Di-Higgs production in bottom quark annihilation at NNLO QCD

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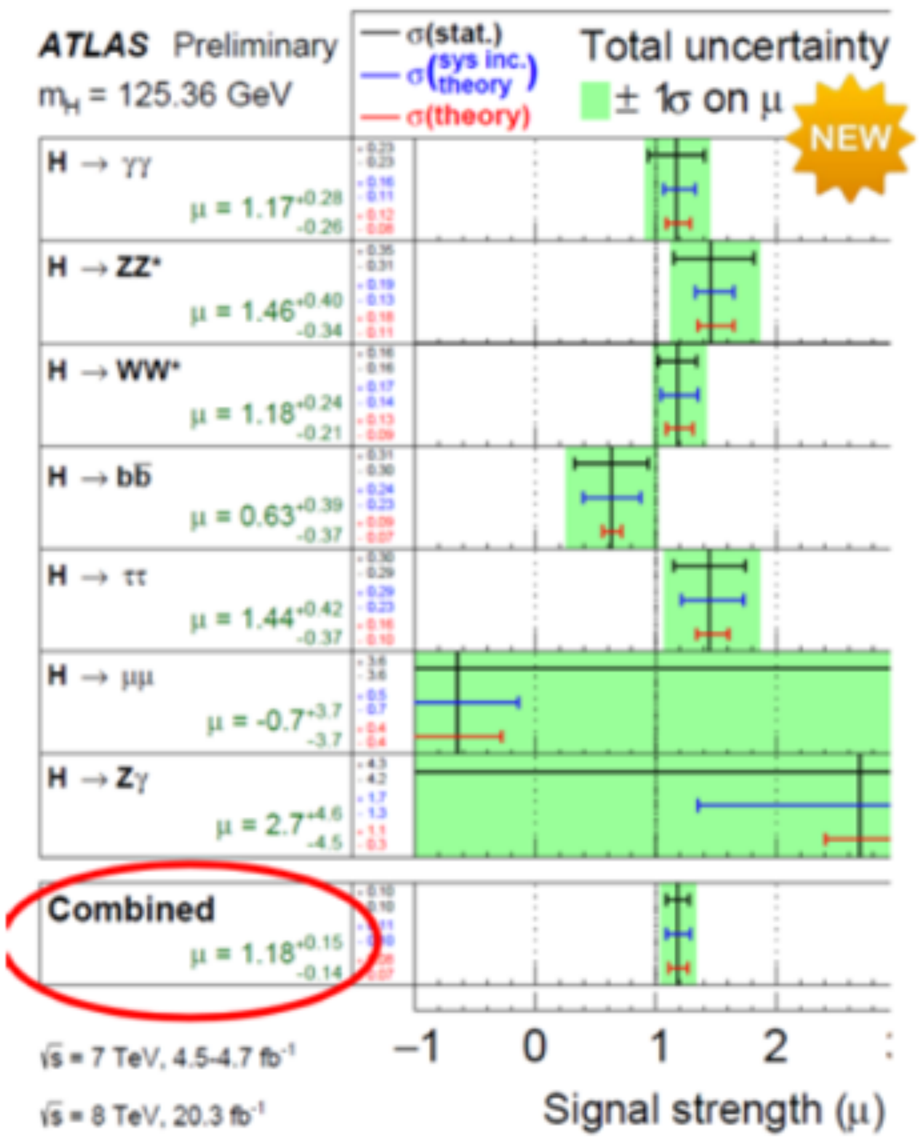
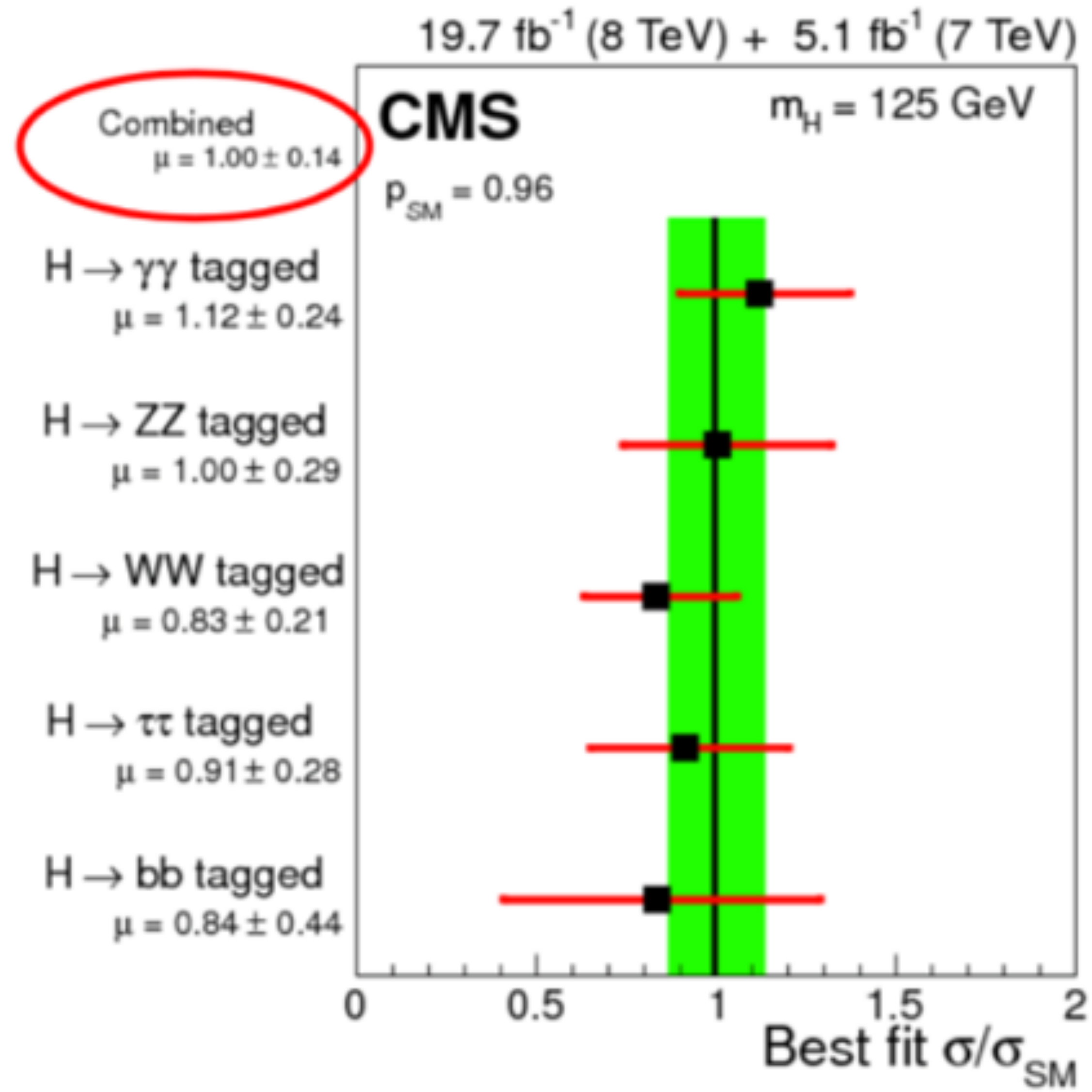
Plan of my talk

- Introduction
- Class-A and Class-B diagrams
- Two loop amplitudes
- Real emissions in the soft limit
- Numerical predictions at NNLO
- Conclusions

Higgs Discovery at the LHC





$$\mu_{ATLAS} = 1.18^{+0.15}_{-0.14}$$

$$\mu_{CMS} = 1.00 \pm 0.14$$

$$\text{stat.} = {}^{+0.10}_{-0.10}$$

$$\text{theory} = {}^{+0.08}_{-0.07}$$

$$\text{sys. (inc. theo.)} = {}^{+0.11}_{-0.10}$$

[M. Dührssen @ Moriond EW 2015]

Single Higgs boson Cross Section

NLO cross section

Dawson ('91), Djouadi, Spira, Zerwas ('91), Graudenz, Spira, Zerwas ('93) , Spira, Djouadi, Graudenz, Zerwas ('95)

NNLO cross section

Harlander ('00), Catani, de Florian, Grazzini ('01), Harlander, Kilgore ('01) Harlander, Kilgore ('02), Anastasiou, Melnikov ('02), Ravindran, Smith, Van Neerven ('03)

Resummation of soft gluons improves fixed order result.

Catani, de Florian, Grazzini, Nason ('03)

Leading soft contribution at NNNLO

Moch, Vogt ('05), Laenen, Magnea ('06), Idilbi, -d. Ji, -P. Ma, Yuan ('06) , Ravindran ('06)

NNNLO production cross section via gluon fusion !

