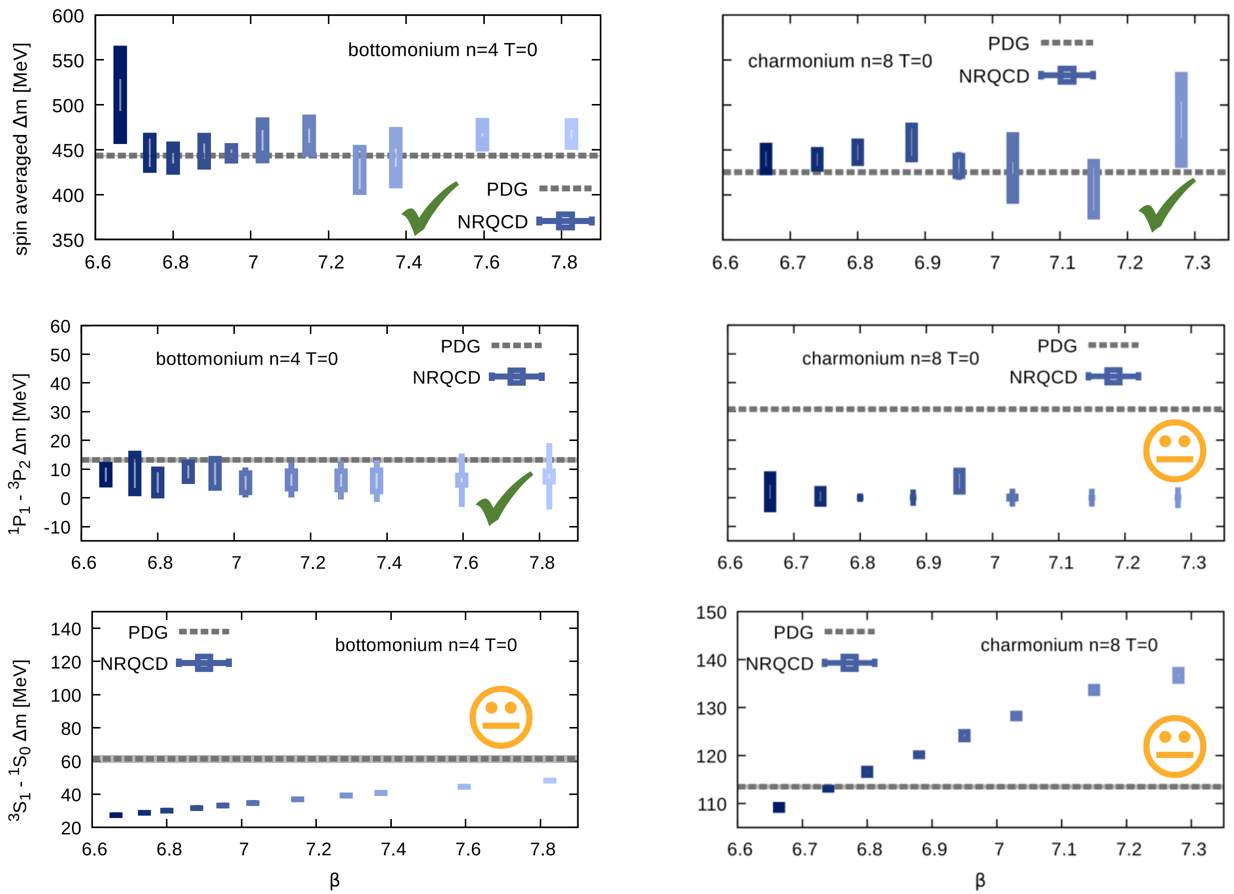

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S-wave splitting (c.f. potential model: HFS)

Max 35MeV deviation. (HFS: NRQCD $O(v^6)$ and $O(\alpha_S v^4)$)

Mass splittings at T=0

University of Stavanger

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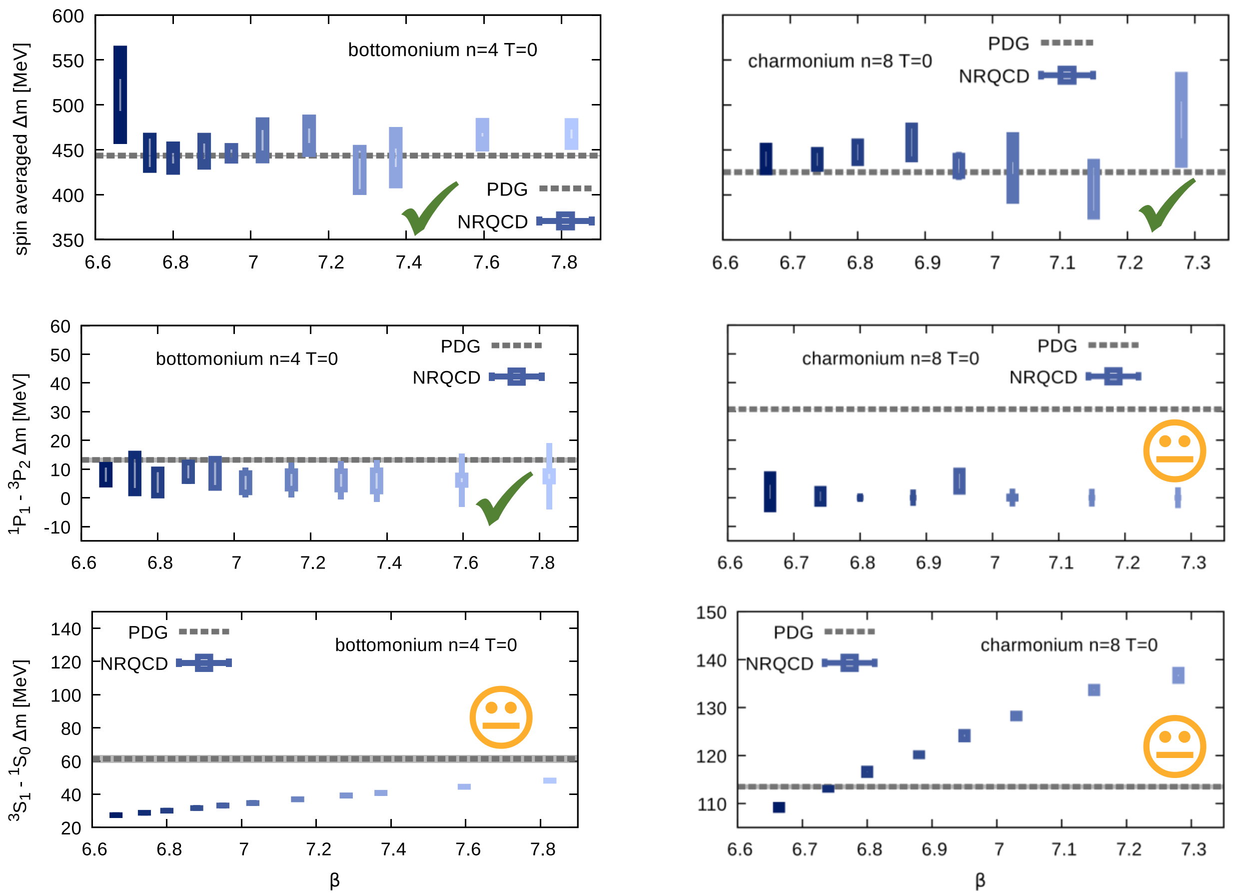

