Open quantum systems approach to the study of quarkonium suppression

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Plan

1. Introduction
2. Quarkonium in thermal equilibrium
3. Open quantum systems combined with effective field theories. The $\frac{1}{r} \gg T$ case
4. Conclusions

Heavy Quarkonium as a multi-scale problem

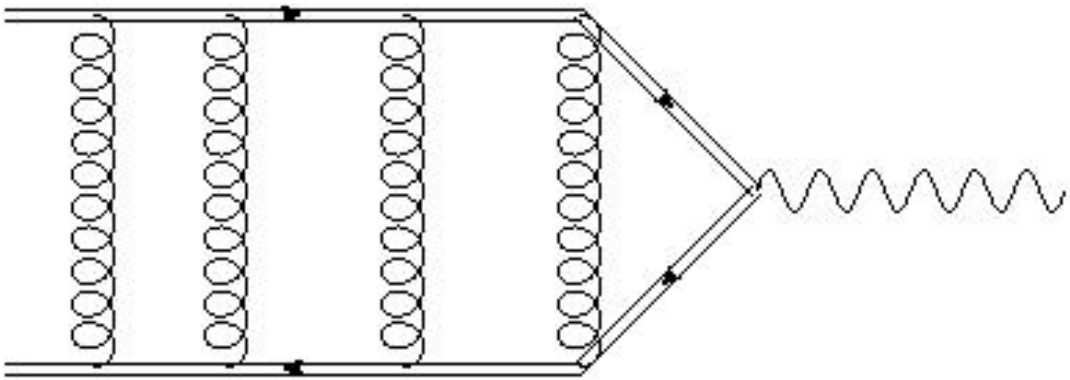
Heavy quarkonium is a non-relativistic system, the typical velocity v of the heavy quark and heavy antiquark around the center of mass is small.

- The creation or annihilation of heavy quark pairs involves energies of order m_Q, hard scale.
- The inverse of the Bohr radius and the typical momentum are of order $m_Q v$, soft scale.
- The binding energy of a non-relativistic system is of order $m_Q v^2$, ultrasoft scale.

Heavy Quarkonium as a multi-scale problem

Why does it matter?

Consider for example the annihilation of a s-wave vector state into a virtual photon (to latter produce leptons).



every rung in the ladder gives a contribution of order $\frac{\alpha_s}{v}$. Break up of naive perturbative expansion if v is small. This is a well-known effect (Sommerfeld factor)

Effective Field Theories (EFTs)

How to know which diagrams need to be resummed? One can use Effective Field Theories (EFTs), introduced by Weinberg (1979).

- A theory which gives the same results as QCD but limited to low energy degrees of freedom.
- In a problem when very separated energy scales are important it is usual to find unexpected suppressions and enhancements. EFT are a very useful tool to deal with this problem.
- The Lagrangian of the EFT can be deduced knowing the symmetries of the full theory and the degrees of freedom active at low energies.

Non-relativistic EFTs at $T = 0$

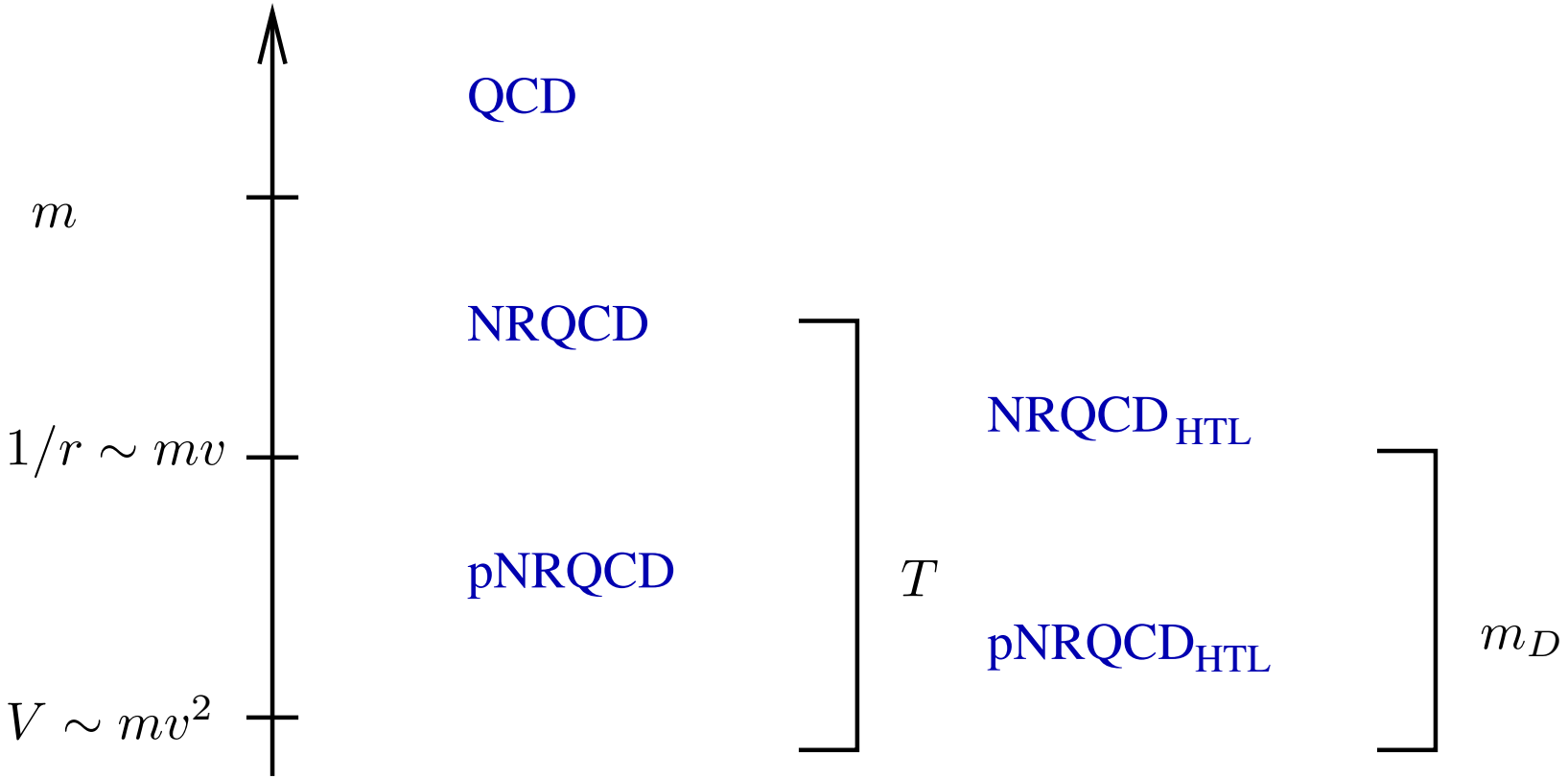
- QCD
  Quarks and gluons.
- NRQCD
  Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994).
  Integrate out hard scale. Non-relativistic quarks and gluons.