Baikov, Chetyrkin, A.V. Smirnov, V.A. Smirnov, Steinhauser ('09) Gehrmann, Glover, Huber, Ikizlerli, Studerus ('10) Anastasiou, Duhr, Dulat, Mistlberger ('13) Anastasiou, Duhr, Dulat, Herzog, Mistlberger ('13) Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger ('14), Anastasiou, Duhr, Dulat, Herzog, Mistlberger ('15)

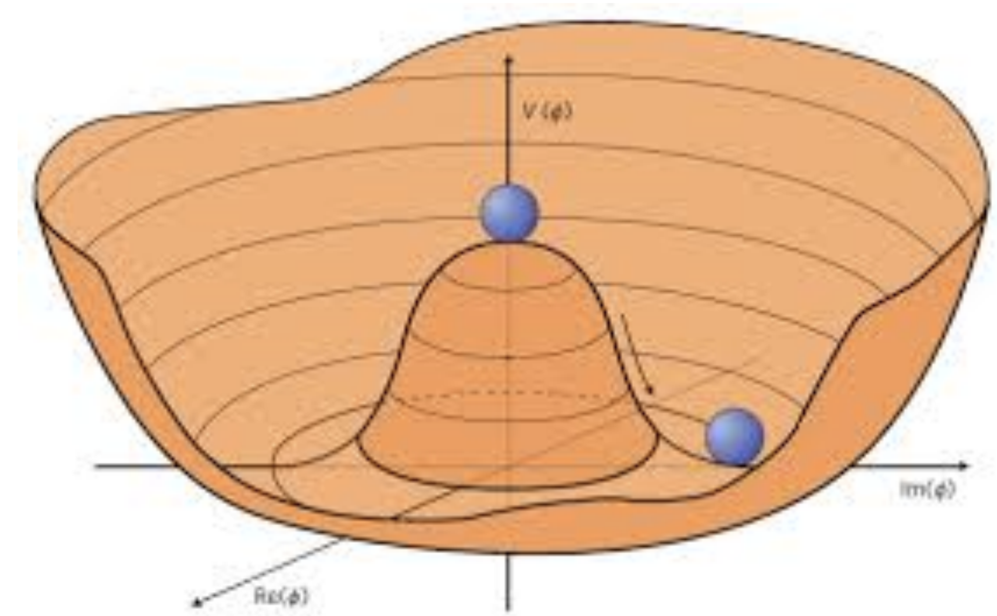
Differential cross section carry more information:

Bozzi, Catani, de Florian, Grazzini ('08), Bozzi, Catani, Ferrera, de Florian, Grazzini ('09) Catani, Grazini ('11), Catani, Grazini, Torre ('13), Grazzini, Kallweit, Rathlev, Wiesemann ('15), Monni, Re, Torrielli ('16), Ebert, Tackman ('17), Ferrera, Pires ('17)

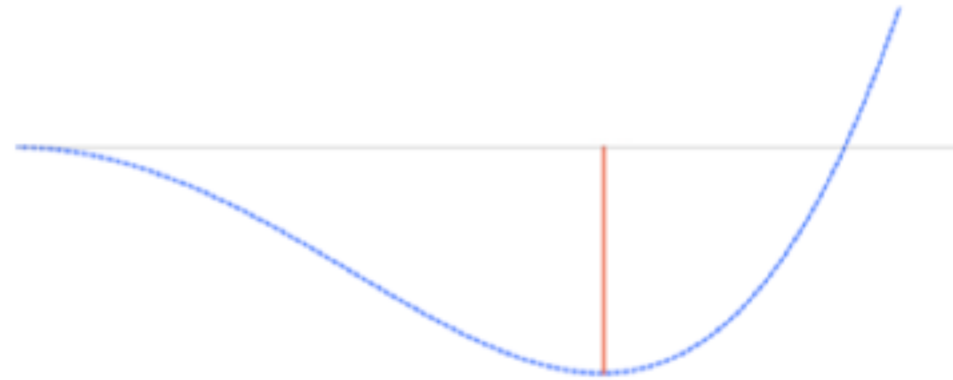
Ahmed, Mandal, Rana, Ravindran ('14), Dulat, Mistleberger, Pelloni ('17)

Di-Higgs Production

Higgs Potential



$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$



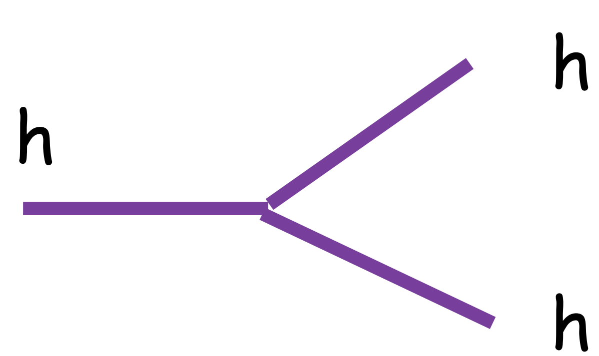
$$\frac{\mu}{\sqrt{\lambda}} \equiv v \quad 246\ \mathrm{GeV}$$

- *Shape of the Potential*

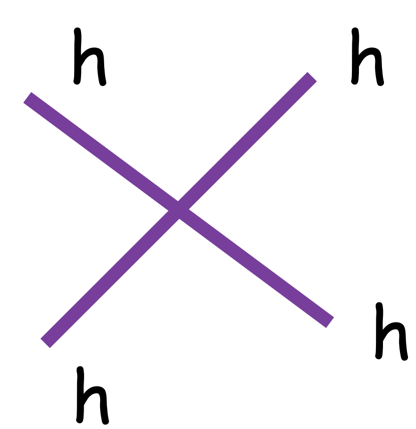
$$\mathcal{L} \supset -\frac{m_h^2}{2}\phi^2(x) - \lambda_3^{\mathrm{SM}} v\phi^3(x) - \lambda_4^{\mathrm{SM}}\phi^4(x),$$

- Test the *Predictions:*

$$\lambda_3^{\mathrm{SM}} = \frac{m_h^2}{2v^2}, \quad \lambda_4^{\mathrm{SM}} = \frac{m_h^2}{8v^2}$$

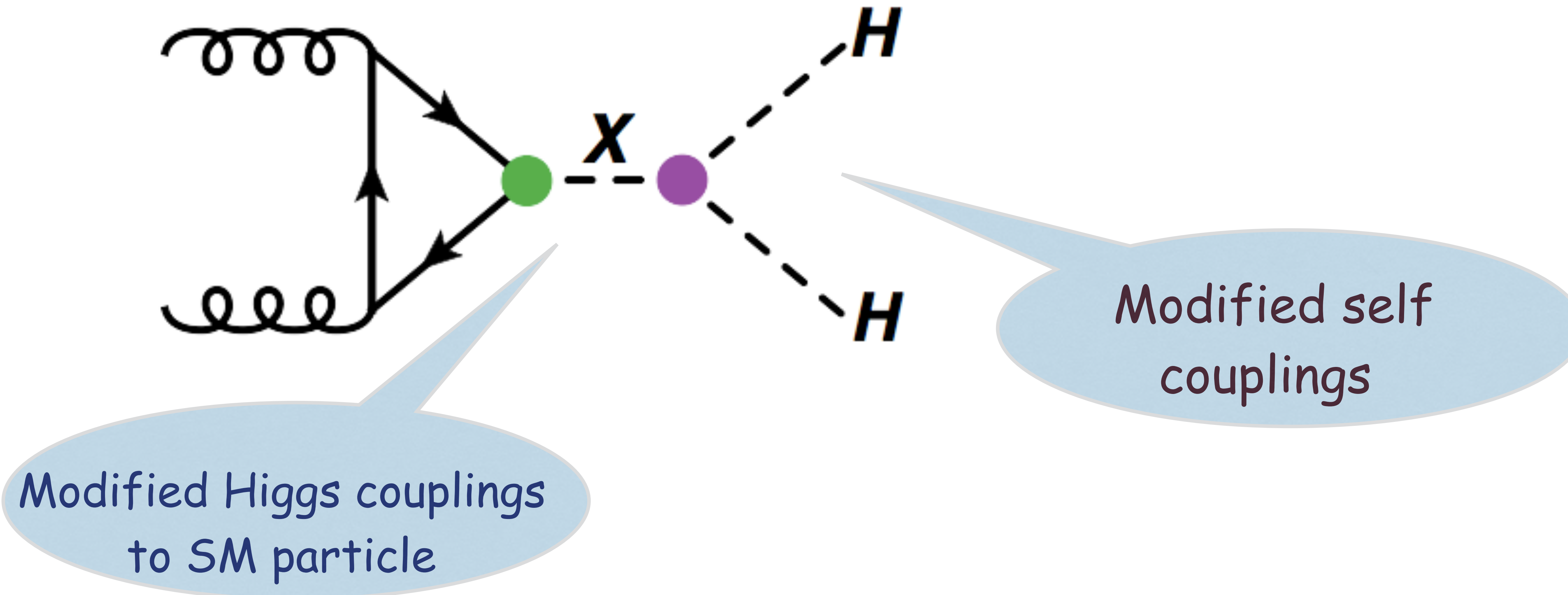


$\lambda_3^{SM} = 0.13$



$\lambda_4^{SM} = 0.03$

In BSM scenarios



- *Non-Resonant production*

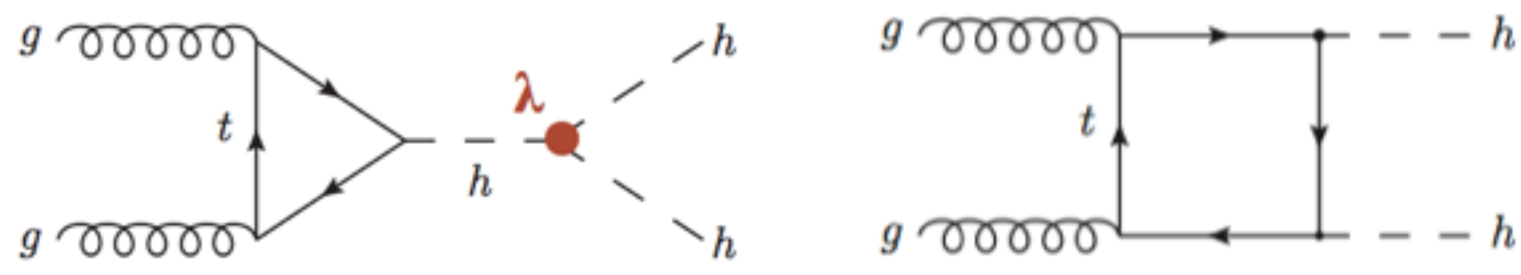
- *Resonant production:*

  Heavy scalars in Two Higgs doublet models,
  Spin-2 resonances from Randal Sundrum Model

Production Cross section

Glover,van der Bij

- Dominant ones:



- Relative Contributions

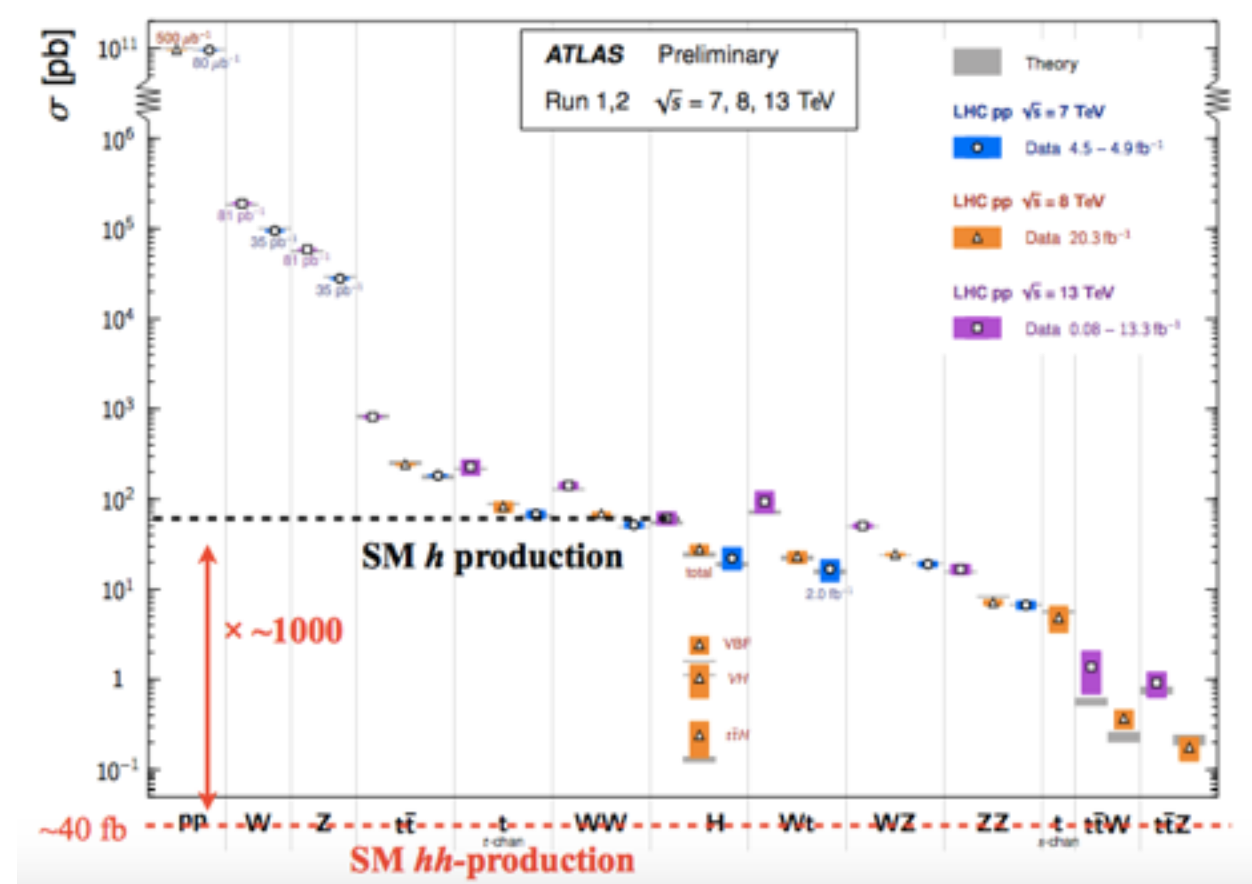
destructively interfere!

$\lambda_3^{SM} = 0.3$

ggF-*hh*	~ 40 fb
VBF-*hh*	~2 fb
V-*hh*	~ 1 fb
tt-*hh*	~ 1 fb

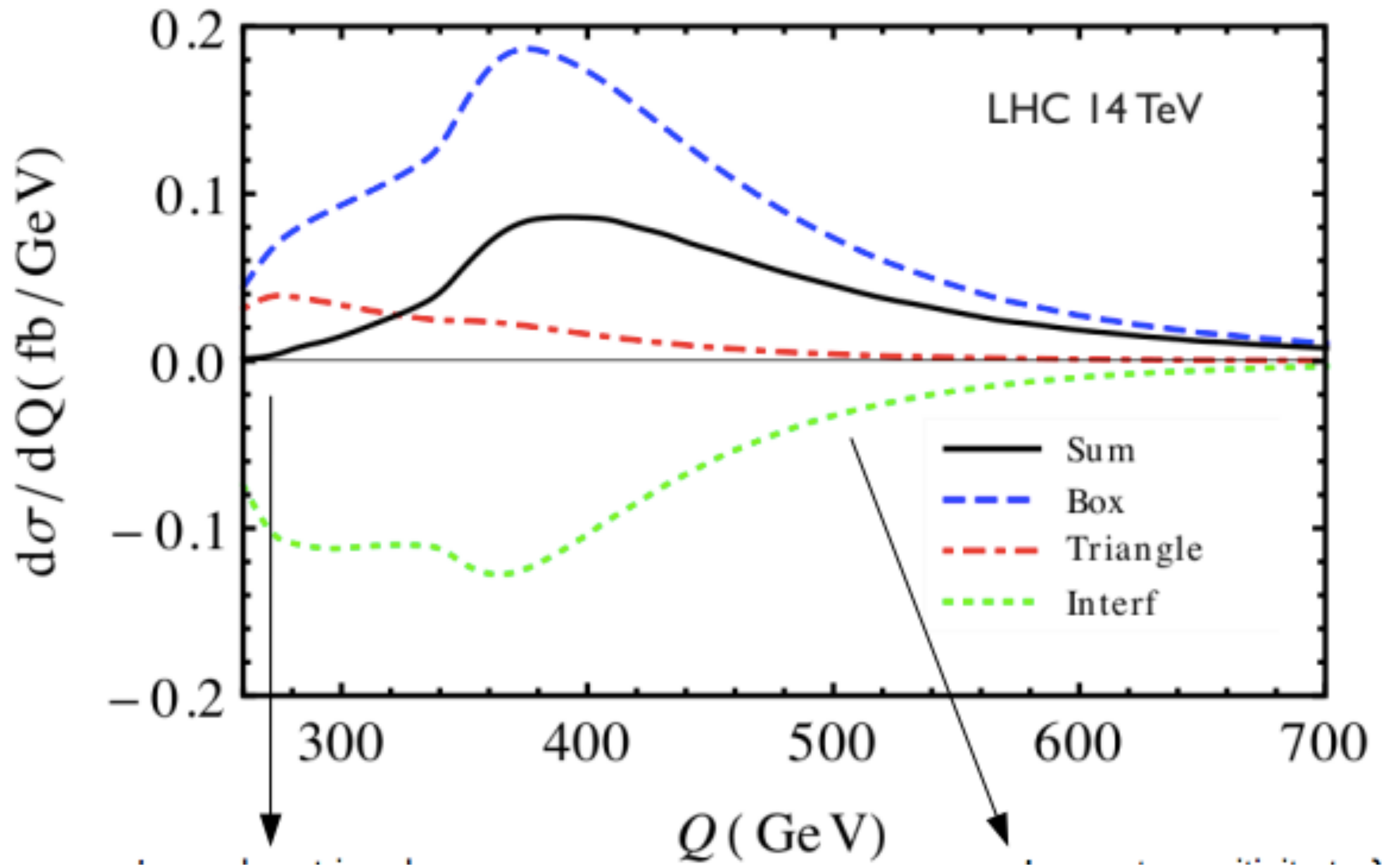
$$b + \bar{b} \to hh \quad \approx 0.1 fb$$

