Heavy-quark form factors

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in collaboration with

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Outline

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Introduction

Considering form factors at three-loop order for the process

$$X \to Q + \overline{Q}$$

coupling through one of the vertices

$$\{1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$$

here:

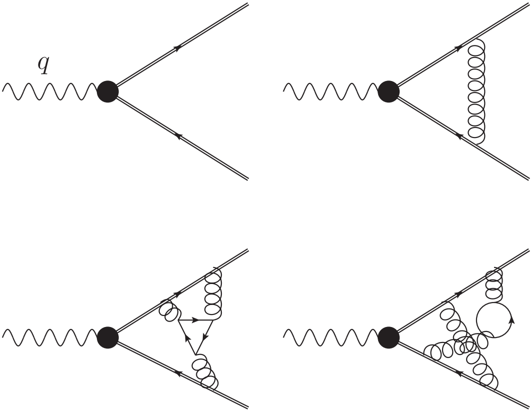
- only non-singlet contributions, i.e. the heavy-quark pair couples directly to the external current.
- at least one heavy-quark loop

Motivation

- heavy quark production
  - continuum production $e^+e^- \to t\bar{t}$
- particle decays
  - $H \to b\bar{b}$
  - $Z \to b\bar{b}$
  - $A \to t\bar{t}$
- technology development

Example: Vector case

$$\bar{\Psi}\Gamma^{\mu}_{V}\Psi = -i\bar{\Psi}\left(\gamma^{\mu} F_{V,1} + \frac{i}{2m}\sigma^{\mu\nu} q_{\nu} F_{V,2}\right)\Psi$$



History / Previous works

- two loop

[Bernreuther,Bonciani,Gehrmann,Heinesch,Leineweber,Mastrolia,Remiddi '05]

[Gluza,Mitov,Moch,Riemann '09]

[Ablinger,Behring,Blümlein,Falcioni,De Freitas,PM,Rana,Schneider '18]

- three loop
  - light-fermionic contributions (HPLs)

    [Lee,Smirnov,Smirnov,Steinhauser'18]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]
  - color-planar contributions (HPLs + cyclotomic HPLs)

    [Henn,Smirnov,Smirnov,Steinhauser '17]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]
  - **NEW** heavy-fermionic contributions
- general infrared and high-energy structure [Ahmed,Henn,Steinhauser '17]

[Blümlein, PM, Rana '18]

[Penin et al '17]

Outline

Calculation

For the calculation of the form factors use the well-established multi-loop toolbox

- ✓ QGRAF for the generation of the diagrams
- ✓ use projectors to obtain scalar integrals
- ✓ FORM for the algebra
- ✓ use integration-by-parts identities [Chetyrkin,Tkachov]
  to reduce to an integral basis using Crusher [Seidel,PM]
  14 families, 104 master integral
- ??? calculate the required master integrals
- ✓ put everything together and renormalize
- ✓ final result still IR divergent – compare with predictions

Calculation / Strategy

Calculation of master integrals problematic since the heavy-fermionic and non-planar contributions contain structures beyond harmonic polylogarithms

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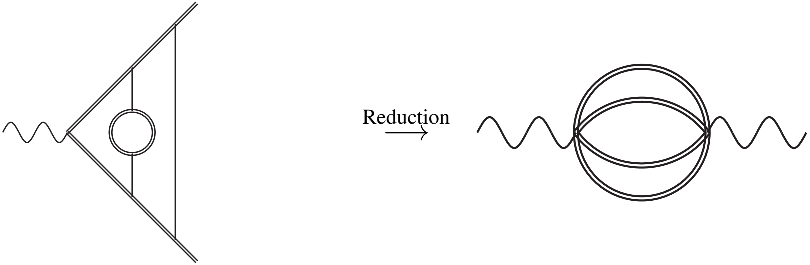