$$\mathcal{L}_{NRQCD} = \sum_n \frac{1}{m_Q^n} \mathcal{L}_n$$

- pNRQCD
  Pineda and Soto (1998), Brambilla, Pineda, Soto and Vairo (2000).
  Integrate out also soft scale. Color singlet field, color octet field and gluons.

$$\mathcal{L}_{pNRQCD} = \sum_{n,m} \frac{1}{m_Q^n} r^m \mathcal{L}_{n,m}$$

Effective Field Theories



Brambilla, Ghiglieri, Vairo and Petreczky (PRD78 (2008) 014017)
M. A. E and Soto (PRA78 (2008) 032520)

pNRQCD Lagrangian at T=0

(Brambilla, Pineda, Soto and Vairo, NPB566 (2000) 275).

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left. + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)\, Tr(O^\dagger \mathbf{r} g\mathbf{E} S + S^\dagger \mathbf{r} g\mathbf{E} O)$$
$$+ \frac{V_B(r)}{2}\, Tr(O^\dagger \mathbf{r} g\mathbf{E} O + O^\dagger O \mathbf{r} g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Degrees of freedom are singlet and octets.
- Allows to obtain manifestly gauge-invariant results. Simplifies the connection with Lattice QCD.
- If $1/r \gg T$ we can use this Lagrangian as starting point. In other cases the matching between NRQCD and pNRQCD will be modified.

What can we learn from the application of EFTs to the study of quarkonium suppression?

From two perspectives:

- Thermal equilibrium studies that can be compared with lattice results.
- Quantities that can be computed in lattice QCD that can be used to make predictions within the open quantum system-EFT approach.

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Information contained in the time-ordered correlator

$$\langle \mathcal{T} S(t, \mathbf{r}, \mathbf{R}) S^{\dagger}(0, \mathbf{0}, \mathbf{0}) \rangle$$

- Tells us about the in-medium dispersion relation. We can obtain binding energy and decay width modifications.
- It can be used to obtain the spectral function. Comparison with Lattice QCD.
- At $T = 0$ it fulfills a Schrödinger equation. At finite temperature this will also be the case in some situations.
- It does not contain information about the number of bound states in the medium.
- If $T \gg E$ it can be described by a Schrödinger equation with a complex potential.[1]
- Survival amplitude of the singlet without altering the medium.

[1] As was found in Laine, Philipsen, Romatschke and Tassler (2006)

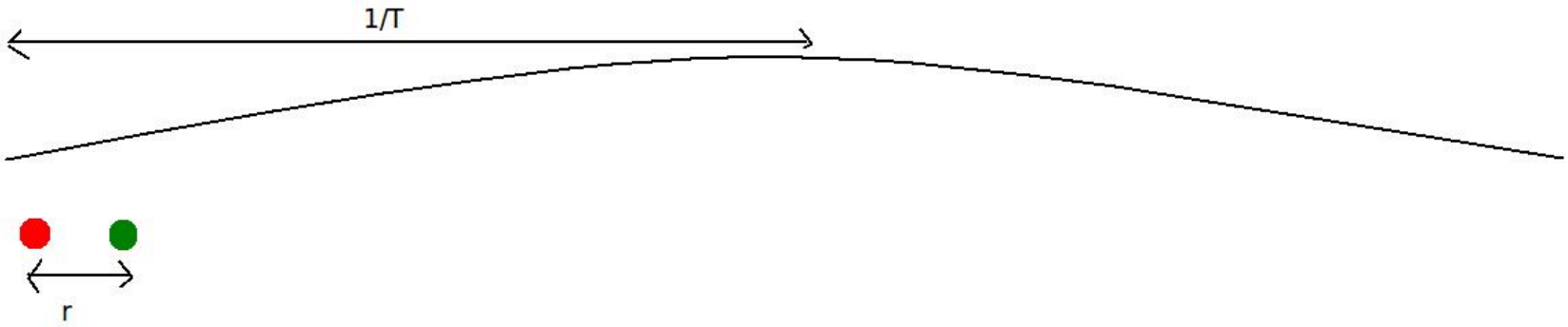
What can be learned from the time-ordered correlator?

- Leading order thermal effects can be encoded in a redefinition of the potential if $T, m_D \gg E$.
- In all cases this potential has both a correction in the real part and an imaginary part.
- In the case $T \gg 1/r \sim m_D$we recover Laine et al. potential.

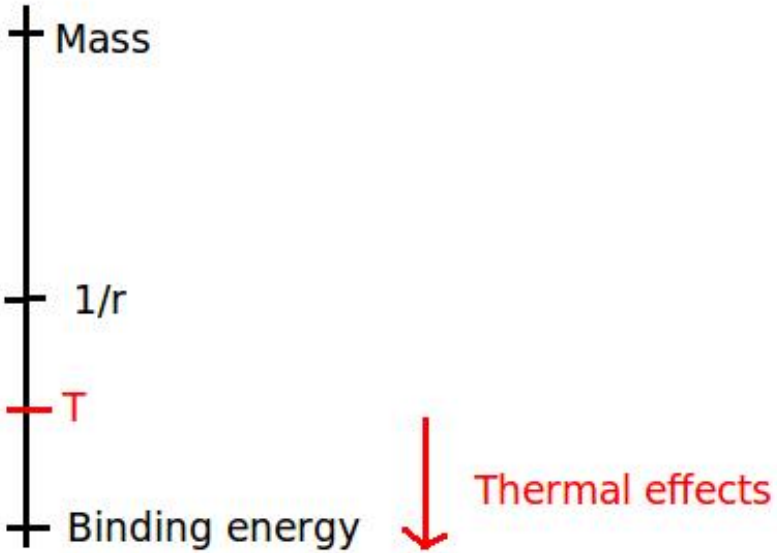
The case $\frac{1}{r} \gg T$. Physical picture.

