Two-Loop QCD amplitudes for di-pseudo scalar Higgs production

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Plan of Talk

- Effective Field Theory with pseudo-scalar
- Operator remormalisation and mixing
- Universal IR structure
- Summary

Motivation

- Di-Higgs is an important channel that can probe the Higgs self coupling, which in turn describes the shape of the Higgs potential
- To go beyond NNLO in di-Higgs production channel, relevant two-loop amplitudes have recently been computed in the gluon fusion process

Pulak talk

- BSM scenarios with pseudo-scalar Higgs boson exists, which can lead to rich phenomenology
- Dominant production channel for single and double pseudo scalar Higgs boson, is gluon fusion, through top quark loop

Motivation

- Effective Field Theory approach can be used to study these processes in the large top quark mass limit
- Unlike the scalar Higgs boson, in the EFT, the pseudo scalar couples to gluons through two composite CP odd operators, *viz.* gluonic and fermionic, dimension four operators
- The production cross section is comparable to the di-Higgs cross section. It is hence important to have predictions at the same level of precision for any phenomenological study

Effective Lagrangian

Interaction of a pseudo-scalar fields $\Phi^A(x)$

$$\mathcal{L}_{eff}^A = \Phi^A(x)\left[-\frac{1}{8}C_G O_G(x) - \frac{1}{2}C_J O_J(x)\right]$$

- $O_G(x)$ pseudo-scalar gluonic operators
- $O_J(x)$ pseudo-scalar fermionic operator, is the derivative of the singlet axial vector current
- C_G and C_J are the Wilsons coefficient as a result of integrating out the heavy top quark degrees of freedom

Operators

- Pseudo-scalar gluonic & fermionic operator

  $O_G(x) = G^{a\mu\nu}\widetilde{G}^a_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma} \qquad O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma^5\psi\right)$

  Gluonic field strength tensor

  $$G^{a\mu\nu} = \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + gf^{abc}G^\mu_b G^\nu_c$$

  ψ and $\bar{\psi}$ represent only the light quark field
- Wilson coefficients

  $$\begin{aligned} C_G &= -a_s 2^{\frac{5}{4}} G_F^{\frac{1}{2}} \cot\beta \\ C_J &= -\left[a_s C_F\left(\frac{3}{2} - 3\ln\frac{\mu_R^2}{m_t^2}\right) + a_s^2 C_J^{(2)} + \ldots\right] C_G \end{aligned}$$

- $\cot\beta$ ratio of the two Higgs doublets vacuum expectation values in a generic two-Higgs doublet model
- Adler-Bardeen theorem ensures, there are no QCD corrections to C_G beyond one-loop, while C_J begins at two-loop order

Tensor decomposition of amplitudes

Production of a pair of pseudo-scalar Higgs boson A of mass m_A

$$g(p_1) + g(p_2) \to A(p_3) + A(p_4)$$

Amplitude, can be decomposed in terms of two second rank Lorentz tensors $\mathcal{T}_i^{\mu\nu}$:

$$\mathcal{M}_{ab}^{\mu\nu}\epsilon_\mu(p_1)\epsilon_\nu(p_2) = \delta_{ab}\left(\mathcal{T}_1^{\mu\nu}\ \mathcal{M}_1 + \mathcal{T}_2^{\mu\nu}\ \mathcal{M}_2\right)\epsilon_\mu(p_1)\epsilon_\nu(p_2)$$

Second rank tensors are given by

$$\begin{aligned}
\mathcal{T}_1^{\mu\nu} &= g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \\
\mathcal{T}_2^{\mu\nu} &= g^{\mu\nu} + \frac{1}{p_1 \cdot p_2\ p_T^2}\Big(m_A^2\ p_2^\mu p_1^\nu - 2p_1 \cdot p_3\ p_2^\mu p_3^\nu - 2p_2 \cdot p_3\ p_3^\mu p_1^\nu \\
&\qquad + 2p_1 \cdot p_2\ p_3^\mu p_3^\nu\Big)
\end{aligned}$$

$\mathcal{T}_2^{\mu\nu}$ symmetric under the interchange $p_3 \leftrightarrow p_4$

Projectors

- Scalar functions $\mathcal{M}_{1,2}$ can be obtained from $\mathcal{M}_{ab}^{\mu\nu}$, by using appropriate d-dim projectors

$$
\begin{aligned}
P_1^{\mu\nu} &= \frac{\delta_{ab}}{N^2-1}\left(\frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_1^{\mu\nu} - \frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_2^{\mu\nu}\right) \\
P_2^{\mu\nu} &= \frac{\delta_{ab}}{N^2-1}\left(-\frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_1^{\mu\nu} + \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_2^{\mu\nu}\right)
\end{aligned}
$$

Glover & Bij 1998

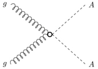
- Using projector method one can compute amplitudes which are much easier to handle analytically and numerically
- Use these amplitudes in other pseudo scalar processes
- Amplitudes are useful for studying spin asymmetries involving pseudo scalar particles.