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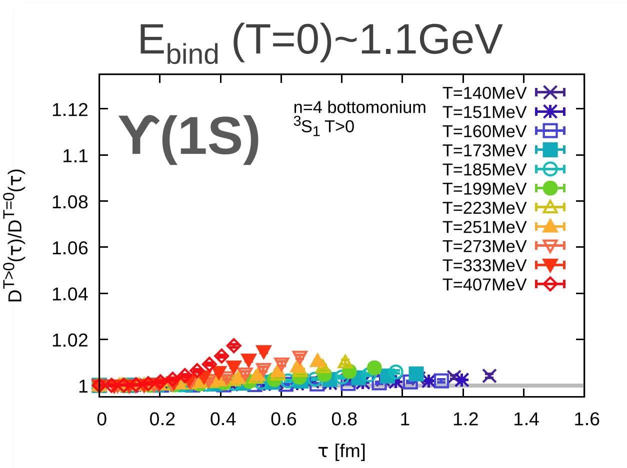
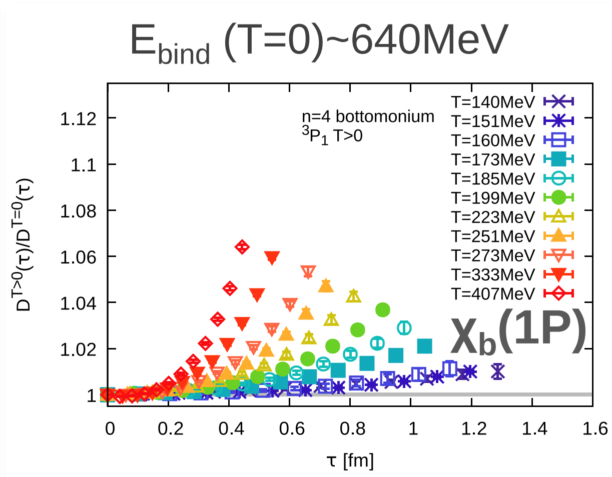
S-wave splitting (c.f. potential model: HFS)

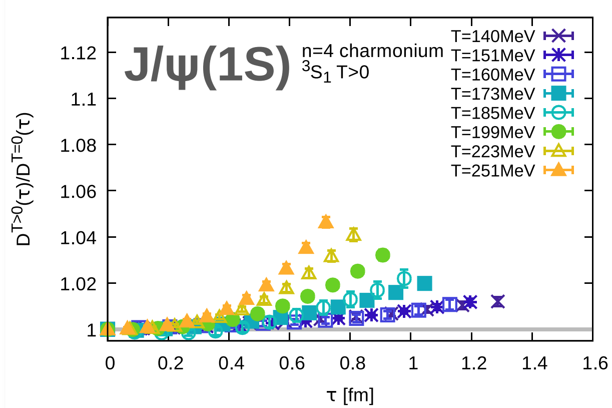
Max 35MeV deviation. (HFS: NRQCD $O(v^6)$ and $O(\alpha_S v^4)$)

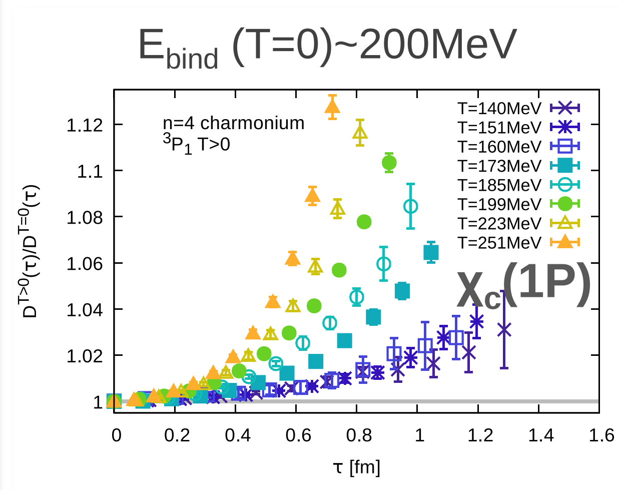
- Reasonable agreement but no competition with high-prec. T=0 NRQCD