N. Brambilla, M. A. E., J. Ghiglieri, J. Soto and A. Vairo, JHEP 1009 (2010) 038

The medium sees heavy quarkonium as a color dipole.

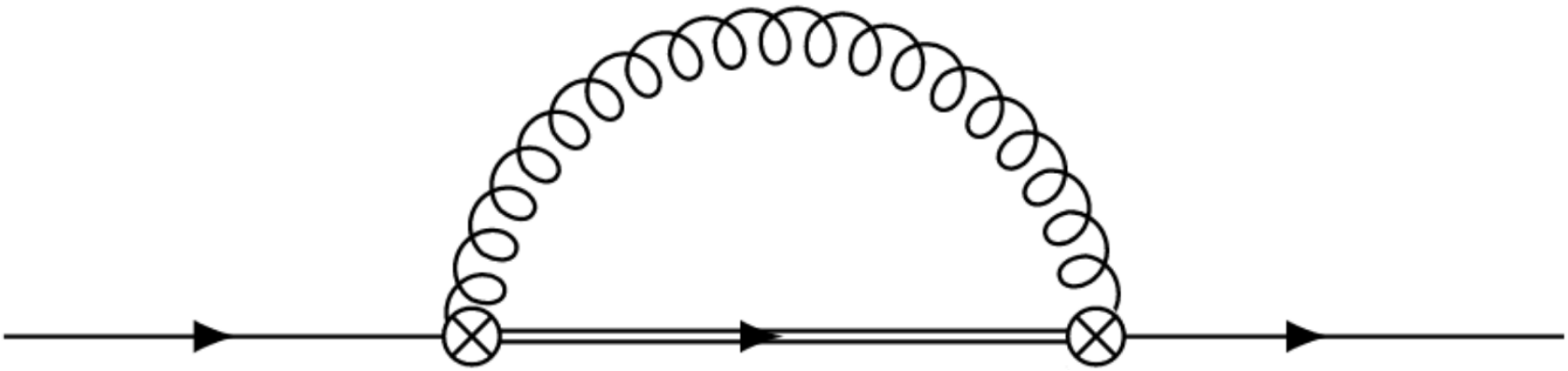


The case $\frac{1}{r} \gg T$. The EFT framework



- The starting point can be pNRQCD at $T = 0$.
- Matching from pNRQCD to $\text{pNRQCD}_{\text{HTL}}$. Effects of the scale T are encoded in a modification of the potential.
- Computation of the scale E effects in $\text{pNRQCD}_{\text{HTL}}$. Modifications to the decay width and binding energy. Not necessarily described by an interaction local in time (aka potential).

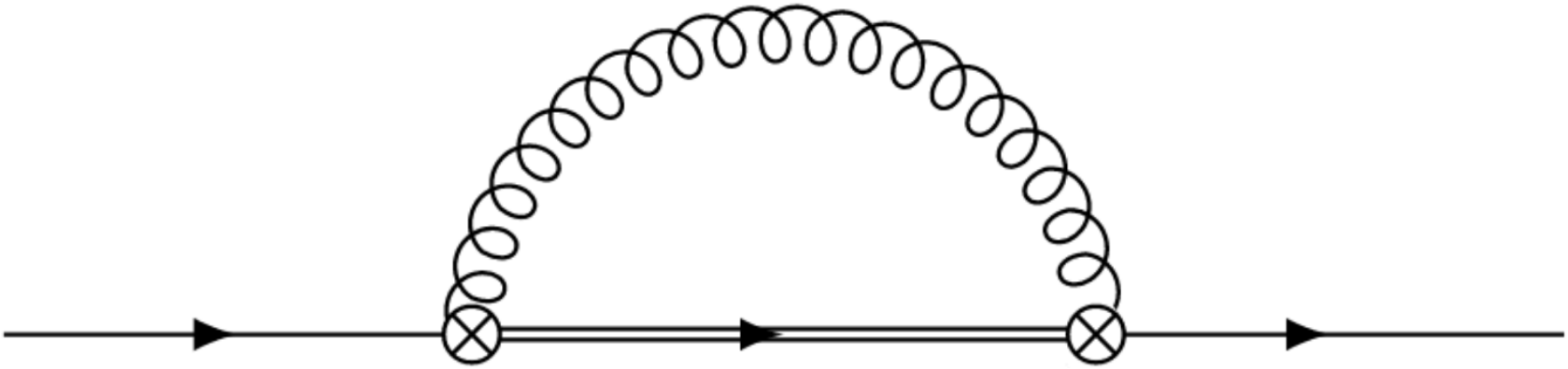
The singlet self-energy



$$\Sigma_s = r^i \frac{1}{2N_c} \int_0^\infty \mathrm{d}\tau \left\langle gE^{a,i}(\tau, \mathbf{0})\, gE^{a,j}(0, \mathbf{0}) \right\rangle e^{-i(h_o - h_s)\tau} r^j$$

We do not write explicitly the gauge links that make the expression gauge invariant.In the static limit $h_o - h_s \to V_o - V_s$.

