Higher order corrections to Higgs boson pair production

RADCOR 2019, Avignon, France, September 9-13, 2019

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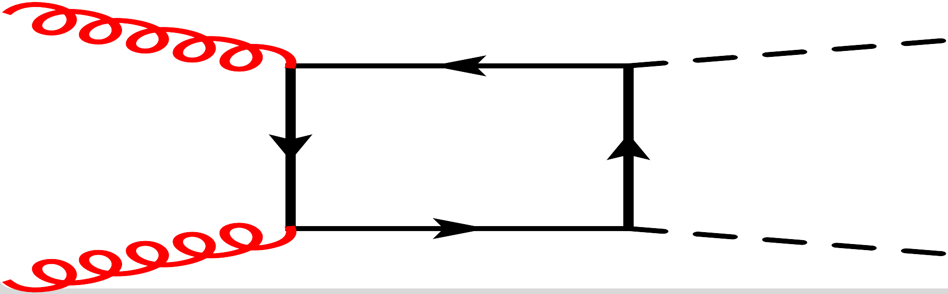
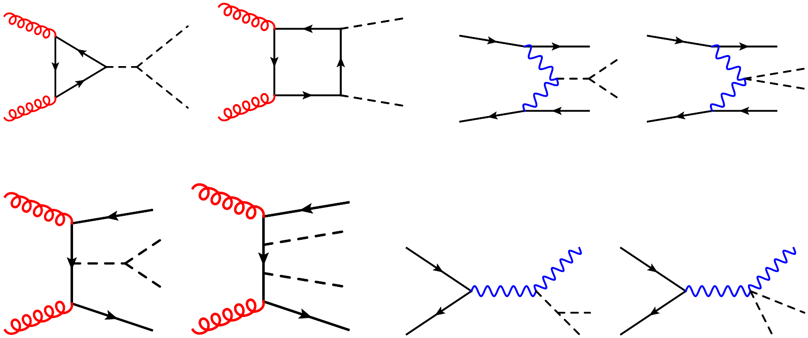
TTP KARLSRUHE



$gg \to HH$

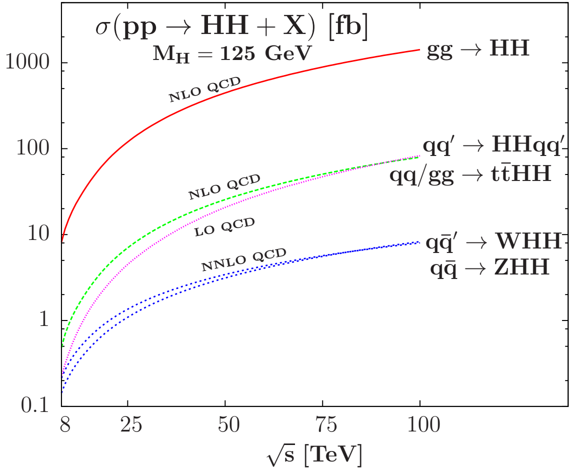
- NLO: approximations vs. exact
- NNLO: large m_t
- N^3LO: $m_t \to \infty$

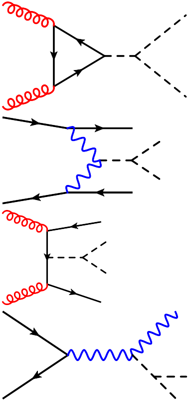
Double Higgs production in SM



Double Higgs production in SM (2)

[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira'12]





$K^{\mathrm{NLO}} \approx 1.9$

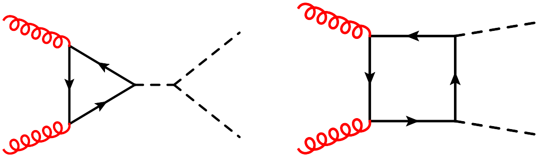
$K^{\mathrm{NNLO}} \approx 1.2$

λ_{HHH} from $gg \to HH$

Standard Model:

gauge	$\alpha, \alpha_{\mathrm{weak}}, \alpha_s$
Yukawa	$y_f \sim m_f$
Higgs	$V = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$
	$m_H^2 = 2\lambda v^2$ ⇨ $\lambda = 0.13\ldots$
	not yet measured

λ from H pair production (?)