Correlator ratios





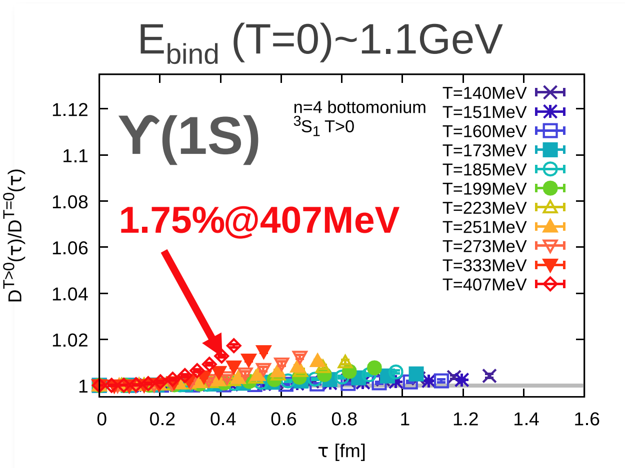


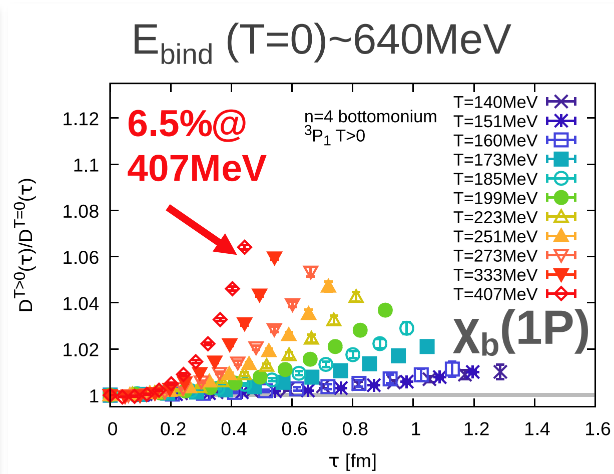


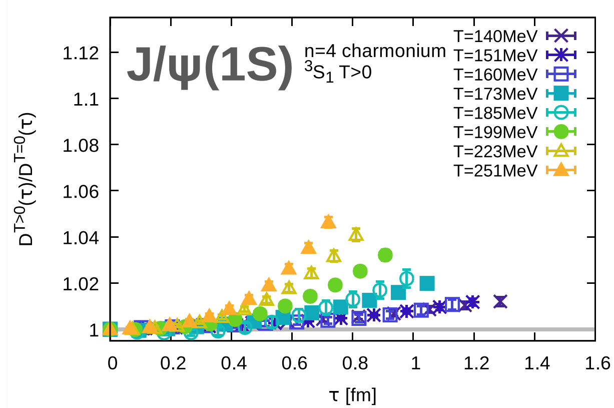
- Significant upward bent in the QGP phase
- Already in hadronic phase deviations from unity

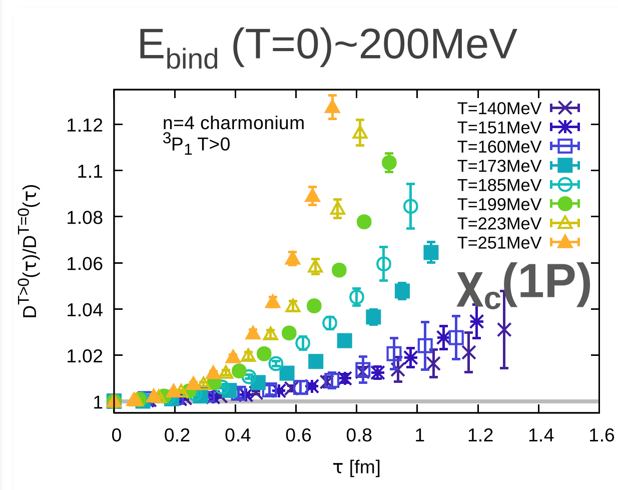
- In-medium modification hierarchically ordered with vacuum binding energy

Correlator ratios









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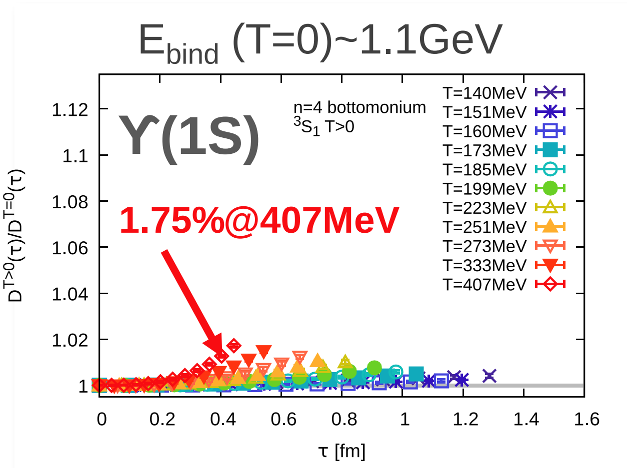

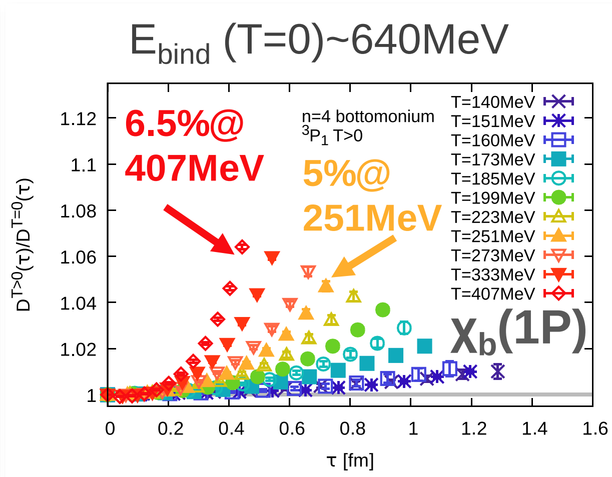

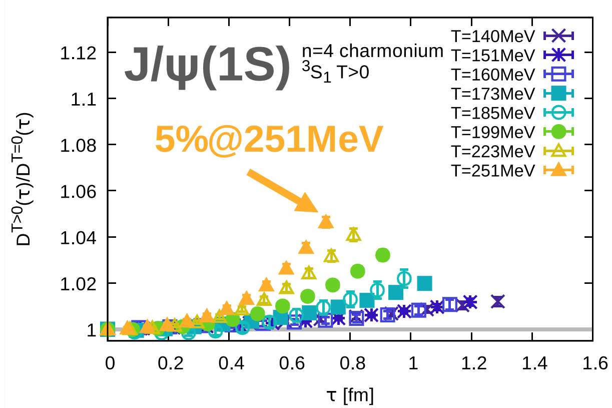

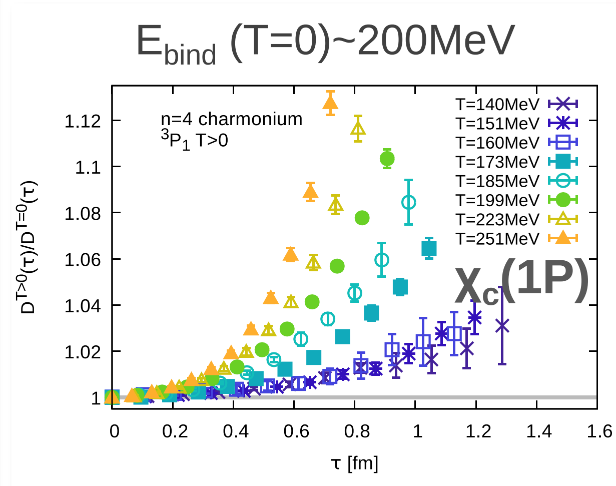

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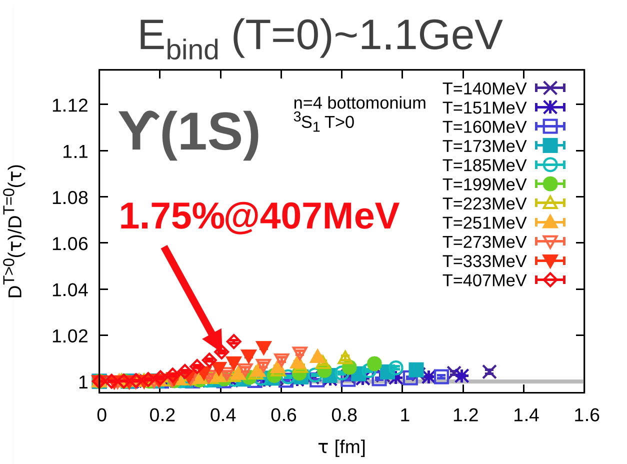