The singlet self-energy



In some situations we can consider that chromoelectric fields are only correlated for times such that $e^{-i(h_o-h_s)\tau} \sim 1$. In this case

$$\Sigma_s = \frac{r^2}{2}(\kappa + i\gamma)$$

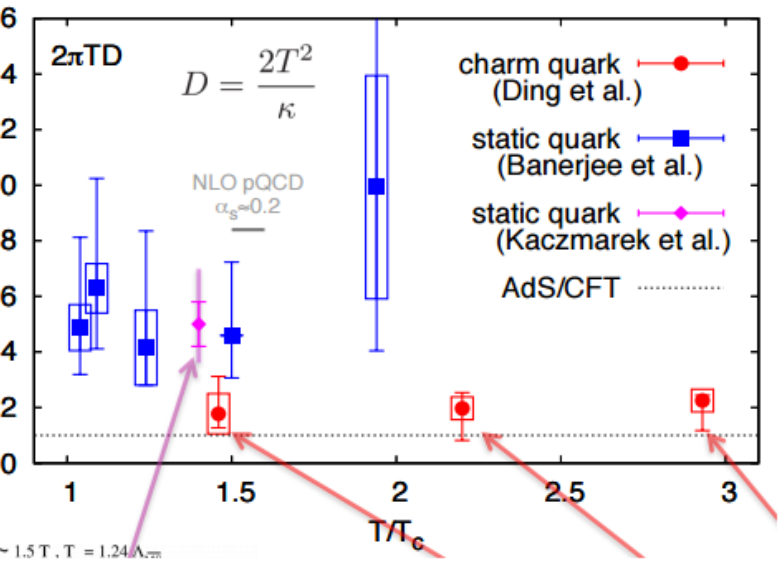
- In perturbation theory, the medium induces a family of energy scales T, $m_D \sim gT$, $g^2 T$. We can make this approximation if the thermal energy scales that we consider are bigger than the binding energy.
- In a strongly-coupled plasma, all thermal scales are of the order of the temperature. We can do this approximation if $T \gg E$.

The parameter κ

$$\kappa = \frac{1}{6N_c}\int_0^\infty \mathrm{d}t \left\langle \{gE^{a,i}(t,\mathbf{0}), gE^{a,i}(0,\mathbf{0})\} \right\rangle$$

quantity also appearing in heavy particle diffusion, recent lattice QCD evaluation in Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)

$$1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$$



Picture taken from O. Kaczmarek talk in "30 years in J/Ψ suppression".

The parameter γ

$$\gamma = \frac{-i}{6N_c}\int_0^\infty \mathrm{d}t \left\langle \left[gE^{a,i}(t,\mathbf{0}), gE^{a,i}(0,\mathbf{0})\right] \right\rangle$$

No lattice QCD information on this but we observe that we reproduce data better if γ is small. In pQCD

$$\gamma = -2\zeta(3)\, C_F \left(\frac{4}{3}N_c + n_f\right) \alpha_s^2(\mu_T)\, T^3 \approx -6.3\, T^3$$

It represents a correction to the real part of the potential proportional to r^2. Therefore, it produces a mass shift

$$\delta M_n = \frac{\gamma}{2}\langle n|r^2|n\rangle$$

More on these parameters in next talk.

The case $\frac{1}{r} \gg T$. Physical picture.

Because $rT \ll 1$, thermal corrections are a perturbation.

- Mass shift.
- Thermal decay width.

Even if the decay width is smaller than the binding energy, it can have an important phenomenological impact if the fireball exists for long enough time. There are two different processes:

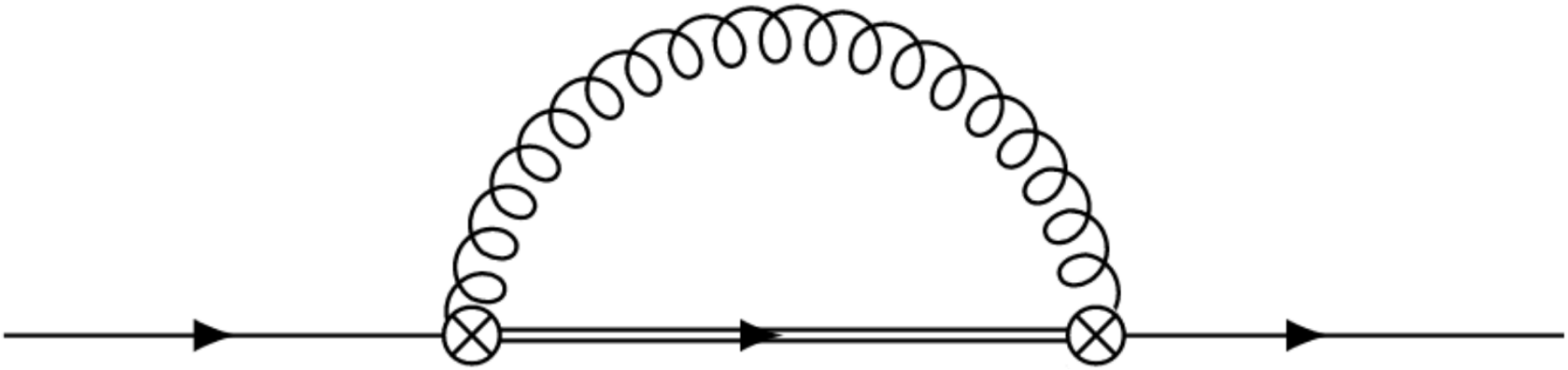
- Gluo-dissociation. Goes like $\alpha_s T E^2$. Dominant if $E \gg m_D$. Contribution of light-like or time-like gluons.
- Scattering with partons in the medium. Goes like $\alpha_s^2 T^3 \sim \alpha_s T m_D^2$. Dominant if $m_D \gg E$. Contribution of space-like gluons.

Gluo-dissociation

N. Brambilla, M. A. E, J. Ghiglieri and A. Vairo, JHEP 1112 (2011) 116

$$HQ + g \to Q + \bar{Q}$$

Imaginary part of the diagram



- The imaginary part is the cut contribution. Gluon must be on-shell with an energy of the order of the binding energy.
- Bose-enhancement (factor $\frac{T}{E}$). Low energy gluons are enhanced in a thermal medium.
- $\Gamma \propto \alpha_s r^2 T E^2$.

Gluo-dissociation in pNRQCD

- Computed for $T \gg E$ in HQ. Brambilla, MAE, Ghiglieri, Soto and Vairo (2010)

$$\delta\Gamma_n = \frac{1}{3} N_C^2 C_F \alpha_s^3 T - \frac{16}{3m} C_F \alpha_s T E_n + \frac{4}{3} N_C C_F \alpha_s^2 T \frac{2}{m n^2 a_0}$$

  where E_n is the binding energy and a_0 the Bohr radius.

- Computed for $T \sim E$ in the hydrogen atom. MAE and Soto (2008).

