Renormalization schemes for mixing angles

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Outline

Relevance of mixing angles

- Standard Model contains mixing angles in fermion sector: parameters of quark-mixing matrix
- Extended Higgs models $\Rightarrow$ mixing between different Higgs fields
- Extended gauge models $\Rightarrow$ mixing between different gauge fields
- Mixing angles as independent parameters
  $\Rightarrow$ renormalization of mixing angles needed

Renormalization of quark-mixing matrix

- Many recipes in literature: Denner, Sack 1990;
  Gambino, Grassi, Mandricardo 1998; Balzereit, Mannel, Plümper 1998;
  Barroso, Brucher, Santos 2000; Diener, Kniehl 2001; Yamada 2001; Pilaftsis 2002;
  Denner, Kraus, Roth 2004; Kniehl, Sirlin 2006, 2009, . . .
  often cumbersome, singular limits, restricted to one-loop order
  gauge-independent, physical, symmetric renormalization condition
  Denner, Kraus, Roth 2004
- Practical relevance limited owing to small effects in SM
  (suppression $\propto m_{\mathrm{b}}/M_{\mathrm{W}}$)

Investigation of Higgs sector of prime importance!

Precision calculations in BSM models require
renormalization of mixing angles in extended Higgs sectors

- Minimal Supersymmetric Standard Model (MSSM): $\tan\beta$
  Chankowski, Pokorski, Rosiek 1994; Dabelstein 1995; Freitas, Stöckinger 2002; Baro, Boudjema, Semenov 2008; Baro, Boudjema 2009
- Two-Higgs-Doublet model (THDM): α, β
  Kanemura et al. 2004, 2014, 2015; Lopez-Val, Solà 2009; Krause et al. 2016; Denner et al. 2016, 2017; Altenkamp et al. 2017; Jenniches et al. 2018
- Higgs-Singlet-Extended-Standard Model (HSESM): α
  Denner et al. 2017; Kanemura et al. 2015, 2017; Bojarski et al. 2015; Altenkamp et al. 2018
- Next-to-Two-Higgs-Doublet model (N2HDM): Krause et al. 2017

Problems of previously existing schemes:

- $\overline{\mathrm{MS}}$ scheme $\Rightarrow$ perturbatively unstable
- on-shell definitions $\Rightarrow$ perturbatively unstable, symmetries violated
- off-shell definitions $\Rightarrow$ gauge dependent

Freitas, Stöckinger 2002: renormalization of $\tan\beta$ in MSSM

Mixing-angle renormalization should be   Freitas, Stöckinger 2002

- gauge independent
  $\hookrightarrow$ gauge-independent S-matrix elements
- symmetric with respect to mixing degrees of freedom
  $\hookrightarrow$ independent of a specific physical process
- numerically stable in perturbative calculations
  $\hookrightarrow$ higher-order corrections not artificially large

in addition   Denner, Dittmaier, Lang 2018

- non-singular for degenerate masses of mixing particles
  or extreme mixing angles (0, π, $\pi/2$)
  $\Rightarrow$ valid in full parameter space

Aim of talk:
Renormalization prescriptions for mixing angles fulfilling these properties

Outline

Appearance of tadpoles

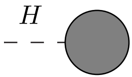
Field theories with spontaneous symmetry breaking: $\langle \Phi \rangle = v \neq 0$
introduce shifted field with vanishing vev: $\Phi(x) = \bar{v} + H(x)$, $\langle H \rangle = 0$
$\Rightarrow$ terms in $\mathcal{L}$ linear in H $\qquad [\mathcal{L} = \ldots + \mu_0^2 \Phi\Phi^\dagger - (\lambda_0/4)(\Phi\Phi^\dagger)^2]$

$$\mathcal{L} = \ldots + t_0 H + \ldots, \qquad t_0 = \bar{v}\left(\mu_0^2 - \frac{1}{4}\lambda_0 \bar{v}^2\right)$$

Leading order:

$$\langle H \rangle = 0 \quad \leftrightarrow \quad t_0 = 0 \quad \rightarrow \quad \bar{v} = v_0 = \frac{2\mu_0}{\sqrt{\lambda_0}}$$

Higher orders: tadpole diagrams

$T^H =$ 

$\neq 0 \quad \leftrightarrow \quad \langle H \rangle \neq 0$

eliminate tadpoles via expansion about true vev $\bar{v} = v = v_0 + \Delta v$

$$\delta t = v\left(\mu_0^2 - \frac{1}{4}\lambda_0 v^2\right), \qquad \delta t + T^H \stackrel{!}{=} 0 \quad \leftrightarrow \quad \Delta v = -\frac{\delta t}{M_{\mathrm{H}}^2}$$

- used in many calculations in the on-shell scheme since 't Hooft 1971, Passarino and Veltman 1979
- uses consistently true vev v
- no explicit tadpole diagrams, few tadpole counterterms in scalar sector
- simple Ward identities (without explicit tadpoles)
- bare gauge-boson masses: $M_{0,\mathrm{W}} = \frac{1}{2}g_2 v$
- T^H gauge dependent $\Rightarrow$ δt, Δv, v gauge dependent
  $\Rightarrow$ $M_{0,\mathrm{W}}$ gauge dependent $\Rightarrow$ δM_{W}^2 gauge dependent
- gauge dependence cancels
  - in renormalized quantities in on-shell scheme
  - in all schemes based on momentum subtraction
  - in relations between physical observables
- $\overline{\mathrm{MS}}$ scheme: renormalized quantities and S-matrix gauge dependent
- $\overline{\mathrm{MS}}$-renormalized mixing angles gauge dependent Denner et al. 2016
- different incarnations = different definitions of bare Higgs mass
  Aoki et al. 1982; Denner 1993; Actis et al. 2006

Fleischer, Jegerlehner 1981

- fully consistent treatment of tadpoles
- uses consistently bare vev $v_0 = \frac{2\mu_0}{\sqrt{\lambda_0}}$
- bare gauge-boson masses: $M_{0,\mathrm{W}} = \frac{1}{2} g_2 v_0$ gauge independent
- $\overline{\mathrm{MS}}$ scheme: renormalized quantities and S-matrix gauge independent
- $\overline{\mathrm{MS}}$-renormalized mixing angles are gauge independent
- explicit tadpoles $\langle H \rangle \neq 0$ appear almost everywhere: in Ward identities, in definition of renormalized masses, in Feynman rules

Variant of FJTS: shift Higgs field by Δv such that $\langle H \rangle = 0$

- explicit tadpoles replaced by contributions from Δv
  $\Rightarrow$ shifted to counter terms of other vertex functions
- Δv appears in Ward identities etc.

