Non-leptonic B-decays at two loops in QCD Factorisation

Tobias Huber
Universität Siegen

TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

G. Bell, TH	1410.2804 (JHEP)
G. Bell, M. Beneke, X.-Q. Li, TH	1507.03700 (PLB)
G. Bell, M. Beneke, X.-Q. Li, TH	in preparation, results preliminary

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Outline

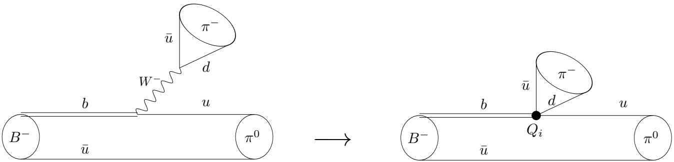
Introduction / Motivation

- Non-leptonic B decays, structure of amplitude

$$\mathcal{A}(\bar{B} \to f) = \lambda_u^{(D)} A_f^u + \lambda_c^{(D)} A_f^c = \sum_i \left[\lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED}\right]_i$$

- Interplay between
  - Wilson coefficients C of tree ($C \sim 1$) or penguin ($C \sim 0.1$) operator
  - CKM factors $\lambda_p^{(D)} = V_{pb} V_{pD}^*$. Hierarchy of CKM elements, weak phase
  - Hadronic matrix elements $\langle f|\mathcal{O}|\bar{B}\rangle$. Contain strong phases.
- Interplay offers rich and interesting phenomenology for non-leptonic decays
- Plethora of data, numerous observables:
  Branching ratios, CP asymmetries, polarisations, Dalitz distributions, . . .
- Test of CKM mechanism and indirect search for New Physics
- Challenging QCD dynamics, effects from many different scales !!

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark
- Effective Hamiltonian: [Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]

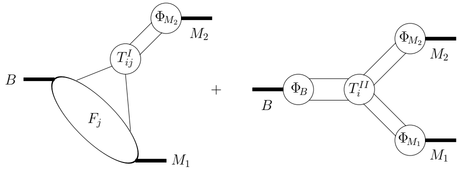
$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b \, \bar{d}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

QCD factorisation



- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ [Beneke,Buchalla,Neubert,Sachrajda'99-'04]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \;\; T_i^I(u) \, \phi_{M_2}(u) \;\; + \;\; (M_1 \leftrightarrow M_2)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^\infty d\omega \int_0^1 dv \, du \;\; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

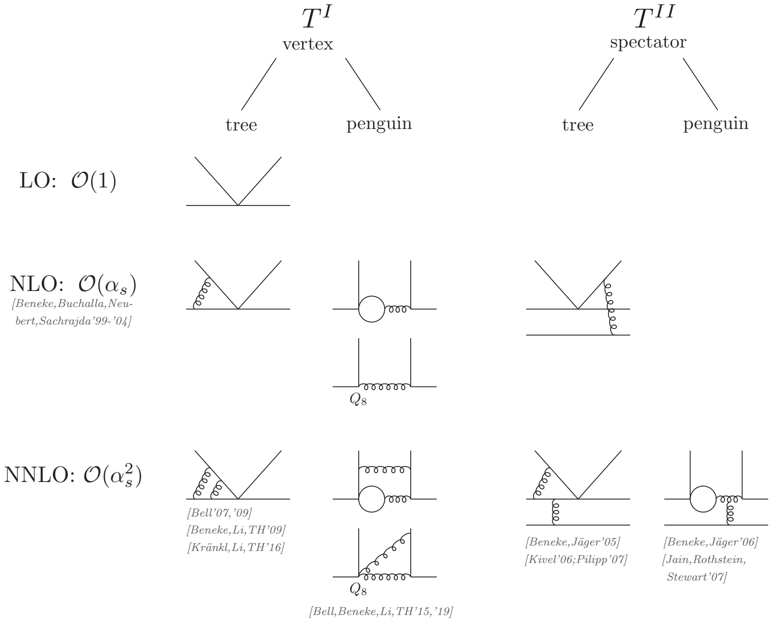
- $T^{I,II}$: Hard scattering kernels, perturbatively calculable

- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

  } Universal. From Sum Rules, Lattice

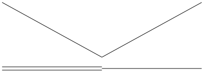
- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

