ETH zürich

Orbital dynamics from double copy and EFT

Talk at Radcor Conference, Avignon, France, 11 Sep 2019,

with Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, arXiv:1901.04424 (PRL), arXiv:1908.01493

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11 Sep, 2019

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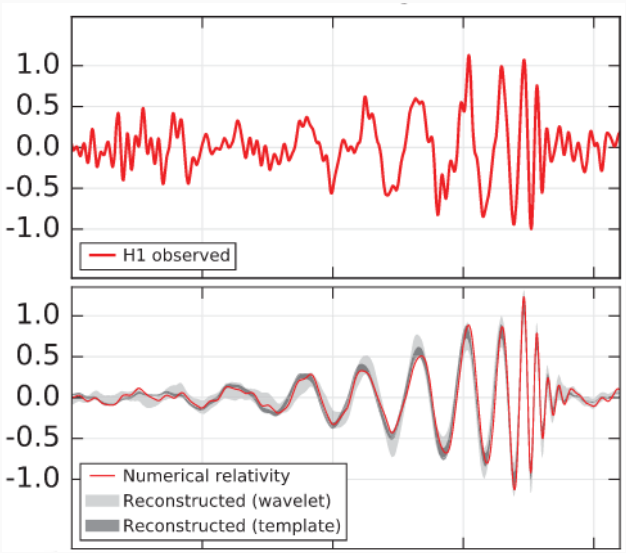
Outline

1. Introduction
2. Two-loop amplitudes
3. Effective potential between black holes
4. Discussions

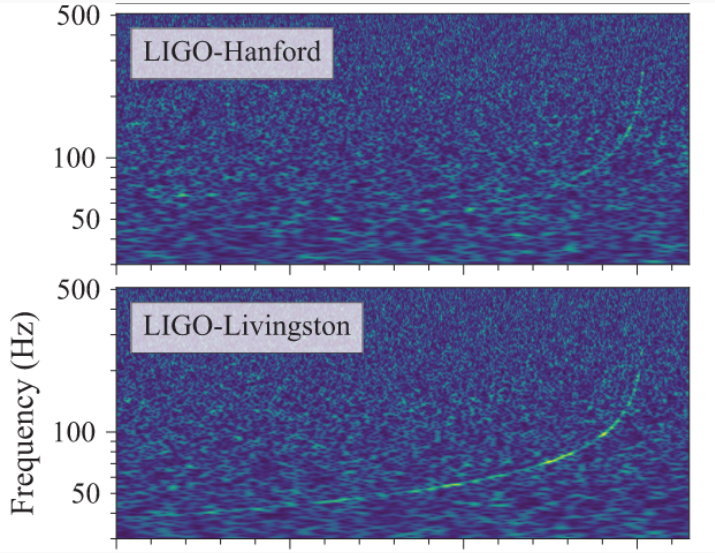
Introduction

BIRTH OF AN ERA

- LIGO / VIRGO detected gravitational waves: BH-BH (2015), BH-NS (2017), NS-NS (2019?)