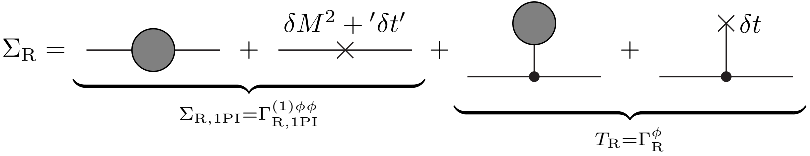
fully consistent, but somewhat cumbersome in practice

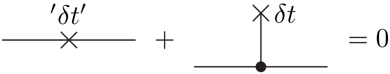
Tadpoles in Green functions

Path integral does not depend on shift $H \to H - \Delta v$

- (connected) Green functions and S-matrix independent of Δv (except 1-point function)
- vertex functions $\Gamma^{\cdots}$ depend on Δv

example: self-energy (connected 2-point function)





gauge-independent mass renormalization constant

$$\delta M^2_{\mathrm{FJTS}} = \Sigma(M^2) = \Sigma_{\mathrm{1PI}}(M^2) + g\frac{T^H}{M_{\mathrm{H}}^2} = \delta M^2_{\mathrm{PRTS}} + g\frac{T^H}{M_{\mathrm{H}}^2}$$

Mixing of two CP even scalar bosons: (generic for extended Higgs sectors)
fields in symmetric basis: η_1, η_2
fields in physical mass-eigenstate basis H_1, H_2

$$\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = R(\alpha)\mathbf{H}, \qquad R(\alpha) = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

$c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$, α mixing angle

Renormalization:

$$\alpha_0 = \alpha + \delta\alpha$$

$$\mathbf{H}_0 = (Z^H)^{1/2}\mathbf{H}$$

with

$$(Z^H)^{1/2} = \begin{pmatrix} (Z^H)^{1/2}_{11} & (Z^H)^{1/2}_{12} \\ (Z^H)^{1/2}_{21} & (Z^H)^{1/2}_{22} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z^H_{11} & \frac{1}{2}\delta Z^H_{12} \\ \frac{1}{2}\delta Z^H_{21} & 1 + \frac{1}{2}\delta Z^H_{22} \end{pmatrix}$$

in complete on-shell scheme (no wave-function renormalization)

Aoki et al. 1982, Denner 1993

$$\delta Z^H_{ij} = \frac{2}{M^2_{\mathrm{H}_i} - M^2_{\mathrm{H}_j}} \Sigma_{ij}(M^2_{\mathrm{H}_j}), \quad i \neq j$$

Higgs doublet Φ and real singlet σ with vevs $v_2/\sqrt{2}$ and v_1

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + \mathrm{i}\chi) \end{pmatrix}, \qquad \sigma = v_1 + \eta_1$$

Higgs potential with $\mathbb{Z}_2$ symmetry $\sigma \to -\sigma$

Schabinger, Wells 2005; Patt, Wilczek 2006; Bowen, Cui, Wells 2007; Pruna, Robens 2013

$$V_{\mathrm{HSESM}} = -\mu_2^2 \Phi^\dagger\Phi + \frac{\lambda_2}{4}\left(\Phi^\dagger\Phi\right)^2 - \frac{1}{2}\mu_1^2\sigma^2 + \frac{\lambda_1}{16}\sigma^4 + \frac{\lambda_3}{2}\Phi^\dagger\Phi\,\sigma^2$$

vector-boson masses

$$M_{\mathrm{W}} = \frac{1}{2} g_2 v_2, \qquad M_{\mathrm{Z}} = \frac{1}{2}\sqrt{g_2^2 + g_1^2}\, v_2$$

free parameters in the symmetric basis

$$g_1, g_2, g_f, \mu_2^2, \lambda_2, \mu_1^2, \lambda_1, \lambda_3$$

free parameters in the physical basis (including mixing angle α)

$$\underbrace{\alpha, M_{\mathrm{W}}, M_{\mathrm{Z}}, m_f, M_h, M_H}_{\text{renormalized on shell}}, \alpha, \underbrace{[\lambda_1 \text{ or } v_1 \text{ or } \tan\beta = v_2/v_1]}_{\to \text{ Higgs self couplings}}$$

Two-Higgs Doublet Model (THDM)

Higgs doublets

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + \mathrm{i}\chi_i) \end{pmatrix}, \quad i = 1,2$$

Higgs potential (CP conserving)

Gunion, Haber 2003; Branco et al. 2011

$$\begin{aligned} V_{\mathrm{THDM}} = {} & m_2^2 \Phi_2^\dagger \Phi_2 + m_1^2 \Phi_1^\dagger \Phi_1 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ & + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\ & + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right] \end{aligned}$$

with $\mathbb{Z}_2$ symmetry $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$ softly broken by m_{12}^2

vector-boson masses

$$M_{\mathrm{W}} = \frac{1}{2} g_2 v, \qquad M_{\mathrm{Z}} = \frac{1}{2} \sqrt{g_2^2 + g_1^2}\, v, \qquad v = \sqrt{v_1^2 + v_2^2}$$