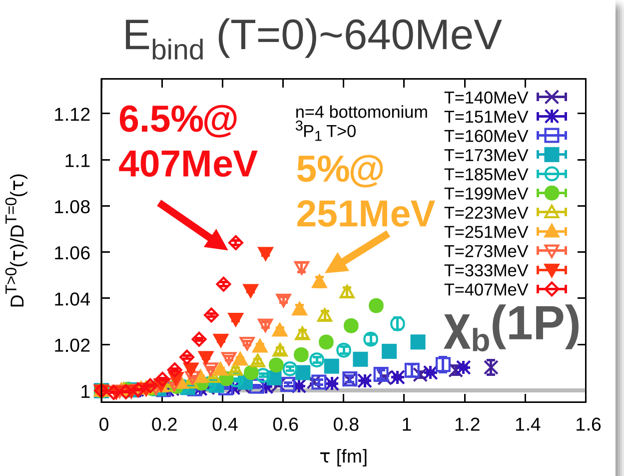

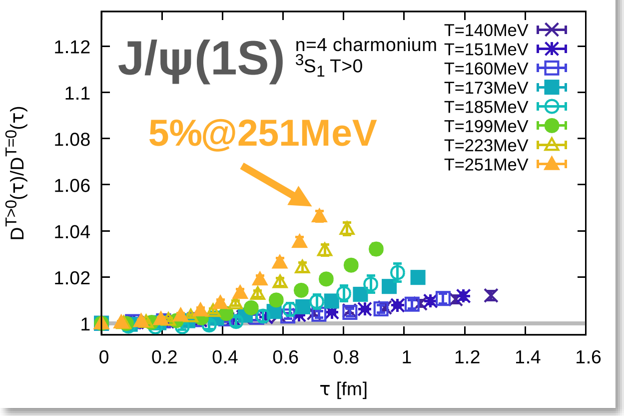

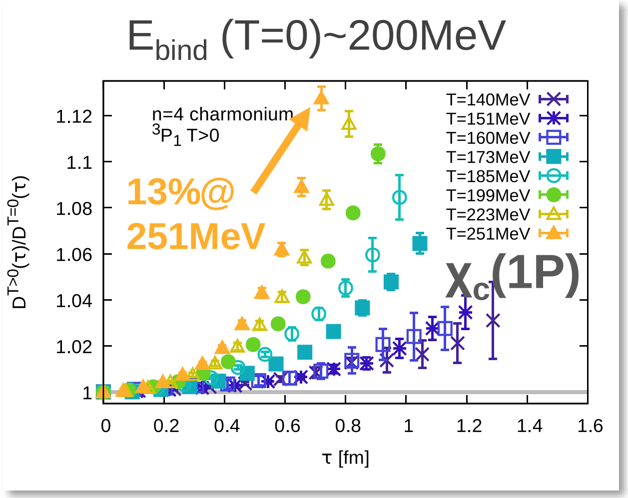

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Bottomonium P-wave melting at T>0

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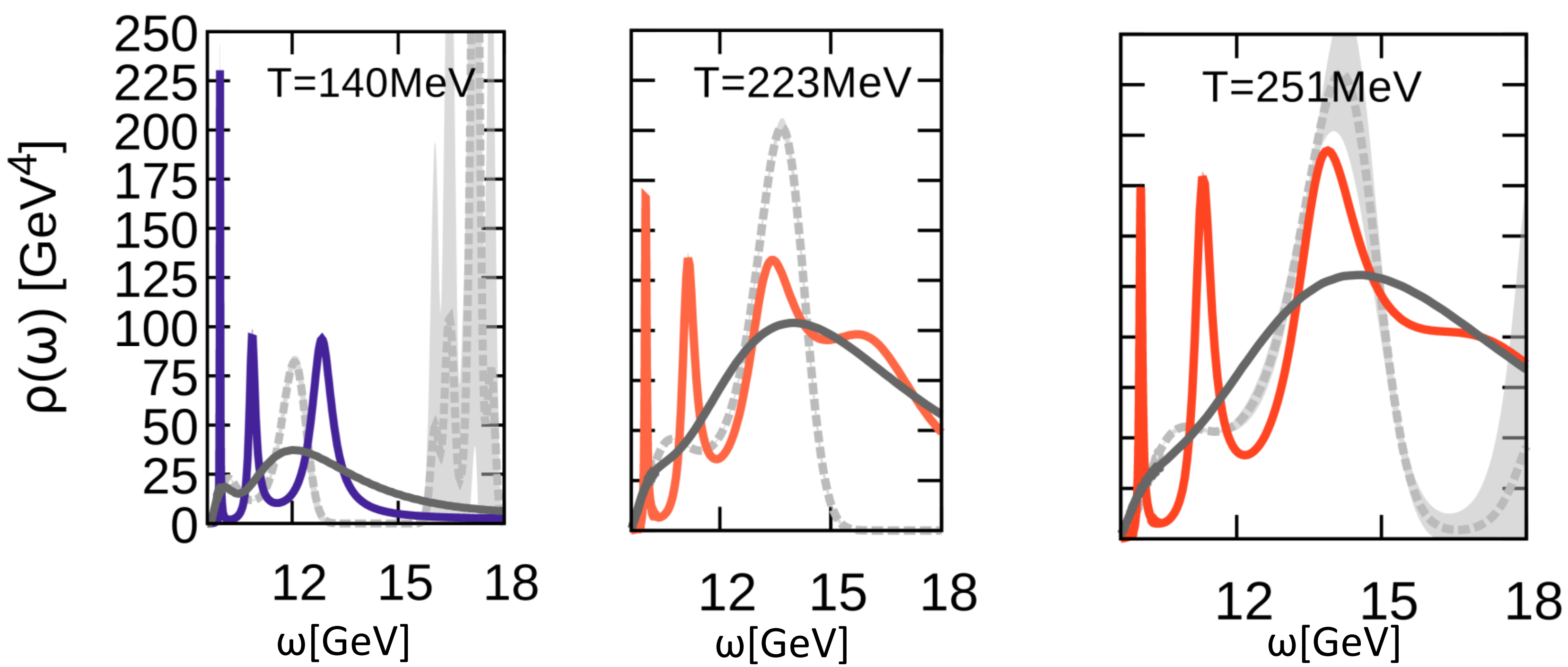
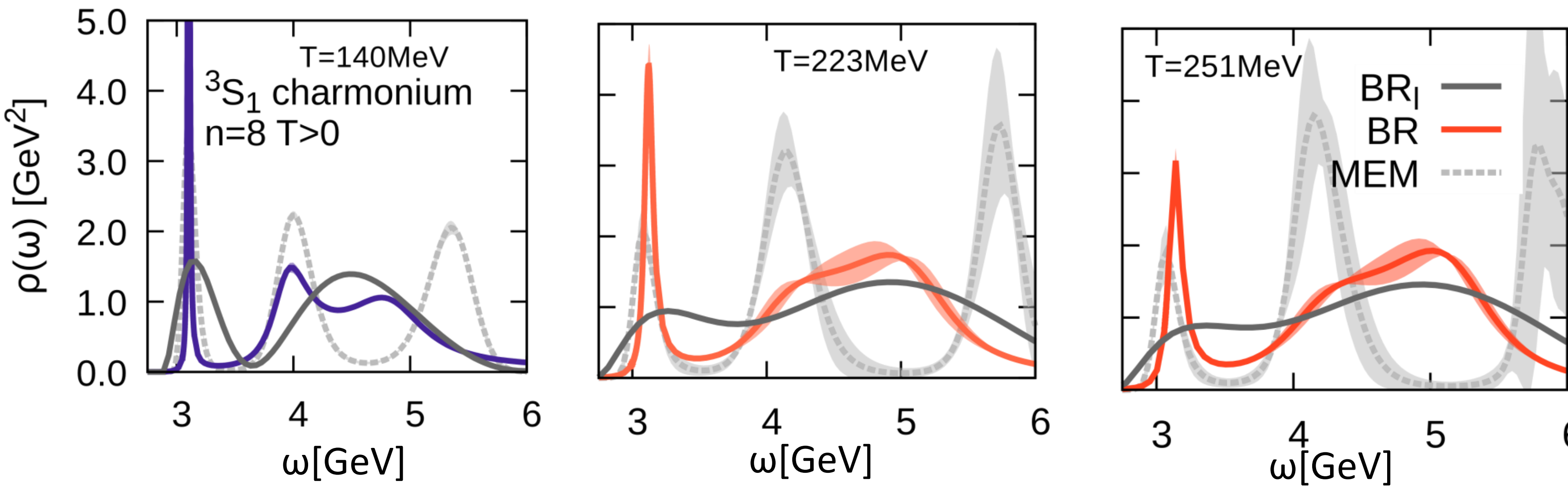