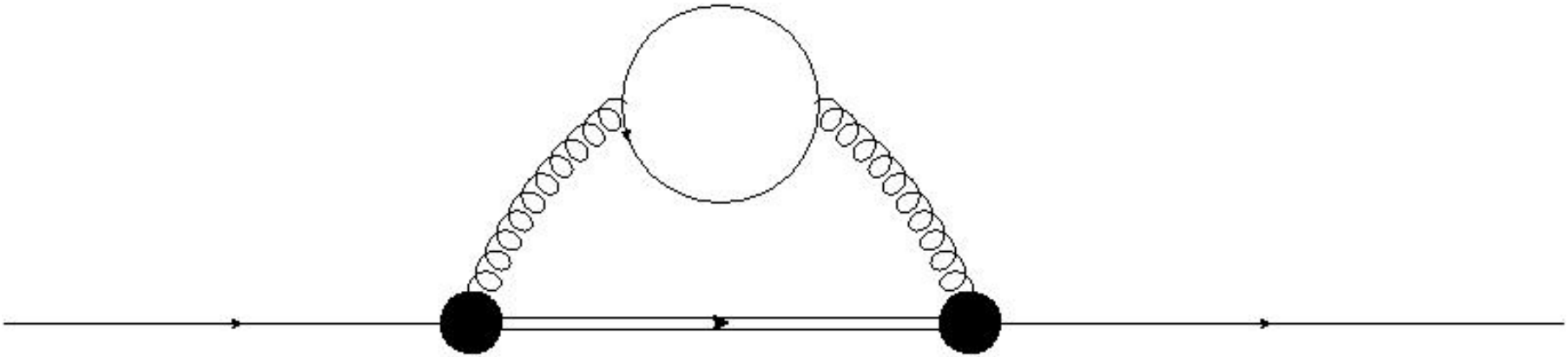
$$\delta\Gamma_n = \frac{4}{3} \alpha_s C_F T \langle n | r_i \frac{|E_n - h_o|^2}{e^{\beta |E_n - h_o|} - 1} r_i | n \rangle$$

Scattering with thermal partons

N. Brambilla, M. A. E, J. Ghiglieri and A. Vairo, JHEP 1305 (2013) 130

$$HQ + p \to Q + \bar{Q} + p$$

Cut contribution of space-like gluons.



- Gluon energy of the order of the binding energy but tri-momentum can be bigger.
- Bose-enhancement still applies.
- $\Gamma_n = \kappa \langle n|r^2|n\rangle$. (In perturbation theory $\kappa \propto \alpha_s^2 T^3$). This process dominates in thermal regimes in which interaction with the medium can be reproduced with a potential.

The case $\frac{1}{r} \sim T \gg m_D \gg E$

- We can use NRQCD as a starting point.
- Integrating out the scale $\frac{1}{r}$, T gives as result $\text{pNRQCD}_{\text{HTL}}$.
- We can also integrate out m_D. As a result we obtain $\text{pNRQCD}_{\text{HTL}}$ with a modified potential. From the point the view of the physics at the scale m_D quarkonium is again a color dipole.

Modification of the potential

$$V = V_c + \delta V_r + \delta V_{m_D}$$

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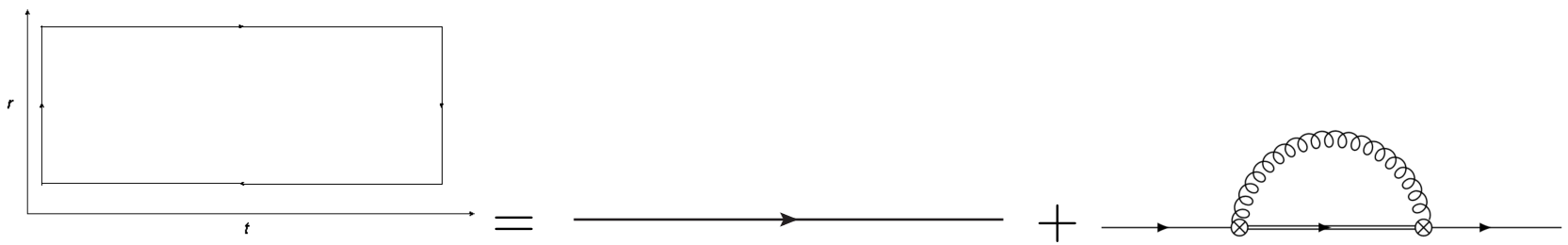
- δV_{m_D} is a correction coming from the scale m_D. Ultraviolet divergence $r^2 \log(m_D \mu)$.

Non-perturbative matching between NRQCD and pNRQCD



$$\langle W \rangle = Z_s e^{-iV_s T} \times$$

$$\times \left(1 - \frac{1}{2N_c} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \int_{-\frac{T}{2}}^{t} dt' r^i \left\langle g E^{a,i}(t, \mathbf{0})\, g E^{a,j}(t', \mathbf{0}) \right\rangle r^j e^{-i(V_o - V_s)(t-t')} \right)$$

The case $T \gg \frac{1}{r} \sim m_D$

- Integrate out scale T. From NRQCD to $\text{NRQCD}_{\text{HTL}}$.
- Integrate out scales $\frac{1}{r}$ and m_D. From $\text{NRQCD}_{\text{HTL}}$ to $\text{pNRQCD}_{\text{HTL}}$.
- Similar to previous case. Most important difference, thermal effects are not a perturbation even at small coupling.

Summary. The thermal scattering cross-section.