Anatomy of QCD factorisation

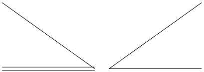


Classification of amplitudes

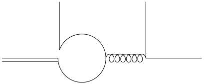
- α_1 : colour-allowed tree amplitude



- α_2 : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$: QCD penguin amplitudes



$$
\begin{aligned}
\sqrt{2}\,\langle \pi^- \pi^0 | \mathcal{H}_{eff}\, | B^- \rangle &= A_{\pi\pi}\; \lambda_u \Big[\alpha_1(\pi\pi) + \alpha_2(\pi\pi)\Big] \\
\langle \pi^+ \pi^- | \mathcal{H}_{eff}\, | \bar{B}^0 \rangle &= A_{\pi\pi}\; \Big\{\lambda_u \Big[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)\Big] + \lambda_c\, \alpha_4^c(\pi\pi)\Big\} \\
-\, \langle \pi^0 \pi^0 | \mathcal{H}_{eff}\, | \bar{B}^0 \rangle &= A_{\pi\pi}\; \Big\{\lambda_u \Big[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)\Big] - \lambda_c\, \alpha_4^c(\pi\pi)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\langle \pi^- \bar{K}^0 | \mathcal{H}_{eff}\, | B^- \rangle &= A_{\pi\bar{K}}\; \Big[\lambda_u^{(s)}\; \alpha_4^u + \lambda_c^{(s)}\; \alpha_4^c\Big] \\
\langle \pi^+ K^- | \mathcal{H}_{eff}\, | \bar{B}^0 \rangle &= A_{\pi\bar{K}} \Big[\lambda_u^{(s)}\, (\alpha_1 + \alpha_4^u) + \lambda_c^{(s)}\, \alpha_4^c\Big]
\end{aligned}
$$

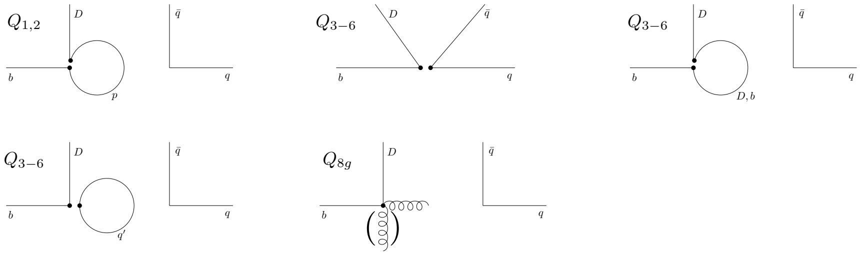
[Beneke,Neubert'03]

- Tree amplitudes α_1 and α_2 known analytically to NNLO

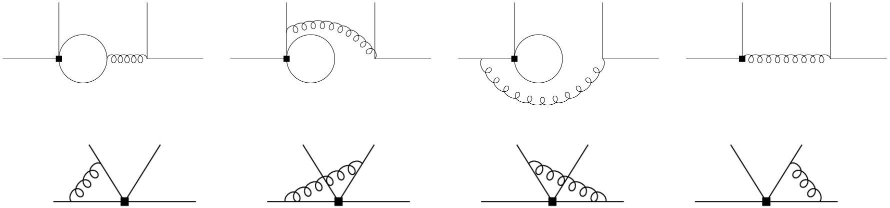
[Bell'07'09; Beneke,Li,TH'09]

Penguin amplitudes

- Various insertions of operators contribute

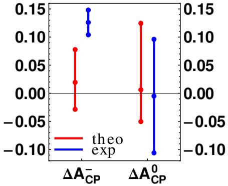


- NLO:



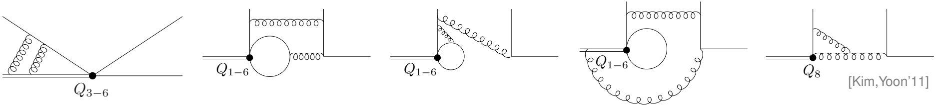
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$
  - Large (scale) uncertainties
  - NNLO is only first perturbative correction
  - NNLO is NLO for direct CP asymmetries!