Transformation of pseudoscalar and charged Higgs fields to physical basis

$$\begin{pmatrix} \phi_1^\pm & \chi_1 \\ \phi_2^\pm & \chi_2 \end{pmatrix} = R(\beta) \begin{pmatrix} G^\pm & G_0 \\ H^\pm & A_0 \end{pmatrix}, \qquad R(\beta) = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}$$

mixing angle β: $\tan\beta = v_2/v_1$, $c_\beta = \cos\beta$, $s_\beta = \sin\beta$
charged and neutral would-be Goldstone fields: $G^\pm$, G_0
charged and pseudoscalar physical Higgs fields: $H^\pm$, A_0

free parameters in the symmetric basis

$$g_1, g_2, g_f, \mu_2^2, \lambda_2, \mu_1^2, m_{12}^2, \lambda_1, \lambda_3, \lambda_4, \lambda_5$$

free parameters in the physical basis

$$\underbrace{\alpha, M_\mathrm{W}, M_\mathrm{Z}, m_f, M_h, M_H, M_{H^\pm}, M_{A_0}}_{\text{renormalized on shell}}, \alpha, \beta, \underbrace{\lambda_5}_{\to\text{ Higgs self couplings}}$$

$\overline{\mathrm{MS}}$ renormalization of mixing angles

Denner et al. 2016, 2017; Altenkamp et al. 2017, 2018; Jenniches et al. 2018

$$\delta\alpha_{\overline{\mathrm{MS}}}(\mu_\mathrm{r}) = \frac{\Sigma_{12}(M_{\mathrm{H}_1}^2) + \Sigma_{12}(M_{\mathrm{H}_1}^2)\big|_{\mathrm{UV}}}{2(M_{\mathrm{H}_1}^2 - M_{\mathrm{H}_2}^2)}$$

- simple, symmetric, process independent
- allows to estimate perturbative stability via scale variation
- gauge independence requires FJTS Denner et al. 2016
- perturbatively unstable, in particular in THDM and FJTS
  Denner et al. 2016; Altenkamp et al. 2017, 2018; Jenniches et al. 2018
- large corrections for degenerate masses
  from finite terms of the form (divergences cancelled by $\delta\alpha$)

$$\delta Z_{12}^{H} = \frac{2}{M_{\mathrm{H}_1}^2 - M_{\mathrm{H}_2}^2}\Sigma_{12}(M_{\mathrm{H}_2}^2)$$

$\overline{\mathrm{MS}}$ renormalization of λ_i instead of α Altenkamp et al. 2017
$\Rightarrow$ dead corners in parameter space, $\lambda_i = \lambda_i(\alpha)$ not invertible

Process specific on-shell conditions: e.g. $\Gamma^{h\to XY} = \Gamma^{h\to XY}_{\text{LO}}$
Krause et al. 2016; Jenniches et al. 2018

- gauge independent
- process dependent ⇒ symmetries violated
- S-matrix element with charged particles ⇒ IR singularities
- observables depend on further parameters (typically $g \times f(\alpha)$)

$$\frac{\delta\Gamma}{\Gamma} = \frac{\delta g}{g} + \frac{\delta f(\alpha)}{f(\alpha)} + \delta_{\text{loops}}$$

⇒ $\delta\alpha$ absorbs corrections unrelated to α (from coupling g)
perturbatively unstable in THDM   Krause et al. 2016

Renormalization conditions based on $\Sigma_{12}(p^2)$ at some momentum transfer
Kanemura et al. 2004, '14, '15; Lopez-Val, Solá 2009; Krause et al. 2016; Jenniches et al. 2018

- process independent
- gauge dependent since not based on observable
- ad hoc, justification?

Rigid symmetry = symmetry under global $\mathrm{SU}(2)\times\mathrm{U}(1)$ transformations
theory renormalizable in unbroken phase: 't Hooft 1971; Lee, Zinn-Justin 1972-74

$$\alpha_0 = \alpha + \delta\alpha, \qquad \eta_0 = (Z^\eta)^{1/2}\eta$$

$$(Z^\eta)^{1/2} = \begin{pmatrix} (Z_1^\eta)^{1/2} & 0 \\ 0 & (Z_2^\eta)^{1/2} \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_1^\eta & 0 \\ 0 & 1+\frac{1}{2}\delta Z_2^\eta \end{pmatrix}$$

consistency with renormalization in complete on-shell scheme requires:

$$(Z^H)^{1/2}\big|_{\mathrm{UV}} = R^{\mathrm{T}}(\alpha+\delta\alpha)(Z^\eta)^{1/2}R(\alpha)\big|_{\mathrm{UV}}$$

$\Rightarrow$

$$\begin{aligned}
\delta Z_{11}^H\big|_{\mathrm{UV}} &= c_\alpha^2\delta Z_1^\eta\big|_{\mathrm{UV}} + s_\alpha^2\delta Z_2^\eta\big|_{\mathrm{UV}} \\
\delta Z_{22}^H\big|_{\mathrm{UV}} &= s_\alpha^2\delta Z_1^\eta\big|_{\mathrm{UV}} + c_\alpha^2\delta Z_2^\eta\big|_{\mathrm{UV}} \\
\delta Z_{12}^H\big|_{\mathrm{UV}} + \delta Z_{21}^H\big|_{\mathrm{UV}} &= 2c_\alpha s_\alpha(\delta Z_2^\eta - \delta Z_1^\eta)\big|_{\mathrm{UV}} \\
\delta Z_{12}^H\big|_{\mathrm{UV}} - \delta Z_{21}^H\big|_{\mathrm{UV}} &= 4\delta\alpha\big|_{\mathrm{UV}}
\end{aligned}$$

$\Rightarrow$ UV divergences of mixing angle are related to UV divergences of δZ_{ij}^H