LIGO & VIRGO, arXiv:1602.03837

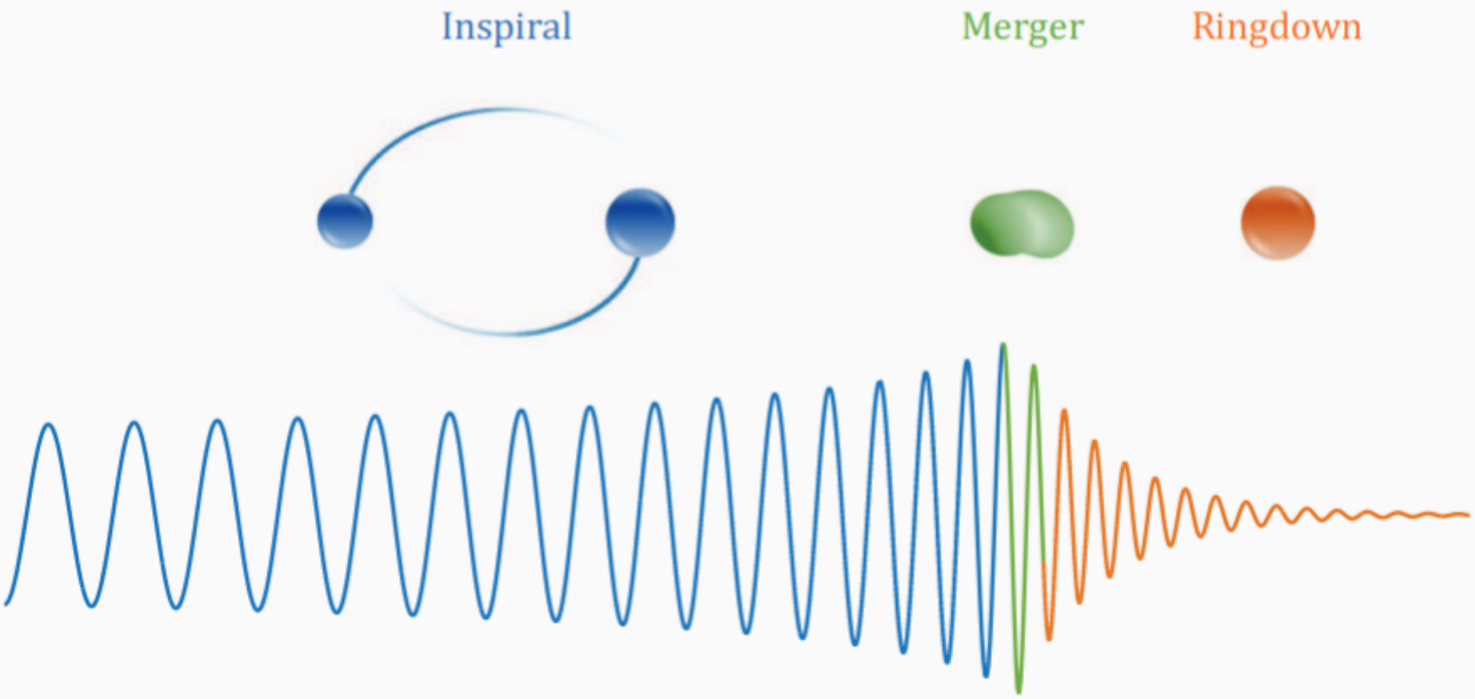


LIGO & VIRGO, arXiv:1710.05832

- Next-gen. experiments (LISA, CE, ET...): high S-N ratio, dominated by *theory uncertainty*.

- **Precision** predictions necessary.

ANATOMY OF GRAVITATIONAL WAVE SIGNAL



[Picture: Antelis, Moreno, 1610.03567]

Inspiral	Post-Newtonian / Post-Minkowskian / EOB
Merger	Numerical relativity / EOB resummation
Ringdown	Perturbative quasi-normal modes

POST-NEWTONIAN EXPANSION

Potential in post Newtonian expansion, e.g. in c.o.m. frame:

$$V = \underbrace{\boxed{-\frac{Gm_1m_2}{r}}}_{\text{0PN},\ \sim G}\left(1 + \underbrace{\boxed{\frac{\vec{p}^2}{m_1m_2}}}_{\text{1PN},\ \sim Gv^2}\left(1 + \frac{3(m_1+m_2)^2}{2m_1m_2}\right)\right) + \underbrace{\boxed{\frac{G^2m_1m_2(m_1+m_2)}{2r^2}}}_{\text{1PN},\ \sim G^2}\left(1 + \frac{m_1m_2}{(m_1+m_2)^2}\right) + \dots$$

[Holstein, Ross, '08; Neill, Rothstein, '13]

1PN [Einstein, Infeld, Hoffman '38]. **2PN** [Ohta *et al.*, '73]. **3PN** [Jaranowski, Schaefer, '97; Damour, Jaranowski, Schaefer, '97; Blanchet, Faye, '00; Damour, Jaranowski, Schaefer, '01] **4PN** [Damour, Jaranowski, Schäfer, Bernard, Blanchet, Bohe, Faye, Marsat, Marchand, Foffa, Sturani, Mastrolia, Sturm, Porto, Rothstein...] **5PN static** [Foffa, Mastrolia, Sturani, Sturm, Bodabilla, '19; Blümlein, Maier, Marquard, '19] **5PN approximate** [Bini, Damour, Geralico, '19]

[Talks by Christian Sturm, Andreas Maier]

POST-MINKOWSKIAN EXPANSION

Bound orbit: $GM/r \sim v^2$. Hyperbolic orbit / scattering: expand with $GM/r \leq v \sim c$. [Bertotti, Kerr, Plebanski, Portilla, Westpfahl, Goller, Bel, Damour, Deruelle, Ibanez, Martin, Ledvinka, Schaefer, Bicak...]

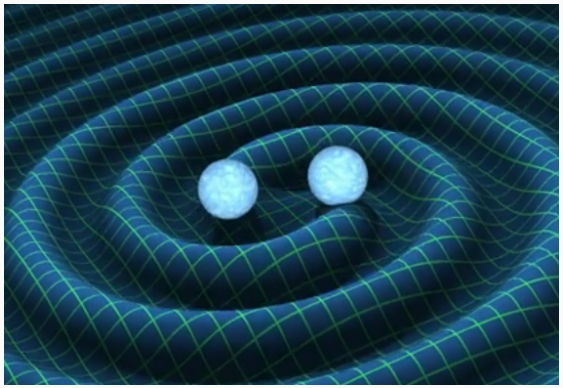
	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \ldots)$	G^1
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ \ldots)$	G^2
3PM			$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ \ldots)$	G^3
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ \ldots)$	G^4
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ \ldots)$	G^5
6PM						$(1$	$+\ v^2$	$+\ v^4$	$+\ \ldots)$	G^6
							$\vdots$			