- Few tensions persist in two-body charmless non-leptonic decays
  - $\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0)$
  - $K\pi$ puzzle



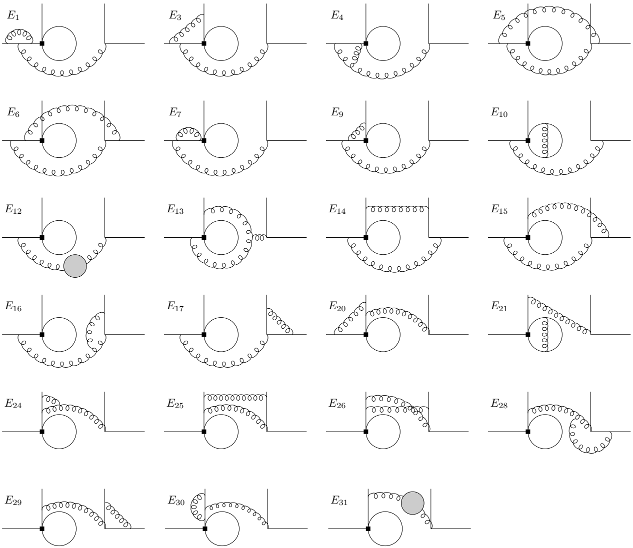
[Hofer,Vernazza'12]

- Sample diagrams at NNLO.



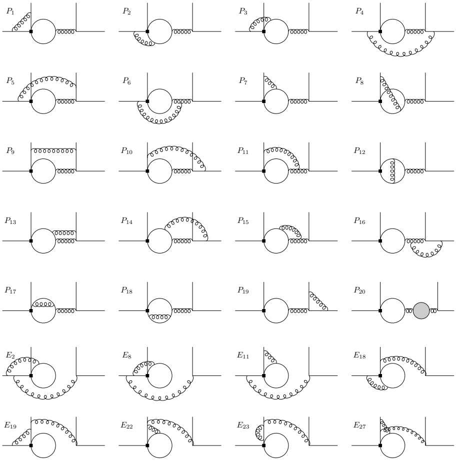
[Kim,Yoon'11]

- Observation: Sum of following diagrams vanishes for all insertions



Penguin amplitudes at two loops

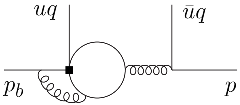
- Contributing two-loop penguin diagrams



- Plus: Q_{3-6} insertion into tree topology (66 diagrams, known)
  and one-loop diagrams involving Q_8 (18 diagrams, simple)

Kinematics

- Kinematics:



$$p^2 = q^2 = 0$$
$$p_b^2 = m_b^2$$

- Fermion loop with either $m = 0$, $m = m_c$ or $m = m_b$.
- Genuine two-scale problem: $\bar{u}$, $z_c \equiv m_c^2/m_b^2$
- Threshold at $\bar{u} = 4m_c^2/m_b^2$
- Choice of suitable kinematic variables crucial

$$s = \sqrt{1-4z_c/\bar{u}}\,,\; r = \sqrt{1-4z_c} \quad \longleftrightarrow \quad \bar{u}\,,\; z_c = \frac{m_c^2}{m_b^2} \quad \longleftrightarrow \quad s_1 = \sqrt{1-4/\bar{u}}\,,\; r$$

$$\updownarrow$$

$$p = \frac{1-\sqrt{u^2+4\bar{u}z_c}}{\bar{u}}\,,\; r$$

Two-loop calculation

- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- Reduction: IBP relations, Laporta algorithm in FIRE

[Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Smirnov'08]

  - Obtain a set of 36 master integrals

- Use differential equations in canonical form [Henn'13]

$$d\,\vec{M}(\epsilon, x_n) \;=\; \epsilon\;\, d\tilde{A}(x_n)\;\, \vec{M}(\epsilon, x_n)$$

- Found canonical basis for all masters, including boundary conditions [Bell,TH'14]
  - First example of canonical basis in case of 2 different internal masses
  - Analytic solution in terms of iterated integrals (GPLs) over alphabet

$$\left\{0\,,\; \pm 1\,,\; \pm 3\,,\; \pm i\sqrt{3}\,,\; \pm r\,,\; \pm\frac{r^2+1}{2}\,,\; \pm(1+2\sqrt{z_c})\,,\; \pm(1-2\sqrt{z_c})\right\}$$

  - Catalyses analytic convolution with LCDA

Matching and hard scattering kernels

[Bell,Beneke,Li,TH, in prep.]