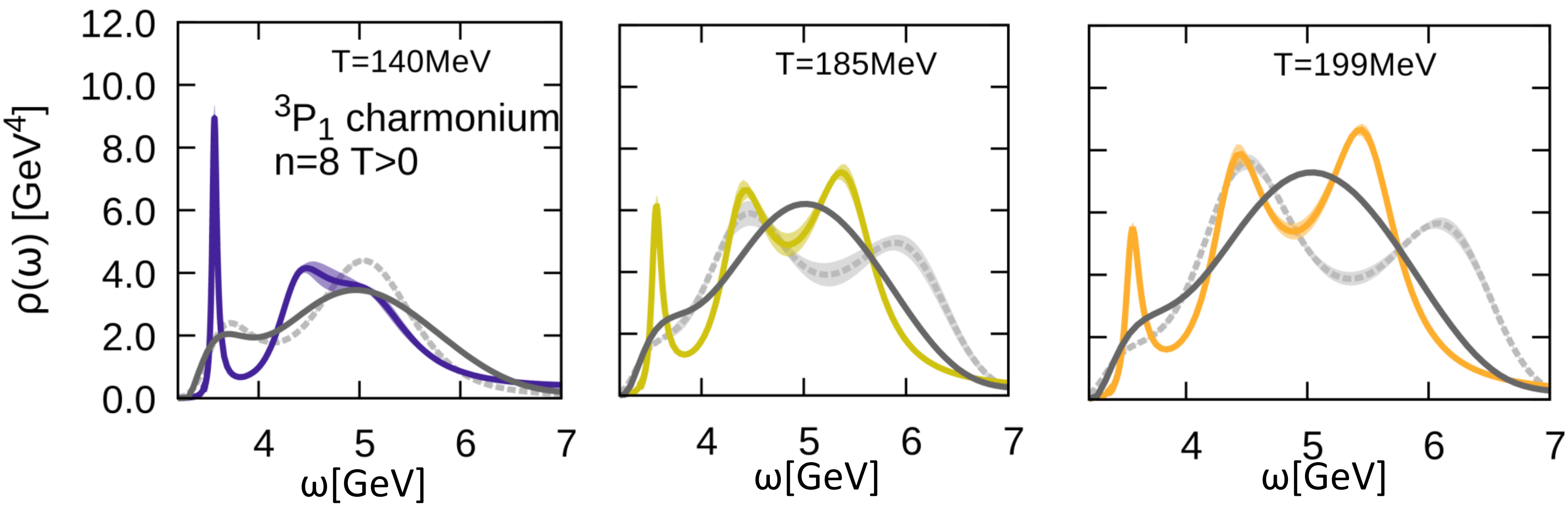
- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - At T=199MeV: all methods show remnant structure (threshold enhancement?)
  - For 199<T<233MeV: only original BR shows peak (amplitude higher than next structure)
  - For T>233MeV: lowest peak in original BR smaller than next, most likely ringing

Charmonium S-wave melting at T>0



- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - MEM and BR find peak at T=251MeV, smooth BR washed out at T>200MeV