Our new 3PM result: [Bern, Cheung, Roiban, Shen, Solon, MZ, PRL '19; 1908.01493 (long paper)] See also talk by Mikhail Solon

HOW QFT HELPS

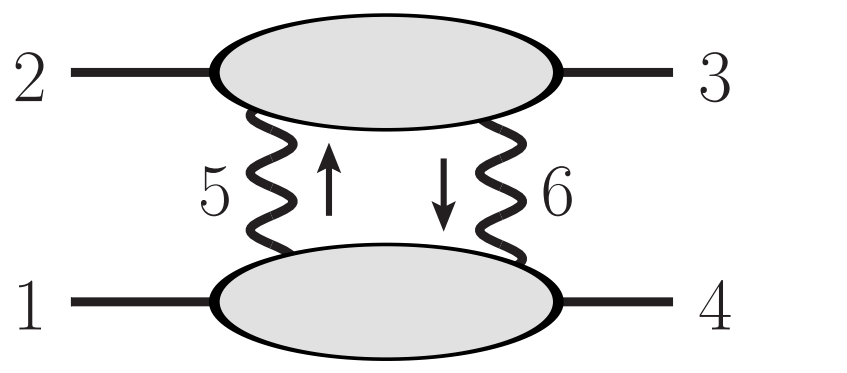
Hierarchy of scales in bound state systems:

effective field theory: [NRGR: Goldberger, Rothstein, '04; Porto, '06; relativistic formulation: Damgaard, Haddad, Helset, '19]



[picture: LIGO]

?



Manifest *gauge invariance* through **scattering amplitudes**, with carefully defined *classical limit* [Iwasaki '71; Gupta, Radford, '79; Donoghue, '94; Holstein, Donoghue, '04; Neill, Rothstein, '13; Vaidya, '14; Kosower, Maybee, O'Connell, '18 ...]

Two-loop amplitude

GRAVITY = (YANG MILLS)^2

Infinite tower, even 3-graviton vertex has ~ 100 terms.

$$\mathcal{A}_3(1^-2^-3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}, \quad \mathcal{M}_3(1^-2^-3^+) = \frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2}$$

Simplification from "square" / double copy!

Kawai-Lewellen-Tye (KLT) relations from string theory:

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}\, A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)$$

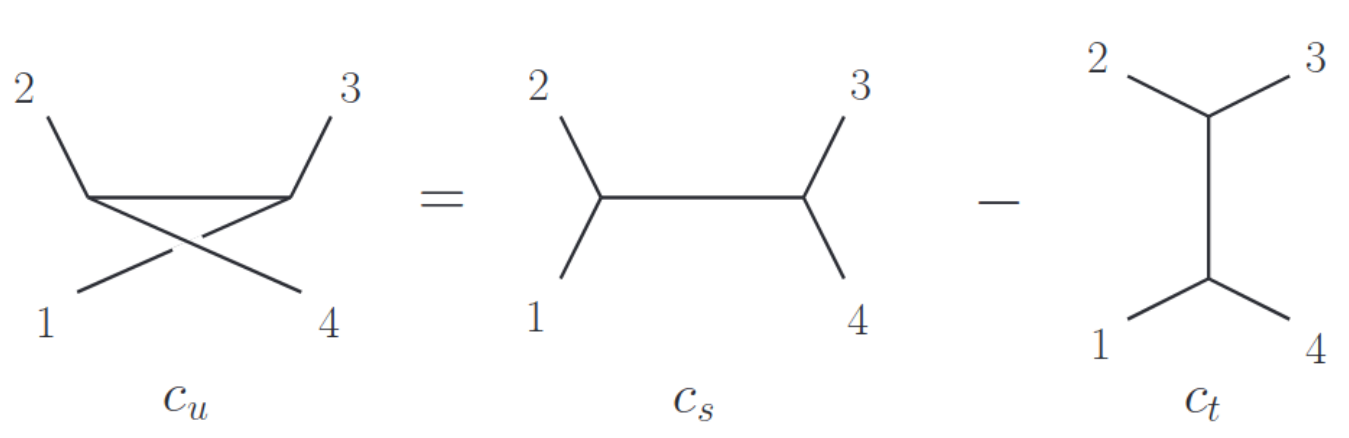
$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}\, A_5^{\text{tree}}(1,2,3,4,5)\, A_5^{\text{tree}}(2,1,4,3,5) + is_{13}s_{24}\, A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5)$$

4 dimensions: $g^\pm \otimes g^\pm \sim h^{\pm\pm}$.