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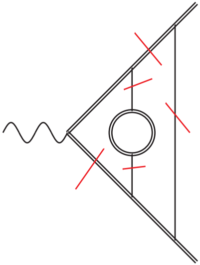

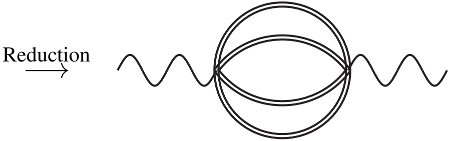

Calculation / Strategy

- Strategy: Sum simpler then the individual parts!
- turn everything into recurrences by considering the expansion around $q^2 = 0$
- try to derive a recurrence for the whole form factor and find a analytic solution for that [Blümlein,Schneider '17]

Method

- choose a more appropriate variable

$$\frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

$q^2 \to \pm\infty \equiv x \to 0_{\mp}$
$q^2 \to 0 \equiv x \to 1$

- around $q^2 = 0$, i.e. $x = 1$ the non-singlet form factors can be expanded in a simple power series

$$\mathcal{F} = \sum_{n=0} C_n \left(\frac{q^2}{m^2}\right)^n \quad \Leftrightarrow \quad \mathcal{F} = \sum_{n=0} D_n (1-x)^n = \sum_{n=0} D_n y^n$$

Method

- start from the coupled system of diff. eqn. for the master integrals
- insert the power series ansatz

$$\mathcal{M}_i = \sum_{j=0} M_j^{(i)} y^j$$

  and obtain recurrences for the coefficients $M_j^{(i)}$
- calculate 2,000 - 8,000 terms in the expansion for the master integrals
- use these to obtain 2,000 - 8,000 terms in the expansions of the full form factors
- as initial condition we need the values at $x = 1$,
  i.e. on-shell propagators [Melnikov,v.Ritbergen]

Method

- the final expansion for the form factors has the form

$$\mathcal{F} = 1(\ldots) + \zeta_2(\ldots) + \zeta_3(\ldots) + \ln(2)(\ldots) + \mathrm{Li}_4(\tfrac{1}{2})(\ldots) + \cdots$$

  where $(\ldots)$ denote power series in y with rational coefficients
- this representation is unique
- can we do better?
  - Guess a recurrence [Kauers,Jaroschek,Johansson '15]
  - and try to solve it using `Sigma` [Schneider '07]
- if recurrence can be solved, i.e. first-order factorizing, one obtains (generalized) harmonic sums, which can be resummed using `HarmonicSums` [Ablinger '13]

Example

- start with the sequence for C_i in $\sum C_i y^i$

$$-2, 0, -\frac{1}{6}, -\frac{1}{6}, -\frac{3}{20}, -\frac{2}{15}, -\frac{5}{42}, -\frac{3}{28}, -\frac{7}{72}, -\frac{4}{45}, -\frac{9}{110}, -\frac{5}{66}, -\frac{11}{156}, -\frac{6}{91}, -\frac{13}{210}, -\frac{7}{120}, -\frac{15}{272}, -\frac{8}{153}, -\frac{17}{342}, -\frac{9}{190}, -\frac{19}{420}, \ldots$$

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  $n^2 C_n - (n-1)(n+2)C_{n+1} = 0$

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- guess recurrence
  $n^2 C_n - (n-1)(n+2)C_{n+1} = 0$
- solve the recurrence
  $C_n = -\frac{n-1}{n(n+1)}$
- sum it
  $-2 - \sum_{n=1}^{\infty} \frac{n-1}{n(n+1)} y^n = -\frac{(y-2)\log(1-y)}{y} \overset{y \to 1-x}{=} \frac{(1+x)\log(x)}{1-x}$

more details, see: talk by C. Schneider, Thursday morning

Outline

Results

We could find analytic results for all terms but for n_h $n_h\zeta_2$ $n_h\zeta_3$

		degree	order	remaining order
F_V	$g_1 n_h$	1288	54	15
	$g_1 n_h\zeta_3$	409	29	10
	$g_1 n_h\zeta_2$	295	24	6
	$g_2 n_h$	1324	55	15
	$g_2 n_h\zeta_3$	430	30	10
	$g_2 n_h\zeta_2$	273	23	6
F_S	n_h	1114	50	15
	$n_h\zeta_3$	350	27	10
	$n_h\zeta_2$	230	22	6