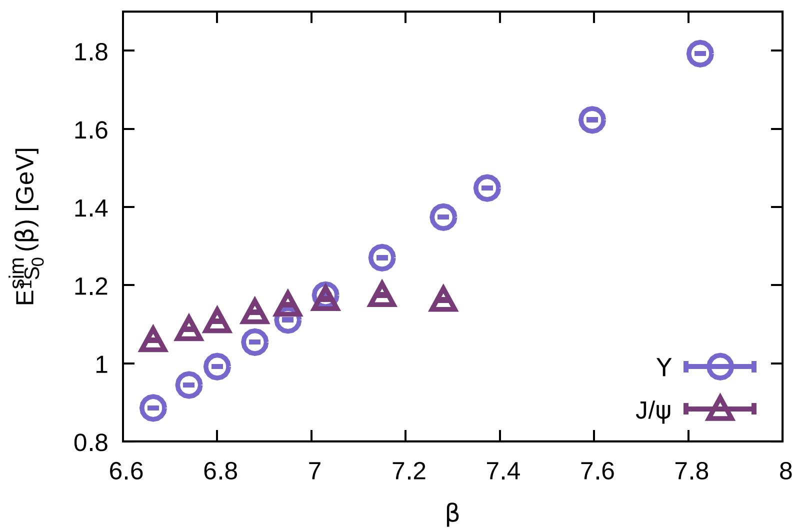
Charmonium P-wave melting at T>0



- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - At T<185MeV: all methods show remnant structure (threshold enhancement?)
  - For T=185MeV: only original BR shows peak (amplitude higher than next structure)
  - For T>185MeV: lowest peak in original BR smaller than next, most likely ringing

Energy calibration

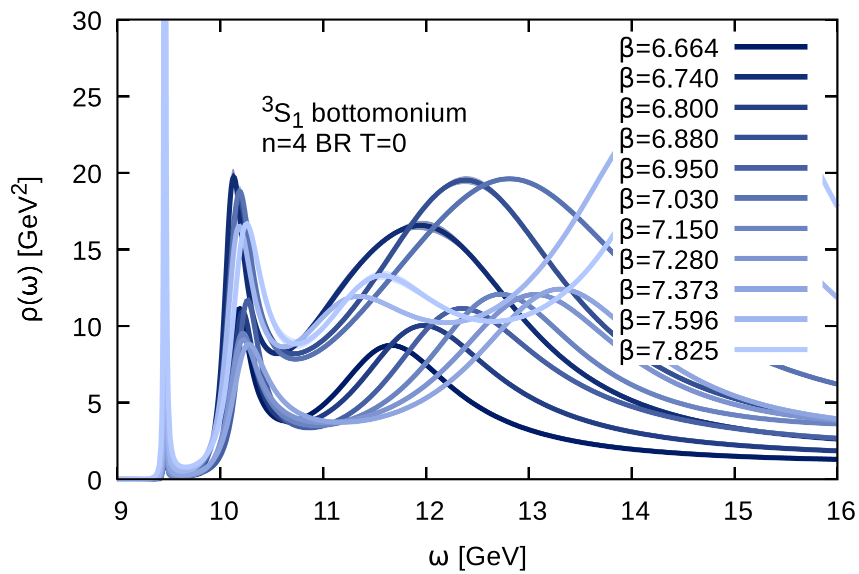
- In NRQCD energy needs extra calibration (shift by $2m_q$):



$$M^{exp}_{^sX_J} = E^{sim}_{^sX_J} + 2(Z_{M_q} M_q - E_0)$$

Use PDG value for
S-wave ground state

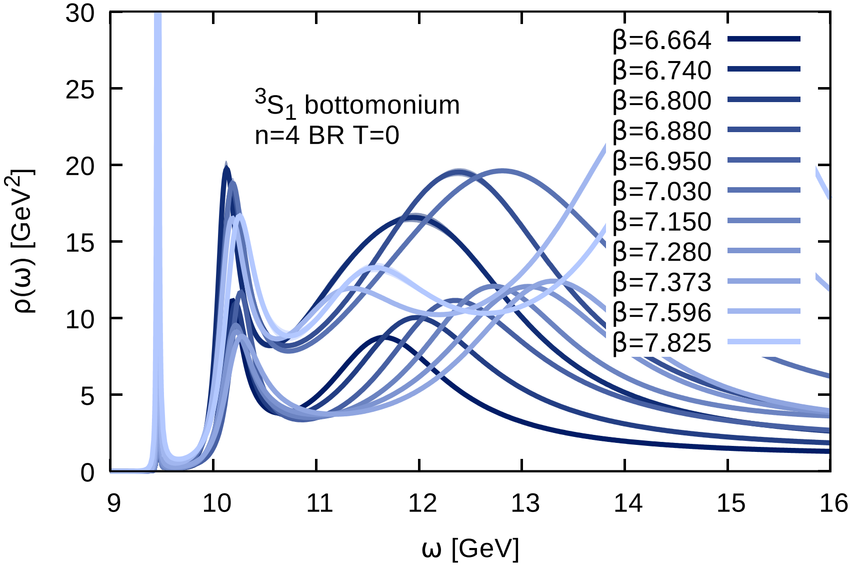
T=0 spectral reconstructions

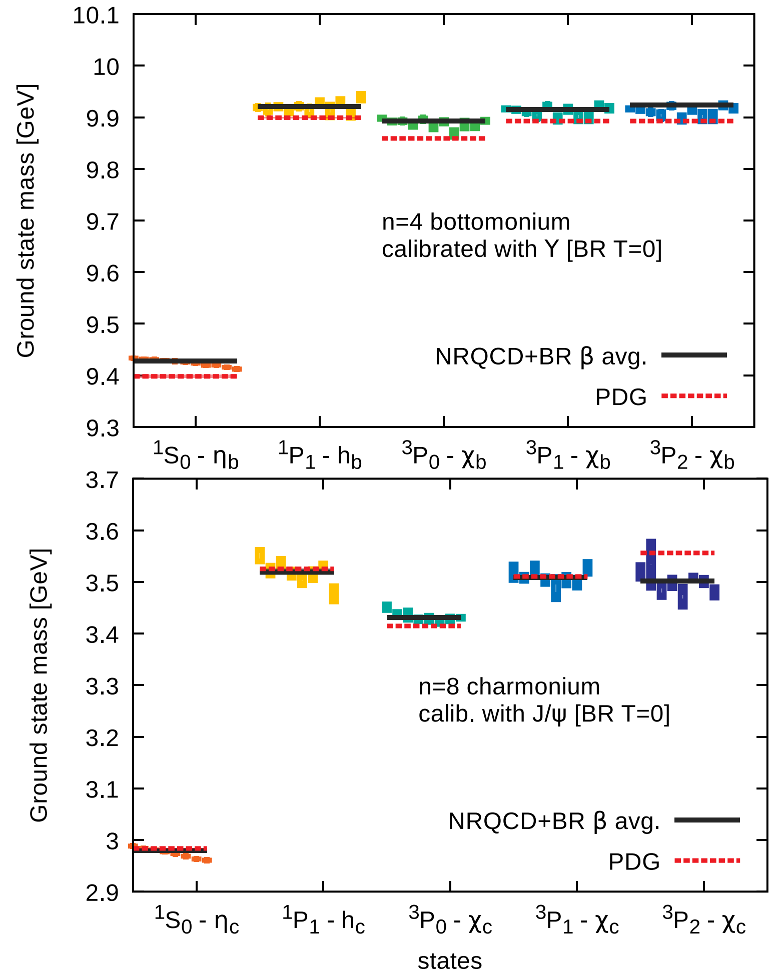


Reconstruction parameters:
N_ω=2000 frequency bins, ωa=[-5,20]
N_ω=600 bin high res. interval around GS peak
m(ω)=const. statistical errors from 10-bin jackknife
systematic errors from variation of default model