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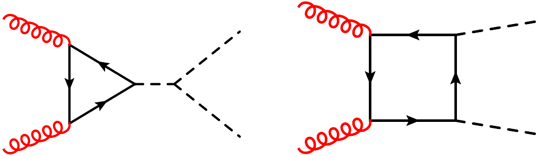
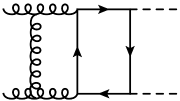


very challenging; combine $b\bar{b}b\bar{b}$, $b\bar{b}\tau^+\tau^-$, $b\bar{b}\gamma\gamma$, $b\bar{b}VV$

[CMS: arXiv:1811.09689]: $-11.8 < \lambda/\lambda_{\rm SM} < 18.8$

[ATL-PHYS-PUB-2019-009]: $-3.2 < \lambda/\lambda_{\rm SM} < 11.9$

$gg \to HH$ at NLO: known results



LO [Glover, van der Bij'88; Plehn,Spira,Zerwas'96]

NLO for $m_t \to \infty$ [Dawson,Dittmaier,Spira'98]

NLO incl. $1/m_t$ terms [Grigo,Hoff,Melnikov,Steinhauser'13; Degrassi,Giardine,Gröber'16]

NLO exact (real rad.): [Maltoni,Vryonidou,Zaro'14]

NLO Padé: [Gröber,Maier,Rauh'17]

NLO small-p_T: [Bonciani,Degrassi,Giardino,Gröber'18]

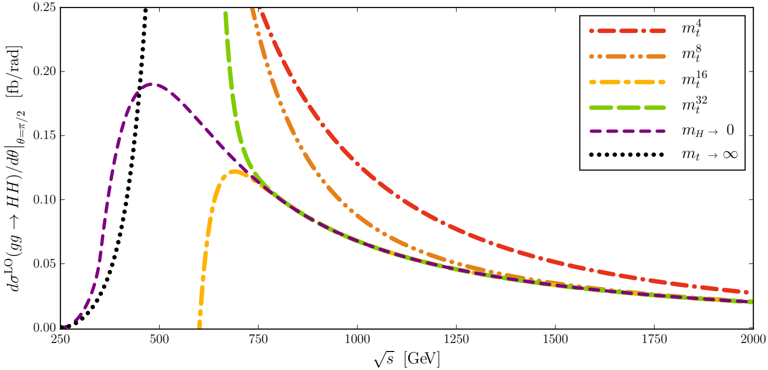
NLO high energy: [Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

NLO exact (numerical): [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

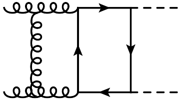
[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher'18]

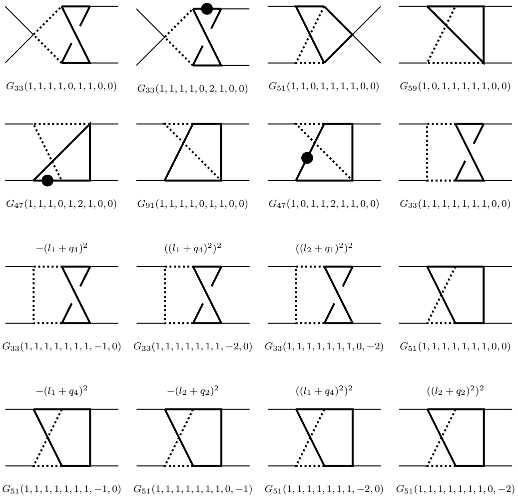
LO: exact vs $s, t \gg m_t^2 > m_H^2$ expansion

$$\frac{d\sigma}{d\theta}(s)$$



Non-planar MIs





total: 161 MIs$(s, t \gg m_t^2)$

Compute MIs

- differentiate MIs ($X = s, t, m_t^2$)

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}$$

- expand in m_t^2 ➪ ansatz

see, e.g., [Melnikov,Tancredi,Wever'16]

$$J \;=\; \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log\left(m_t^2\right)^k$$