Define Kanemura et al. 2004; Krause et al. 2016; Denner, Dittmaier, Lang 2018

$$\delta\alpha = \frac{1}{4}\left(\delta Z^H_{12} - \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\mathrm{H}_2}) + \Sigma^H_{12}(M^2_{\mathrm{H}_1})}{2(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})} \tag{1}$$

- symmetric, process independent
- if $\alpha \to 0$ protected by symmetry: $\delta\alpha \to 0$ for $\alpha \to 0$
- smooth limit for degenerate masses $\Rightarrow$ numerically stable
- generalizable to mixing angle β (mixing with unphysical fields)

$\delta\alpha$ appears in S-matrix elements only in the combinations

$-\delta\alpha + \frac{1}{2}\delta Z^H_{12}$,	$\delta\alpha + \frac{1}{2}\delta Z^H_{21}$	from field rotation $R(\alpha)$
$(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})\delta\alpha$		from parameter relations (λ_i)

with (1) $\Rightarrow$

$$\frac{1}{2}\delta Z^H_{12} - \delta\alpha = \delta\alpha + \frac{1}{2}\delta Z^H_{21} = \frac{1}{4}\left(\delta Z^H_{12} + \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\mathrm{H}_2}) - \Sigma^H_{12}(M^2_{\mathrm{H}_1})}{2(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})}$$

$$\delta\alpha = \frac{1}{4}\left(\delta Z^H_{12} - \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\mathrm{H}_2}) + \Sigma^H_{12}(M^2_{\mathrm{H}_1})}{2(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})} \quad (1)$$

gauge dependent via δZ^H_{ij}, $\Sigma^H_{12}(M^2_{\mathrm{H}_i})$

Solution: (compare Yamada 2001; Pilaftsis 2002; Kanemura et al. 2015)

- choose gauge to calculate $\delta\alpha$ and fix renormalization constant
- variation of gauge for fixed $\delta\alpha \Rightarrow$ gauge-independent S matrix (in FJTS)
- suitable gauge for $\delta\alpha$: 't Hooft–Feynman gauge ($\xi = 1$)
  $\Rightarrow$ no artificially large parameters

Argument holds also for renormalization of quark-mixing matrix of Denner, Sack 1990 in PRTS (not tadpoles in fermion vertices).

Background field method (BFM)

Origin: DeWitt 1967; Kluber-Stern, Zuber 1975; 't Hooft 1976
QCD: Abbott 1981, 1982; Abbot, Grisaru, Schaefer 1983
SM: Denner, Dittmaier, Weiglein 1994; Denner, Dittmaier 1996

- Split fields in Lagrangian into quantum Φ and background $\hat{\Phi}$ fields:
  $\mathcal{L}_{\text{class.}}(\Phi) \to \mathcal{L}_{\text{class.}}(\hat{\Phi} + \Phi)$
- add gauge-fixing term that maintains invariance under gauge transformation of background fields
- background-field effective action $\hat{\Gamma}[\hat{\Phi}] = \Gamma[\tilde{\Phi} = 0, \hat{\Phi}]$ invariant under gauge transformation of background fields
  $\Rightarrow$ ghost-free Ward identities for vertex functions, rigid invariance, simple relations between renormalization constants
  (e.g. $Z_e = Z_{AA}^{-1/2}$ as in QED)
  $\Rightarrow$ use relations to fix $\delta\alpha$, $\delta\beta$ ($\delta\alpha$ as from rigid invariance)
- S matrix can be constructed from $\hat{\Gamma}[\hat{\Phi}]$ and extra gauge fixing for $\hat{\Phi}$
- practical calculations almost as usual (modified Feynman rules):
  quantum fields Φ in loops, background fields $\hat{\Phi}$ on tree lines

Application to HSESM and THDM

Denner, Dittmaier, Lang 2018

Relations involving $\delta\beta$ in THDM

$$\delta Z_1^{\hat{\eta}} = -2\delta Z_e - \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\frac{\delta c_\mathrm{w}^2}{c_\mathrm{w}^2} + \frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + 2\frac{\delta c_\beta}{c_\beta} + \mathrm{tadpoles}$$

$$\delta Z_2^{\hat{\eta}} = -2\delta Z_e - \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\frac{\delta c_\mathrm{w}^2}{c_\mathrm{w}^2} + \frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + 2\frac{\delta s_\beta}{s_\beta} + \mathrm{tadpoles}$$

$\Rightarrow$ with above relations between δZ_i^{η} and δZ_{ij}^{H}

$$\delta\beta = \frac{1}{2}c_\beta s_\beta\left[(s_\alpha^2 - c_\alpha^2)\left(\delta Z_{11}^{\hat{H}} - \delta Z_{22}^{\hat{H}}\right) + 2s_\alpha c_\alpha\left(\delta Z_{12}^{\hat{H}} + \delta Z_{21}^{\hat{H}}\right)\right] + \mathrm{tadpoles}$$

similar results by Krause et al. 2016 using the pinch technique

- symmetric, process independent
- smooth limits for c_α, s_α, c_β, or $s_\beta \to 0$
- smooth limits for degenerate masses
- to be fixed in specific gauge (e.g. 't Hooft–Feynman gauge of BFM)

Idea: consider combinations of observables that depend exclusively on mixing angle (like for quark mixing matrix in Denner, Kraus, Roth 2004)