Diagrams

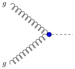
- Higher order corrections to the amplitude, calculated in massless QCD using the effective Lagrangian:

  Two type of diagrams contribute:
  - TYPE-I:
    (a) one AAgg effective vertex
    (b) one Agg effective vertex and one AAA vertex
  - TYPE-II:
    (a) two Agg effective vertex
    (b) one Agg effective vertex and one $Aq\bar{q}$ effective vertex

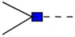
$AAgg$ proportional to C_G of $\mathcal{O}(a_s)$



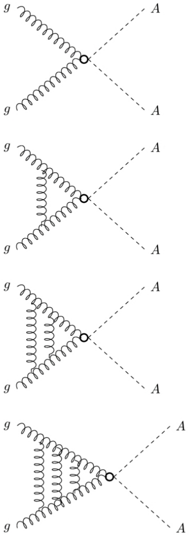
Agg proportional to C_G of $\mathcal{O}(a_s)$



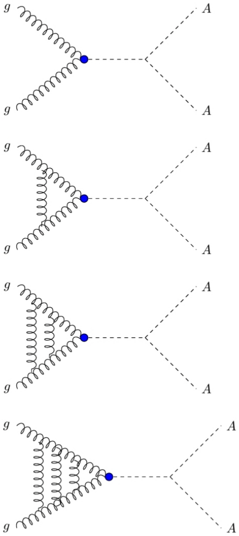
$Aq\bar{q}$ proportional to C_J of $\mathcal{O}(a_s^2)$



Type-I: Form factor type



Type-Ia



Type-Ib

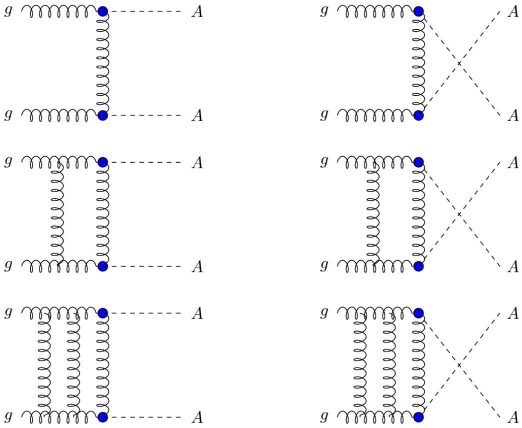
- effective vertex $AAgg$ same as HHgg effective vertex, in turn same as effective Hgg vertex
- effective Agg vertex
- both, contribute at $\mathcal{O}(a_s)$, tree level, LO for this amplitude.
- It has been calculated to $\mathcal{O}(a_s^4)$ (3-loop)

  Baikov *et. al.* 2009;

  Gehrmann *et. al.* 2010

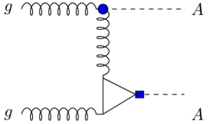
  Ahmed *et. al.* 2015

Type-IIa C_G^2



- two effective vertex Agg
- proportional to C_G
- tree level begins at $\mathcal{O}(a_s^2)$
- evaluated to $\mathcal{O}(a_s^4)$ (2-loop)

Type-IIb C_G C_J



- Due to the axial anomaly, pseudo-scalar operator for the gluonic field strength mixes with the divergence of the singlet axial vector current
- effective $Aq\bar{q}$ vertex, proportional to C_J, begins at $\mathcal{O}(a_s^2)$
- effective Agg vertex, proportional to C_G at $\mathcal{O}(a_s)$
- contribution starts at $\mathcal{O}(a_s^4)$

$$\mathcal{M}_i = \mathcal{M}_i^{\mathrm{I}} + \mathcal{M}_i^{\mathrm{II}} \qquad i = 1, 2$$

$\mathcal{M}_i^{\mathrm{I}}$ Hgg and Agg FF type $\mathcal{M}_i^{\mathrm{II}}$ to order $\mathcal{O}(a_s^4)$, two loop

Renormalisation

- dimensional regularisation $d = 4 + \epsilon$ to regularise both UV and IR singularities which appear as poles in ϵ in the UV, soft and collinear regions
- Levi-Civita tensor in O_G operator and γ_5 in O_J operator are constructs, inherently 4-dimensional
- a consistent method to deal with them in $4 + \epsilon$ dimensions is essential
- renormalisation and mixing of composite operators O_G & O_J
- contact terms?