Some notation

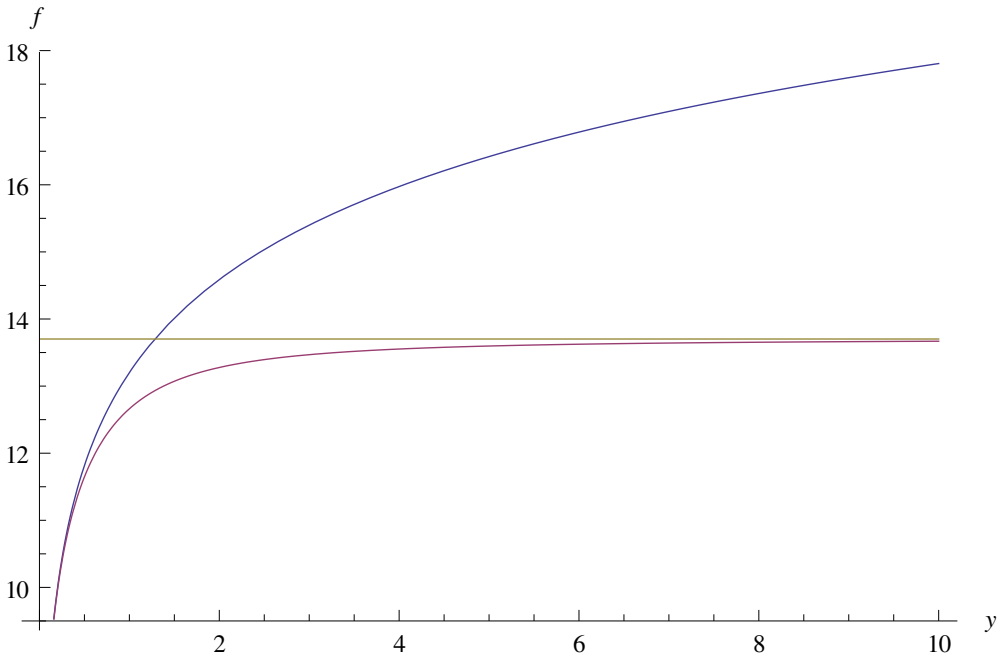
$$\sigma(k, m_D) = \sigma_R f(x, y)$$

where

$$\sigma_R = 8\pi C_F \alpha_s^2 N_F a_0^2$$
$$x = m_D a_0$$
$$y = k a_0$$

Summary. The thermal scattering cross-section.

$m_D a_0 = 0.001$



$\frac{1}{r} \gg T \gg m_D$, $T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

Towards a computation of R_{AA} in EFT

- Until now we have discussed the decay width and corrections to the binding energy.

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- Both the decay width and the binding energy can be found in the EFT by studying the pole of the singlet time-ordered propagator.
- To compute R_{AA} we need to know how the state of the heavy quarks in the medium changes with time.
- In the pNRQCD framework the derivative of the probability to find a quarkonium state will unavoidably depend on off-diagonal elements of the density matrix. No a priori reason to neglect quantum coherence.

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Section based on work done in collaboration with N. Brambilla, J. Soto and A. Vairo (PRD 97 no.7 074009 and PRD 96 no.3 034021)

Why the $\frac{1}{r} \gg T$ case?

- Multipole expansion ensures that thermal corrections are small even if the medium is strongly-coupled.
- The initial temperature depends on the impact parameter.
- At the most central collisions of LHC the higher temperature is around 450 MeV, therefore $\frac{1}{a_0} \sim \pi T$ where a_0 is the Bohr radius of Upsilon 1S.
- As time passes the system cools down.
- The $\frac{1}{r} \gg T$ case is realized in heavy-ion collisions for most collisions during some time.

Evolution of the number of singlets

$$f_s(x,y) = Tr(\rho S^\dagger(x) S(y))$$

We can use perturbation theory but expanding in r instead of α_s. In the interaction picture

$$i\partial_t S = [S, H_0]$$
$$i\partial_t \rho = [H_I, \rho]$$

Evolution of the number of singlets

$$\partial_t f_s = -i(H_{s,eff} f_s - f_s H^{\dagger}_{s,eff}) + \mathcal{F}(f_o)$$

- $H_{s,eff} = h_s + \Sigma_s$ where Σ_s corresponds with the self-energy that can be obtained in pNRQCD by computing the time-ordered correlator.
- $\mathcal{F}(f_o)$ is a new term that takes into account the process $O \to g + S$. In ensures that the total number of heavy quarks is conserved.
- $\mathcal{F}(f_o)$ is a complicated function of $Tr(\rho O^{\dagger} O)$ and $\langle E^i E^j \rangle$. The information about the medium enters only in the chromoelectric field correlator.

Evolution of the octet

Very similar reasoning.

$$f_o^{ab}(x,y) = Tr(\rho O^{\dagger,a}(x) O^b(y))$$

$$\partial_t f_o = -i(H_{o,eff} f_o - f_o H_{o,eff}^\dagger) + \mathcal{F}_1(f_s) + \mathcal{F}_2(f_o)$$

The $\frac{1}{r} \gg T, m_D \gg E$ regime

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form.

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

there is a transition singlet-octet

$$C_i^{so} = \sqrt{\frac{\kappa}{N_c^2 - 1}} r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix}$$

and octet to octet

$$C_i^{oo} = \sqrt{\frac{(N_c^2 - 4)\kappa}{2(N_c^2 - 1)}} r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The $\frac{1}{r} \gg T, m_D \gg E$ regime

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form.

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

Initial conditions and hydrodynamics

In order to understand well the underlying mechanism we worked in the simplest possible conditions. However it is possible and straightforward to couple our theory to the full hydro evolution and we will do it in the future

