Towards completion of the four-body contributions to $B \to X_s\gamma$ at NLO

Lars-Thorben Moos

with Tobias Huber

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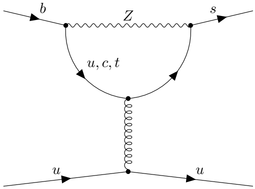
$B \to X_s\gamma$

$B \to X_s\gamma$ is one of the most suitable processes for the search for new physics in the quark flavor sector

$b \to s\gamma$ forbidden at tree-level, dominant contributions loop induced by weak decays
$\to$ small Standard Model rate
$\to$ sensitive to new particles running in the loop



$\Rightarrow$ Calculate the branching ratio for this process in terms of generic Wilson coefficients using the framework of the weak effective theory.

State of the art

- NLO QCD completed in 2002

  [Buras et al., hep-ph/0203135]

- Estimate of corrections $\mathcal{O}(\alpha_s^2)$

  [Misiak et al., hep-ph/0609232]

- Multi-parton contributions

  - Completion of BLM corrections

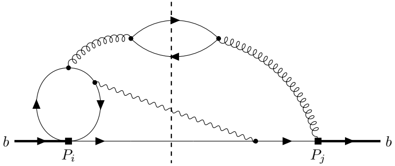
    [Misiak, Poradziński, arXiv:1009.5685]

  - Tree level contributions

    [Kamiński et al., arXiv:1209.0965]

  - Most NLO four-body corrections $B \to s\gamma + q\bar{q}$

    [Huber et al., arXiv:1411.7677]



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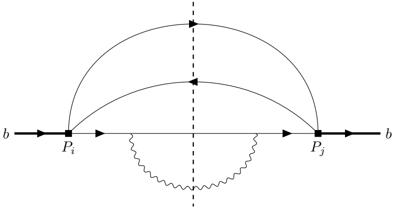

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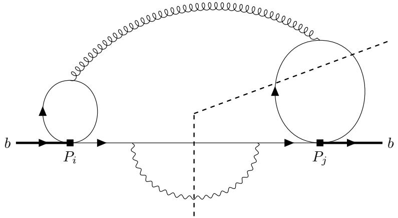


Currently, the theoretical and experimental values of the CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$ (with $E_\gamma > 1.6$ GeV) do agree well within their uncertainties:

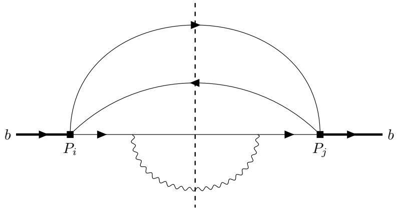
$$\mathcal{B}_{s\gamma}^{SM} = (3.36 \pm 0.23) \cdot 10^{-4}$$

[Misiak et al., arXiv:1503.01789]

$$\mathcal{B}_{s\gamma}^{exp} = (3.32 \pm 0.15) \cdot 10^{-4}$$

[HFLAV, arXiv:1612.07233]

Four-body contributions to $B \to X_s + \gamma$



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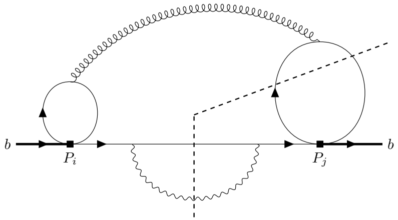

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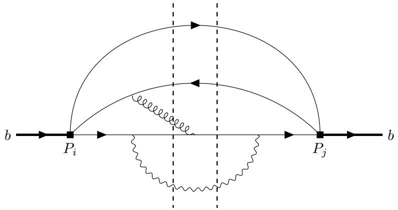


this talk

- diagrams as above only contain 4-particle cuts
- uncalculated until now: diagrams on the left, where additional 5-particle cuts have to be taken into account

SMEFT Operators

The relevant processes in our case are described by a subset of dimension-6 operators from the weak effective theory:

$$\mathcal{L}_{eff} = \mathcal{L}_{QED+QCD} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \Big[\sum_{i=1}^{2} \mathcal{C}_i^u P_i^u + \sum_{i=3}^{6} \mathcal{C}_i P_i \Big]$$

$$P_1^u = (\bar{s}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \qquad P_2^u = (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

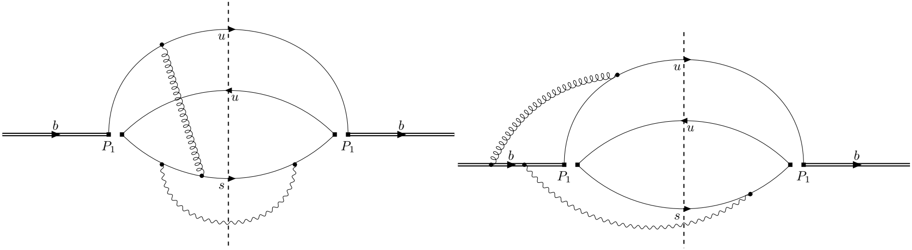
$$P_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q) \qquad P_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q)$$

$$P_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho q) \quad P_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho T^a q)$$