➪ system of linear equations for $C_{ijk}(s,t)$

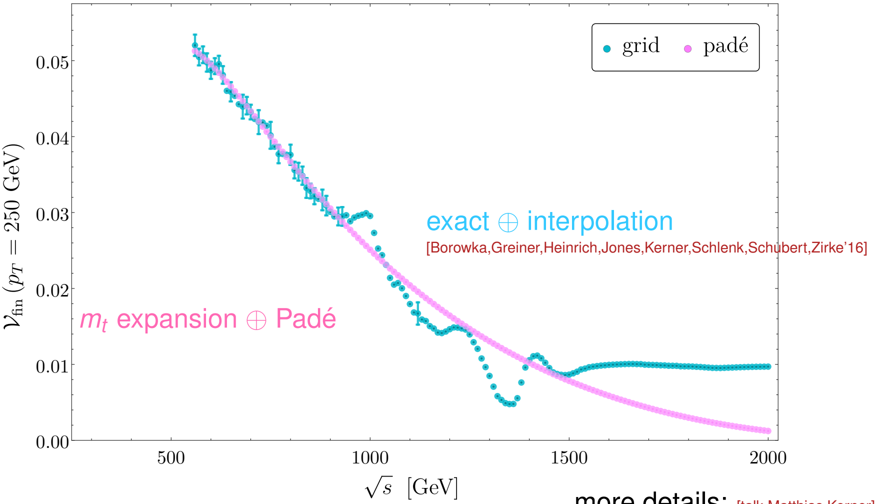
- solution requires BCs for $m_t \to 0$
- compute MIs such that F_{tri}, F_{box1}, F_{box2} are available up to m_t^{32}

[Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

$$\mathcal{M} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\left(\mathcal{M}_1 A_1^{\mu\nu} + \mathcal{M}_2 A_2^{\mu\nu}\right) \qquad \mathcal{M}_1 \sim \frac{3m_H^2}{s-m_H^2}F_{\mathrm{tri}} + F_{\mathrm{box1}} \qquad \mathcal{M}_2 \sim F_{\mathrm{box2}}$$

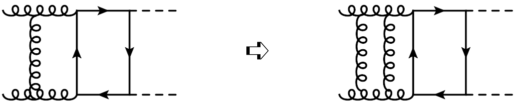
NLO $\mathcal{V}_{\text{fin}}$ for $p_T = 250$ GeV

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



more details: [talk Matthias Kerner]

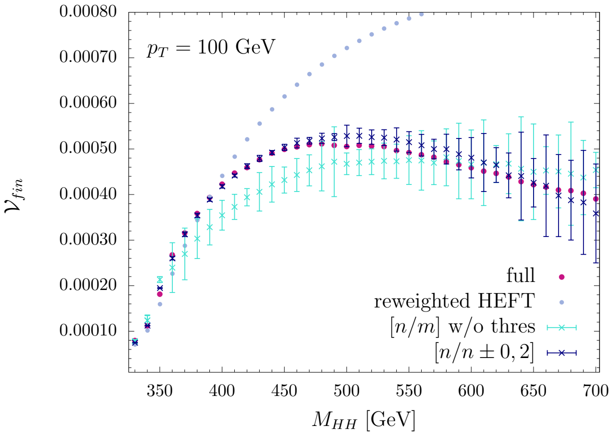
From NLO to NNLO



- exact: NO
- high-energy: box integrals with s, t and m_t
  ➪ 3 scales: not yet at 3 loops (NNLO)
- large-m_t: tadpoles + massless triangles (+ 1-loop massless box)
  ➪ 1-scale integrals: doable at 3 loops
- threshold: some non-analytic terms [Gröber,Maier,Rauh'17]
  $(s \approx 4m_t^2)$

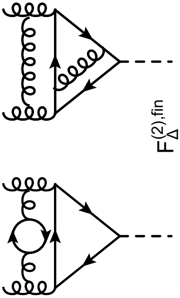
NLO approximation for $gg \to hh$: large-m_t + threshold + Padé

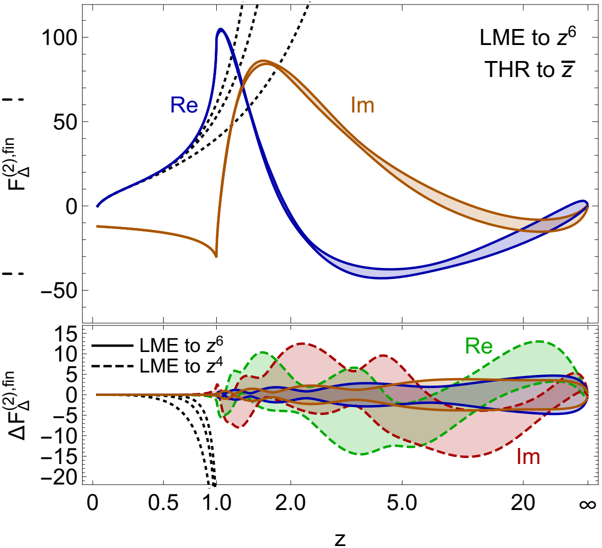
[Gröber,Maier,Rauh'17]



input for Padé: large-m_t: up to $1/m_t^8$
threshold: non-analytic terms ("logs")

