Two-loop evolution for the leading-twist B-meson distribution amplitude

Alexander Manashov

based on: V. Braun, Yao Ji, A.M., PRD100 (2019) 014023

Hamburg University

Outline

- Motivations
- Definition of the leading-twist B-meson LCDA
- One-loop evolution of the leading-twist DA
- Two-loop evolution kernel
- Summary

Motivations

Amplitudes of high energy processes can be written as a convolution of perturbative and nonperturbative parts. Schematically,

$$\mathcal{A} = [C] \otimes [\phi]$$

where $[C]$ corresponds to a high energy process which can be computed perturbatively, $[\phi]$ encodes the nonperturbative information of the hadronic state and is called a distribution amplitude.
At present, $[\phi]$ is not directly accessible theoretically.

Definition of the leading-twist B-meson LCDA

The two-particle B-meson LCDAs are defined by the matrix element of the renormalized nonlocal operator involving a heavy quark field $h_v(0)$ and a light (anti)quark with a light-like separation: [A. Grozin and M. Neubert, (1997)]

$$\langle 0|\, \bar q(nz)\Gamma[nz,0]h_v(0)\,|\bar B(v)\rangle =$$
$$= -\frac{i}{2}F_B(\mu)\,\mathrm{Tr}\left\{\gamma_5\Gamma P_+\left[\Phi_+(z,\mu)-\frac{1}{2}\not{n}\,(\Phi_+(z,\mu)-\Phi_-(z,\mu))\right]\right\}$$

where $n^\mu=(1,0,0,-1)$ is the light-cone vector, $v^\mu=(1,0,0,0)$ is the heavy quark velocity, $P_+=\frac{1}{2}(1+\not{v})$, $F_B(\mu)$ is the B-meson decay constant in HQET. $\Phi_+(z,\mu)$ and $\Phi_-(z,\mu)$ are called leading and sub-leading-twist two-particle B-meson DAs. [M. Beneke and T. Feldmann, (2001)]

Nonlocal operator ($z\neq 0$)

$$\mathcal{O}(z)=\bar q(nz)[nz,0]h_v(0)=\bar q(nz)[nz,0][0,v\infty]$$

Evolution Equation

$$\Big(\mu\frac{\partial}{\partial\mu}+\beta(a)\frac{\partial}{\partial a}+\mathcal{H}(a)\Big)\Phi_+(z,\mu)=0\,,$$

where $\mathcal{H}$ is the evolution kernel, usually presented as an integral operator.
The one-loop evolution kernel takes the form:

$$\mathcal{H}^{(1)}\Phi_+(z)=4C_F\Big\{\Big(\ln(i\widetilde{\mu}z)+\frac{1}{2}\Big)\Phi_+(z)+\int_0^1 du\frac{\bar u}{u}\left[\Phi_+(z)-\Phi_+(\bar u z)\right]\Big\},$$

where $\widetilde{\mu}=\mathrm{e}^{\gamma_E}\mu^{\overline{\mathrm{MS}}}$. [B. Lange and M. Neubert, (2003); V. Braun, D. Ivanov and G. Korchemsky, (2004)]

The corresponding solution for the RGE reads, [G. Bell, T. Feldmann, Y.-M. Wang and M. W. Y. Yip, (2013), V. Braun, A. M. (2014)]

$$\Phi_+(z,\mu)=\int_0^\infty ds\, s\, Q_s(z)\,\eta_+(s,\mu)\,,\qquad Q_s(z)=-\frac{1}{z^2}\mathrm{e}^{is/z}$$

$$\eta_+(s,\mu)=R(s,\mu,\mu_0)\,\eta_+(s,\mu_0)\,,\qquad R(s,\mu,\mu_0)\propto s^{\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}}$$

Conformal symmetry

Light-quark kernels commute with the generators of the $SL(2,R)$ collinear subgroup of conformal group: translations, dilatations and special conformal transformations

$$\mathcal{O}(z_1,z_2) = \bar{q}(z_1 n)\gamma_+ q(z_2,n)$$

Symmetry generators

$$S_+ = z_1^2\partial_{z_1} + 2z_1 + z_2^2\partial_{z_2} + 2z_2, \qquad S_0 = z_1\partial_{z_1} + z_2\partial_{z_2} + 2, \qquad S_- = -\partial_{z_1} - \partial_{z_2}$$

The evolution kernel commutes with $S_{\pm,0}$, $[S_{\pm,0},\mathcal{H}_{q\bar{q}}] = 0$.

$$\mathcal{H}_{q\bar{q}} = \mathcal{H}(J_{12}) \sim \psi(J_{12}+1) + \psi(J_{12}-1) + \text{const}$$

$J_{12}(J_{12}-1) = S_+S_- + S_0(S_0-1)$ – the quadratic Casimir operator.