GRAVITY = (YANG MILLS)^2

D dimensions: more convenient to use **color-kinematics duality**, i.e. double-copy construction. [Bern, Carrasco, Johansson, '08]

Gauge theory amplitude: $\mathbb{A}_m^{\text{tree}} = g^{m-2} \sum_j \frac{c_j n_j}{D_j}$, in "BCJ form" if n_j satisfies same Jacobi identities as c_j.

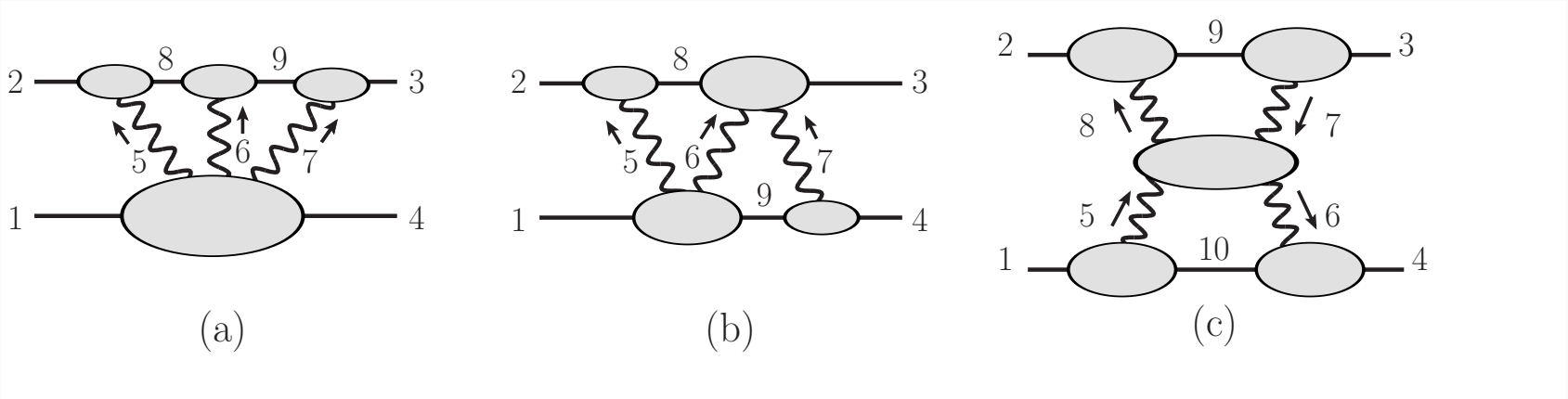


Then gravity amplitude $M_m^{\text{tree}} = i \sum_j \frac{\tilde{n}_j n_j}{D_j}$, from $c_j \to n_j$.

TWO-LOOP CUTS

[Bern, Cheung, Roiban, Shen, Solon, MZ, PRL '19; 1908.01493 (long paper)]

$$S = \int d^D x \sqrt{g} \left[-\frac{1}{2} R + \frac{1}{2} \sum_{i=1,2} \left(D^\mu \phi_i D_\mu \phi_i - m_i \phi_i^2 \right) \right]$$



From KLT: $M_4^{\text{tree}}(1,2,3,4) = -i s_{12} \mathcal{A}_4^{\text{tree}}(1,2,3,4) A_4^{\text{tree}}(1,2,4,3)$.

$$C_{\text{GR}}^{(c)} = -i \left\{ 2t^2 m_1^4 m_2^4 + \frac{1}{t^6} \left[Tr[(\not{7}\not{8}\not{6}\not{1})^4] + (7 \leftrightarrow 8) \right] \right\} \left(\frac{1}{(k_5 - k_8)^2} + \frac{1}{(k_6 + k_8)^2} \right)$$

Very compact expression at 2 loops. Higher loops within reach!

TWO-LOOP INTEGRAND

Cuts *merged* into an integrand with diagrams & numerators:

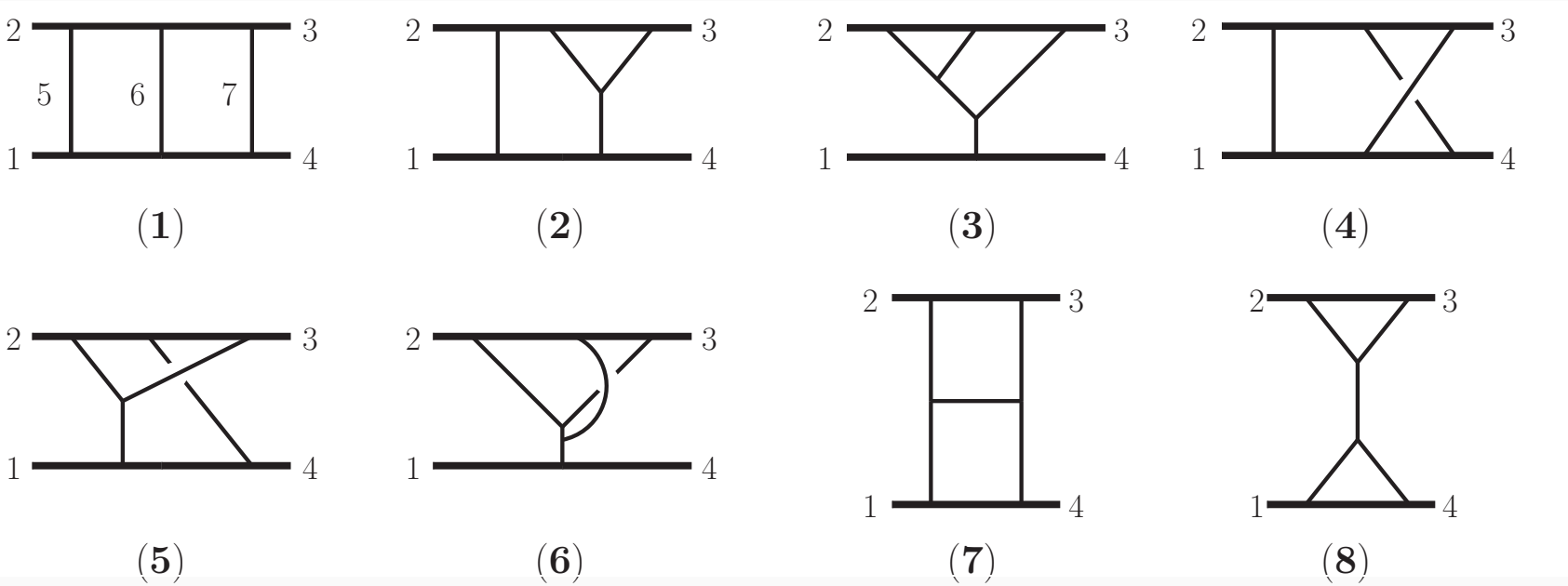
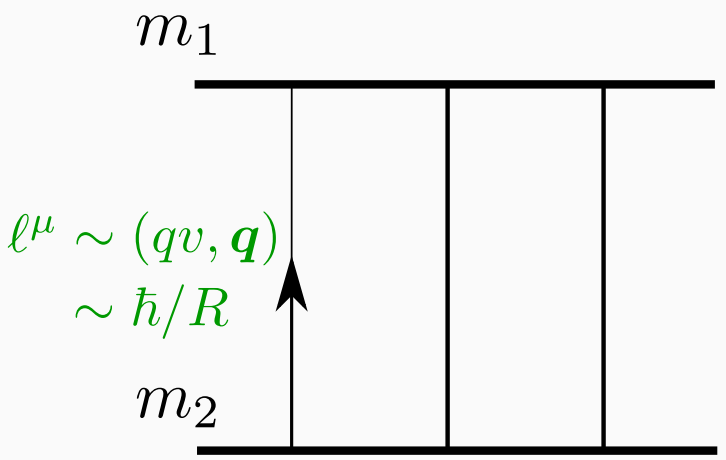


Diagram symmetries imposed. $\sim$ 90KB file.

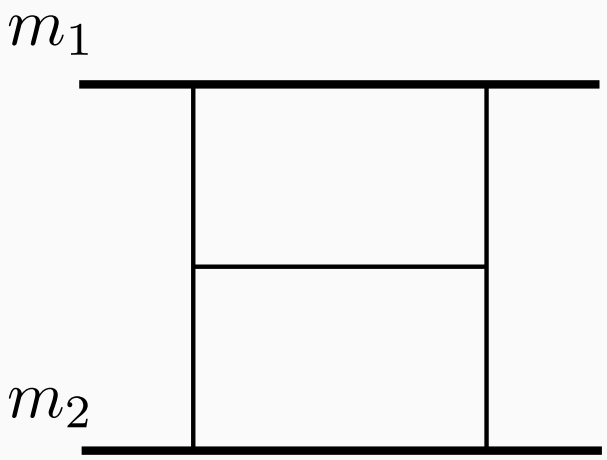
4 and D dimensional results agree, up to "μ" terms with no classical effects.

INTEGRATING THE AMPLITUDE

- $m_1 \neq m_2$, even planar master integrals unknown!