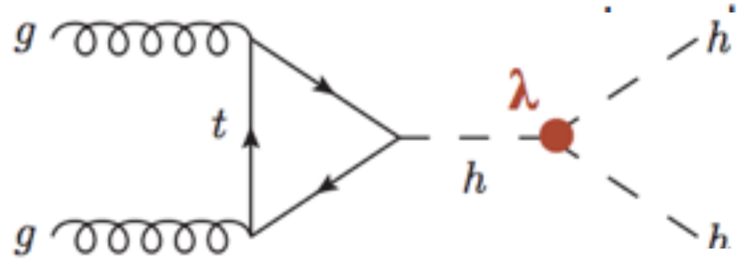
Tough Task

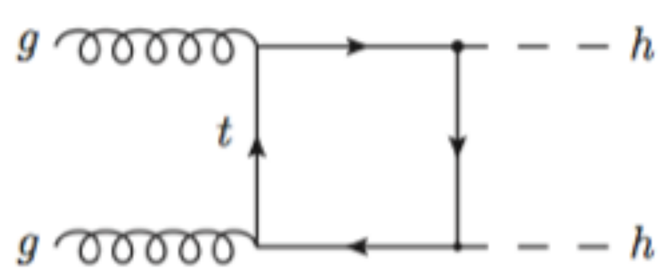


Dominant Production

- Dominant channel at LO: $g + g \to hh$





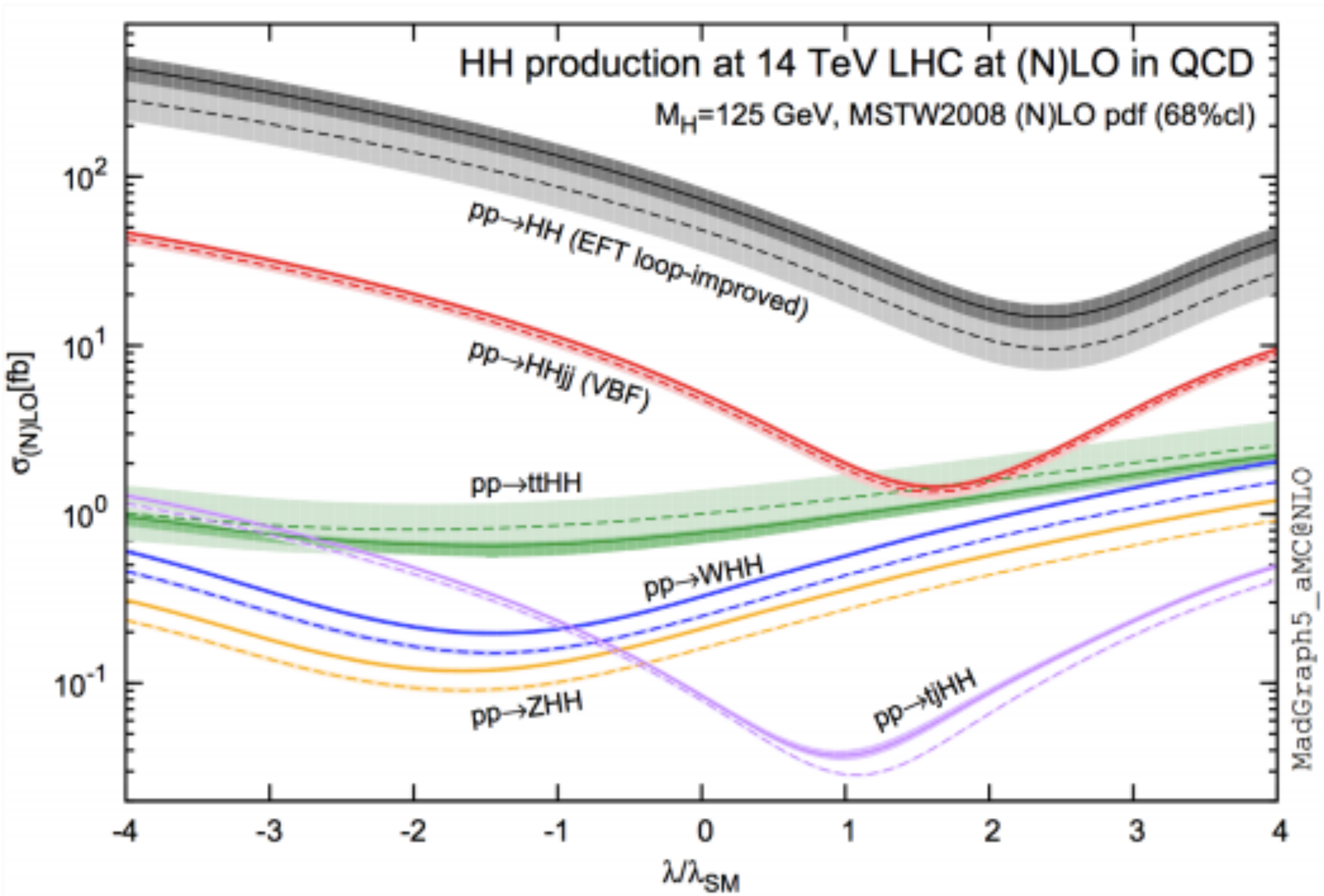


Interference

Javier 2018

Relative contributions at NLO

- Di-Higgs boson at NLO in QCD

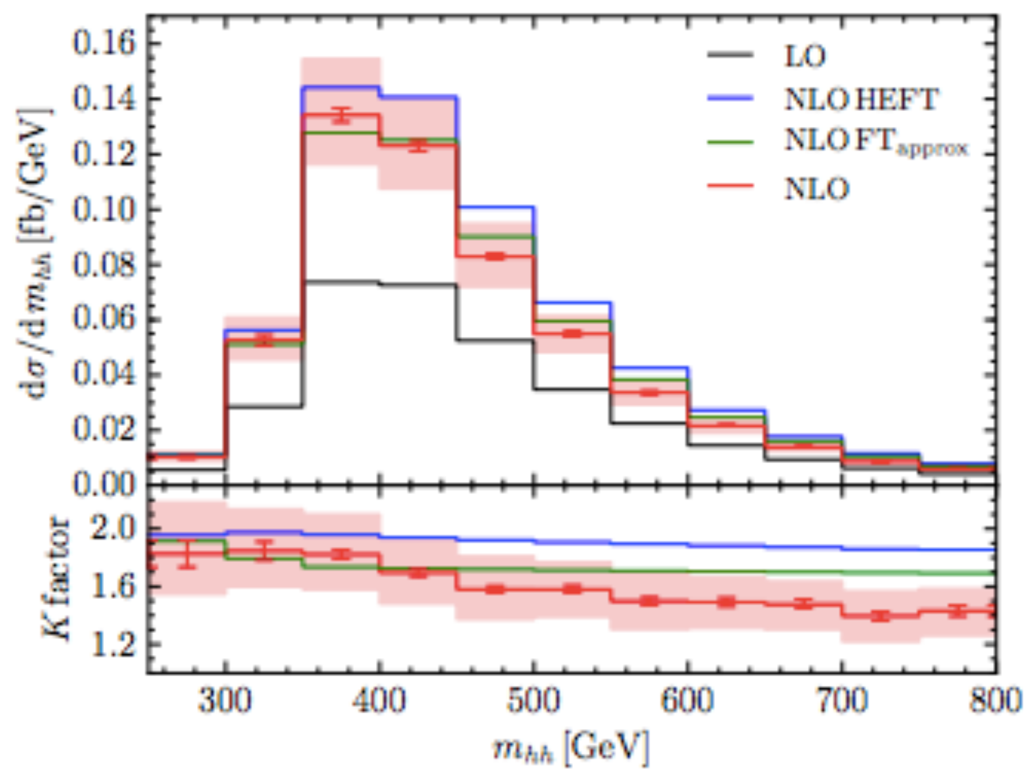


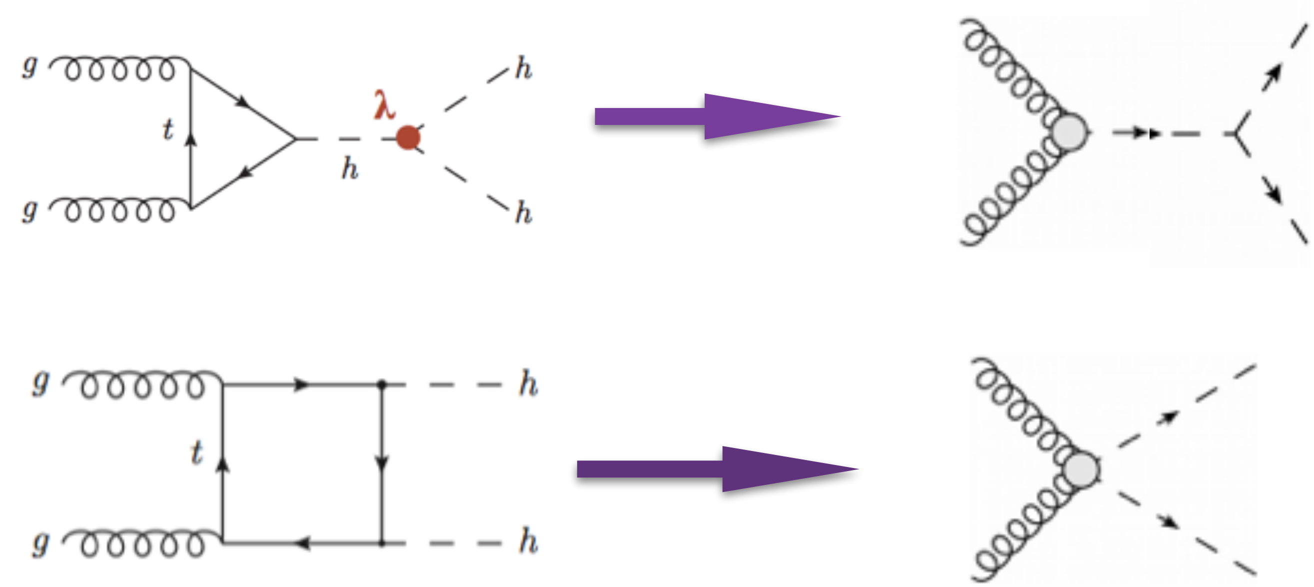
Beyond LO from gluon fusion

Exact top mass

Effective Field theory
$m_t \to \infty$

$$\sigma^{\mathrm{NLO}}(pp \to hh) = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{virt}} + \sum_{i,j\in\{g,q,\bar{q}\}} \sigma^{\mathrm{real}}_{ij}$$