γ_5: dimensional regularisation

- Computing higher order corrections with chiral quantities, involve inherently $d = 4$ dimensional objects *viz.* γ_5 & $\epsilon_{\mu\nu\rho\sigma}$: warrants a prescription in going to $d = 4 + \epsilon$
- Prescription

$$\gamma_5 = \frac{i}{4!} \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4}$$

where Levi-Civita tensor is purely 4-dimensional, while the Lorentz indices on the γ^{μ_i} are in $d = 4 + \epsilon$ dimensions

't Hooft & Veltman 1972

- To maintain anticommuting nature of γ_5 with d-dimensional γ_{μ_i}, the form of the axial current to be used

$$J^5_\mu = \frac{1}{2} \overline{\psi} (\gamma_\mu \gamma_5 - \gamma_5 \gamma_\mu) \psi$$

- operators

$$O_G(x) = \varepsilon_{\mu\nu\rho\sigma} G^{a\mu\nu} G^{a\rho\sigma} \qquad O_J(x) = \frac{i}{3!} \epsilon_{\mu\nu_1\nu_2\nu_3} \partial^\mu \left(\bar{\psi} \gamma^{\nu_1} \gamma^{\nu_2} \gamma^{\nu_3} \psi \right)$$

Larin 1993

- Contraction of two Levi-Civita tensor as a result of two O_G operators or due to O_G and O_J operators

$$\varepsilon_{\mu_1\nu_1\rho_1\sigma_1}\varepsilon^{\mu_2\nu_2\rho_2\sigma_2} = \begin{vmatrix} \delta^{\mu_2}_{\mu_1} & \delta^{\nu_2}_{\mu_1} & \delta^{\rho_2}_{\mu_1} & \delta^{\sigma_2}_{\mu_1} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\nu_2}_{\nu_1} & \delta^{\rho_2}_{\nu_1} & \delta^{\sigma_2}_{\nu_1} \\ \delta^{\mu_2}_{\rho_1} & \delta^{\nu_2}_{\rho_1} & \delta^{\rho_2}_{\rho_1} & \delta^{\sigma_2}_{\rho_1} \\ \delta^{\mu_2}_{\sigma_1} & \delta^{\nu_2}_{\sigma_1} & \delta^{\rho_2}_{\sigma_1} & \delta^{\sigma_2}_{\sigma_1} \end{vmatrix}$$

- Lorentz indices in determinant, could now be considered as d-dim and consequence would be, addition of only the inessential $\mathcal{O}(\epsilon)$ terms to the renormalisated quantity
- This prescription though is not without consequence, a finite renormalisation of the axial vector current is required

Larin 1993

UV renormalisation $d = 4 + \epsilon$

Bare coupling $\hat{a}_s$ related to renormalized coupling a_s

$$\hat{a}_s S_\epsilon = \left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon/2} Z_{a_s} a_s$$

$$Z_{a_s} = 1 + a_s \left[\frac{2}{\epsilon}\beta_0\right] + a_s^2 \left[\frac{4}{\epsilon^2}\beta_0^2 + \frac{1}{\epsilon}\beta_1\right] + a_s^3 \left[\frac{8}{\epsilon^3}\beta_0^3 + \frac{14}{3\epsilon^2}\beta_0\beta_1 + \frac{2}{3\epsilon}\beta_2\right] .$$

Type-I diagrams which begins to contribute at LO, Z_{a_s} upto order $\mathcal{O}(a_s^3)$ is needed, while for Type-II diagrams, one order lower is sufficient

Tarasov, Vladimirov, Zharkov

Operator renormalization

- Amplitudes require renormalisation of composite operators O_G and O_J in the effective Lagrangian
- Renomalisation of O_J is related to the renormalisation of the singlet axial vector current J_μ^5 which needs the standard overall UV renormalisation constant $Z^s_{\overline{MS}}$
- Even with $Z^s_{\overline{MS}}$, the 1-loop character of the opr relation of axial anomaly can not be maintained

$$[\partial_\mu J_5^\mu] = a_s \frac{n_f}{2} \left[G \tilde{G} \right] \qquad \text{i.e.} \quad [O_J] = a_s \frac{n_f}{2} [O_G]$$

  which is true in Pauli-Villars, a 4-dim regularsation
- A multiplicative finite renormalisation constant Z_5^s is required, in dim regularisation