NNLO approximation for $gg \to h$: large-m_t + threshold + Padé [Davies,Gröber,Maier,Rauh,Steinhauser'19][talk Thomas Rauh]





$z = \frac{s}{4m_t^2}$

$\bar{z} = 1 - z$

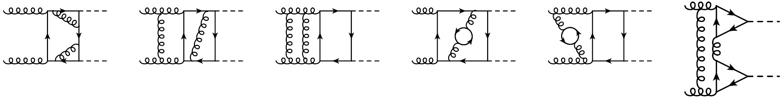
(analytic n_l terms: [Harlander,Prausa,Usovitsch'19])

NNLO approximation for $gg \to hh$: large-m_t + threshold + Padé

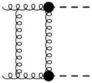
- large m_t
  - expansion up to order $1/m_t^4$ for cross section [Grigo,Hoff,Steinhauser'15]
  - form factors $F_{\rm box1}$, $F_{\rm box2}$ up to order $1/m_t^8$ [Davies,Steinhauser'19]
    $F_{\rm tri}$ up to order $1/m_t^{14}$
    - Expansion: $\approx$ 10 days wall time
      (a few $\times \geq$ 96 GB RAM, 12 cores; stored expression 324 GB)
    - Projection/integration: $\approx$ 1 month wall time
      (tasks are simpler: $\geq$ 8 GB RAM, 4 cores; total time $\approx$ 1,600 days)
    - heavy use of modern `(T)FORM` commands: [Ruijl,Ueda,Vermaseren'17]
      e.g. `ArgToExtraSymbols`
- theshold input from [Gröber,Maier,Rauh'17]

⇨ promising to obtain stable Padé results $F_{\rm tri}$: [Davies,Gröber,Maier,Rauh,Steinhauser'19]

$F_{\rm box1}$,$F_{\rm box2}$: [Davies,Gröber,Maier,Rauh,Steinhauser, WIP]



NNLO real corrections for large m_t

- $m_t \to \infty$: use NNLO $gg \to H$ +  + C_{HH}

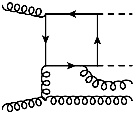
  [de Florian,Mazzitelli'14; Grigo,Melnikov,Steinhauser'14]

- Motivation for $1/m_t^2$ terms: input for Padé approximation (or similar)

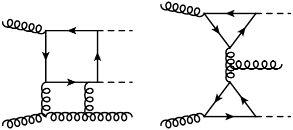
- known numerically: $gg \to HH + gg$ and $gg \to HH + q\bar{q}$

  [Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli'18]

NNLO real corrections for large m_t

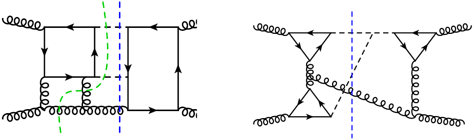
- 

  1-loop $2 \to 4$

  2-loop $2 \to 3$

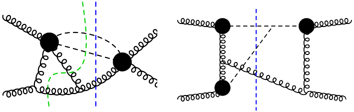
- use optical theorem

- asymptotic expansion for $m_t^2 \gg m_H^2, s$ ➪

tadpoles $\otimes$ "phase-space" integrals

1 and 2 loops



m_t^2 m_H^2, s

3-loop 3- and 4-particle cuts [DHMS WIP]
and 2-loop 3-particle cut [DHMS'19]

[DHMS = Davies,Herren,Mishima,Steinhauser]

NNLO real corrections for large m_t (2)

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	amplitude	$\rightarrow$	map to integral family	$\rightarrow$	IBP reduction `FIRE` [Smirnov]	$\rightarrow$	minimal set of MIs
$\rightarrow$	diff. eqs. in $x = m_H^2/s$	$\rightarrow$	Fuchsian form `epsilon` [Prausa'17]	$\rightarrow$	ϵ form `LIBRA` [Lee] [Henn'13]	$\rightarrow$	solution Goncharov polylogs `ginac` [Vollinga,Weinzierl'05]