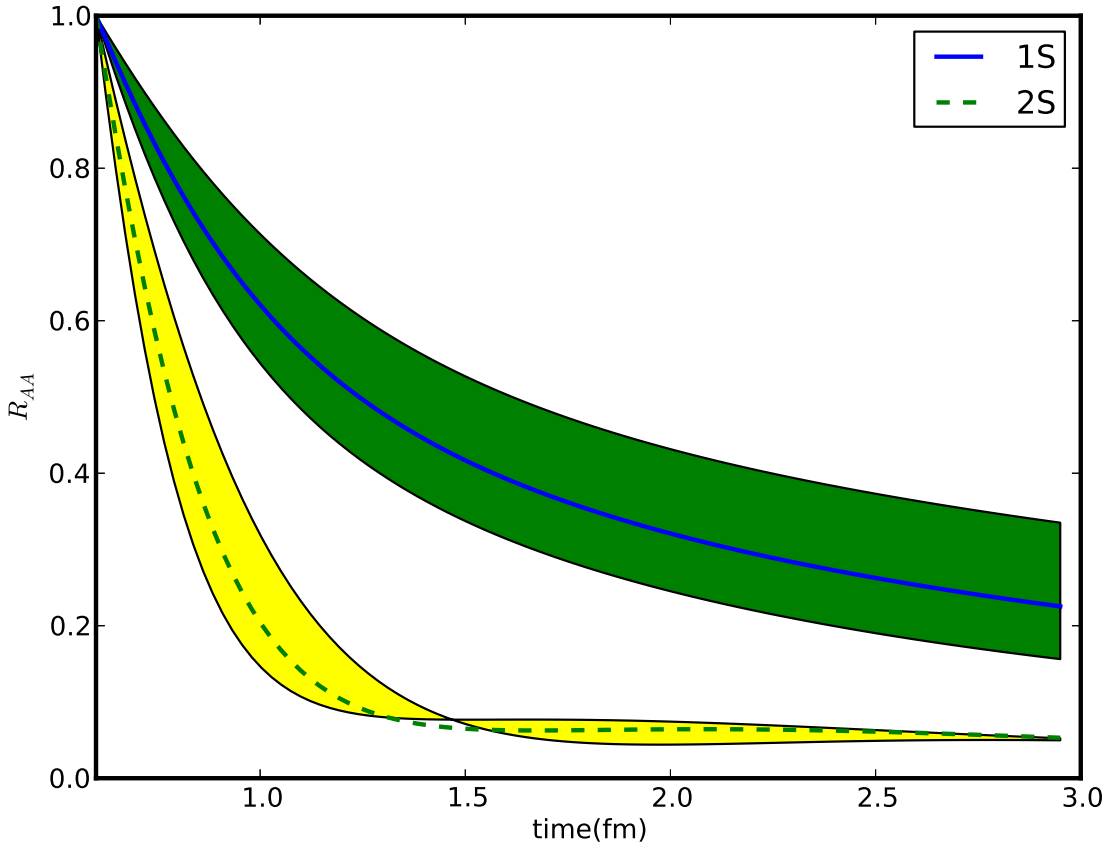
Initial conditions

- To create a pair of heavy particles is a high energy process $\rightarrow$ pair initially created in a Dirac delta state in the relative coordinate.
- Naively (without taking into account P_T dependence) it is α_s suppresses to create a spin 1 singlet compared to an octet.

Hydrodynamics

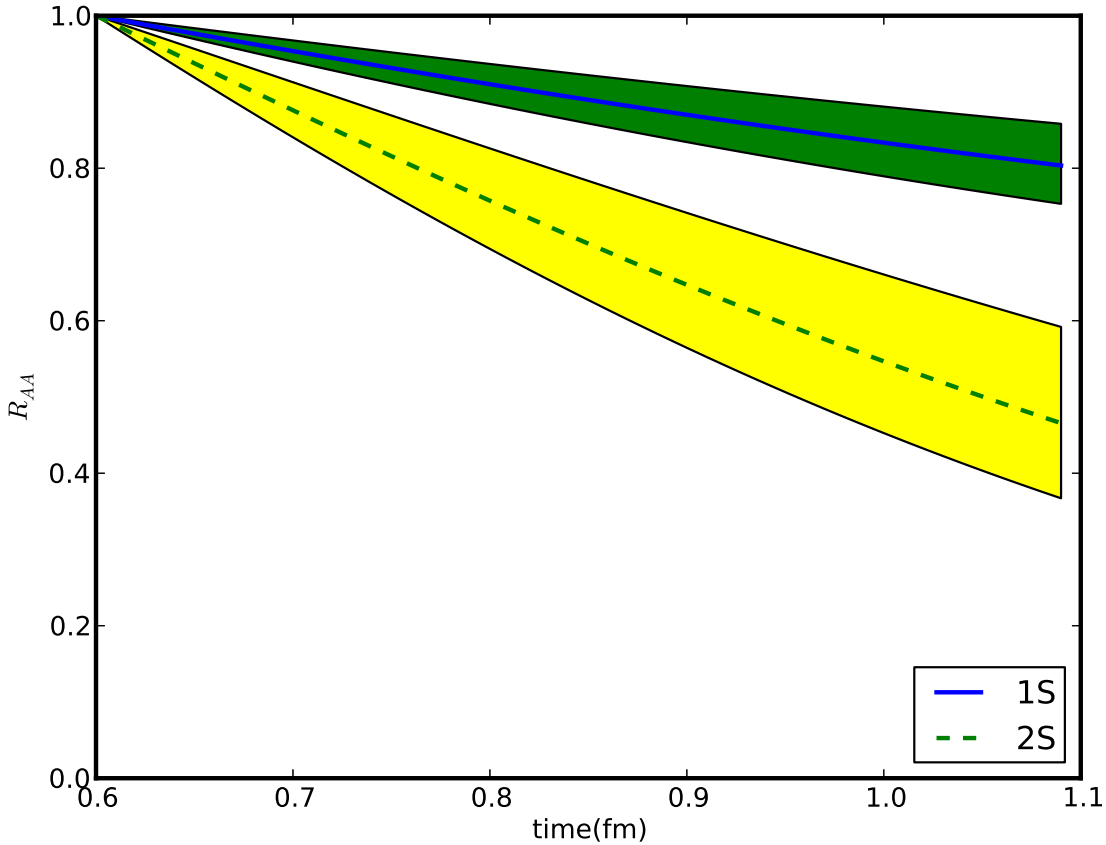
- Bjorken expansion.
- Optical Glauber model to compute dependence of initial temperature with centrality.
- Quarkonium propagates in the vacuum from $t = 0$ to $t_0 = 0.6\, fm$, then the plasma is created.

Results. $30 - 50\%$ centrality. $\sqrt{s_{NN}} = 2.76\ TeV$



Error bars only take into account uncertainty in the determination of κ. γ is set to zero.