- To obtain finite hard scattering kernel T
  - Renormalisation of UV divergencies
  - Subtraction of IR divergencies via matching onto SCET
    - Subtlety: Evanescent operators both on QCD and SCET side.
    - Physical SCET operator: $O_1 = \bar{\chi} \frac{\not{n}_-}{2}(1-\gamma_5)\chi \, \bar{\xi}\not{n}_+(1-\gamma_5)h_v$
    - SCET operators of penguin contraction have different Fierz ordering

$$\tilde{O}_n = \bar{\xi}\, \gamma_\perp^\alpha \gamma_\perp^{\mu_1} \gamma_\perp^{\mu_2} \ldots \gamma_\perp^{\mu_{2n-2}} \chi \, \bar{\chi}(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}} \cdots \gamma_{\perp\mu_1} h_v$$

    - All but $\tilde{O}_1$ are evanescent. In $D=4$, have $\tilde{O}_1 = O_1/2$. Treat $\tilde{O}_1 - O_1/2$ as additional evanescent operator.
- Tree-level and one-loop master formula

$$\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)}$$

$$\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{21}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{\sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(1)}}_{\mathcal{O}(\epsilon)}$$

Matching and hard scattering kernels

[Bell,Beneke,Li,TH, in prep.]

- Two-loop master formula

$$
\begin{aligned}
\widetilde{T}_i^{(2)} \quad = \quad & \widetilde{A}_{i1}^{(2)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\text{nf}} \\
& + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}'^{(1)\text{nf}} + Z_{ext}^{(1)}\,\left[\widetilde{A}_{i1}^{(1)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\right] \\
& - \widetilde{T}_i^{(1)}\left[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\right] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\text{f}} - A_{21}^{(2)\text{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}'^{(1)\text{f}} - A_{21}'^{(1)\text{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)})\,[\widetilde{A}_{i1}^{(1)\text{f}} - A_{21}^{(1)\text{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}
\end{aligned}
$$

- All poles in ϵ cancel analytically, also checked numerically.
- Obtain hard scattering kernel completely analytically.

Convolution with LCDA

- Penguin amplitude requires convolution $\int_0^1 du \; T_i(u)\, \phi_M(u)$
  - Expand LCDA of light meson in Gegenbauer polynomials

$$\phi_M(u) = 6u\bar{u}\left[1 + \sum_{n=1}^{\infty} a_n^M\, C_n^{(3/2)}(2u-1)\right]$$

  - Re-write in terms of $s = \sqrt{1 - 4z_c/\bar{u}}$ and $r = \sqrt{1 - 4z_c}$
  - Integrate from $s = r \ldots + i\infty$
  - Upper integration limit requires argument inversion in GPLs, e.g.

$$\lim_{s\to i\infty} H_{+,r-}(s) = -2H_{+,(1/r)-}(-i) + 2H_{+,r-}(i) - i\pi H_{(1/r)-}(-i) + i\pi H_{r-}(i) - 8iC$$

- Use GiNaC for numerical evaluation of GPLs up to weight five.

[Bauer,Frink,Kreckel'02; Vollinga,Weinzierl'04]

- Obtain a_4^u completely analytically
- In a_4^c a few terms known only as interpolation in z_c.

Results

[Bell,Beneke,Li,TH, in prep.]

- Penguin amplitudes through to NNLO — All numbers and plots preliminary

$$\begin{aligned}
a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\rm V_1} + [0.49-1.32i]_{\rm P_1} - [0.32+0.71i]_{\rm P_2,\,Q_{1,2}} + [0.33+0.38i]_{\rm P_2,\,Q_{3-6,8}} \\
&\quad + \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14+0.12i]_{\rm HV} - [0.01-0.05i]_{\rm HP} + [0.07]_{\rm tw3}\Big\} \\
&= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,
\end{aligned}$$