For leading color we could also solve the term $\propto N_c^2 n_h\zeta_2$

Results – Scalar form factor

$$
\begin{aligned}
F_S \;=\; & -\frac{1}{\varepsilon^3}\frac{1}{2(1+x)^2}\Bigg\{ n_h^2\left[-\frac{64}{27}(1+x)^2+\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right] \\
& +n_h\left[\frac{4}{27}\left(997+1418x+997x^2\right)-\frac{32H_0P_8^{(5)}}{27\left(1-x^2\right)}\right] \\
& -n_l\left[\frac{32}{9}(1+x)^2-\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right]+\frac{256\left(1+x^2\right)^2}{27(1-x)^2}H_0^2\Bigg]\Bigg\}
\end{aligned}
$$

Results – Scalar form factor cont'd

$$
-\frac{1}{\varepsilon^2}\frac{1}{2(1+x)^2}\Bigg\{n_h^2\Bigg[-\frac{832}{81}(1+x)^2-\frac{256x(1+x)H_0}{27(1-x)}-\frac{128(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0
+\frac{32(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{128(1+x)(1+x^2)}{27(1-x)}H_{0,-1}-\frac{64(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]
+n_h\Bigg[\frac{16}{27}(897+1786x+897x^2)+n_l\Bigg[-\frac{64}{3}(1+x)^2+\frac{64(1+x)(5-24x+5x^2)}{81(1-x)}H_0
-\frac{256(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0+\frac{64(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{256(1+x)(1+x^2)}{27(1-x)}H_{0,-1}
-\frac{128(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]+\left(\frac{128H_{-1}P_7^{(5)}}{27(1-x^2)}-\frac{16P_{13}^{(5)}}{27(1-x^2)}\right)H_0+\Bigg(\frac{64P_{26}^{(5)}}{27(1-x)^2(1+x)}
-\frac{1024(1+x^2)^2}{27(1-x)^2}H_{-1}\Bigg)H_0^2-\frac{128(-2+x^2)(1+x^2)}{27(1-x)^2}H_0^3-\frac{128(1+x)(1+x^2)}{3(1-x)}H_0H_1
+\left(\frac{128(1+x)(1+x^2)}{3(1-x)}-\frac{128(1+x^2)^2}{3(1-x)^2}H_0\right)H_{0,1}-\left(\frac{128P_7^{(5)}}{27(1-x^2)}-\frac{2176(1+x^2)^2}{27(1-x)^2}H_0\right)
\times H_{0,-1}+\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,1}-\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,-1}+\Bigg(\frac{64P_5^{(5)}}{27(1-x^2)}
-\frac{64(1+x^2)(-1+35x^2)}{27(1-x)^2}H_0\Bigg)\zeta_2-\frac{64(1+x^2)^2}{3(1-x)^2}\zeta_3\Bigg]\Bigg\}
$$

Results – Scalar form factor – unsolved recurrences

$$F_S = \ldots + n_h F_{S,1}^{(0)}(x) + n_h \zeta_2 F_{S,2}^{(0)}(x) + n_h \zeta_3 F_{S,3}^{(0)}(x)$$