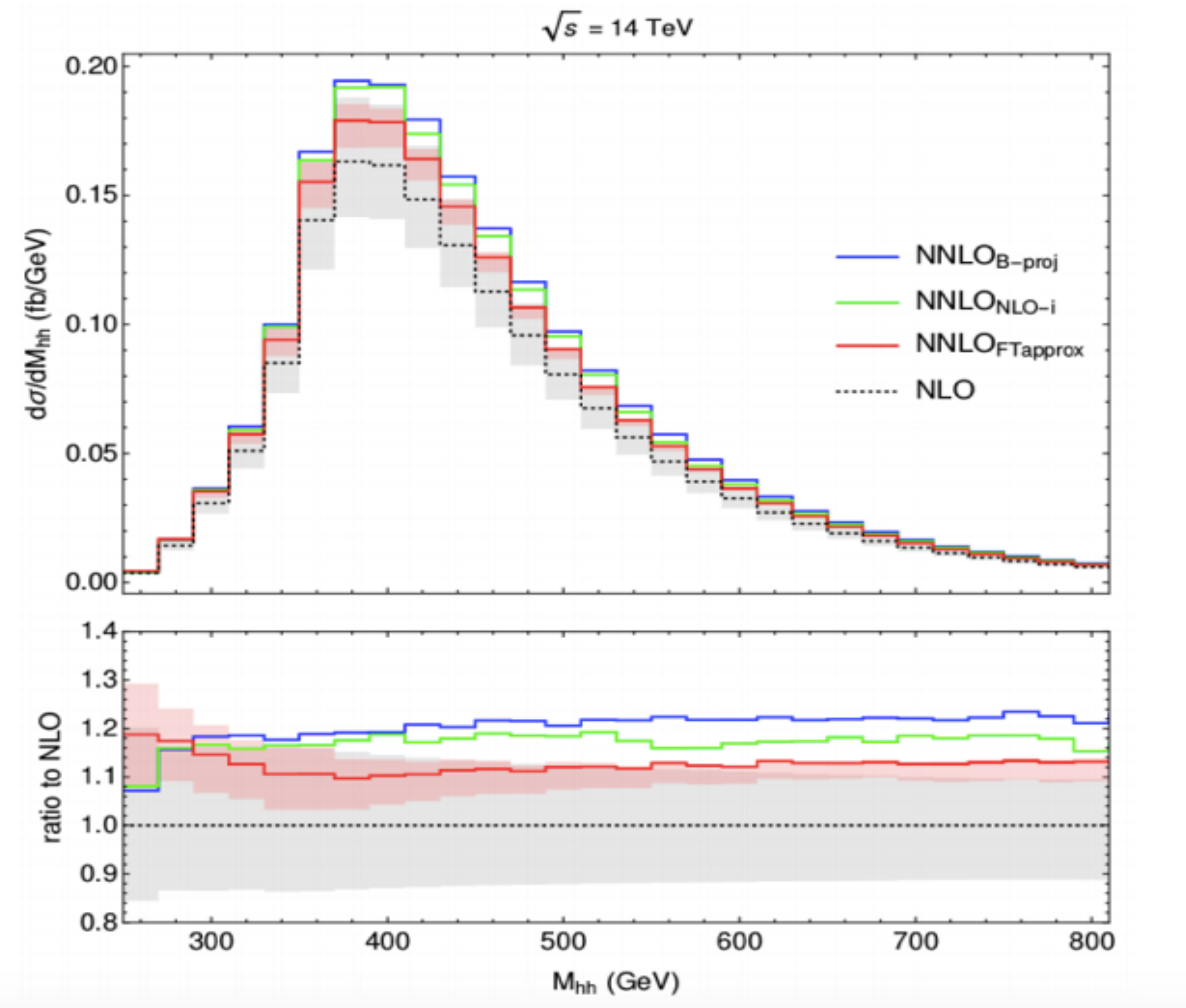


NNLO-large mt
J. Grigo, K. Melnikov, and M. Steinhauser;
D. de Florian and J. Mazzitelli,
Resummed:
D. Y. Shao, C. S. Li, H. T. Li, and J. Wang
D. de Florian and J. Mazzitelli,

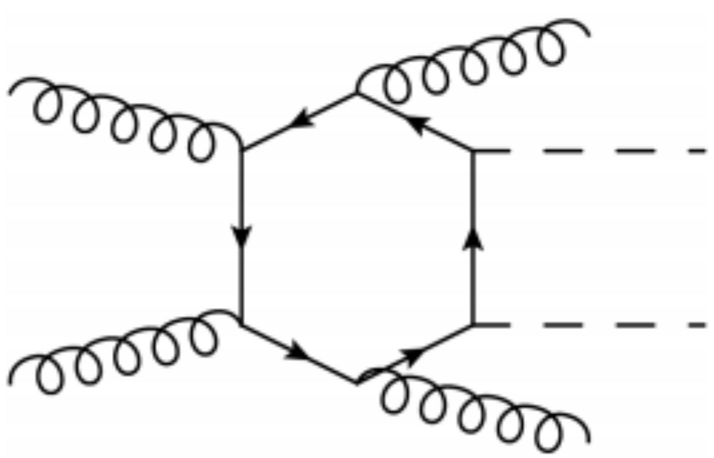
LO: Glover and van der Bij, Plen, Spira, Zerwas,
NLO: Dawson, Dittmair, Spira,
NLO-mt exp: J. Grigo, J. Hoff, K. Melnikov, and M. Steinhauser R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, and M. Zaro; J. Grigo, K. Melnikov, and M. Steinhauser; F. Maltoni, E. Vryonidou, and M. Zaro; J. Grigo, J. Hoff, and M. SteinhauserG. Degrassi, P. P. Giardino, and R. Groeber
NLO-exact mt: S. Borowka, N. Greiner, G. Heinrich, S. P. Jones, M. Kerner, J. Schlenk, U. Schubert, and T. Zirke

At NNLO (approx) from gluon fusion

Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert,Javier Mazitelli



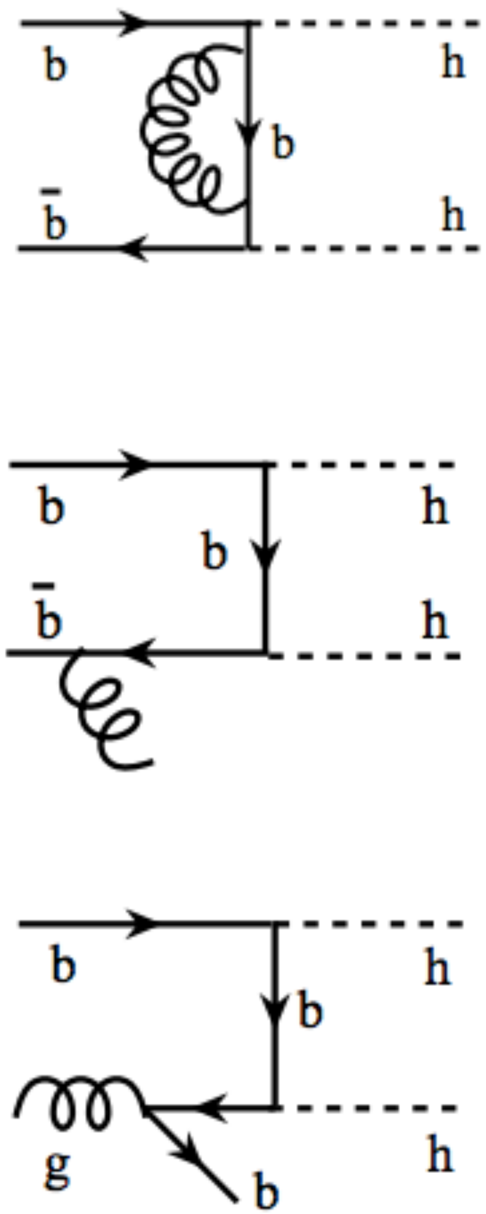
- Corrections of the order of a 12%
- Uncertainties are small
- Better convergence

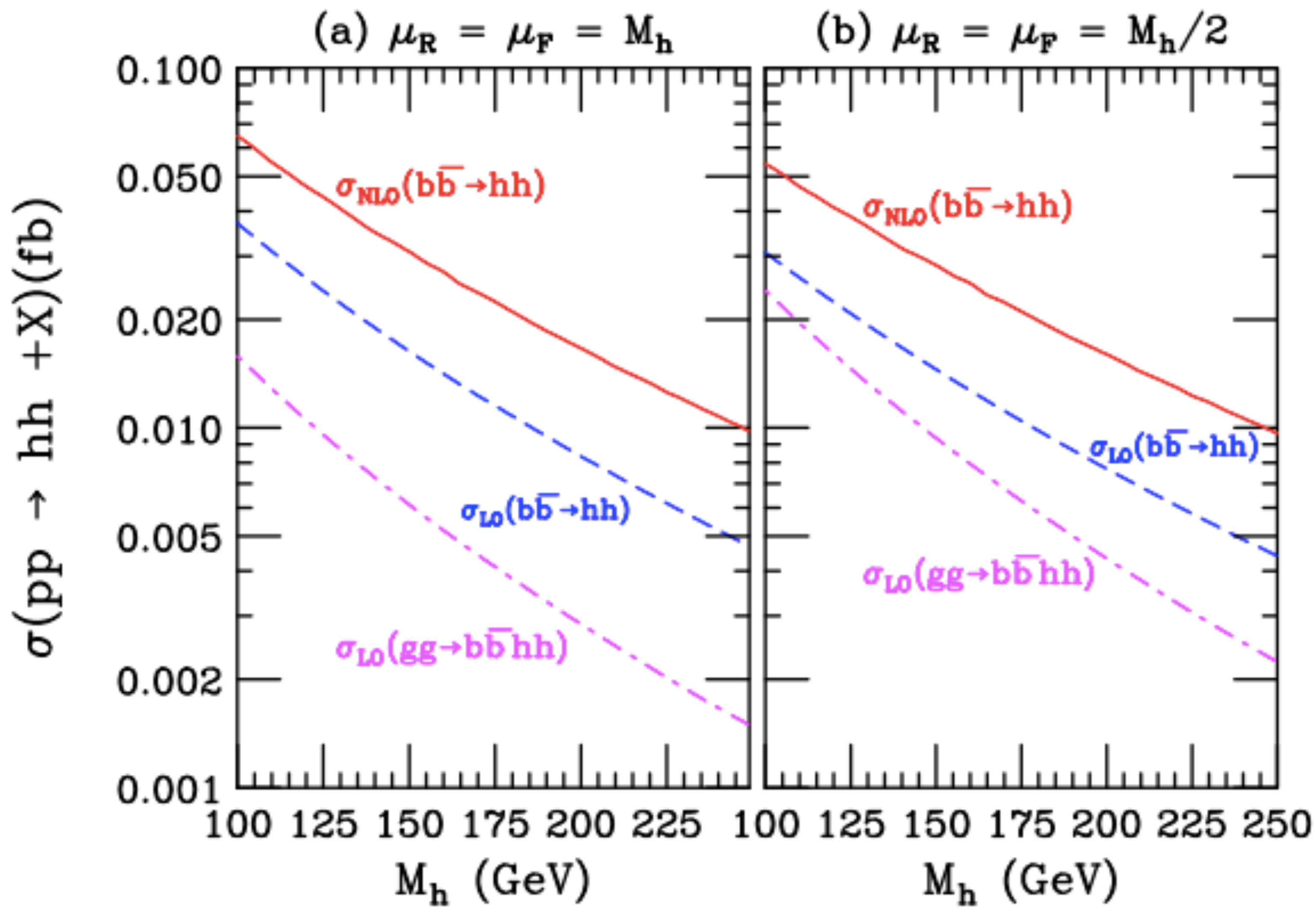


Bottom quark annihilation

Dawson et al.