$$[O_J] = Z_5^s \; Z^s_{\overline{MS}} \; O_J$$

Larin 1993

Operator mixing

- bare pseudo-scalar gluon operator O_G mixes under the renormalisation with O_J

$$[O_G] = Z_{GG}\ O_G + Z_{GJ}\ O_J$$

[ ] denoted renormalised quantity

- mixing matrix

$$\begin{pmatrix} [O_G] \\ [O_J] \end{pmatrix} = \begin{pmatrix} Z_{GG} & Z_{GJ} \\ Z_{JG} & Z_{JJ} \end{pmatrix} \begin{pmatrix} O_G \\ O_J \end{pmatrix}$$

$Z_{JG} = 0$ all orders

$Z_{JJ} \equiv Z_5^s Z_{\overline{MS}}^s$

Z_{GG} to $\mathcal{O}(a_s^3)$

Z_{GJ} to $\mathcal{O}(a_s^3)$

Larin 1993 $\mathcal{O}(a_s^2)$

Zoller 2013 $\mathcal{O}(a_s^3)$

Ahmed *et. al.* 2015 $\mathcal{O}(a_s^3)$

$g + g \to A + A$ amplitude to $\mathcal{O}(a_s^4)$

- amplitude can be obtained *via* the insertion of any two renormalised operators $[O_G]$, $[O_J]$ for each A

$$\langle g|[O_G O_G]|g\rangle \qquad \langle g|[O_G O_J]|g\rangle \qquad \langle g|[O_J O_J]|g\rangle$$

- renormalised operators are related to bare ones. O_G couples to gluons at $\mathcal{O}(a_s)$ and O_J couples to quarks at $\mathcal{O}(a_s^2)$

$$\begin{aligned}
[O_G O_G] &= Z_{GG}^2 O_G^2 + 2 Z_{GG} Z_{GJ} O_G O_J + Z_{GJ}^2 O_J^2 \\
[O_G O_J] &= Z_{GG} Z_{JJ} O_G O_J + Z_{GJ} Z_{JJ} O_J^2
\end{aligned}$$

- O_G^2 contributes at tree level i.e. $\mathcal{O}(a_s^2)$, $O_G O_J$ contributes at 1-loop level at $\mathcal{O}(a_s^4)$ while O_J^2 at $\mathcal{O}(a_s^6)$

$$\begin{aligned}
\mathcal{M}_{GG,g}^{II} &= Z_{GG}^2 \left[\hat{\mathcal{M}}_{GG,g}^{II,(0)} + \hat{a}_s \hat{\mathcal{M}}_{GG,g}^{II,(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{GG,g}^{II,(2)} \right] \\
&\quad + 2 Z_{GG} Z_{GJ} \left[\hat{a}_s \hat{\mathcal{M}}_{GJ,g}^{II,(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{GJ,g}^{II,(2)} \right] \\
&\quad + Z_{JJ}^2 \left[\hat{a}_s \hat{\mathcal{M}}_{JJ,g}^{II,(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{JJ,g}^{II,(2)} \right]
\end{aligned}$$

$g + g \to A + A$ amplitude to $\mathcal{O}(a_s^4)$

- Relation between UV finite amplitudes and bare amplitudes

$$\mathcal{M}_{GG,g} = \mathcal{M}^{(0)}_{GG,g} + a_s \mathcal{M}^{(1)}_{GG,g} + a_s^2 \mathcal{M}^{(2)}_{GG,g} + \mathcal{O}\left(a_s^3\right)$$

$$\begin{aligned}
\mathcal{M}^{(0)}_{GG,g} &= \hat{\mathcal{M}}^{(0)}_{GG,g} \\
\mathcal{M}^{(1)}_{GG,g} &= \frac{1}{\mu_R^{\varepsilon}} \left[\hat{\mathcal{M}}^{(1)}_{GG,g} + \mu_R^{\varepsilon} \frac{4\beta_0}{\varepsilon} \hat{\mathcal{M}}^{(0)}_{GG,g} \right] \\
\mathcal{M}^{(2)}_{GG,g} &= \frac{1}{\mu_R^{2\varepsilon}} \left[\hat{\mathcal{M}}^{(2)}_{GG,g} + \mu_R^{\varepsilon} \frac{6\beta_0}{\varepsilon} \hat{\mathcal{M}}^{(1)}_{GG,g} \right. \\
&\quad + \mu_R^{2\varepsilon} \left(\frac{12\beta_0^2}{\varepsilon^2} + \frac{2\beta_1}{\varepsilon} \right) \hat{\mathcal{M}}^{(0)}_{GG,g} \\
&\quad \left. - \mu_R^{\varepsilon} \frac{48 C_F}{\varepsilon} \hat{\mathcal{M}}^{(1)}_{GJ,g} \right]
\end{aligned}$$

- UV singularities at 1-loop and 2-loop are regularised using coupling constant renormalisation Z_{a_s} and operator renormalisation Z_{ij} with $i, j = G, J$.