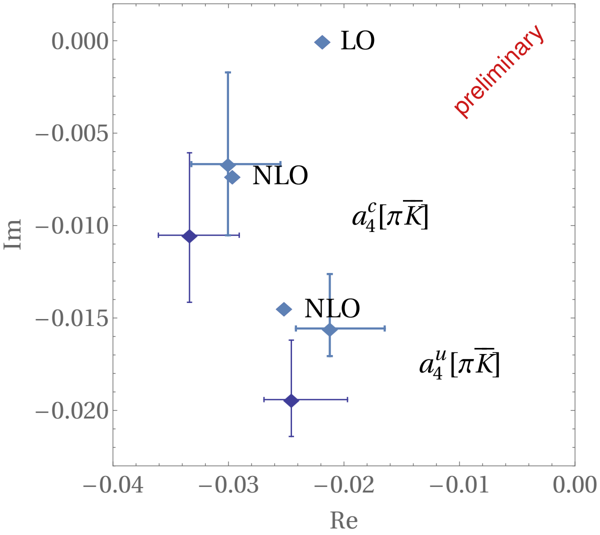
$$\begin{aligned}
a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\rm V_1} + [0.05-0.62i]_{\rm P_1} - [0.77+0.50i]_{\rm P_2,\,Q_{1,2}} + [0.33+0.38i]_{\rm P_2,\,Q_{3-6,8}} \\
&\quad + \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14+0.12i]_{\rm HV} + [0.01+0.03i]_{\rm HP} + [0.07]_{\rm tw3}\Big\} \\
&= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.
\end{aligned}$$

- NNLO corrections from QCD penguin and chromomagnetic dipole operator tend to cancel those of current-current operators

$$r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)} \qquad \lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)$$

- Penguin amplitudes: Anatomy of corrections

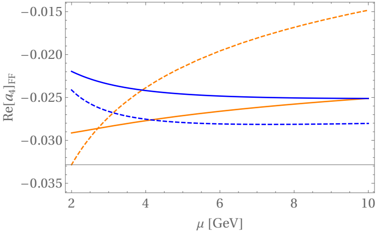
[Bell,Beneke,Li,TH, in prep.]

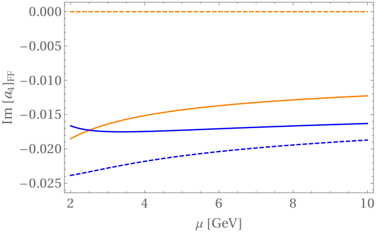


Results

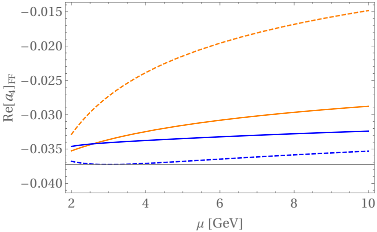
- Penguin amplitudes: Scale dependence. [Bell,Beneke,Li,TH, in prep.]
  - Only form factor term, no spectator scattering. preliminary

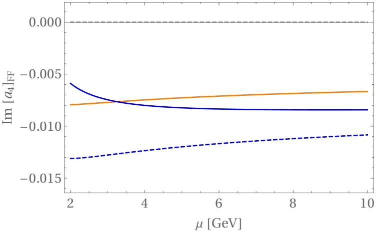
- a_4^u





- a_4^c





- Orange: LO (dashed) and NLO (solid) Blue: NNLO$_{Q_{1,2}}$ (d) and NNLO$_{all}$ (s)

Conclusion and Outlook

- We computed the QCD penguin amplitudes a_4^u and a_4^c in QCD factorisation through to NNLO
  - Genuine two-loop two-scale problem
  - Matching from QCD onto SCET is involved ($\to$ evansecent operators)
- Obtained hard scattering kernel completely analytically as iterated integrals over generalised weights
- NNLO shift in amplitudes is rather small
- Future plans, e.g.
  - Phenomenology based on NNLO results:
    Branching ratios, direct CP asymmetries, . . .
  - Treatment of power corrections in QCDF framework

Backup slides

Some definitions

$$A_{\pi\pi} = i\,\frac{G_F}{\sqrt{2}}\,m_B^2 F_+^{B\to\pi}(0) f_\pi$$

$$r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)}$$

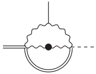
$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)$$

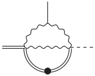
$$\Delta A_{\rm CP}^{-}(\pi K) = A_{\rm CP}(B^- \to \pi^0 K^-) - A_{\rm CP}(\bar{B}^0 \to \pi^+ K^-) = \Delta A_{\rm CP}(\pi K)$$