varying reconstructed peak heights since N_τ different

T=0 spectral reconstructions





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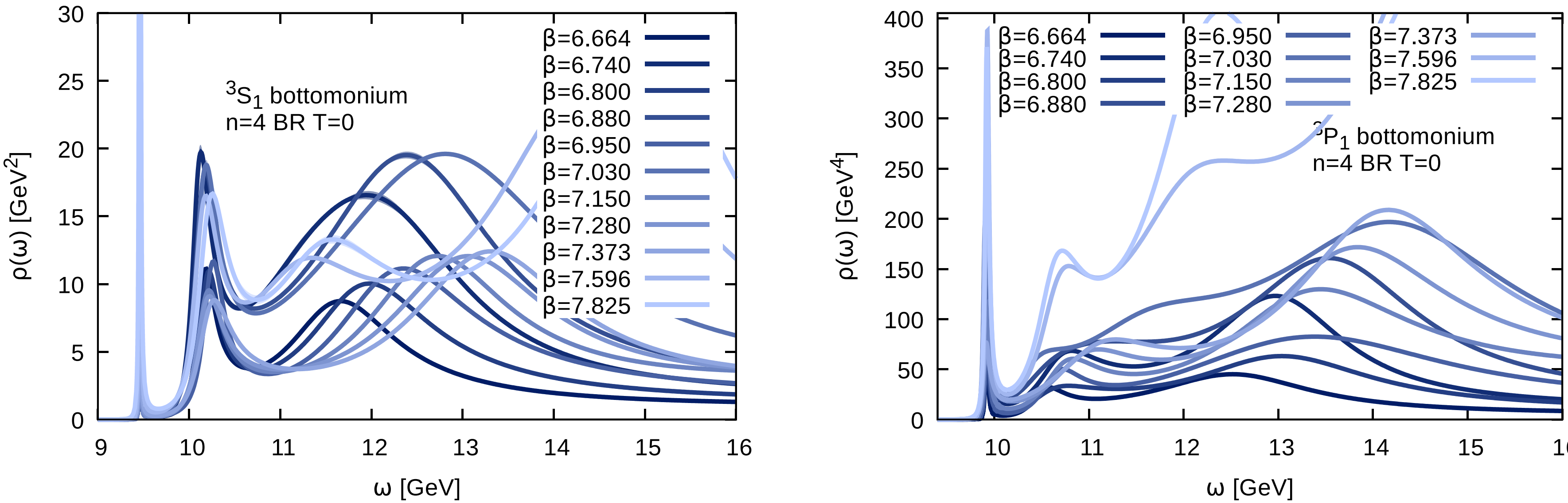
varying reconstructed peak heights since N_τ different

- GS peak position agree among effective mass fit and BR reconstruction

T=0 spectral reconstructions



Reconstruction parameters:
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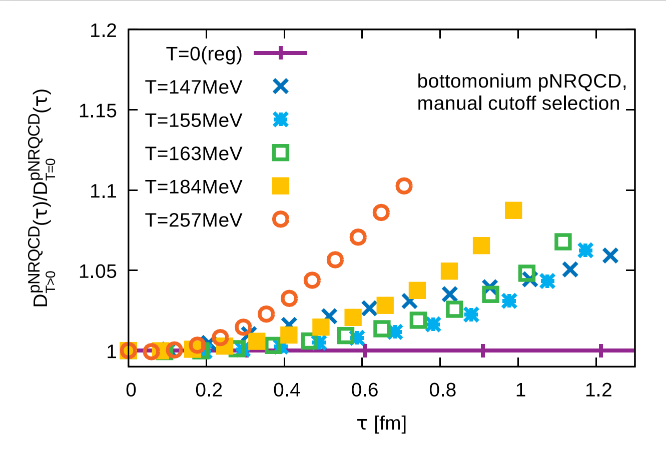
m(ω)=const. statistical errors from 10-bin jackknife
systematic errors from variation of default model

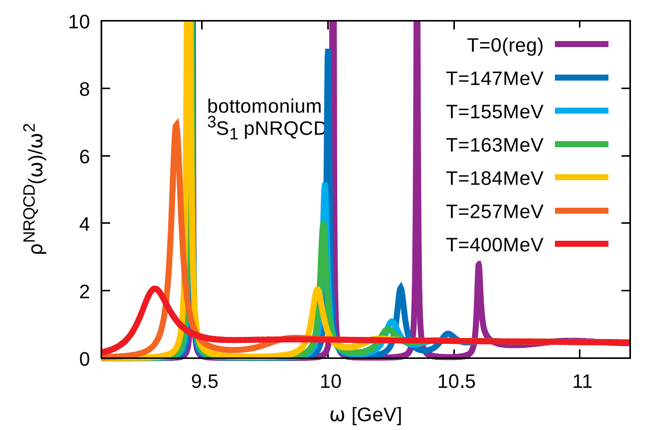
- Combined Euclidean and imaginary frequency data: less ringing at large ω
- GS peak position agree among effective mass fit and BR reconstruction

Interpreting the ratios

- Behavior in ratios qualitatively reproduced by potential based computation
  - Strong upward bend: ground state moves to smaller mass