Example: LO matrix elements for $H_{1,2} \to ZZ$ in HSESM

$$\mathcal{M}_0^{H_1 \to \mathrm{ZZ}} = \frac{e s_\alpha}{s_\mathrm{w} c_\mathrm{w}^2} M_\mathrm{W} (\varepsilon_1^* \cdot \varepsilon_2^*) \qquad \mathcal{M}_0^{H_2 \to \mathrm{ZZ}} = \frac{e c_\alpha}{s_\mathrm{w} c_\mathrm{w}^2} M_\mathrm{W} (\varepsilon_1^* \cdot \varepsilon_2^*)$$

renormalization condition (on-shell, physical)

$$\frac{\mathcal{M}^{H_1 \to \mathrm{ZZ}}}{\mathcal{M}^{H_2 \to \mathrm{ZZ}}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1 \to \mathrm{ZZ}}}{\mathcal{M}_0^{H_2 \to \mathrm{ZZ}}} = \frac{s_\alpha}{c_\alpha} = f(\alpha) = \text{function of } \alpha \text{ only}$$

resulting renormalization constant

$$\delta\alpha = c_\alpha s_\alpha (\delta_{H_2 ZZ} - \delta_{H_1 ZZ}) + \frac{1}{2} c_\alpha s_\alpha (\delta Z_{22}^H - \delta Z_{11}^H) + \frac{1}{2} (\delta Z_{12}^H s_\alpha^2 - \delta Z_{21}^H c_\alpha^2)$$

$\delta_{H_i ZZ} = \delta_{H_i ZZ}(M_{\mathrm{H}_i}^2)$: unrenormalized relative corrections to decays

Properties

- gauge independent (based on S-matrix elements)
- symmetric with respect to H_1 and H_2
- numerically stable for degenerate masses $M_{\mathrm{H}_1} \sim M_{\mathrm{H}_2}$
- smooth limits for $c_\alpha \to 0$ or $s_\alpha \to 0$.
- restricted to processes with neutral external particles
  (charged particles $\Rightarrow$ IR singularities)
- requires S-matrix elements at unphysical phase-space points
  ($H_2(125) \to ZZ$ kinematically not possible)
  at one-loop analytic continuation to unphysical points available

Drawbacks can be lifted

- introduce extra neutral fields with simple coupling structure
- send extra couplings to zero

Add additional fermion singlet ψ to HSESM Denner, Dittmaier, Lang 2018

$$\mathcal{L}_\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - y_\psi \sigma \bar{\psi}\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - y_\psi (v_1 + H_1 c_\alpha - H_2 s_\alpha)\bar{\psi}\psi$$

and consider limit of vanishing Yukawa coupling y_ψ (singlet decouples).

renormalization condition

$$\frac{\mathcal{M}^{H_1\to\psi\psi}}{\mathcal{M}^{H_2\to\psi\psi}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to\psi\psi}}{\mathcal{M}_0^{H_2\to\psi\psi}} \propto -\frac{c_\alpha}{s_\alpha}$$

renormalization constant

$$\delta\alpha = \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22}) c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12} c_\alpha^2 - \delta Z^H_{21} s_\alpha^2)$$

Vertex corrections vanish for $y_\psi \to 0$ owing to simple structure of model.

$y_\psi \to 0 \quad \Rightarrow \quad m_\psi \to 0 \quad \Rightarrow$ decays kinematically allowed

Add two right-handed fermion singlets to THDM Denner, Dittmaier, Lang 2018

$\mathbb{Z}_2$ symmetry: $\nu_{1\mathrm{R}} \to -\nu_{1\mathrm{R}}, \nu_{2\mathrm{R}} \to \nu_{2\mathrm{R}}$

$$\begin{aligned}\mathcal{L}_{\nu_\mathrm{R}} &= \mathrm{i}\bar{\nu}_{1\mathrm{R}}\partial\!\!\!/\nu_{1\mathrm{R}} + \mathrm{i}\bar{\nu}_{2\mathrm{R}}\partial\!\!\!/\nu_{2\mathrm{R}} - \left[y_{\nu_1}\bar{L}_{1\mathrm{L}}(\mathrm{i}\sigma_2\Phi_1^*)\nu_{1\mathrm{R}} + y_{\nu_2}\bar{L}_{2\mathrm{L}}(\mathrm{i}\sigma_2\Phi_2^*)\nu_{2\mathrm{R}} + \mathrm{h.c.}\right] \\ &= -\frac{1}{\sqrt{2}}\left[y_{\nu_1}\bar{\nu}_{1\mathrm{L}}\nu_{1\mathrm{R}}(vc_\beta + H_1c_\alpha - H_2s_\alpha + \mathrm{i}A_0s_\beta - \mathrm{i}G_0c_\beta) + \mathrm{h.c.}\right] \\ &\quad -\frac{1}{\sqrt{2}}\left[y_{\nu_2}\bar{\nu}_{2\mathrm{L}}\nu_{2\mathrm{R}}(vs_\beta + H_1s_\alpha + H_2c_\alpha - \mathrm{i}A_0c_\beta - \mathrm{i}G_0s_\beta) + \mathrm{h.c.}\right] + \ldots\end{aligned}$$

and consider limit $y_{\nu_i} \to 0$.

renormalization condition for α (alternative using ν_2)

$$\frac{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_2\to\nu_1\bar{\nu}_1}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}_0^{H_2\to\nu_1\bar{\nu}_1}} \propto -\frac{c_\alpha}{s_\alpha}$$

renormalization constant

$$\delta\alpha = (\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{H_2\nu_1\bar{\nu}_1})c_\alpha s_\alpha + \frac{1}{2}(\delta Z_{11}^H - \delta Z_{22}^H)c_\alpha s_\alpha + \frac{1}{2}(\delta Z_{12}^H c_\alpha^2 - \delta Z_{21}^H s_\alpha^2)$$