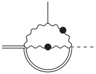
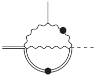
$$\Delta A_{\rm CP}^{0}(\pi K) = A_{\rm CP}(B^- \to \pi^- \bar{K}^0) - A_{\rm CP}(\bar{B}^0 \to \pi^0 \bar{K}^0)$$

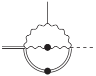
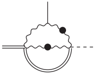
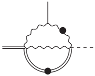
$$\hat{\alpha}_4^p(M_1 M_2) = a_4^p(M_1 M_2) \pm r_\chi^{M_2} a_6^p(M_1 M_2) + \beta_3^p(M_1 M_2)$$

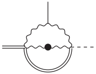
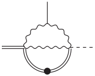
Canonical basis for master integrals I

$\frac{M_{18}}{u\epsilon^3} =$ 

$\frac{M_{19}}{u\epsilon^3} =$ 

$-\frac{2\,M_{20}}{u\bar{u}s\epsilon^2} =$  $+$ 

$\frac{M_{21}}{\epsilon^2} = \frac{2[(1+\bar{u})^2 z_c - \bar{u}^2]}{\bar{u}}$  $- \bar{u}s^2(1+\bar{u})\Big[$  $+$  $\Big]$

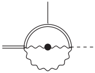
$+\frac{2\epsilon u}{m_b^2}\Big[$  $+$  $\Big]$

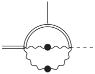
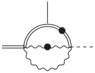
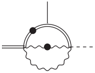
- Differential equation (sample)

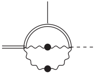
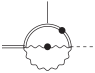
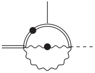
$$\frac{dM_{19}}{ds} = \frac{4\epsilon M_{18}\, r\,\left(r^2+1\right)}{((r^2+1)^2-4s^2)} - \frac{2\epsilon M_{19}\, r\,\left(r^2+s^2-2\right)}{(1-r^2)(r^2-s^2)} + \frac{4\epsilon M_{20}\, r\, s}{\left((r^2+1)^2-4s^2\right)} - \frac{\epsilon M_{21}\, r\,\left(r^2+1\right)}{((r^2+1)^2-4s^2)}$$

- Boundary conditions
  - M_{18} and M_{19} vanish in $s = r$ (i.e. in $u = 0$)
  - M_{20} and M_{21} vanish in $s = +i\infty$ (i.e. in $u = 1$)

Canonical basis for master integrals II

$$\frac{M_{23}}{u\epsilon^3} =$$ 

$$\frac{M_{24}}{\epsilon^2} = \frac{2(1+s_1)\sqrt{1+\frac{8z_c(1-s_1)}{(1+s_1)^2}}}{1-s_1}\Bigg[$$  $+\,2$  $-\frac{2(1+s_1)}{1-s_1}$  $\Bigg]$

$$\frac{M_{25}}{\epsilon^2} = \frac{2(1-s_1)\sqrt{1+\frac{8z_c(1+s_1)}{(1-s_1)^2}}}{1+s_1}\Bigg[$$  $+\,2$  $-\frac{2(1-s_1)}{1+s_1}$  $\Bigg]$

- Differential equation

$$\frac{dM_{23}}{ds_1} = \frac{2\,\epsilon\,M_{23}\,s_1\left(5-s_1^2\right)}{\left(1-s_1^2\right)\,\left(3+s_1^2\right)} - \frac{\epsilon\,M_{24}\,(3-s_1)}{4\left(1-s_1^2\right)\sqrt{1+\frac{8\,z_c(1-s_1)}{(1+s_1)^2}}} + \frac{\epsilon\,M_{25}\,(3+s_1)}{4\left(1-s_1^2\right)\sqrt{1+\frac{8\,z_c(1+s_1)}{(1-s_1)^2}}}$$

- Variable transformation to rationalize irrational factors:

$$t = \frac{1-s_1}{2} + \frac{1+s_1}{2}\sqrt{1+\frac{2(1-r^2)(1-s_1)}{(1+s_1)^2}} \qquad v = \frac{1+s_1}{2} + \frac{1-s_1}{2}\sqrt{1+\frac{2(1-r^2)(1+s_1)}{(1-s_1)^2}}$$