$m_1 = m_2$ Smirnov, '01; Lower topologies: Henn & Smirnov, '13; Duhr, Amplitudes '18; Heller, von Manteuffel, Schabinger, '19



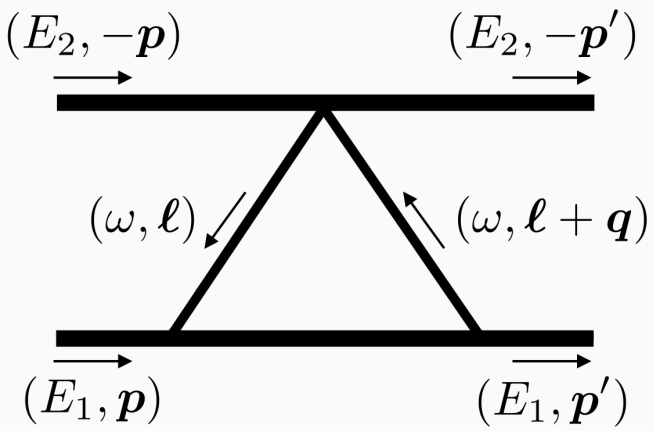
$m_1 = m_2$ Bianchi, Leoni, 1612.05609

- Simplification 1: expand in small $q \sim \hbar/R \leq m_i, \sqrt{s}$.
- Simplification 2: Expand in $v \ll 1$ from potential region. ($\int d^4\ell$) localized on $+$ve energy matter poles.

NR INTEGRATION / VELOCITY EXPANSION

Plan: Series expansion around static limit, then resum by matching to simple functions.

Step 1: determine integrand in potential region



$$\mathcal{I}_{\mathrm{T}} = \frac{1}{(E_1+\omega)^2 - (\boldsymbol{p}+\boldsymbol{l})^2 - m_1^2} \times \frac{1}{\omega^2 - \boldsymbol{l}^2} \frac{1}{\omega^2 - (\boldsymbol{l}+\boldsymbol{q})^2}$$

$$= \boxed{\frac{1}{\omega - \omega_{P_1}}} \frac{1}{2m_1 \boldsymbol{l}^2 (\boldsymbol{l}+\boldsymbol{q})^2} + \dots$$

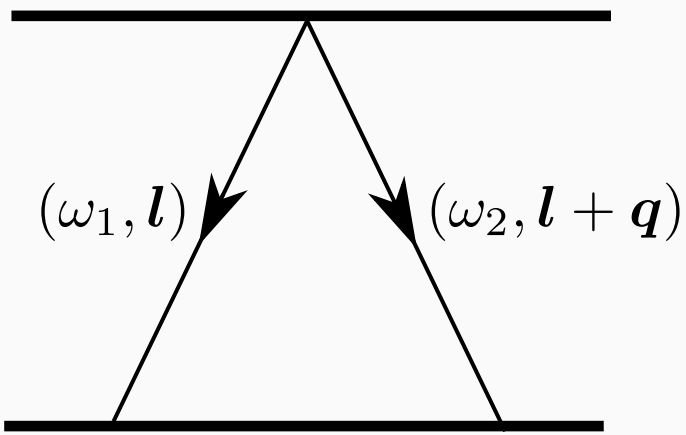
$$\frac{1}{(E_1+\omega)^2 - (\boldsymbol{p}+\boldsymbol{l})^2 - m_1^2} = \frac{1}{(\omega - \omega_{P_1})(\omega - \omega_{A_1})},$$

$$\omega_{P_1}, \omega_{A_1} = -E_1 \pm \sqrt{E_1^2 + 2\boldsymbol{p}\boldsymbol{l} + \boldsymbol{l}^2}\,. \quad \omega_{P_1} \ll |\boldsymbol{l}|,\ \omega_{A_1} \approx -2m_1$$

NR INTEGRATION / VELOCITY EXPANSION

Step 2: Energy integration, keeping only residues from +ve energy matter poles.

$$\frac{1}{2\pi}\int d\omega_1 \int d\omega_2\, \delta(\omega_1+\omega_2)\,\frac{1}{2}\Big[\frac{1}{\omega_1-\omega_{P_1}+i0}+\omega_1\leftrightarrow\omega_2\Big]$$
$$=\frac{1}{2\pi}\int d\omega\,\frac{1}{2}\Big[\frac{1}{\omega-\omega_{P_1}+i0}+\frac{1}{-\omega-\omega_{P_1}+i0}\Big]=-\frac{i}{2}$$

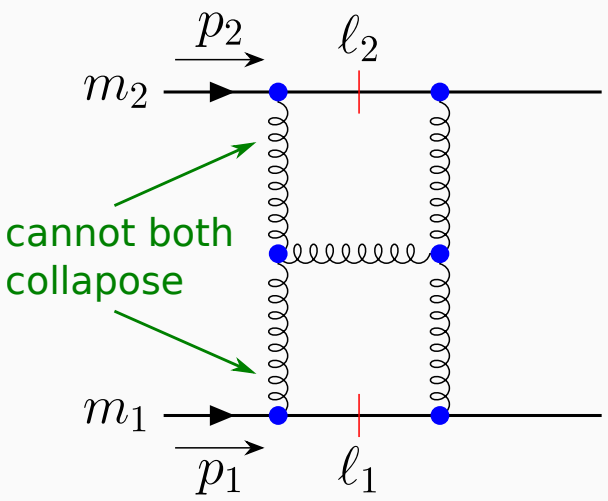


Step 3: Spatial integration

$$\int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\left(-\frac{i}{2}\right)\frac{1}{2m_1\,\boldsymbol{l}^2(\boldsymbol{l}+\boldsymbol{q})^2}=-\frac{i}{32m_1|\boldsymbol{q}|}\,.$$