$$b + \bar{b} \to hh$$





Our Goal is to go beyond NLO

$$b + \bar{b} \to hh$$

Di-Higgs at NNLO in QCD

b + B -> h + h beyond NLO

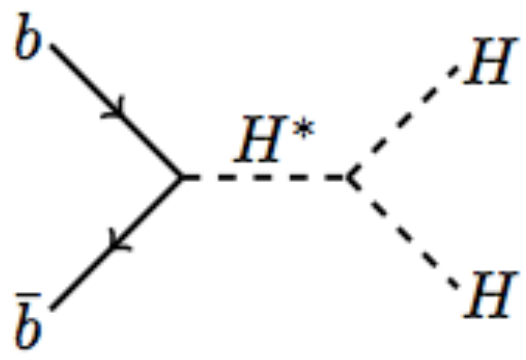
- Hadron Cross section

Variable 5 Flavour scheme

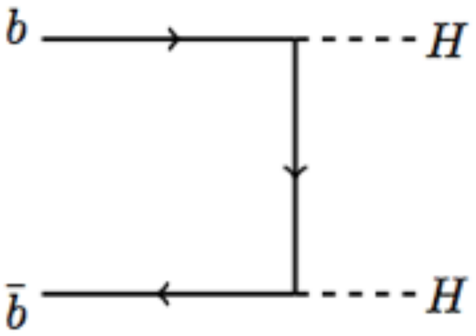
$$\sigma^{HH} = \sum_{a_1,a_2} \int dx_1 \hat{f}_{a_1}(x_1) \int dx_2 \hat{f}_{a_2}(x_2) \hat{\sigma}^{HH}_{a_1 a_2}(x_1, x_2, m_h^2)$$

No interference !

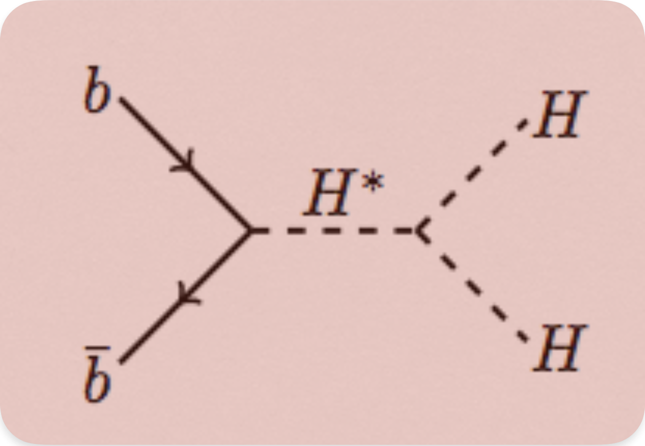
$$\hat{\sigma}^{HH}_{a_1 a_2} = \hat{\sigma}^{HH}_{A a_1 a_2} + \hat{\sigma}^{HH}_{B a_1 a_2}$$



Class-A



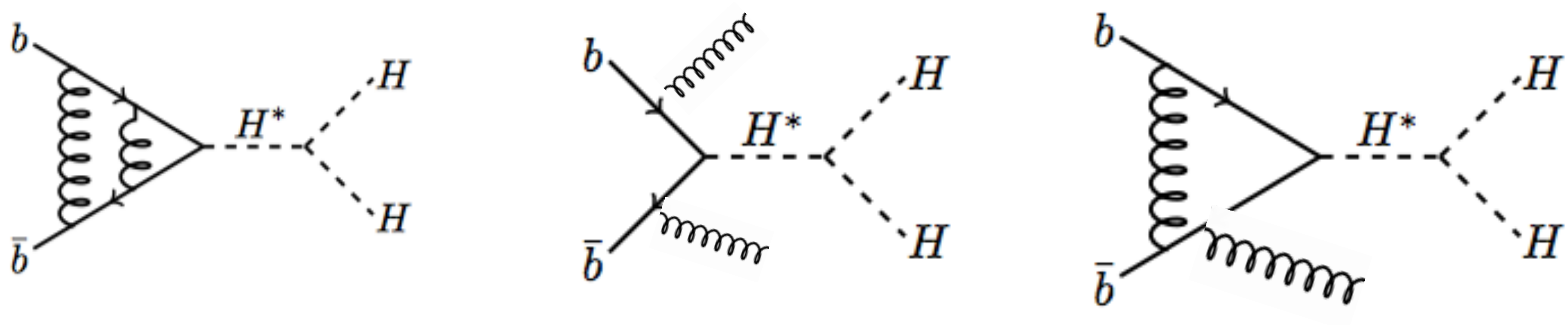
Class-B



Class - A processes

NNLO from Class-A

- Class-A



$$\sigma_A^{HH} = \int \frac{dQ^2}{2\pi} \sigma_A^{H^*}(Q^2)\, D_H(Q^2)$$

Production of
off-shell Higgs boson

Off-shell
Higgs Decay

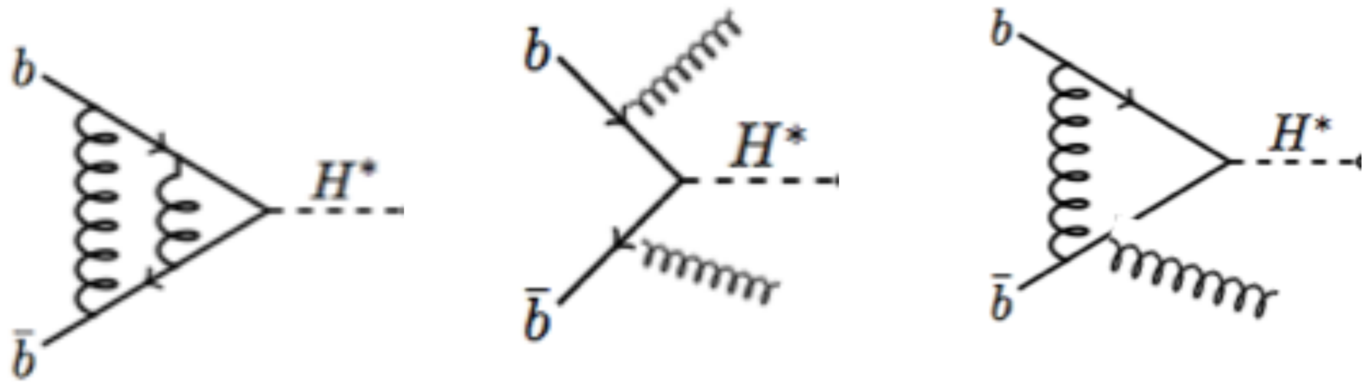
$$D_H(Q^2) = 2Q\Gamma_A^{H^* \to HH}(Q^2)\, |\mathcal{P}_H(Q^2)|^2$$

Single Off-Shell Higgs production

- Off-shell Higgs X-section

Harlander, Kilgore, VR, Prakash Mathews, Majhi

$$\sigma_A^{H^*}(Q^2) = \sum_{ab} \int_{\tau^*}^{1} \frac{dy}{y} \Phi_{ab}(y, \mu_F^2)\ \Delta_{ab}\left(\frac{\tau^*}{y}, Q^2, \mu_F^2\right)$$



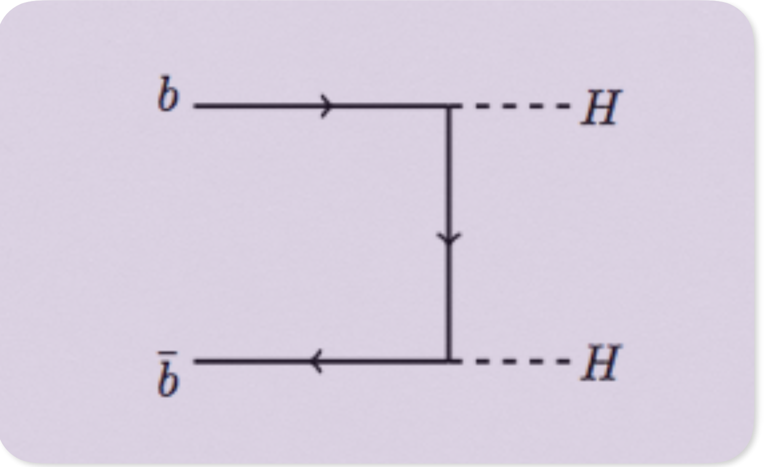
- Partonic Flux

$$\Phi_{a\bar{a}}(y, \mu_F^2) = \int_y^1 \frac{dz}{z} f_a(z, \mu_F^2) f_b\left(\frac{y}{z}, \mu_F^2\right)$$

- Partonic X-section

$$\Delta_{ab}(w, Q^2, \mu_F^2) = \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \Delta_{ab}^{(i)}\ (w, Q^2, \mu_F^2, \mu_R^2)$$

Known to three loops



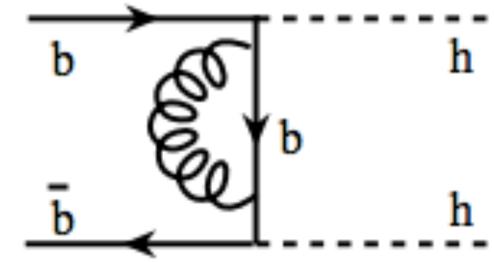
Class - B processes

NLO for Class-B

Dawson et al.