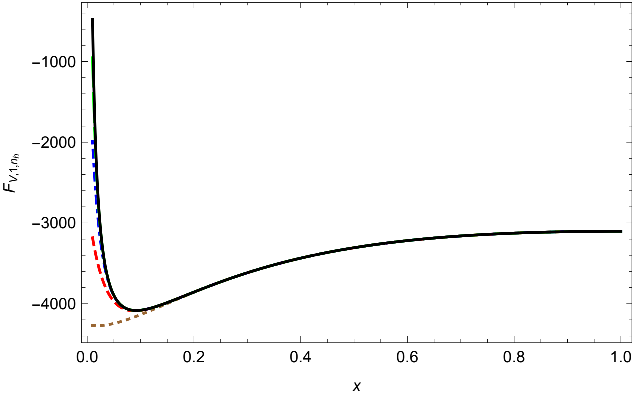
$$F_{S,1}^{(0)}(x) = -\frac{874750}{243} - \frac{731018}{729}y^2 - \frac{731018}{729}y^3 - \frac{316061833}{437400}y^4 - \frac{96756433}{218700}y^5 + \cdots + O(y^{2001})$$

$$F_{S,2}^{(0)}(x) = \frac{343864}{81} + \frac{2421832}{3645}y^2 + \frac{2421832}{3645}y^3 + \frac{16041283}{36450}y^4 + \frac{3932123}{18225}y^5 + \cdots + O(y^{2001})$$

$$F_{S,3}^{(0)}(x) = \frac{62968}{27} - \frac{22516}{81}y^2 - \frac{22516}{81}y^3 - \frac{21262303}{97200}y^4 - \frac{7752703}{48600}y^5 + \cdots + O(y^{2001}).$$

Results – High-Energy region

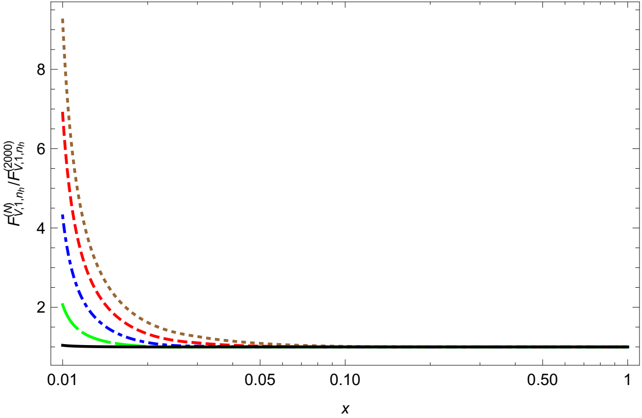
Expansion about $x = 1$, log singularity for $x \to 0$



shown: 20, 50, 100, 200, 500 terms in the expansion

Results – High-Energy region

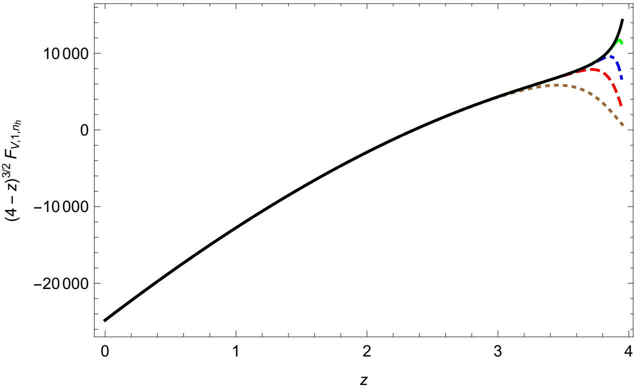
Ratios with respect to the result obtained with 2,000 terms



shown: 20, 50, 100, 200, 500 terms in the expansion

Results – Threshold region

Expansion about $z = q^2/m^2 = 0$, square-root singularity for $z \to 4$



shown: 20, 50, 100, 200, 500 terms in the expansion

Conclusions

- Calculated the heavy-fermionic corrections to the heavy-quark form factors in an expansion about $q^2 = 0$
- Many parts can be resummed and are available analytically
- For some parts only recurrences and thus deep expansions exist
- Results for pole terms agree with predictions
- ToDo: full color
- ToDo: singlet contributions
- ToDo: find a solution for the non-first-order factorizing recurrences