Only need 3D propagator integrals. More in Mikhail Solon's talk

RELATIVISTIC INTEGRATION



Conservative contribution: localize on matter poles

$$I_i = \int d^d\ell_1 \int d^d\ell_2 \frac{\delta(\ell_1^2 - m_1^2)\delta(\ell_2^2 - m_2^2)\,\mathcal{N}_i}{\prod_{i=3}^{7} \ell_i^2}$$

Exact v **result** from differential equations $\boxed{\partial I_i/\partial s = M_{ij}\, I_j.}$

IBP done by Kira [Maierhoefer, Usovitsch, Uwer, '17]. 9 master integrals (H & N topology) rescaled to $\mathcal{O}(t^0)$. Take $t \to 0$ limit in M_{ij}.

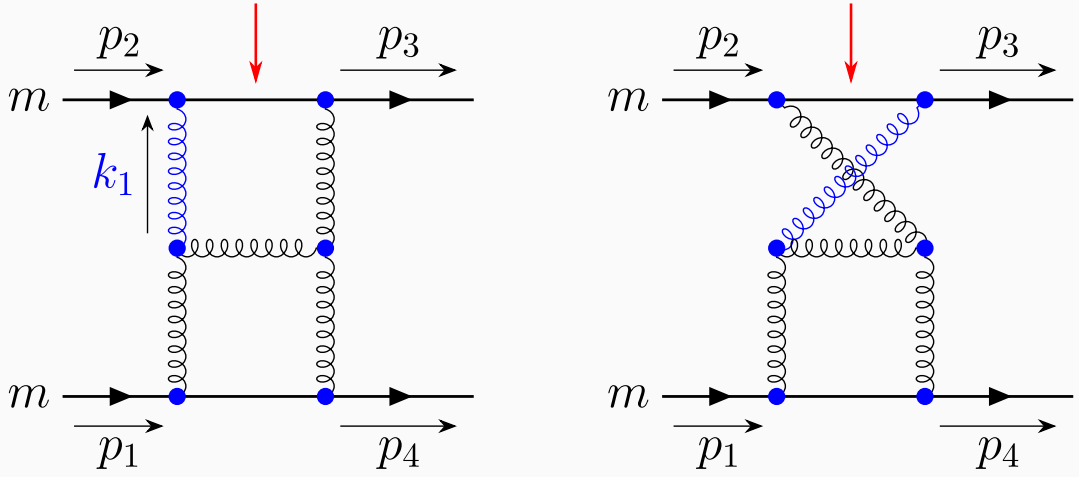
Boundary condition: regular at threshold $s = (m_1 + m_2)^2$.

Scalar result: H + xH $= \dfrac{\log(-t)}{128\pi^2} \dfrac{\text{arcsinh}\left(|\boldsymbol{p}|/m_1\right) + \text{arcsinh}\left(|\boldsymbol{p}|/m_2\right)}{|\boldsymbol{p}|\,(E_1 + E_2)}.$

COMPARISON WITH UNEXPANDED INTEGRAL

$m_1 = m_2$ results in [Bianchi, Leoni, 1612.05609], thanks to *Loopedia.org*.

$$\frac{1}{(k_1+p_2)^2+i0} \approx \frac{1}{2k_1\cdot p_2+i0} \qquad \frac{1}{(k_1-p_3)^2+i0} \approx \frac{1}{-2k_1\cdot p_2+i0}$$



$$s = (p_1+p_2)^2 = -\frac{(1-x)^2}{x}$$

25 master integrals

$$(I_H)\big|_{\epsilon^0} = -\frac{4}{3}\log(-t)\frac{x}{1-x^2}\left(\pi^2\log x + \log^3 x\right) + (\text{non-singular in } t)$$

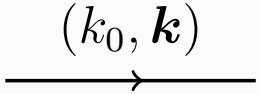
$$(I_{xH})\big|_{\epsilon^0} = -\frac{4}{3}\log(-t)\frac{-x}{1-x^2}\left(\pi^2\log(-x) + \log^3(-x)\right) + (\text{non-singular in } t)\,.$$

$$\log(x) \to \log(-x) + i\pi, \quad (I_H + I_{xH})\big|_{\epsilon^0} = 4\pi^2\log(-t)\log(-x) + \text{imaginary}$$