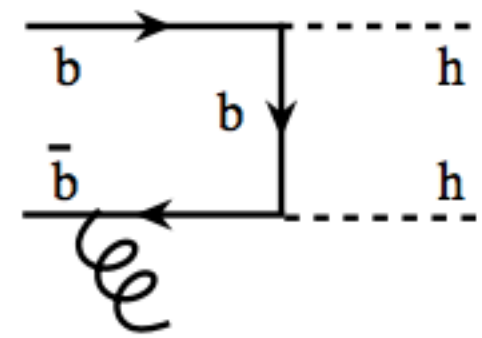
- Phase Space Slicing

Using $\delta_c, \delta_s -$ Slicing parameters

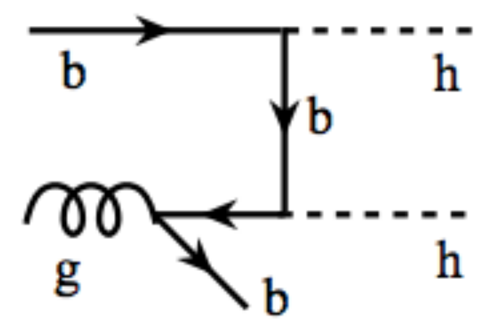


$$d\sigma^{HH}_{\rm NLO}(\mu_F) = \lim_{\delta_s,\delta_c\to 0}\left(d\sigma^{\mathcal{S}+\mathcal{V}+\mathcal{HC}+\rm CT}(\delta_s,\delta_c) + d\sigma^{HH,\overline{\mathcal{HC}}}(\delta_s,\delta_c)\right)$$

Hard
non-collinear



Soft + Virtual +
Hard Collinear + Mass Fact.CT

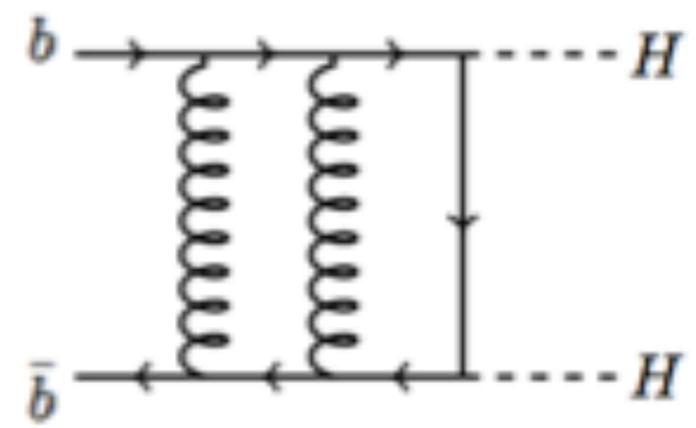


SUM = free of δ_c, δ_s

NNLOsv from Class-B

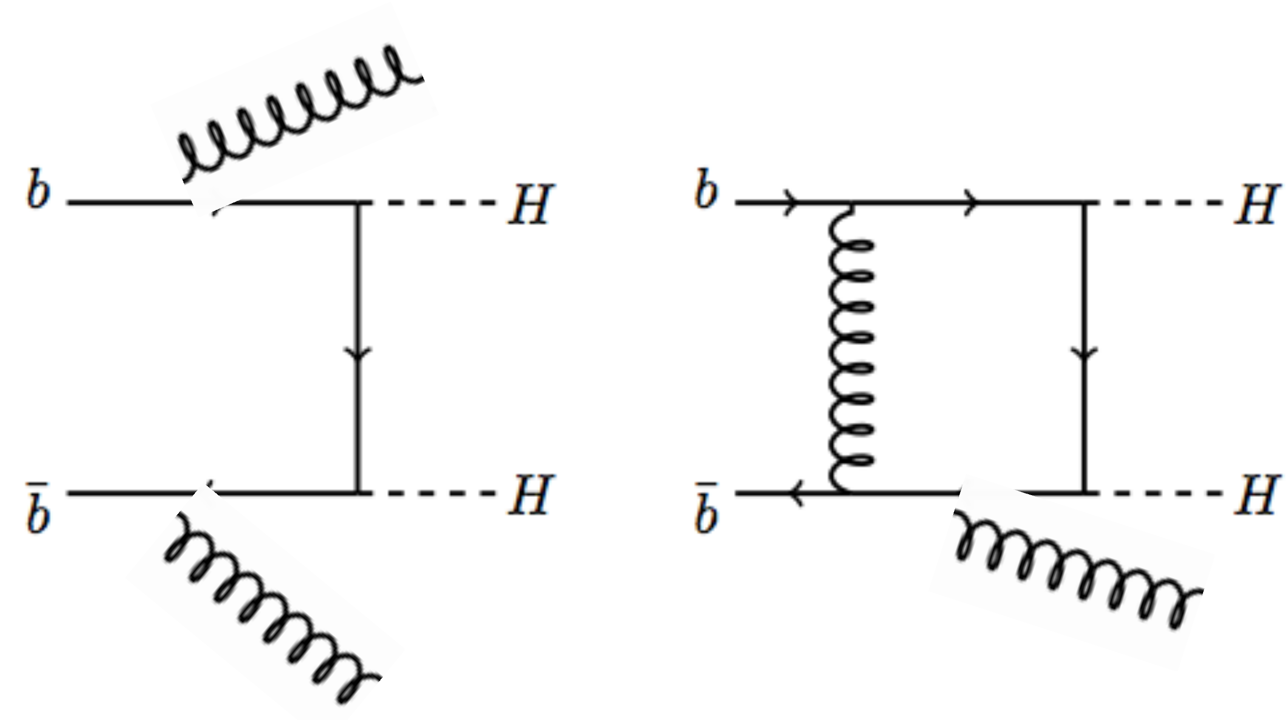
- Two Loop Virtual

  (Exact result)



- Real Emissions

  (Soft gluons only)



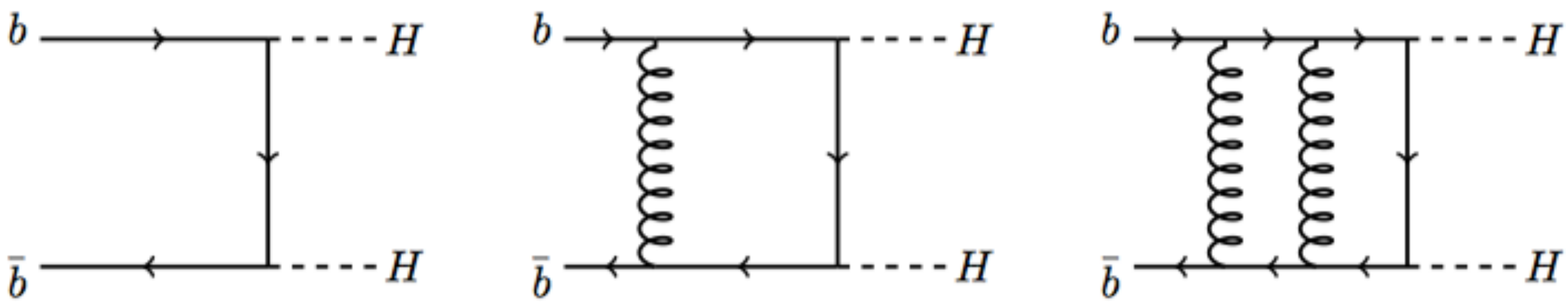
- New result :

Exact Virtual + Soft emissions = NNLOsv

Two loop Virtual corrections

- Class-B diagrams

$$b(p_1) + \bar{b}(p_2) \to h(p_3) + h(p_4)$$



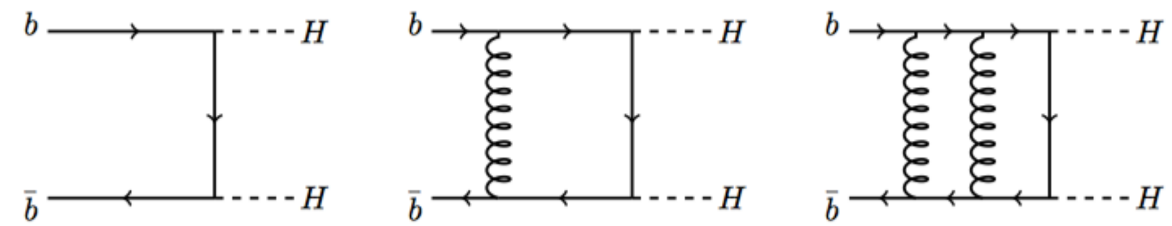
$$\mathcal{A}_{ij} = \bar{v}(p_2)\Big(\mathcal{C}_1 + \mathcal{C}_2 \, \not{p}_3\Big)u(p_1)\delta_{ij}$$

$$\mathcal{C}_1 = 0$$

$$\mathcal{C}_2 = \left(\frac{\lambda_b}{\mu_R^{\epsilon/2}}\right)^2 \left[\mathcal{C}_2^{(0)} + a_s\, \mathcal{C}_2^{(1)} + a_s^2\, \mathcal{C}_2^{(2)} + \mathcal{O}(a_s^3)\right]$$

Two loop Virtual corrections

- Class-B Virtual:



Evaluation: 2 + 10 + 153 diagrams:

- Feynman Diagrams: Qgraf — Nogueira
- Momentum shifts: REDUZE-2 — Manteuffel, Studerus
- Integration by Parts, Lorentz Invariance Identities — LiteRed — Chetyrkin, Tkachov, Gehrmann, Remiddi, Lee, Laporta
- Master Integral: 10 1-loop + 149 2-loops — Gehrmann et.al

see Prakash and Pulak talks

Infrared Structure

Catani; Becher,Neubert; Gardi,Magnea

- ## Catani's Iterative IR Structure

In dimensional regularization
$d = 4 + \epsilon$

Amplitudes can be expanded in terms of
Universal IR operators : $\mathcal{I}_b^{(i)}$

$\mathcal{I}_b^{(1)}$ *and* $\mathcal{I}_b^{(2)}$
Contain poles in ϵ

$$\mathcal{C}_2^{(0)}(\epsilon) = \mathcal{C}_2^{(0),\text{fin}}(\epsilon)\,,$$
$$\mathcal{C}_2^{(1)}(\epsilon) = 2\mathcal{I}_b^{(1)}(\epsilon)\mathcal{C}_2^{(0)}(\epsilon) + \mathcal{C}_2^{(1),\text{fin}}(\epsilon),$$
$$\mathcal{C}_2^{(2)}(\epsilon) = 4\mathcal{I}_b^{(2)}(\epsilon)\mathcal{C}_2^{(0)}(\epsilon) + 2\mathcal{I}_b^{(1)}(\epsilon)\mathcal{C}_2^{(1)}(\epsilon) + \mathcal{C}_2^{(2),\text{fin}}(\epsilon).$$

