Orbital dynamics from double copy and EFT

Talk at Radcor Conference, Avignon, France, 11 Sep 2019,
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Outline

1. Introduction
2. Two-loop amplitudes
3. Effective potential between black holes
4. Discussions
Introduction
BIRTH OF AN ERA

• LIGO / VIRGO detected gravitational waves: BH-BH (2015), BH-NS (2017), NS-NS (2019?)

• Next-gen. experiments (LISA, CE, ET...): high S-N ratio, dominated by theory uncertainty.

• Precision predictions necessary.
ANATOMY OF GRAVITATIONAL WAVE SIGNAL

- **Inspiral**: Post-Newtonian / Post-Minkowskian / EOB
- **Merger**: Numerical relativity / EOB resummation
- **Ringdown**: Perturbative quasi-normal modes

[Picture: Antelis, Moreno, 1610.03567]
POST-NEwTONIAN EXPANSION

Potential in post Newtonian expansion, e.g. in c.o.m. frame:

\[ V = -\frac{G m_1 m_2}{r} \left(1 + \frac{\vec{p}^2}{m_1 m_2} \left(1 + \frac{3(m_1 + m_2)^2}{2m_1 m_2}\right)\right) \]

\[ + \left(1 + \frac{m_1 m_2}{(m_1 + m_2)^2}\right) \]

\[1PN, \sim G^2 \]

[Holstein, Ross, ’08; Neill, Rothstein, ’13]


[Talks by Christian Sturm, Andreas Maier]
BOUND ORBIT: $GM/r \sim v^2$. Hyperbolic orbit / scattering: expand with $GM/r \leq v \sim c$. [Bertotti, Kerr, Plebanski, Portilla, Westpfahl, Goller, Bel, Damour, Deruelle, Ibanez, Martin, Ledvinka, Schaefer, Bicak...]

Our new 3PM result: [Bern, Cheung, Roiban, Shen, Solon, MZ, PRL '19; 1908.01493 (long paper)] See also talk by Mikhail Solon
HOW QFT HELPS

Hierarchy of scales in bound state systems: effective field theory: [NRGR: Goldberger, Rothstein, ’04; Porto, ’06; relativistic formulation: Damgaard, Haddad, Helset, ’19]

[picture: LIGO]

Manifest gauge invariance through scattering amplitudes, with carefully defined classical limit [Iwasaki ’71; Gupta, Radford, ’79; Donoghue, ’94; Holstein, Donoghue, ’04; Neill, Rothstein, ’13; Vaidya, ’14; Kosower, Maybee, O’Connell, ’18 ...]
Two-loop amplitude
GRAVITY = (YANG MILLS)$^2$

Infinite tower, even 3-graviton vertex has $\sim 100$ terms.

$$\mathcal{A}_3(1^-2^-3^+) = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}, \quad \mathcal{M}_3(1^-2^-3^+) = \frac{\langle 12 \rangle^6}{\langle 23 \rangle^2 \langle 31 \rangle^2}$$

Simplification from “square” / double copy!

Kawai-Lewellen-Tye (KLT) relations from string theory:

$$\mathcal{M}^\text{tree}_4(1, 2, 3, 4) = -is_{12} \, A^\text{tree}_4(1, 2, 3, 4) \, A^\text{tree}_4(1, 2, 4, 3)$$
$$\mathcal{M}^\text{tree}_5(1, 2, 3, 4, 5) = is_{12}s_{34} \, A^\text{tree}_5(1, 2, 3, 4, 5) \, A^\text{tree}_5(2, 1, 4, 3, 5)$$
$$\quad + is_{13}s_{24} \, A^\text{tree}_5(1, 3, 2, 4, 5) \, A^\text{tree}_5(3, 1, 4, 2, 5)$$

4 dimensions: $g^\pm \otimes g^\pm \sim h^{\pm \pm}$.
GRAVITY = (YANG MILLS)²

$D$ dimensions: more convenient to use **color-kinematics duality**, i.e. double-copy construction. [Bern, Carrasco, Johansson, ’08]

Gauge theory amplitude: $\mathcal{A}_m^{\text{tree}} = g^{m-2} \sum_j \frac{c_j n_j}{D_j}$, in “BCJ form” if $n_j$ satisfies same Jacobi identities as $c_j$.

Then gravity amplitude $M_m^{\text{tree}} = i \sum_j \frac{\tilde{n}_j n_j}{D_j}$, from $c_j \rightarrow n_j$. 
TWO-LOOP CUTS

\[ S = \int d^D x \sqrt{g} \left[ -\frac{1}{2} R + \frac{1}{2} \sum_{i=1,2} (D^\mu \phi_i D_\mu \phi_i - m_i \phi_i^2) \right] \]

From KLT: \( M_{4\text{tree}}^{1,2,3,4} = -i s_{12} A_{4\text{tree}}^{1,2,3,4} A_{4\text{tree}}^{1,2,4,3} \).

\[ C_{\text{GR}}^{(c)} = -i \left\{ 2 t^2 m_1^4 m_2^4 + \frac{1}{t^6} \left[ Tr[(788615)^4] + (7 \leftrightarrow 8) \right] \right\} \left( \frac{1}{(k_5 - k_8)^2} + \frac{1}{(k_6 + k_8)^2} \right) \]

Very compact expression at 2 loops. Higher loops within reach!
TWO-LOOP INTEGRAND

Cuts \textit{merged} into an integrand with diagrams & numerators:

Diagram symmetries imposed. $\sim$ 90KB file.