NNLO real corrections for large m_t (2)

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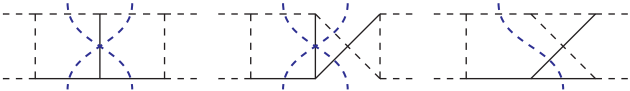
n_h^3:



2 loops, 3-particle cuts

NNLO real corrections for large m_t (2)

	amplitude	$\rightarrow$	map to integral family	$\rightarrow$	IBP reduction FIRE [Smirnov]	$\rightarrow$	minimal set of MIs
$\rightarrow$	diff. eqs. in $x = m_H^2/s$	$\rightarrow$	Fuchsian form epsilon [Prausa'17]	$\rightarrow$	ϵ form LIBRA [Lee] [Henn'13]	$\rightarrow$	solution Goncharov polylogs ginac [Vollinga,Weinzierl'05]

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I}$$

$$x_i = 0, 1, 1/4, -1, -1/4, e^{\pm i\pi/3}, -1/3$$

apply variable transformations, e.g., $x = \frac{y}{(1+y)^2}$

NNLO real corrections for large m_t (2)

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amplitude	$\rightarrow$	map to integral family	$\rightarrow$	IBP reduction `FIRE` [Smirnov]	$\rightarrow$	minimal set of MIs	
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$$x_i = 0, 1, 1/4, -1, -1/4, e^{\pm i\pi/3}, -1/3$$

$$\partial_y \vec{J} = \epsilon \sum_i \frac{\tilde{M}^{(i)}}{y - y_i} \vec{J}$$

NNLO real corrections for large m_t (2)

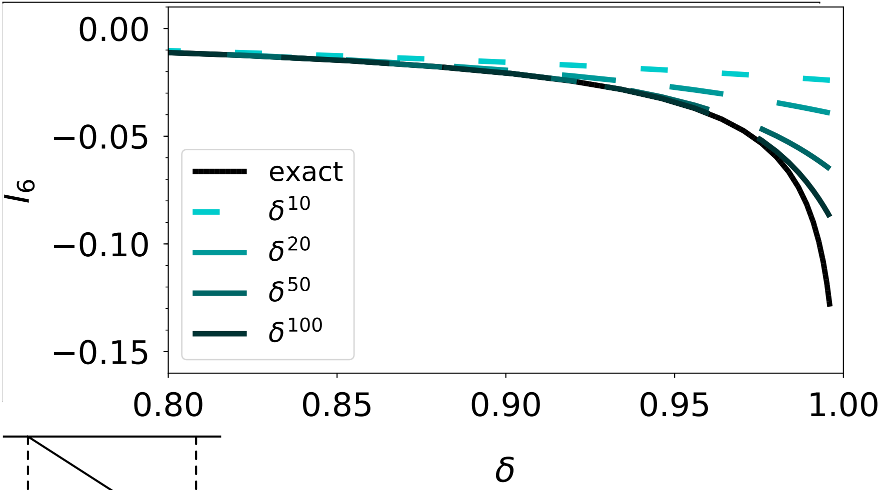
amplitude $\rightarrow$ map to integral family $\rightarrow$ IBP reduction `FIRE` [Smirnov] $\rightarrow$ minimal set of MIs

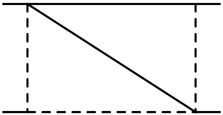
$\rightarrow$ diff. eqs. in $x = m_H^2/s$ $\rightarrow$ Fuchsian form `epsilon` [Prausa'17] $\rightarrow$ ϵ form `LIBRA` [Lee] [Henn'13] $\rightarrow$ solution Goncharov polylogs `ginac` [Vollinga,Weinzierl'05]

$\rightarrow$ expand in $\delta = \sqrt{1 - 4m_H^2/s}$ `PolyLogTools` [Duhr,Dulat'19]

alternative: use diff. eqs. to obtain expansion in δ ($>$500 terms possible)

NNLO real corrections for large m_t (2)





$\delta = 0.9 \Rightarrow \sqrt{s} = 800$ GeV

NNLO real corrections for large m_t (3)