Renormalization condition for β

$$\frac{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_1\to\nu_2\bar{\nu}_2}}\,\frac{\mathcal{M}^{H_2\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_2\to\nu_2\bar{\nu}_2}}\left(\frac{\mathcal{M}^{A_0\to\nu_2\bar{\nu}_2}}{\mathcal{M}^{A_0\to\nu_1\bar{\nu}_1}}\right)^2 \overset{!}{=} -\frac{c_\beta^2}{s_\beta^2}$$

renormalization constant

$$\delta\beta = \frac{1}{2}c_\beta s_\beta\left(\delta_{H_1\nu_1\bar{\nu}_1}+\delta_{H_2\nu_1\bar{\nu}_1}-2\delta_{A_0\nu_1\bar{\nu}_1}-\delta_{H_1\nu_2\bar{\nu}_2}-\delta_{H_2\nu_2\bar{\nu}_2}+2\delta_{A_0\nu_2\bar{\nu}_2}\right)$$
$$-\frac{c_\beta s_\beta}{4c_\alpha s_\alpha}(\delta Z^H_{12}+\delta Z^H_{21})+\frac{1}{2}\delta Z_{G_0A_0},$$

potentially singular for $s_\alpha\to 0$ or $c_\alpha\to 0$
alternative renormalization condition based on form factors $\Rightarrow$

$$\delta\beta = \frac{1}{2}c_\beta s_\beta\left[(c_\alpha^2-s_\alpha^2)(\delta Z^H_{11}-\delta Z^H_{22})-2c_\alpha s_\alpha(\delta Z^H_{12}+\delta Z^H_{21})\right]+\frac{1}{2}\delta Z_{G_0A_0}$$
$$+c_\beta s_\beta\left(\delta_{A_0\nu_2\bar{\nu}_2}+c_\alpha^2\delta_{H_1\nu_1\bar{\nu}_1}+s_\alpha^2\delta_{H_2\nu_1\bar{\nu}_1}-\delta_{A_0\nu_1\bar{\nu}_1}-s_\alpha^2\delta_{H_1\nu_2\bar{\nu}_2}-c_\alpha^2\delta_{H_2\nu_2\bar{\nu}_2}\right)$$

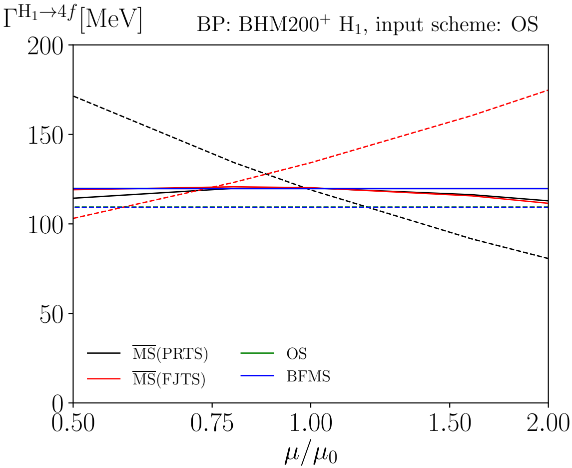
non-singular in all limits $s_\alpha\to 0$, $c_\alpha\to 0$, $s_\beta\to 0$, $c_\beta\to 0$

Outline

Renormalization scale dependence of the decay width $\Gamma^{\mathrm{H}_1\to 4f}$

(heavy HSESM Higgs boson H_1) Denner, Dittmaier, Lang 2018

LO dashed, NLO full lines, parameters converted between schemes



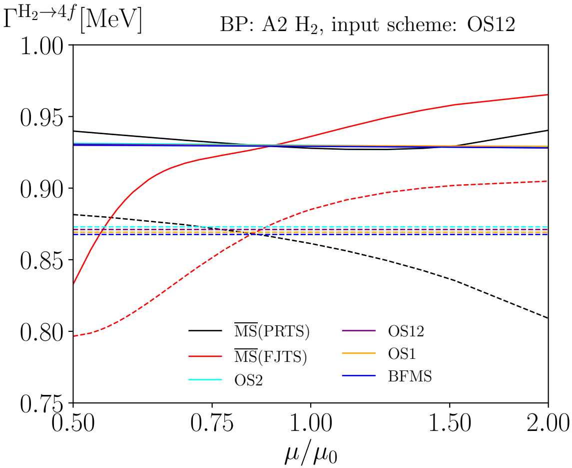
LO $\to$ NLO

- $\overline{\mathrm{MS}}$ schemes: reduction of ren. scale dependence
- reduction of ren. scheme dependence
- uncertainty at NLO $\lesssim 1\%$

Renormalization scale dependence of the decay width $\Gamma^{\mathrm{H}_2\to 4f}$

(light THDM Higgs boson H_2) Denner, Dittmaier, Lang 2018

LO dashed, NLO full lines, parameters converted between schemes



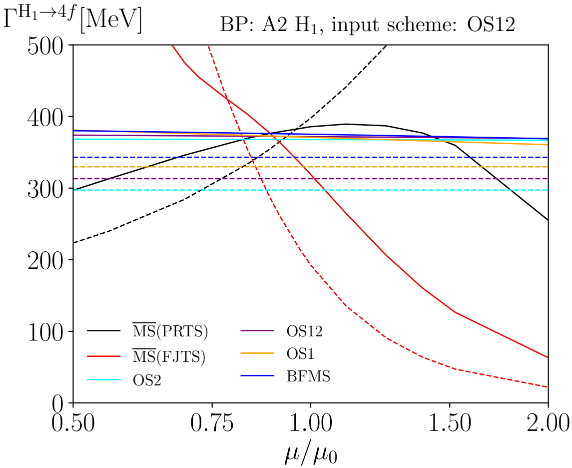
LO → NLO

- $\overline{\mathrm{MS}}$ schemes:
  PRTS: 2% scale dependence
  FJTS: sizeable scale dependence at NLO
- on-shell and BFM schemes agree perfectly
- base uncertainty on scheme dependence of well behaved schemes

Renormalization scale dependence of the decay width $\Gamma^{\mathrm{H}_1\to 4f}$

