Integrability in gauge and string theory

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Plan

- Maximally supersymmetric gauge theory in 4D

- String theory on $AdS_5 \times S^5$

- Deformations preserving integrability
Gauge theory
\( \mathcal{N}=4 \) super Yang-Mills

gauge group \( SU(N) \), coupling constant \( g_{\text{YM}} \)
scalar+fermions+gauge bosons with \textbf{maximal susy in 4D}

\textbf{Conformal:} \quad \beta(g_{\text{YM}}) = 0

\textbf{Planar limit:}
\( g_{\text{YM}} \to 0, \; N \to \infty \)
while 't Hooft coupling
\( \lambda \equiv Ng_{\text{YM}}^2 \) is fixed

picture stolen from Alfonso’s review \[ \text{arXiv:1310.4319} \]
The spin chain

$\mathfrak{su}(2)$ sector $\supset$ scalar fields $\Phi, \bar{\Phi}$ of $\mathcal{N} = 4$ SYM

$$\mathcal{O}(x) = \text{Tr}[\Phi \Phi \bar{\Phi} \Phi \Phi \ldots \Phi \Phi \bar{\Phi} \Phi \Phi]$$

[Minahan, Zarembo 02]

Anomalous dimension at 1-loop: operators mix and the mixing matrix is the Hamiltonian of Heisenberg’s XXX spin chain!

Higher-loop corrections $\Rightarrow$ long-range interactions

For similar methods applied to QCD see [arXiv:1012.4000]
Magnon excitations interact with \textbf{factorised S-matrix}

\[ e^{ip_k L} \prod_{j \neq k} S(p_k, p_j) = 1 \]

S-matrix fixed at \textbf{all loops} from supersymmetry and analyticity

\textbf{Exact spectrum} in $\lambda$ and $L$ (size of the chain) from “Thermodynamic Bethe Ansatz” or “Quantum Spectral Curve”
String theory
\[ \mathcal{N} = 4 \text{ SYM} \]

\[ \text{AdS}_5 \times S^5 \]

\[ -X_0^2 + \sum_{i=1}^{4} X_i^2 - X_5^2 = -1 \]

\[ \sum_{i=1}^{6} Y_i^2 = 1 \]

\( \lambda \ll 1 \, \text{weakly-coupled gauge theory} / \lambda \gg 1 \, \text{classical string} \)
\[ S = -\frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} + \text{fermions} \]

\[ ds^2 = G_{MN} \, dX^M dX^N = ds_{\text{AdS}_5}^2 + ds_{S^5}^2 \]

Hamiltonian in light-cone gauge for 8 bosons + 8 fermions

\[ H = H_2 + \frac{1}{\lambda} H_4 + \frac{1}{\lambda^2} H_6 + \ldots \]

**Same S-matrix** of spin-chain but expanded at \( \lambda \sim \infty \)

(Classical integrability)
Deformations
Integrability **beyond** spectrum of $AdS_5/CFT_4$

- Higher point-functions

- **Lower dimensional** dualities
  e.g. $AdS_4/CFT_3$, $AdS_3/CFT_2$

- **Deformations** of $AdS_5/CFT_4$
Break isometries of target space of string

Some deformations $\sim$ twisted boundary conditions for the string

Deformations of the gauge theory?

On the gauge theory we can break e.g. supersymmetry, conformal invariance

In certain cases, deformations correspond to non-commutative gauge theories

Extension of the integrability methods to the deformed models?
Classical integrability: **Lax connection** $L_\alpha(z, \tau, \sigma)$, $\alpha = \tau, \sigma$

**Flatness condition**

$$\partial_\alpha L_\beta - \partial_\beta L_\alpha + [L_\alpha, L_\beta] = 0 \iff \text{EOM} \quad \frac{\delta S}{\delta X^M} = 0$$

**Monodromy matrix:**

$$T(z) = \text{Pexp} \int d\sigma L_\sigma(z)$$

Generating function of conserved quantities