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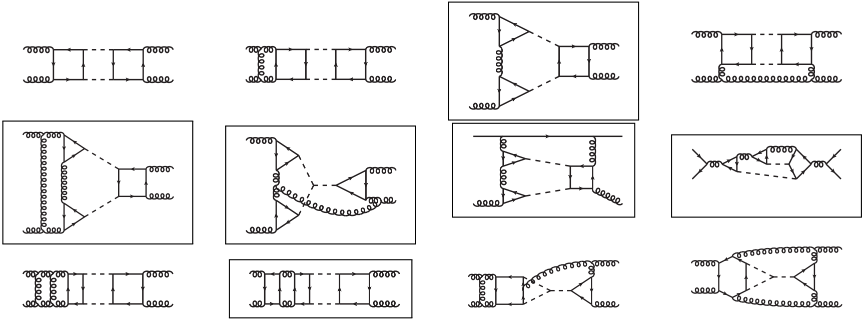
Cross section for n_h^3 diagrams



- expansion up to $1/m_t^8$
- combine with virtual corrections [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]

NNLO real corrections for large m_t (3)

Cross section for n_h^3 diagrams

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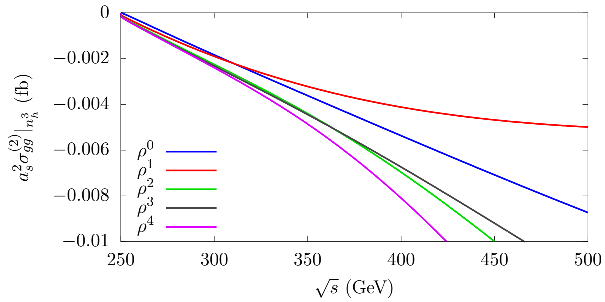
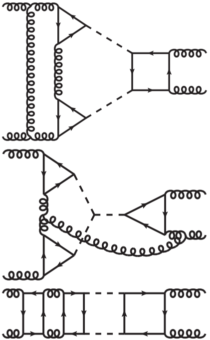
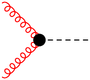
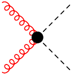




$\rho = m_H^2/m_t^2$

N^{3}LO: $m_t \to \infty$

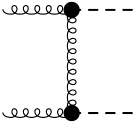
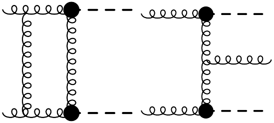
$$\mathcal{L}_{\rm eff} = -\frac{H}{v} C_H \mathcal{O}_1 + \frac{1}{2}\frac{H^2}{v^2} C_{HH} \mathcal{O}_1 \qquad \mathcal{O}_1 = -G^2_{\mu\nu}/4$$

 and  have same operator

➪ most parts can be taken over from $gg \to H$

N^{3}LO: [Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Mistlberger'14]

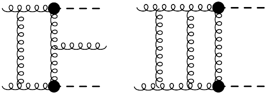
NEW for $gg \to HH$:

- NLO: 
- NNLO: 

[de Florian,Mazzitelli'14]

$C_{HH} \neq C_H$

[Grigo,Melnikov,Steinhauser'14]

- N^{3}LO: 

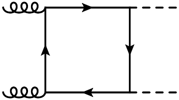
[Banerjee,Borowka,Dhani,Gehrmann,Ravindran'18]

$C_{HH} \neq C_H$ [Spira'16; Gerlach,Herren,Steinhauser'18]

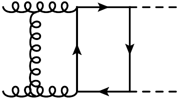
C_{HH} to 4 loops

$$\mathcal{L}_{\text{eff}} = -\frac{H}{v} C_H \mathcal{O}_1 + \frac{1}{2}\frac{H^2}{v^2} C_{HH} \mathcal{O}_1$$

$$C_{HH} = C_H + \Delta_{HH}$$



$$\Delta_{HH} = 0$$



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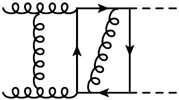
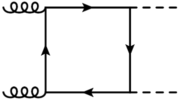


$$\Delta_{HH} = \frac{7}{8} C_A^2 - \frac{11}{8} C_A C_F - \frac{5}{6} C_A T_F + \frac{1}{2} C_F T_F + n_l C_F T_F$$

[Grigo,Melnikov,Steinhauser'14]

C_{HH} to 4 loops

$$\mathcal{L}_{\rm eff} = -\frac{H}{v} C_H \mathcal{O}_1 + \frac{1}{2}\frac{H^2}{v^2} C_{HH} \mathcal{O}_1 \qquad C_{HH} = C_H + \Delta_{HH}$$