In the Limit $\epsilon \to 0$

$$\mathcal{C}^{(1),fin}(\epsilon) \text{ and } \mathcal{C}^{(2),fin}(\epsilon) \to \text{Finite}$$

Soft plus Virtual at NNLO

Ajjath, P. Mukherjee, VR

- UV finite cross section

$$\hat{\sigma}(z, Q^2, \epsilon) = \frac{1}{2\hat{s}} \int d\phi(k_i) |\overline{M}(\{p_i\}, \{k_i\}, Q, \epsilon)|^2$$

- Mass Factorisation

$$\Delta(z, Q^2) = \Gamma^T \left(\mu_F^2, \frac{1}{\epsilon}\right) \otimes \hat{\sigma}(z, Q^2, \epsilon) \otimes \Gamma \left(\mu_F^2, \frac{1}{\epsilon}\right)$$

- Factor out Virtual

$$|\overline{M}(\{p_i\}, \{k_i\}, Q, \epsilon)|^2 = |M^V(\{k_i\}, Q, \epsilon)|^2 |\overline{M}^{\overline{V}}(p_i, k_i, Q, \epsilon)|^2$$

Pure Virtual

Real Emissions

Known up to Two loops

Factorization of real emissions

Ajjath, P. Mukherjee, VR

$$|\overline{M}(\{p_i\},\{k_i\},Q,\epsilon)|^2 = |M^V(\{k_i\},Q,\epsilon)|^2 |\overline{\overline{M}^V}(p_i,k_i,Q,\epsilon)|^2$$

- Soft emissions Factorise:

$$\lim_{soft} |\overline{\overline{M}^V}(p_i,k_i,Q,\epsilon)|^2 = S(p_i,k_i,Q,\epsilon) \otimes H(p_i,k_i,Q,\epsilon)$$

Soft emissions

Hard emissions

- Phase Space Factorise:

$$\lim_{soft}\ d\phi(k_i) = d\phi^{soft} \otimes d\phi^{hard}$$

Factorization of real emissions

Ajjath, P. Mukherjee, VR

- All order factorization

$$\Delta^{SV} = \Gamma^T \otimes \sigma^V \otimes \sigma^S \otimes \sigma^H \otimes \Gamma$$

Soft

Virtual

Hard

Mass Fact.

- Exponentiation:

$$\Delta^{SV}(p_i, k_i, Q^2, z) = \mathcal{C} \exp\left(\Psi(p_i, k_i, Q^2, z, \epsilon)\right)\Big|_{\epsilon=0}$$

$$\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

$$\delta(1-z)$$

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z) \otimes f(z) + \cdots .$$

Soft + Virtual (SV)

Ajjath,P.Mukherjee,A.Charkroborty,VR

$$\Delta^{SV} = \Delta_\delta \delta(1-z) + \sum_{j=0}^{2j-1} \Delta_{\mathcal{D}_j} \left(\frac{\log^j(1-z)}{1-z} \right)_+$$

$$\Delta_\delta = \Delta_\delta(s,t,Q^2)$$

$$\Delta_{\mathcal{D}_j} = \Delta_{\mathcal{D}_j}(s,t,Q^2)$$

$$\Delta_I = \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \Delta_I^{(i)}(\mu_R^2)$$

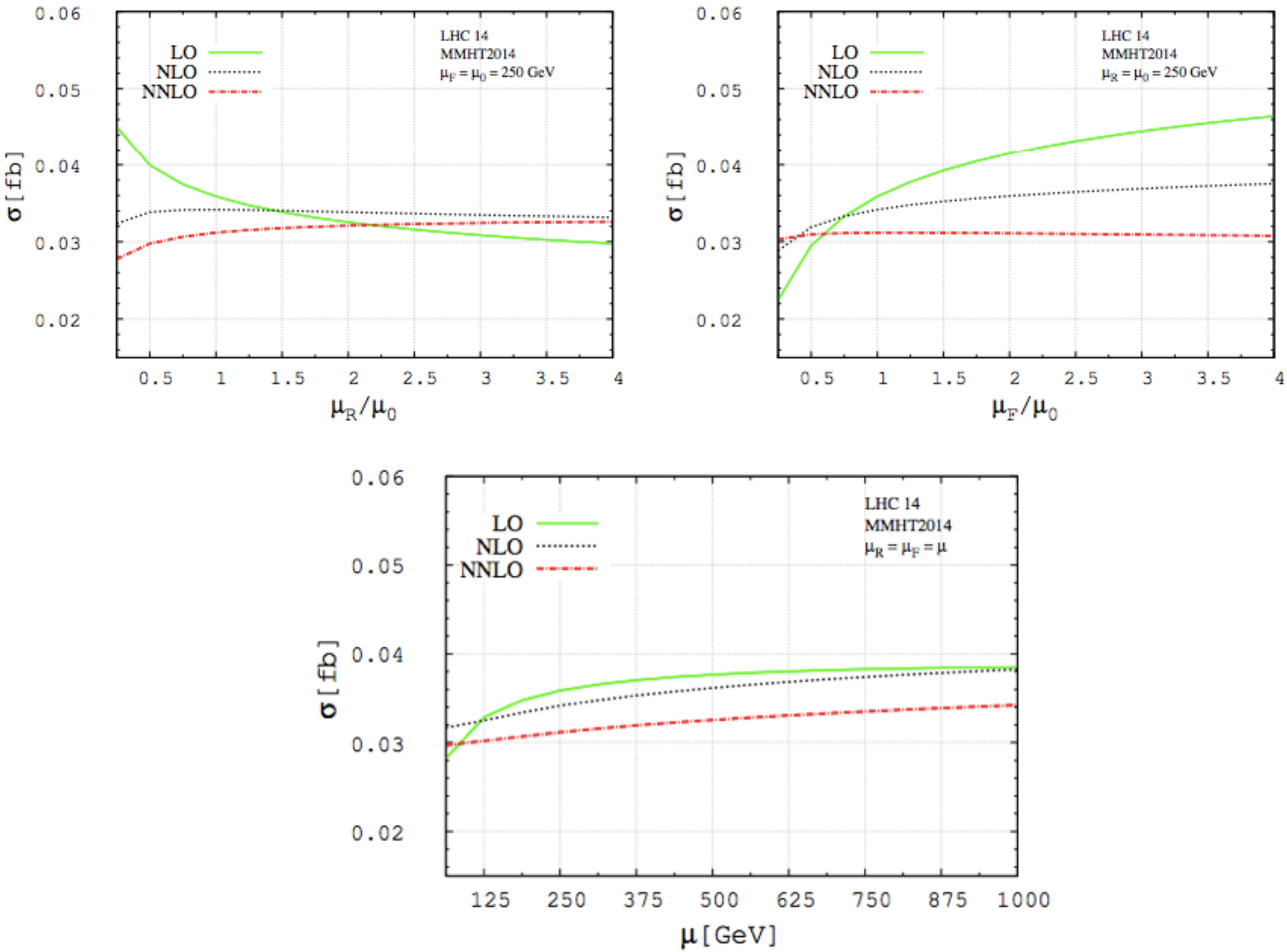
t,u-Mandelstem variables

$$I = \delta, \mathcal{D}_j$$

- All order factorization

$$\sigma^{HH,SV} = \int d\phi(l_i)\ \Phi_{b\bar{b}} \otimes \Delta^{SV}(l_i)$$

Scale Variations



Scale Variations

$\left(\frac{\mu_R}{\kappa m_h}, \frac{\mu_F}{\kappa m_h}\right)$	LO[fb]$\times 10^{-1}$	NLO[fb]$\times 10^{-1}$	NNLO[fb]$\times 10^{-1}$
(2,2)	0.3587	0.3416	0.3119
(2,1)	0.2951	0.3191	0.3098
(1,2)	0.3994	0.3384	0.2976
(1,1)	0.3286	0.3250	0.3020
(1,1/2)	0.2502	0.3032	0.3031
(1/2,1)	0.3704	0.3246	0.2879
(1/2,1/2)	0.2821	0.3169	0.2970

Table 1. 7-point scale variation for central scale at $m_h = 125\text{GeV}, \ \ \kappa = 1$

$\left(\frac{\mu_R}{\kappa m_h}, \frac{\mu_F}{\kappa m_h}\right)$	LO[fb]$\times 10^{-1}$	NLO[fb]$\times 10^{-1}$	NNLO[fb]$\times 10^{-1}$
(2,2)	0.3765	0.3617	0.3256
(2,1)	0.3254	0.3384	0.3210
(1,2)	0.4150	0.3594	0.3110
(1,1)	0.3587	0.3416	0.3119
(1,1/2)	0.2951	0.3191	0.3098
(1/2,1)	0.3994	0.3384	0.2976
(1/2,1/2)	0.3286	0.3250	0.3020

Table 2. 7-point scale variation for central scale at $m_h = 125\text{GeV}, \ \ \kappa = 2$

Scale Variations

Central Scale(GeV)	LO[fb]$\times 10^{-1}$	NLO[fb]$\times 10^{-1}$	NNLO[fb]$\times 10^{-1}$
125	$0.3286^{+21.546\%}_{-23.859\%}$	$0.3250^{+5.108\%}_{-6.708\%}$	$0.3020^{+3.278\%}_{-4.669\%}$
250	$0.3587^{+15.696\%}_{-17.731\%}$	$0.3416^{+5.210\%}_{-6.587\%}$	$0.3119^{+4.392\%}_{-4.585\%}$

Table 3. % scale uncertainty at LO, NLO and NNLO

Conclusions

- Production of Pair of Higgs bosons is one of most important observables at the LHC
- Two classes Class-A and Class-B contribute
- Class-A is related to Single off-shell Higgs boson production
- Real emissions in Class-B is difficult to compute, hence Soft+Virtual
- Reduces the scale uncertainties

Thank you