For this calculation, the sum over the quarks includes up-, down- and strange-quark (bottom is kinematically forbidden and final states including charm are excluded from $B \to X_s \gamma$ per definition)

Example Processes: Virtual contributions

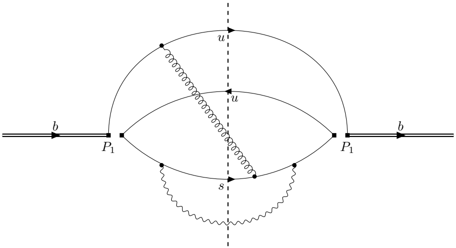
11968 virtual contributions
$\rightarrow$ related by color factors, reducing to $\sim$3000

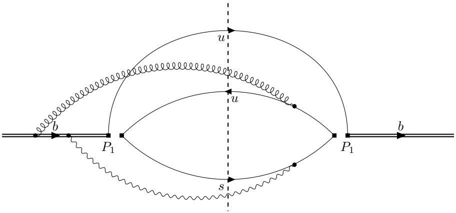


$\Rightarrow$ 4-particle-cuts with up to three massive propagators, up to two of them contained in the loop

Example Processes: Real contributions

14400 contributions from real emissions
$\rightarrow$ 3600 after use of color identities





$\Rightarrow$ 5-particle-cuts of with up to four massive propagators

Steps of the calculation

a) Evaluation of the cut-diagrams

b) Integration over the four- and five-particle phase space

c) Renormalization of UV divergences

d) Treatment of IR divergences

a) Evaluation of the diagrams

Program Setup:

a) Feynman Rules: FeynRules 2.3
[Alloul et al., arXiv:1310.1921]

b) Generation of the Diagrams: FeynArts 3.10
[T. Hahn, hep-ph/0012260]

c) Simplification of Dirac-, color- and Lorentz structures: FormCalc 9.8
[T. Hahn et al., arXiv:1604.04611]

d) Passarino-Veltman Reduction: FeynCalc 9.2
[Shtabovenko et al., arXiv:1601.01167]

$$\Rightarrow |\mathcal{M}(s_{ij})|^2$$

b) Phase space

Need to integrate the Kernels $\mathcal{K}(s_{ij})$ $(= |\mathcal{M}(s_{ij})|^2)$ from a) over the four- and five-particle phase space in $d = 4 - 2\epsilon$ dimensions

For the four-particle-cuts this looks the following:

$$\int [ds_{ij}]\ \delta(1 - \sum s_{ij})\mathcal{K}(s_{ij})(-\Delta_4)^{\frac{d-5}{2}}\Theta(-\Delta_4)$$

with:

$s_{ij} = 2k_i \cdot k_j / m_b^2$, $\left[\text{labelled } b \to q(k_1)\bar{q}(k_2)s(k_3)\gamma(k_4)\right]$

$\delta(1 - \sum s_{ij})$ from momentum conservation,

Δ_4 Gram determinant, can be parametrized as

$$-\Delta_4 = (\bar{z} - s_{34})^2(a^+ - s_{23})(s_{23} - a^-)$$

Factorization of Gram determinant in the case of d-dimensional massless four- and five-particle phase space is known

[Gehrmann-De Ridder et al., hep-ph/0311276][Heinrich, hep-ph/0601062]

Implementation of cut on photon energy E_γ

Subtlety of the calculation: Cut on the photon energy E_γ is imposed
$\Rightarrow$ Integration is not symmmetric in all the s_{ij}

$\Rightarrow$ Express cut on energy as $E_\gamma > \frac{m_b}{2}(1-\delta)$, translating (in the restframe of the bottom-quark) to

$$s_{14} + s_{24} + s_{34} > 1 - \delta.$$

This condition can then be incorporated into the integral:

$$\int_0^\delta dz \int_0^1 [ds_{ij}]\delta(1 - z - s_{14} - s_{24} - s_{34})\delta(z - s_{12} - s_{23} - s_{13}) \times$$

$$\times \mathcal{K}(s_{ij})(-\Delta_4)^{\frac{d-5}{2}}\Theta(-\Delta_4)$$

Sample Kernel

$$\mathcal{I} = \int dPS_4 \int \frac{d^4\ell}{(4\pi)^d} \frac{s_{13}s_{24}}{\ell^2(\ell+k_1+k_2+k_3)^2 s_{34}} (-\Delta_4)^{\frac{d-5}{2}}$$