(heavy THDM Higgs boson H_1) Denner, Dittmaier, Lang 2018

LO dashed, NLO full lines, parameters converted between schemes



LO $\to$ NLO

- $\overline{\mathrm{MS}}$ schemes: no reduction of scale dependence at NLO $\Rightarrow$ schemes useless
- on-shell and BFM schemes agree perfectly
- base uncertainty on scheme dependence of well behaved schemes
- 20% NLO correction

Outline

Renormalization of mixing angles in extended Higgs models

- $\overline{\mathrm{MS}}$ renormalization
  - simple, symmetric, generally applicable
  - perturbatively unstable, in particular for degenerate masses
  - requires careful treatment of tadpoles for gauge independence (FJTS)
  - allows to study scale dependence
- on-shell renormalization from one observable
  - gauge independent, violates symmetries, perturbatively unstable
- New: use rigid invariance/background field gauge invariance
  - simple, symmetric, generally applicable, stable
  - requires careful treatment of tadpoles (FJTS)
  - must fix gauge to define renormalization constants, best generic choice
- New: on-shell renormalization from combinations of observables
  - gauge independent, stable, symmetric
  - model dependent, require spurious fields, best choice if available

Schemes implemented in 2HDECAY Krause, Mühlleitner, Spira 2018 and PROPHECY4F Denner, Dittmaier, Mück in preparation

estimate theoretical uncertainty by comparing stable schemes!

Outline

HSESM scenarios

Scenario	M_{H_1} [GeV]	s_α	$\lambda_3/2$
BHM200$^\pm$	200	± 0.29	± 0.07
BHM400	400	0.26	0.17
BHM600	600	0.22	0.23

Considered processes:

- decays of light and heavy scalar Higgs bosons: $\mathrm{H}_{1,2} \to 4f$
- production of light and heavy scalar Higgs bosons in association with a vector boson: $\mathrm{pp} \to \mathrm{H}_{1,2}\mu^+\nu_\mu$

HSESM input parameters defined in on-shell scheme (OS) for $\delta\alpha$ converted to other schemes with NLO accuracy for $\mu = M_{\mathrm{H}_2} = 125\,\mathrm{GeV}$

Scheme	BHM200$^+$		BHM200$^-$		BHM400		BHM600	
	s_α	$\frac{\lambda_3}{2}$	s_α	$\frac{\lambda_3}{2}$	s_α	$\frac{\lambda_3}{2}$	s_α	$\frac{\lambda_3}{2}$
OS	0.29	0.07	-0.29	-0.07	0.26	0.17	0.22	0.23
$\overline{\mathrm{MS}}$ (PRTS)	0.302	0.073	-0.304	-0.073	0.267	0.175	0.226	0.236
$\overline{\mathrm{MS}}$ (FJTS)	0.321	0.077	-0.316	-0.076	0.264	0.173	0.212	0.222
BFMS	0.290	0.070	-0.290	-0.070	0.258	0.172	0.218	0.237

parameter conversion between schemes 1 and 2 based on

$$p_{0,i} = p_i^{(1)} + \delta p_i^{(1)}(\{p_j^{(1)}\}) = p_i^{(2)} + \delta p_i^{(2)}(\{p_j^{(2)}\}),$$

Decay width $\Gamma^{H_1 \to 4f}$ [MeV] of the heavy HSESM Higgs boson H_1

Scheme	BHM200$^+$ LO	BHM200$^+$ NLO	BHM200$^-$ LO	BHM200$^-$ NLO
$\overline{\text{MS}}$ (PRTS)	$118.976(4)^{-31.9\%}_{+43.6\%}$	$120.240(7)^{-5.9\%}_{-4.7\%}$	$120.397(4)^{-32.4\%}_{+44.6\%}$	$120.114(7)^{-5.7\%}_{-5.6\%}$
$\overline{\text{MS}}$ (FJTS)	$134.126(4)^{+30.1\%}_{-23.2\%}$	$120.403(9)^{-7.1\%}_{-1.0\%}$	$129.570(4)^{+5.6\%}_{-6.0\%}$	$120.132(8)^{-0.1\%}_{+0.6\%}$
OS	$109.430(4)$	$119.847(8)$	$109.430(4)$	$119.812(8)$
BFMS	$109.393(4)$	$119.846(8)$	$109.419(4)$	$119.811(8)$

Scheme	BHM400 LO	BHM400 NLO	BHM600 LO	BHM600 NLO
$\overline{\text{MS}}$ (PRTS)	$1617.26(4)^{-6.2\%}_{+6.3\%}$	$1648.62(8)^{-0.6\%}_{+0.6\%}$	$4530.1(1)^{-2.3\%}_{+2.1\%}$	$4546.0(2)^{-0.4\%}_{+0.6\%}$
$\overline{\text{MS}}$ (FJTS)	$1582.44(4)^{+27.6\%}_{-21.7\%}$	$1646.83(8)^{-1.5\%}_{-3.6\%}$	$4007.1(1)^{+32.5\%}_{-24.8\%}$	$4509.4(3)^{-0.3\%}_{-6.0\%}$
OS	$1533.42(4)$	$1643.86(8)$	$4295.9(1)$	$4532.4(2)$
BFMS	$1505.02(4)$	$1636.86(9)$	$4226.6(1)$	$4493.8(2)$

THDM scenarios

Scenario	M_{H_1} [GeV]	$M_{\mathrm{H}+}, M_{A_0}$ [GeV]	λ_5	$\tan\beta$	$c_{\alpha\beta}$
A1	300	460	-1.9	2	0.1
A2	300	460	-1.9	2	0.2
B1	600	690	-1.9	4.5	0.15
B2	200	420	-2.5746	3	0.3

$c_{\alpha\beta} = \cos(\alpha - \beta)$

alignment limit $c_{\alpha\beta} = 0$:
couplings of H_2 to gauge bosons and fermions as in SM
couplings of H_1 to gauge bosons and fermions vanish