Results. $50 - 100\%$ centrality. $\sqrt{s_{NN}} = 2.76\ TeV$



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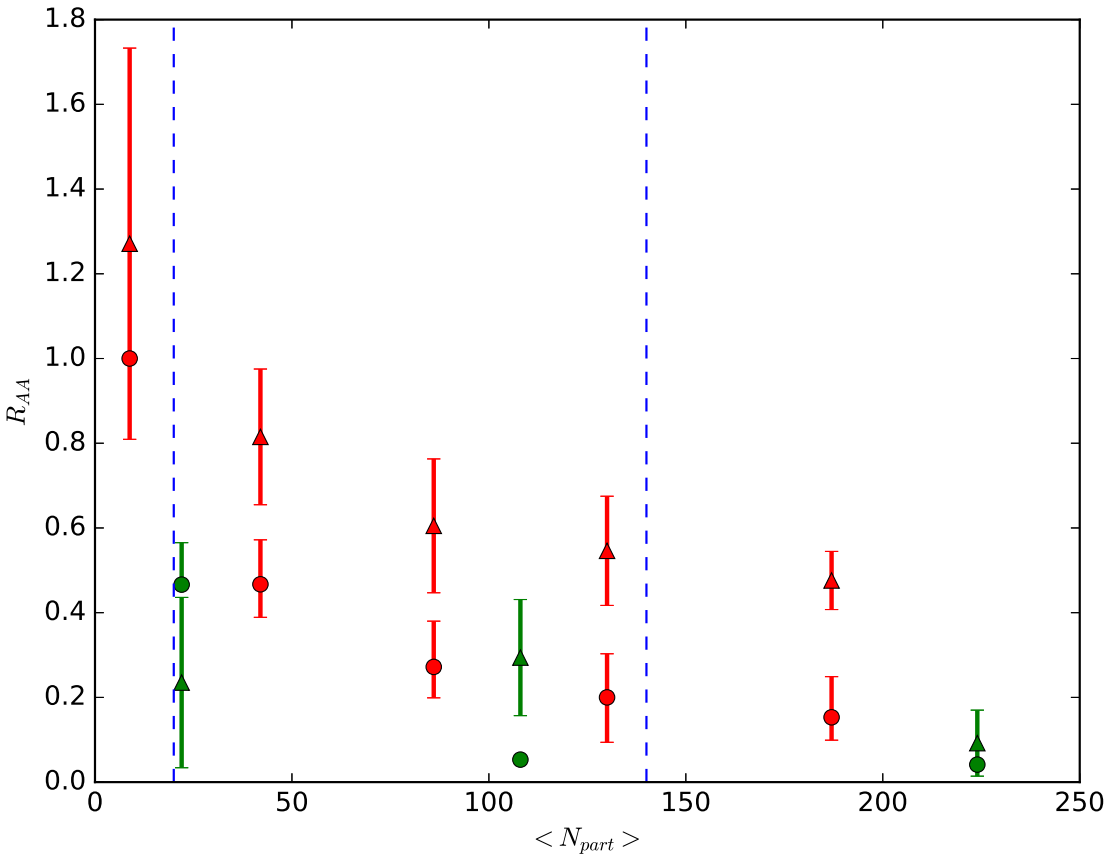
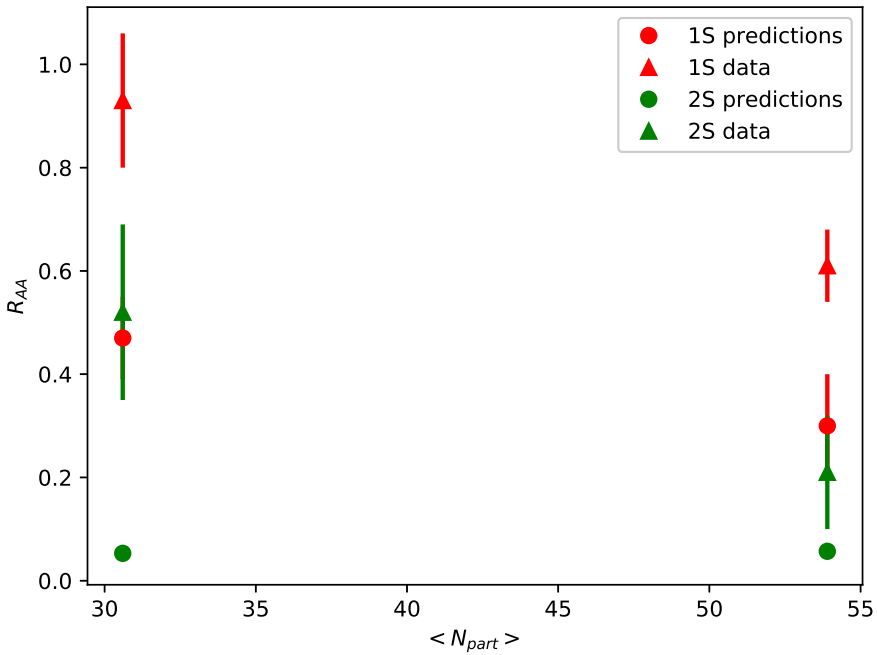


Comparison between the CMS data of 2017 (triangles) and our computation (circles). Upper (red) entries refer to the $\Upsilon(1S)$, lower (green) entries to the $\Upsilon(2S)$.

Results. $\sqrt{s_{NN}} = 5.02\ TeV$



[2]

[2] Data taken from CMS paper (arXiv:1805.09215)

Possible sources of the discrepancy

Due to the assumption 1/r≫T≫E

- The assumption $T \gg E$ was done to simplify the computation and to be able to use lattice input on κ, it can be relaxed in the future.
- Some theoretical work needs to be done to study the cases $T \gtrsim 1/r$ because we could not longer use pNRQCD at $T = 0$ as a starting point.

Due to the computational cost

- Each computation takes a few hours. Each energy and centrality class is represented by just one temperature. This limits our ability to use realistic hydro.
- We are working now in Monte Carlo techniques that reduces the computational cost.
- In some situations a semi-classical equation might be applicable. Blaizot and M.A.E (2017), Yan Zhu's talk in SEWM 2018.

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Conclusions

- We can compute R_{AA} using pNRQCD in a simple setting (initial conditions, hydrodynamics...). The framework can be systematically improved.
- In the limit $\frac{1}{r} \gg T \gg E$ all the information from the medium can be encoded in two non-perturbative parameters.
- The pNRQCD framework allows to connect several non-perturbative quantities (the static Wilson loop, chromoelectric fields correlators like κ and γ etc.) with R_{AA}.
- Work in progress. More realistic scenarios will be explored in the future.