$$\Rightarrow \int dPS_4 \frac{\Gamma(\epsilon)\Gamma(1-\epsilon)^2}{\Gamma(2-2\epsilon)} \frac{s_{13}s_{24}(s_{23}+s_{34}+s_{24})^{-\epsilon}}{s_{34}} (-\Delta_4)^{\frac{d-5}{2}}$$

use cyclicity in momenta of the light quarks $(3 \to 2 \to 1)$ and change of variables:

$$s_{13} = z - s_{23} - s_{12} \qquad s_{24} = \bar{z} - s_{14} - s_{34}$$
$$s_{12} = vwz \qquad s_{34} = \bar{z}\bar{v}$$
$$s_{14} = \bar{z}vx \qquad s_{23} = (a^+ - a^-)u + a^-$$

$$\Rightarrow \int_0^\delta dz (z\bar{z})^{d-3} \int_0^1 du\ dv\ dx\ dw\ (u\bar{u})^{\frac{d-5}{2}} v^{d-3} (\bar{v}w\bar{w}x\bar{x})^{\frac{d-4}{2}} \times$$
$$\Big[(a^+ - a^-)u + a^-\Big] x\bar{x}^{-1} \Big[v(wz+\bar{z})\Big]^{-\epsilon}$$

Sample Kernel

Evaluation of the integral leads to a sum of Hypergeometric functions:

$$c_1\ \bar{z}^{1-2\epsilon} z^{2-2\epsilon}\ {}_2\mathrm{F}_1(2-\epsilon,\epsilon;3-2\epsilon;z) + c_2\ \bar{z}^{1-2\epsilon} z^{2-2\epsilon}\ {}_2\mathrm{F}_1(1-\epsilon,\epsilon;2-2\epsilon;z)$$

where the c_i are functions of ϵ and we are still differential in the photon energy.

- Best case: fully analytic expression to all orders in ϵ, e.g. in terms of Hypergeometric and β-functions (the evaluation of the integral over z and the series in ϵ are interchangeable, if the former does not introduce new poles)
- Second best case: obtain result in terms of e.g. Mellin-Barnes representation, which can then be expanded as a series in ϵ

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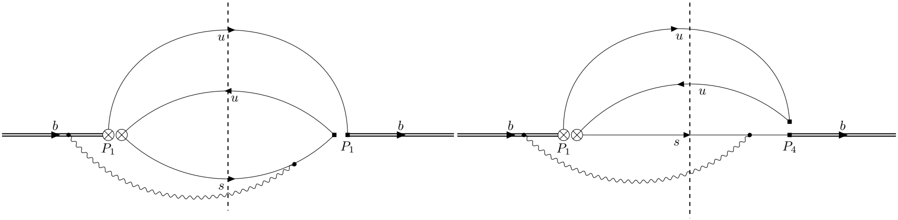
c) Renormalization

To cancel UV divergences, insertions of the bare operators $P_i^{(0)}$ into the tree-level diagrams have to be computed



With this, the renormalization constants δZ_{ij} can be used to cancel the UV-divergences via the relation

$$\sum_{i=1..6} \mathcal{C}_i P_i^{(0)} = \sum_{i=1..6} \mathcal{C}_i P_i + \frac{\alpha_s}{4\pi\epsilon} \sum_{i,j=1..6} \mathcal{C}_i \delta Z_{ij} P_j$$

d) IR regularization

- regions where photon is collinear to light quarks gives rise to collinear divergences
  $\rightarrow$ automatically regularized in DimReg
- divergences are artifact of massless limit
  $\rightarrow$ could more naturally be regulated by light quark masses, but massless case is already quite complicated
- fortunately, amplitudes in the quasi-collinear limit factorize:

$$b \rightarrow q_1 q_2 \bar{q}_3 \gamma \Rightarrow b \rightarrow \sum_i q_1 q_2 \bar{q}_3 \times f_i$$

with f_i a DGLAP splitting function describing emission of γ from q_i

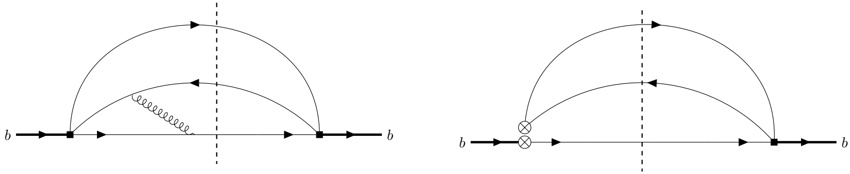
Comparing the splitting functions in the two different schemes of mass regulators and DimReg leads to the relation

$$\frac{d\Gamma_m}{dz} = \frac{d\Gamma_\epsilon}{dz} + \frac{d\Gamma_{shift}}{dz}$$