THDM parameter conversion

THDM input parameters defined in on-shell scheme (OS12) for $\delta\alpha$ and $\delta\beta$ converted to other schemes with NLO accuracy for $\mu = M_{\mathrm{H}_2} = 125\,\mathrm{GeV}$

Scheme	A1		A2		B1		B2	
	$c_{\alpha\beta}$	$\tan\beta$	$c_{\alpha\beta}$	$\tan\beta$	$c_{\alpha\beta}$	$\tan\beta$	$c_{\alpha\beta}$	$\tan\beta$
OS12	0.1	2.0	0.2	2.0	0.15	4.5	0.3	3.0
$\overline{\mathrm{MS}}$ (PRTS)	0.137	1.90	0.225	1.91	0.153	4.40	0.176	2.83
$\overline{\mathrm{MS}}$ (FJTS)	0.090	1.93	0.157	1.91	0.061	3.82	-0.031	3.70
BFMS	0.110	1.91	0.209	1.92	0.149	4.42	0.323	2.82

λ_5 renormalized in $\overline{\mathrm{MS}}$ enters via running

Decay width $\Gamma^{H_1\to 4f}$[MeV] of the light THDM Higgs boson H_2

Scheme	A1 LO	A1 NLO	A2 LO	A2 NLO
$\overline{\text{MS}}$ (PRTS)	$0.89035(3)^{-2.8\%}_{+0.9\%}$	$0.96107(7)^{+1.2\%}_{+0.4\%}$	$0.86130(3)^{-6.1\%}_{+2.3\%}$	$0.92784(7)^{+1.3\%}_{+1.3\%}$
$\overline{\text{MS}}$ (FJTS)	$0.89996(3)^{+0.7\%}_{-7.4\%}$	$0.96286(7)^{+0.8\%}_{-0.2\%}$	$0.88508(3)^{+2.2\%}_{-10.0\%}$	$0.93605(7)^{+3.1\%}_{-11.0\%}$
OS12	$0.89832(3)$	$0.96197(7)^{-0.1\%}_{+0.1\%}$	$0.87110(3)$	$0.92947(7)^{-0.2\%}_{+0.1\%}$
BFMS	$0.89647(3)$	$0.96177(7)^{-0.1\%}_{+0.1\%}$	$0.86764(3)$	$0.92914(7)^{-0.1\%}_{+0.1\%}$

Scheme	B1 LO	B1 NLO	B2 LO	B2 NLO
$\overline{\text{MS}}$ (PRTS)	$0.88609(3)^{-4.9\%}_{+0.4\%}$	$0.94053(7)^{+1.1\%}_{+1.0\%}$	$0.87941(3)^{-42.7\%}_{+2.6\%}$	$0.924184(7)^{-39.2\%}_{+4.2\%}$
$\overline{\text{MS}}$ (FJTS)	$0.90406(3)^{+0.4\%}_{+0.4\%}$	$0.96073(7)^{+1.5\%}_{-1.1\%}$	$0.90654(3)^{-87.0\%}_{-2.5\%}$	$0.96980(7)^{>+100\%}_{-0.0\%}$
OS12	$0.88698(3)$	$0.94074(7)^{-0.5\%}_{+0.2\%}$	$0.82573(3)$	$0.87189(6)^{-0.5\%}_{+0.3\%}$
BFMS	$0.88721(3)$	$0.94113(8)^{-0.2\%}_{+0.1\%}$	$0.81262(3)$	$0.86741(6)^{+0.1\%}_{+0.0\%}$

Decay width $\Gamma^{\mathrm{H}_1\to 4f}$ [MeV] of the heavy HSESM Higgs boson H_1

Scheme	A1 LO	A1 NLO	A2 LO	A2 NLO
$\overline{\mathrm{MS}}$ (PRTS)	$147.102(4)^{>+100\%}_{-47.8\%}$	$104.86(2)^{<-100\%}_{-24.1\%}$	$397.75(1)^{>+100\%}_{-43.8\%}$	$385.72(2)^{-33.9\%}_{-23.0\%}$
$\overline{\mathrm{MS}}$ (FJTS)	$64.096(2)^{-86.9\%}_{>+100\%}$	$92.17(1)^{-81.4\%}_{+5.6\%}$	$192.524(5)^{-88.6\%}_{>+100\%}$	$318.95(5)^{-80.2\%}_{>+100\%}$
OS12	$78.304(2)$	$98.812(8)^{-0.8\%}_{+0.7\%}$	$313.217(8)$	$371.86(3)^{-0.7\%}_{+0.6\%}$
BFMS	$94.265(2)$	$100.117(5)^{-2.2\%}_{+1.6\%}$	$343.049(9)$	$375.22(2)^{-1.7\%}_{+1.3\%}$

Scheme	B1 LO	B1 NLO	B2 LO	B2 NLO
$\overline{\mathrm{MS}}$ (PRTS)	$2083.68(5)^{>+100\%}_{-17.9\%}$	$2162.3(1)^{<-100\%}_{-27.0\%}$	$40.122(1)^{>+100\%}_{-82.7\%}$	$57.73(1)^{>+100\%}_{-82.7\%}$
$\overline{\mathrm{MS}}$ (FJTS)	$325.923(7)^{-99.6\%}_{-100\%}$	$1179.8(3)^{-99.5\%}_{-99.8\%}$	$1.21460(4)^{>+100\%}_{>+100\%}$	$-5.226(3)^{<-100\%}_{>+100\%}$
OS12	$1997.07(5)$	$2155.2(1)^{-1.4\%}_{+0.7\%}$	$117.108(4)$	$131.88(1)^{-2.9\%}_{+2.2\%}$
BFMS	$1973.68(5)$	$2145.0(1)^{-9.5\%}_{+3.6\%}$	$135.904(5)$	$138.729(8)^{-7.7\%}_{+4.7\%}$