Calculation of amplitude to $\mathcal{O}(a_s^4)$

- reduced to computing the Type-II diagrams, with the effective Lagrangian, in $d = 4 + \epsilon$ dim, using QGRAF

  Nogueira 1993

- Type-IIa: tree level: 2 diagrams $\left(\mathcal{O}(a_s^2)\right)$
  1-loop: 35 diagrams $\left(\mathcal{O}(a_s^3)\right)$
  2-loop: 789 diagrams $\left(\mathcal{O}(a_s^4)\right)$
- Type-IIb: 1-loop: 8 diagrams $\left(\mathcal{O}(a_s^4)\right)$
- We use REDUZE2 package to identify the momentum shifts required to express each of these diagrams in terms of auxiliary topology

  Manteuffel, Studerus 2012

- Auxiliary topologies needed for pair production of equal-mass vector bosons are the same

  Gehrmann, *et. al.* 2014; 2013

- Use IBP/LI identities *via* LiteRed package implementing Laporta algorithm to the irreducible set of MIs (two-loop four-point functions with two equal mass external legs)

  Tkachov 81; Chetyrkin *et. al.* 81; Gehrmann *et. al.* 00; Lee 14; Laporta 00

Infrared factorisation

- UV finite amplitudes contain only divergences of IR origin, appear as poles in ϵ
- amplitudes beyond LO: very rich universal structure in the IR

$$
\begin{aligned}
\mathcal{M}_i^{II,(0)} &= \mathcal{M}_i^{II,(0)} \\
\mathcal{M}_i^{II,(1)} &= 2\mathbf{I}_g^{(1)}(\epsilon)\mathcal{M}_i^{II,(0)} + \mathcal{M}_i^{II,(1),fin} \\
\mathcal{M}_i^{II,(2)} &= 4\mathbf{I}_g^{(2)}(\epsilon)\mathcal{M}_i^{II,(0)} + 2\mathbf{I}_g^{(1)}(\epsilon)\mathcal{M}_i^{II,(1)} + \mathcal{M}_i^{B,(2),fin}
\end{aligned}
$$

$\mathbf{I}_g^{(1)}(\epsilon)$, $\mathbf{I}_g^{(2)}(\epsilon)$ are the IR singularity operators

$$
\boldsymbol{I}_g^{(1)}(\varepsilon) = -\frac{e^{-\frac{\varepsilon}{2}\gamma_E}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(\frac{4C_A}{\varepsilon^2} - \frac{\beta_0}{\varepsilon}\right)\left(-\frac{s}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}
$$

$$
\boldsymbol{I}_g^{(2)}(\epsilon) = -\frac{1}{2}\boldsymbol{I}_g^{(1)}(\epsilon)\left[\boldsymbol{I}_g^{(1)}(\epsilon) - \frac{2\beta_0}{\epsilon}\right] + \frac{e^{\frac{\epsilon}{2}\gamma_E}\Gamma(1+\epsilon)}{\Gamma\left(1+\frac{\epsilon}{2}\right)}\left[-\frac{\beta_0}{\epsilon} + K\right]\boldsymbol{I}_g^{(1)}(2\epsilon) + 2\boldsymbol{H}_g^{(2)}(\epsilon)
$$

Catani 1998; Becher,Neubert 2009; Gardi,Magnea 2009

Infrared factorisation

- Two loop case a fully analytical comparison was possible for poles ϵ^{-i} with $i = 2 - 4$
- Full agreement with the predictions of Catani up to two loop level for all the IR poles
- Due to the large file size for the ϵ^{-1} pole term, comparison only at the numerical level
- Finally the finite part can be extracted

Summary

- Compute all the virtual helicity amplitudes to $\mathcal{O}(a_s^4)$ relevant for pair of pseudo scalar Higgs production at the LHC
- The computation is done in the EFT where the top quark degrees of freedom is integrated out
- Larin's prescription for γ_5 and Levi-Civita tension in $4 + \epsilon$ dimensions
- UV renormalisation with operator mixing and finite renormalisation, with no contact counter terms
- Agrees with the universal IR structure