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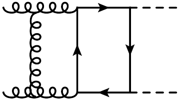

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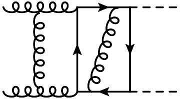
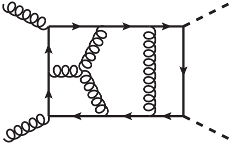


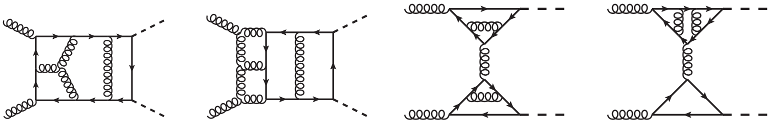
$$\Delta_{HH} = \frac{7}{8}C_A^2 - \frac{11}{8}C_A C_F - \frac{5}{6}C_A T_F + \frac{1}{2}C_F T_F + n_l C_F T_F$$

[Grigo,Melnikov,Steinhauser'14]



$$\Delta_{HH} = ?$$

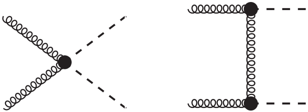
Renormalization and matching



- Tricky renormalization:
  $\langle\mathcal{O}_1\mathcal{O}_1\rangle^{\rm ren} = (Z_{\mathcal{O}_1})^2\langle\mathcal{O}_1\mathcal{O}_1\rangle^{\rm bare} + Z_{11}^L\langle\mathcal{O}_1\rangle^{\rm bare}$ $\quad Z_{11}^L = 1 + \mathcal{O}(\alpha_s^2)$

[Zoller'16]

- Matching



$$(C_{HH}Z_{\mathcal{O}_1} + C_H^2 Z_{11}^L)\mathcal{A}^{\rm eff}_{\rm LO,1PI} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}^{\rm eff}_{\rm LO,1PR,\lambda=0} = \mathcal{A}^h_{\rm 1PI} + \mathcal{A}^h_{\rm 1PR,\lambda=0}$$

[Gerlach,Herren,Steinhauser'18]

Δ_{HH}

[Gerlach,Herren,Steinhauser'18]

$$\Delta_{HH}^{(4)} = \frac{1993}{576} C_A^3 - \frac{1289}{144} C_A^2 C_F - \frac{3191}{864} C_A^2 T_F + \frac{165}{32} C_A C_F^2 + \frac{67}{18} C_A C_F T_F + \frac{5}{72} C_A T_F^2$$
$$- \frac{3}{2} C_F^2 T_F + \frac{1}{9} C_F T_F^2 + \left[\frac{77}{48} C_A^3 - \frac{121}{48} C_A^2 C_F - \frac{7}{12} C_A^2 T_F + \frac{11}{12} C_A C_F T_F\right] \ln\left(\frac{\mu^2}{M_t^2}\right)$$
$$+ n_l T_F \left[-\frac{55}{144} C_A^2 + \frac{55}{18} C_A C_F + \frac{109}{216} C_A T_F - \frac{11}{4} C_F^2 + \frac{19}{36} C_F T_F\right]$$
$$+ n_l^2 T_F^2 \left[\frac{5}{72} C_A + \frac{1}{9} C_F\right] + n_l T_F \left[-\frac{7}{12} C_A^2 + \frac{11}{4} C_A C_F - \frac{2}{3} C_F T_F\right] \ln\left(\frac{\mu^2}{M_t^2}\right)$$
$$- \frac{2}{3} n_l^2 C_F T_F^2 \ln\left(\frac{\mu^2}{M_t^2}\right)$$

[Spira'16] **Low-energy theorem:** $[C_H = -\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s}$ [Chetyrkin,Kniehl,Steinhauser'98]]

$$C_{HH} = \frac{m_t^2}{\zeta_{\alpha_s}} \frac{\partial^2}{\partial m_t^2} \zeta_{\alpha_s} - 2 \left(\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s}\right)^2 \qquad \alpha_s^{(5)} = \zeta_{\alpha_s} \alpha_s^{(6)}$$

Conclusions

- LO, NLO: exact
- NLO: many approximations
  - exact result is expensive! ➪ combine approximations and exact result
  - test bed for NNLO
- NNLO: large-m_t (virt. + real)
- N^3LO: effective-theory ($m_t \to \infty$)
  - C_{HH} to 4 loops
  - cross section in reach