Conformal symmetry

The one loop kernel $\mathcal{H}^{(1)}$ commutes with the generator of special conformal transformations, $S_+ \sim \mathbf{K}^\mu v_\mu$, [M. Knödlseder and N. Offen, (2011)]

$$S_+ = z^2\partial_z + 2z \qquad [S_+, \mathcal{H}^{(1)}] = 0 \qquad \mathcal{H}^{(1)} = f(S_+)$$

$$S_0 = z\partial_z + 1 \qquad [S_0, \mathcal{H}^{(1)}] = 4C_F \qquad xf'(x) = 4C_F$$

It results in [V. Braun, A. M. (2014)]

$$\boxed{\mathcal{H}^{(1)} = 4C_F \ln(S_+\mu) + \text{constant}}$$

The function $Q_s(z) = e^{is/z}/z^2$ is the eigenfunction of S_+:

$\hookrightarrow$ Some progress in solving higher twist B-meson DAs. [V. Braun, S. Derkachov, A. M. (2014), V. Braun, Yao Ji, A. M. (2017), M. Beneke, V.Braun, Y. Ji, Y. Wei (2018)]

Motivations for the two-loop study

- Conjecture

$$\mathcal{H}(a) = \Gamma_{\mathsf{cusp}}(a) \ln\Big(S_+(a)\mu\Big) + \Gamma_+(a)$$

  where $S_+(a)$ is the generator of special conformal transformations.

- The last missing piece for a complete NNLL resummation of QCD corrections for charmless B-decays

symmetry

Conformal symmetry is restored in QCD in $d = 4 - 2\epsilon$ at the critical point $\beta(a_*) = 0$, $a_* = -\epsilon/\beta_0 + \dots$.
But symmetry generators receive quantum corrections.

$$S_+^{(0)} = z^2\partial_z + 2z \longmapsto S_+(a) = S_+^{(0)} + z\Delta(a), \qquad [z\partial_z, \Delta(a)] = 0$$

where $\Delta(a) = -\epsilon + a\Delta^{(1)} + a^2\Delta^{(2)} + \dots$.
At one loop

$$\Delta^{(1)}\mathcal{O}(z) = C_F\left\{3\mathcal{O}(z) + 2\int_0^1 d\alpha\,\omega(\alpha)\big[\mathcal{O}(z) - \mathcal{O}(\bar\alpha z)\big]\right\},$$

where $\omega(\alpha) = 2\bar\alpha/\alpha + \ln\alpha$.

First equation:

$$[S_+(a), \mathcal{H}(a)] = 0 \qquad \longmapsto \qquad \underline{\mathcal{H}(a) = f(S_+(a))}$$

Second equation: $\ln\mu z$ enters the kernel with the coefficient Γ_{cusp}, G. Korchemsky, A. Radyushkin,(1992)

$$[z\partial_z, \mathcal{H}(a)] = \Gamma_{\text{cusp}}(a) \qquad [z\partial_z, S_+(a)] = S_+(a) \longmapsto \qquad \underline{xf'(x) = \Gamma_{\text{cusp}}(a)}\,.$$

The solution is

$$\mathcal{H}(a) = \Gamma_{\mathsf{cusp}}(a) \ln \Big(S_+(a)\mu\Big) + \Gamma_+(a)$$

For applications it is better to represent $\mathcal{H}$ in a different form:

$$\begin{aligned}\mathcal{H}(a)\mathcal{O}(z) =& \boldsymbol{\Gamma}_{\mathsf{cusp}}(a)\Big[\ln(i\mu z)\mathcal{O}(z) + \int d\alpha \frac{\bar{\alpha}}{\alpha}\Big(\mathcal{O}(z) - \mathcal{O}(\bar{\alpha}z)\Big) \\ &+ \int d\alpha\, \frac{\bar{\alpha}}{\alpha} h(\alpha)\Big(\mathcal{O}(z) - \mathcal{O}(\bar{\alpha}z)\Big)\Big] + \gamma_+(a)\end{aligned}$$

$S_+(a) = S_+^{(0)} + z\Delta$: The integral operator Δ commutes with $z\partial_z$, hence $\Delta z^j = \Delta(j) z^j$.

$$\Delta(j) = a\left((\beta_0 + C_F) + 4C_F\left(\psi(j+2) - \psi(2) + \frac{1}{2}\frac{\psi(j+2) - \psi(1)}{j+1}\right)\right)$$

Go over to the Mellin representation $\mathcal{O}(z) = \int dj \mathcal{O}(j) z^j$ and solve two equations $[S_+, \mathcal{H}] = 0$ and $[z\partial_z, \mathcal{H}] = \Gamma_{\text{cusp}}$.