\textbf{4 and $D$ dimensional} results agree, up to “$\mu$” terms with no classical effects.
INTEGRATING THE AMPLITUDE

• $m_1 \neq m_2$, even planar master integrals unknown!

\[
\begin{array}{c}
\ell^\mu \sim (q_v, q) \\
\sim \hbar/R \\
m_2
\end{array}
\quad
\begin{array}{c}
\ell^\mu \sim (q_v, q) \\
\sim \hbar/R \\
m_2
\end{array}
\]

$m_1 = m_2$ Smirnov, '01; Lower topologies: Henn & Smirnov, '13; Duhr, Amplitudes '18; Heller, von Manteuffel, Schabinger, '19

$m_1 = m_2$ Bianchi, Leoni, 1612.05609

• Simplification 1: expand in small $q \sim \hbar/R \leq m_i, \sqrt{s}$.

• Simplification 2: Expand in $v \ll 1$ from potential region. \((\int d^4\ell)\) localized on +ve energy matter poles.
NR INTEGRATION / VELOCITY EXPANSION

Plan: Series expansion around static limit, then resum by matching to simple functions.

Step 1: determine integrand in potential region

\[
\mathcal{I}_T = \frac{1}{(E_1 + \omega)^2 - (p + l)^2 - m_1^2} \times \frac{1}{\omega^2 - l^2} \frac{1}{\omega^2 - (l + q)^2}
\]

\[
= \frac{1}{\omega - \omega_{P_1}} \frac{1}{2m_1 l^2(l + q)^2} + \ldots
\]

\[
\frac{1}{(E_1 + \omega)^2 - (p + l)^2 - m_1^2} = \frac{1}{(\omega - \omega_{P_1})(\omega - \omega_{A_1})},
\]

\[
\omega_{P_1}, \omega_{A_1} = -E_1 \pm \sqrt{E_1^2 + 2pl + l^2}. \quad \omega_{P_1} \ll |l|, \; \omega_{A_1} \approx -2m_1
\]
NR INTEGRATION / VELOCITY EXPANSION

**Step 2:** Energy integration, keeping only residues from +ve energy matter poles.

\[
\frac{1}{2\pi} \int d\omega_1 \int d\omega_2 \delta(\omega_1 + \omega_2) \frac{1}{2} \left[ \frac{1}{\omega_1 - \omega_{P_1} + i0} + \frac{1}{\omega_2 - \omega_{P_1} + i0} \right] + \omega_1 \leftrightarrow \omega_2
\]

\[
= \frac{1}{2\pi} \int d\omega \frac{1}{2} \left[ \frac{1}{\omega - \omega_{P_1} + i0} + \frac{1}{-\omega - \omega_{P_1} + i0} \right] = -\frac{i}{2}
\]

**Step 3:** Spatial integration

\[
\int \frac{d^3l}{(2\pi)^3} \left( -\frac{i}{2} \right) \frac{1}{2m_1 l^2 (l + q)^2} = -\frac{i}{32m_1|q|}.
\]

Only need 3D propagator integrals. More in Mikhail Solon’s talk.
RELATIVISTIC INTEGRATION

Conservative contribution: localize on matter poles

\[ l_i = \int d^d \ell_1 \int d^d \ell_2 \frac{\delta(\ell_1^2 - m_1^2)\delta(\ell_2^2 - m_2^2)N_i}{\prod_{i=3}^{7} \ell_i^2} \]

Exact result from differential equations

\[ \frac{\partial l_i}{\partial s} = M_{ij} l_j. \]

IBP done by Kira [Maierhoefer, Usovitsch, Uwer, ’17]. 9 master integrals (H & N topology) rescaled to \( O(t^0) \). Take \( t \to 0 \) limit in \( M_{ij} \).

Boundary condition: regular at threshold \( s = (m_1 + m_2)^2 \).

Scalar result: \( H + xH = \frac{\log(-t) \arcsinh(|p|/m_1) + \arcsinh(|p|/m_2)}{128\pi^2 |p| (E_1 + E_2)} \).
COMPARISON WITH UNEXPANDED INTEGRAL

$m_1 = m_2$ results in [Bianchi, Leoni, 1612.05609], thanks to Loopedia.org.

\[
\frac{1}{(k_1 + p_2)^2 + i0} \approx \frac{1}{2k_1 \cdot p_2 + i0} \quad \text{and} \quad \frac{1}{(k_1 - p_3)^2 + i0} \approx \frac{1}{-2k_1 \cdot p_2 + i0}
\]

\[
s = (p_1 + p_2)^2 = -\frac{(1 - x)^2}{x}
\]