Lattice QCD potential based computation



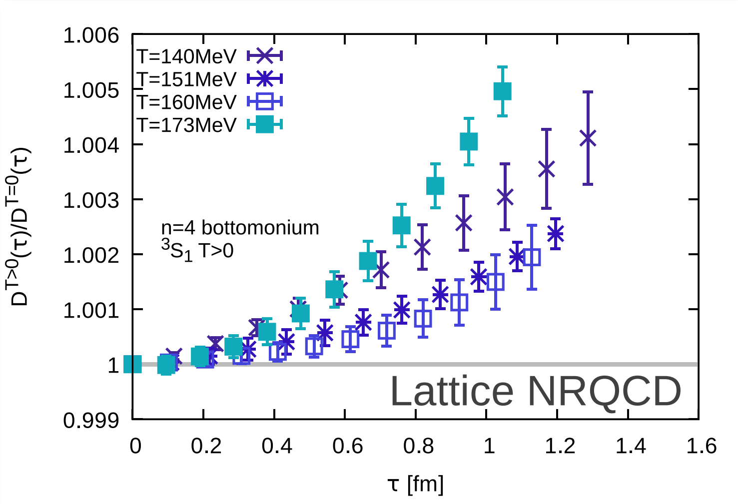


Interpreting the ratios

University of Stavanger

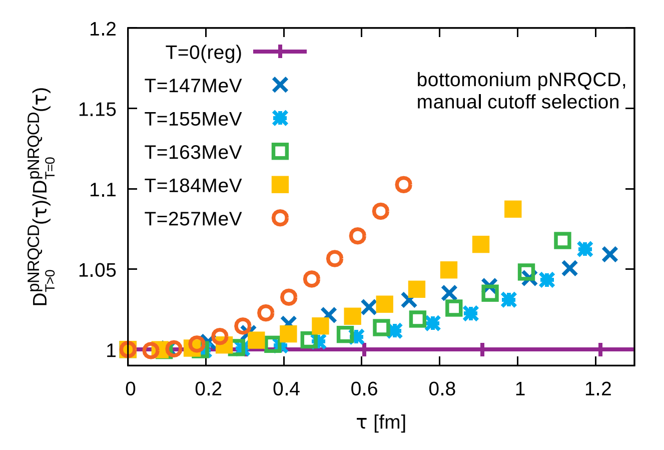
- Behavior in ratios qualitatively reproduced by potential based computation

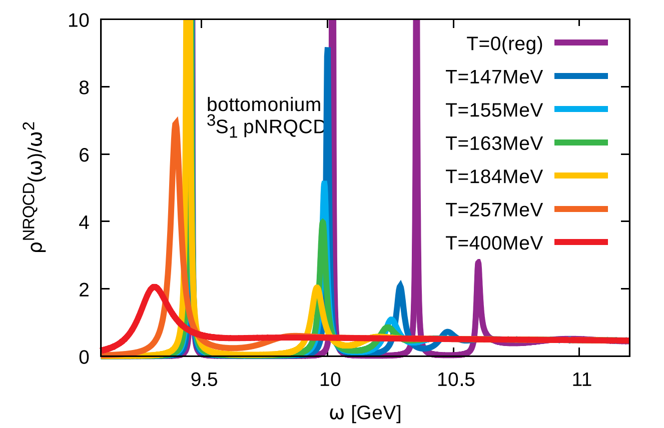
  - Strong upward bend: ground state moves to smaller mass



  - Non-monotonicity only in the Bottomonium S-wave: destabilization of excited states

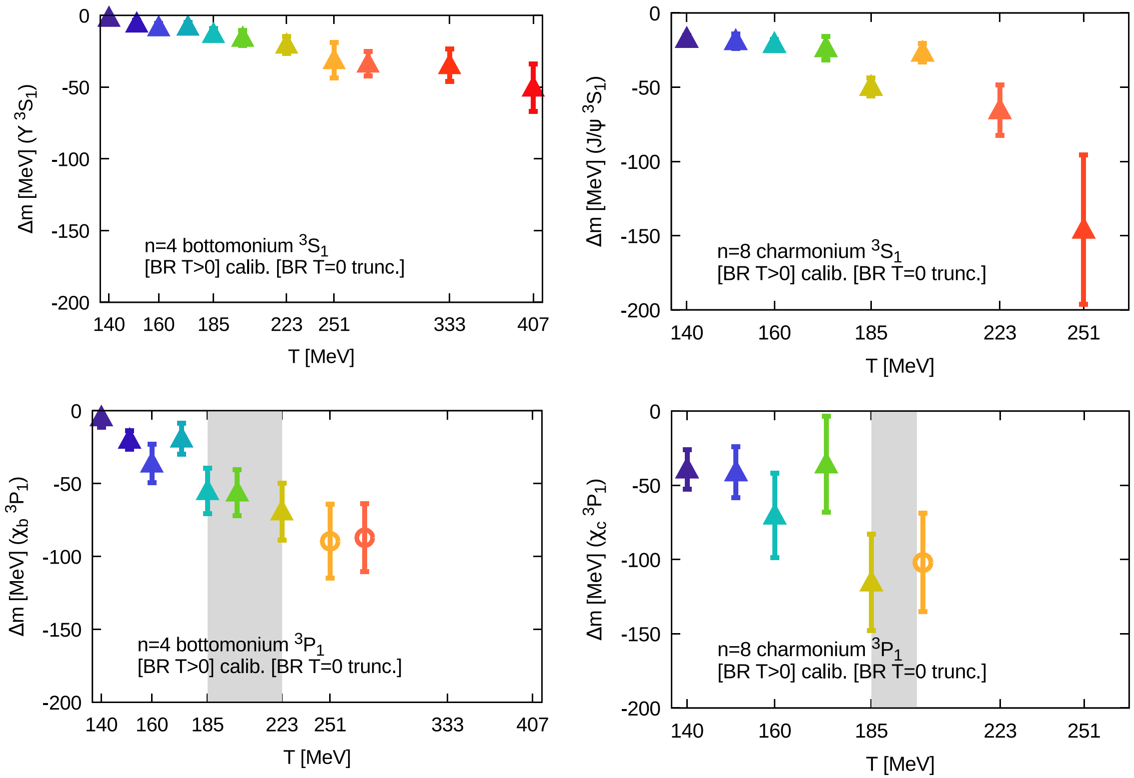
Lattice QCD potential based computation



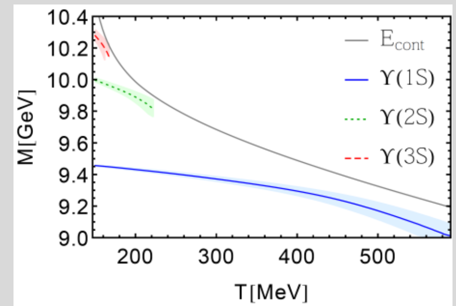


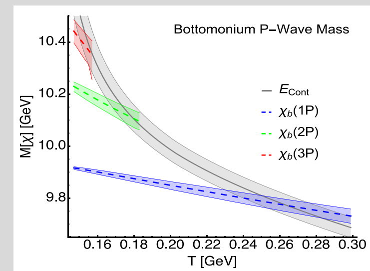
In-medium mass shifts II

Our direct lattice NRQCD result for T>0 mass shifts



Lattice QCD potential based computation





- Negative mass shifts in qualitative agreement with potential based computation
  (i.e. with a non-perturbative implementation of the EFT pNRQCD)