The expression for $h(\alpha)$ takes the form:

$$h(\alpha) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dj\, \bar{\alpha}^{-j-2}\, \partial_j \ln\left(1 + \frac{\Delta(j)}{j+2}\right) = \frac{\ln \bar{\alpha}}{2\pi i} \int_{-i\infty}^{i\infty} dj\, \bar{\alpha}^{-j-2}\, \ln\left(1 + \frac{\Delta(j)}{j+2}\right)$$

At the leading order

$$h(\alpha) = a_s \ln \bar{\alpha} \left\{ \beta_0 - 2C_F \left(\frac{3}{2} + \ln \frac{\alpha}{\bar{\alpha}} + \frac{\ln \alpha}{\bar{\alpha}} \right) \right\} + O(a_s^2).$$

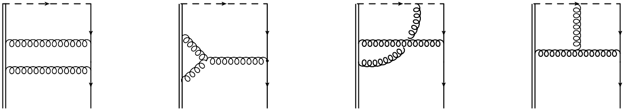
The constant $\gamma_+(a)$ requires explicit calculation:

$$\gamma_+(a) = -a_s C_F + a_s^2 C_F \left\{ 4C_F \left[\frac{21}{8} + \frac{\pi^2}{3} - 6\zeta_3 \right] + C_A \left[\frac{83}{9} - \frac{2\pi^2}{3} - 6\zeta_3 \right] + \beta_0 \left[\frac{35}{18} - \frac{\pi^2}{6} \right] \right\}.$$

Diagrams

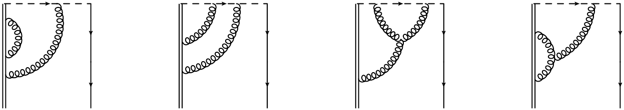
There are ~ 30 diagrams in three categories :

- Exchange diagrams



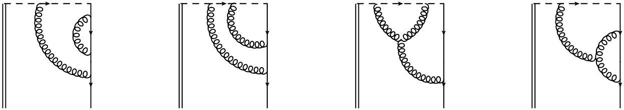
. . .

- Cusp diagrams



. . .

- Light vertex



. . .

Diagrams

- Light vertices: The answers for these diagrams are known, [V. Braun, A. Manashov, S. Moch, and M. Strohmaier (2016)]. These diagrams contribute only to the function h, and do not contribute to the constant γ_+.

- Cusp diagrams have to be calculated. They are responsible for $\ln i\mu z$ term and contribute to the constant γ_+

- Exchange diagrams contribute both to h and γ_+.

  They are related to the contribution for the light quark operator, $\mathcal{O}(z_1, z_2) = \bar{q}(z_1 n)\gamma_+ q(z_2 n)$.

  The counterterm for a diagram with light quarks operator can be written as

$$Z \simeq C \cdot \mathcal{O}(z_1, z_2) + \int d\alpha h_1(\alpha)\mathcal{O}(z_{12}^{\alpha}, z_2) + \int d\alpha h_2(\alpha)\mathcal{O}(z_1, z_{21}^{\alpha}) + \int d\alpha \int d\beta \chi(\alpha, \beta)\mathcal{O}(z_{12}^{\alpha}, z_{21}^{\beta})$$

Diagrams

- Light vertices: The answers for these diagrams are known, [V. Braun, A. Manashov, S. Moch, and M. Strohmaier (2016)]. These diagrams contribute only to the function h, and do not contribute to the constant γ_+.

- Cusp diagrams have to be calculated. They are responsible for $\ln i\mu z$ term and contribute to the constant γ_+

- Exchange diagrams contribute both to h and γ_+.

  They are related to the contribution for the light quark operator, $\mathcal{O}(z_1, z_2) = \bar{q}(z_1 n)\gamma_+ q(z_2 n)$.

  The counterterm for a diagram with light quarks operator can be written as

$$Z \simeq C \cdot \mathcal{O}(z_1, z_2) + \int d\alpha h_1(\alpha)\mathcal{O}(z_{12}^{\alpha}, z_2)\Big|_{z_2 = z_h \to 0}$$

$$+ \cancel{\int d\alpha h_2(\alpha)\mathcal{O}(z_1, z_{21}^{\alpha})} + \cancel{\int d\alpha \int d\beta \chi(\alpha, \beta)\mathcal{O}(z_{12}^{\alpha}, z_{21}^{\beta})}$$

Evolution of the coefficient function at two-loop

The RGE of ϕ_+ to $\mathcal{O}(a^2)$ can be cast into an integro-differential equation over the coefficient function $\eta_+(s,\mu)$ as the eigenfunctions of the one-loop kernel form a complete orthonormal basis: [V. Braun, YJ and A. Manashov (2019)]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} + \Gamma_{\text{cusp}}(a)\ln(\widetilde{\mu}\mathrm{e}^{\gamma_E}s) + \gamma_\eta(a)\right)\eta_+(s,\mu) = 4C_F a^2\int_0^1 du\frac{\bar{u}}{u}h(u)\eta_+(\bar{u}s,\mu)\,,$$

with

$$\gamma_\eta = \gamma_+ - \Gamma^{(2)}_{\text{cusp}}\left[1 - a\left(C_F\left(\frac{\pi^2}{6}-3\right) + \beta_0\left(1-\frac{\pi^2}{6}\right)\right)\right].$$

The two-loop corrections turn out to be rather small that provides a theoretical explanation that the scale dependence of the $B\to\gamma\ell\nu_\ell$ decay is rather weak. [M. Beneke, V. Braun, YJ and Y-B. Wei, (2018)]

Summary

- The two-loop evolution kernel of the leading twist DA has a simple form and is directly related to the special conformal generator.
- Numerical analysis shows that the two-loop effect is small.