25 master integrals

\[
(I_H)\big|_{\epsilon_0} = -\frac{4}{3} \log(-t) x \frac{x}{1 - x^2} \left( \pi^2 \log x + \log^3 x \right) + (\text{non-singular in } t)
\]

\[
(I_{xH})\big|_{\epsilon_0} = -\frac{4}{3} \log(-t) \frac{-x}{1 - x^2} \left( \pi^2 \log(-x) + \log^3(-x) \right) + (\text{non-singular in } t).
\]

\[
\log(x) \rightarrow \log(-x) + i\pi, \quad (I_H + I_{xH})\big|_{\epsilon_0} = 4\pi^2 \log(-t) \log(-x) + \text{imaginary}
\]
Effective potential between black holes
PM POTENTIAL AS EFT MATCHING COEFFICIENT

\[(k_0, k) = \frac{i}{k_0 - \sqrt{k^2 + m_{A,B}^2 + i0}}, \]

- kinetic term with only +ve energy pole.

\[= -iV(k, k'), \]

- \(k^0\)-independent vertex.

Matching at \(L\) loops gives \(V(k, k')\) with smooth \(\hbar \rightarrow 0\) limit. Uncalculated IR divergent integrals cancel in matching.

\[M^{L-\text{loop}}_{\text{EFT}} = \frac{p}{-p} \frac{k_1}{-k_1} \cdots \frac{k_L}{-k_L} \frac{p'}{-p'} = M^{L-\text{loop}}_{\text{full}}\]

Alternative QM treatment: [Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove, ’19]
RESULT: 3PM CONSERVATIVE POTENTIAL

[Bern, Cheung, Roiban, Shen, Solon, MZ, PRL ’19; 1908.01493 (long paper)]

\[ H^{3PM}(p, r) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V^{3PM}(p, r) \]

\[ V^{3PM}(p, r) = \sum_{n=1}^{3} \left( \frac{G}{|r|} \right)^n c_n(p^2) \]

\[ m = m_1 + m_2, \quad \nu = \frac{m_1 m_2}{m^2}, \quad E = E_1 + E_2, \quad \xi = \frac{E_1 E_2}{E^2}, \quad \gamma = \frac{E}{m}, \quad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2} \]

\[ c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} \left( 1 - 2\sigma^2 \right), \quad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi} \left[ \frac{3}{4} \left( 1 - 5\sigma^2 \right) - \frac{4\nu\sigma \left( 1 - 2\sigma^2 \right)}{\gamma \xi} - \frac{\nu^2 \left( 1 - \xi \right) \left( 1 - 2\sigma^2 \right)^2}{2\gamma^3 \xi^2} \right], \]

\[ c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi} \left[ \frac{1}{12} \left( 3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu^3 \right) - \frac{4\nu \left( 3 + 12\sigma^2 - 4\sigma^4 \right) \arcsinh \sqrt{\frac{\sigma - 1}{\sigma + 1}}}{\sqrt{\sigma^2 - 1}} \right. \]

\[ - \frac{3\nu\gamma \left( 1 - 2\sigma^2 \right) \left( 1 - 5\sigma^2 \right)}{2(1 + \gamma)(1 + \sigma)} - \frac{3\nu\sigma \left( 7 - 20\sigma^2 \right)}{2\gamma \xi} + \frac{2\nu^3 \left( 3 - 4\xi \right)\sigma \left( 1 - 2\sigma^2 \right)^2}{\gamma^4 \xi^3} \]

\[ - \frac{\nu^2 \left( 3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2 \right) \left( 1 - 2\sigma^2 \right)}{4\gamma^3 \xi^2} + \frac{\nu^6 \left( 1 - 2\xi \right) \left( 1 - 2\sigma^2 \right)^3}{2\gamma^6 \xi^4} \]
VALIDATION

• 4PN part of Hamiltonian agrees with [Jaranowski, Schäfer, 1508.01016]. New 5PN part subsequently verified [Bini, Damour, Geralico, ’19].
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  2. Gauge-invariant EFT amplitudes.

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• Apparent collinear divergence due to expansion in small $q \ll m$; small-$q$ limit does not commute with small-$m$ regge limit of [Amati, Ciafaloni, Veneziano, '90].

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PREDICTIONS FOR BINDING ENERGY

Comparision with state-of-the-art numerical relativity ("truth"), PN, and EOB approximations. [Antonelli et al., 2019]
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Comparision with state-of-the-art numerical relativity ("truth"), PN, and EOB approximations. [Antonelli et al., 2019]

- Clear improvement over lower PM orders.
- Not reaching accuracy of 4PN.
- Hyperbolic orbit comparison would be more illuminating.
FUTURE OUTLOOK

• **Higher orders** within reach. Double copy and EFT integration methods expected to scale well.
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• Relativistic integration to be further explored to bypass velocity resummation → Canonical differential equations for integrals in soft expansion.

• Spin, finite-size effects in PM expansion [Bini, Damour, ’17; Vines, ’17, Bini, Damour, ’18; Guevera, Ochirov, Vines, ’18; Vines, Steinhoff, Buonanno, ’18; Chung, Huang, Kim, Lee, ’18; Maybee O’Connell, Vines, ’19; Guevera, Ochirov, Vines, ’19]
Thank you!