Shifting part can be calculated from three-particle-cut diagrams:

$$\frac{\Gamma_{shift}}{dz} = \frac{1}{2m_b}\frac{1}{2N_c}\int dPS_3\mathcal{K}_3(s_{ij})\frac{\alpha_e}{2\pi\bar{z}}\left\{Q_1^2\left[1+\frac{(z-s_{23})^2}{(1-s_{23})^2}\right]\right\}\times$$
$$\times\left[\frac{1}{\epsilon}-1+2\log\frac{(1-s_{23})\mu}{m_{q_1}(1-z)}\Theta(z-s_{23})+(\text{cyclic})\right]$$

$\Rightarrow$ trade the surviving $1/\epsilon$ terms for $\log(\frac{m_q}{m_b})$ terms



Status and Outlook

Current status: Setup for the automatic generation and evaluation is completed
$\rightarrow$ kernels that were obtained need to be integrated over the phase space

Problem: Very large number of different integrals

$\Rightarrow$ Goal: reduce number of numerators through application of integration-by-parts identities

IBP Reduction

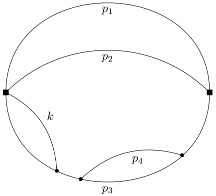
One subtlety: IBP relations do not know about the cut on the photon energy
$\rightarrow$ have to put it in by hand, using

$$\delta(p^2) \rightarrow \frac{1}{p^2 + i\varepsilon} - \frac{1}{p^2 - i\varepsilon}$$

for the relation from the cut in $E_\gamma : (z - s_{12} - s_{13} - s_{23}) = 0$.

[Anastasiou, Melnikov, arXiv:hep-ph/0207004]

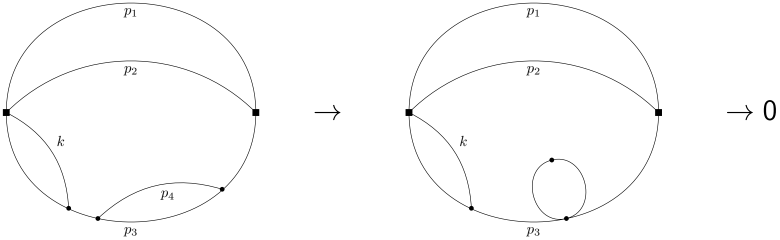
This relation is also used for the final state particles with $k_i^2 = 0$, leading to four-loop topologies that are getting reduced:



The reduction does not care about the ε prescription, so one can just use $\frac{1}{p^2}$ and turn it back to $\delta(p^2)$ after the reduction.

Advantage: if during the reduction, the power of one of the 5 propagators becomes 0 in a certain term, we can throw away that contribution:

$$\frac{1}{(p^2)^0} = 1 = \frac{p^2}{p^2} \to p^2\delta(p^2) = 0$$



Thank you for your attention!

5-particle phase space

$$s_{1345}/q^2 = t_7$$
$$s_{134}/q^2 = t_6 t_7$$
$$s_{13}/q^2 = t_6 t_7 \bar{t}_2$$
$$s_{23}/q^2 = t_3 \bar{t}_7 (1 - t_2 t_4)(t_6 \bar{t}_9 + t_9)$$
$$s_{14}/q^2 = t_2 t_4 t_6 t_7$$
$$s_{24}/q^2 = y_5^- + (y_5^+ - y_5^-) t_5)$$
$$s_{34}/q^2 = t_2 t_6 t_7 \bar{t}_4$$
$$s_{15}/q^2 = t_7 \bar{t}_6 [1 - t_9(1 - t_2 t_2)] - y_{10}$$
$$s_{25}/q^2 = y_8^- + (y_8^+ - y_8^-) t_8$$
$$s_{35}/q^2 = t_7 t_9 \bar{t}_6 (1 - t_2 t_4)$$
$$s_{45}/q^2 = y_{10}^- + (y_{10}^+ - y_{10}^-) t_{10}$$

$$\int d\Phi^D_{1\to 5} = \mathcal{K}_\Gamma^{(5)} (q^2)^{2D-5} \int_0^1 \prod_{j=2}^{10} dt_j [t_5 \bar{t}_5]^{-1-\epsilon} [t_8 \bar{t}_8 t_{10} \bar{t}10]^{-\frac{1}{2}-\epsilon}$$
$$\times [t_2 t_6 \bar{t}_6 \bar{t}_7]^{1-2\epsilon} [(\bar{t}_2 t_3 \bar{t}_3 t_4 \bar{t}_4 t_9 \bar{t}_9)]^{-\epsilon} t_7^{2-3\epsilon}$$