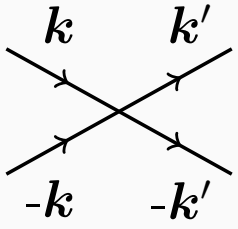
Effective potential between black holes

PM POTENTIAL AS EFT MATCHING COEFFICIENT

[From Cheung, Rothstein, Solon, '18 PRL]



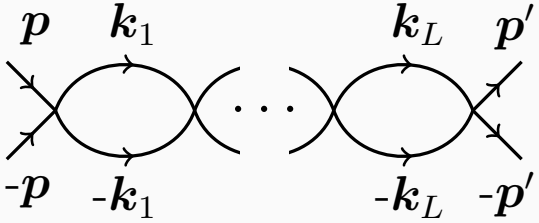
$$= \frac{i}{k_0 - \sqrt{\boldsymbol{k}^2 + m_{A,B}^2} + i0}\,,$$



$$= -iV(\boldsymbol{k}, \boldsymbol{k}')\,,$$

- kinetic term with only +ve energy pole.
- k^0-independent vertex.

Matching at L loops gives $V(\boldsymbol{k}, \boldsymbol{k}')$ with smooth $\hbar \to 0$ limit.
Uncalculated IR divergent integrals cancel in matching.

$\mathcal{M}_{\text{EFT}}^{L\text{-loop}} =$ 

$= \mathcal{M}_{\text{full}}^{L\text{-loop}}$

Alternative QM treatment: [Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove, '19]

RESULT: 3PM CONSERVATIVE POTENTIAL

[Bern, Cheung, Roiban, Shen, Solon, MZ, PRL '19; 1908.01493 (long paper)]

$$H^{3\mathrm{PM}}(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{3\mathrm{PM}}(\boldsymbol{p},\boldsymbol{r})$$

$$V^{3\mathrm{PM}}(\boldsymbol{p},\boldsymbol{r}) = \sum_{n=1}^{3} \left(\frac{G}{|\boldsymbol{r}|}\right)^n c_n(\boldsymbol{p}^2) \longrightarrow \text{EFT matching coefficients}$$

$$m = m_1 + m_2,\ \nu = \frac{m_1 m_2}{m^2},\ E = E_1 + E_2,\ \xi = \frac{E_1 E_2}{E^2},\ \gamma = \frac{E}{m},\ \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right), \quad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}}\right.$$

$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3}$$

$$\left. - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right].$$

VALIDATION

- 4PN part of Hamiltonian agrees with [Jaranowski, Schäfer, 1508.01016]. New 5PN part subsequently verified [Bini, Damour, Geralico, '19].

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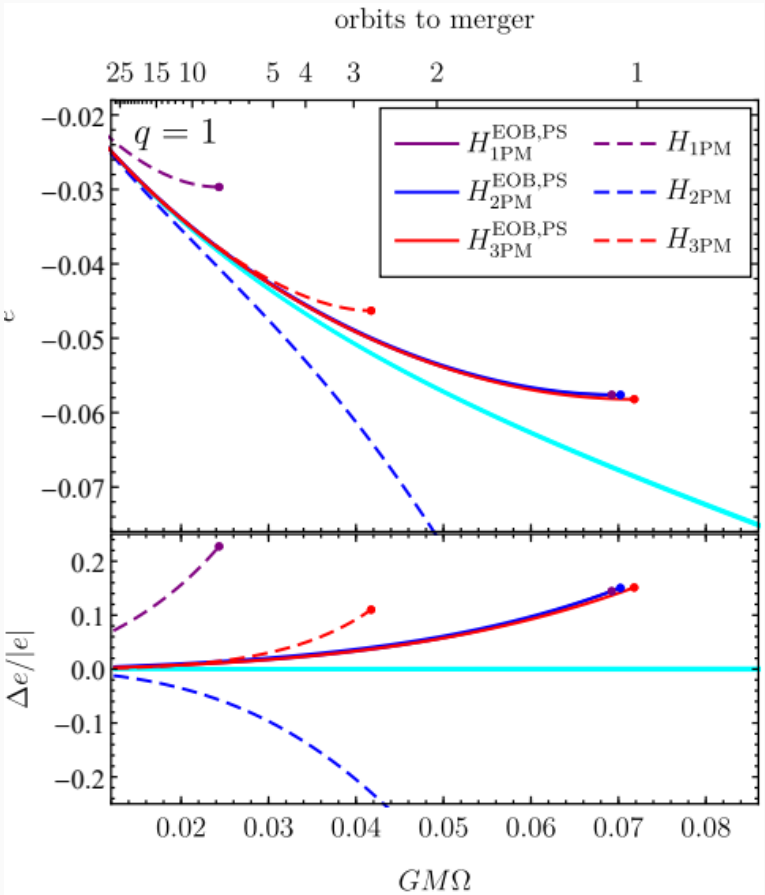
  4. Binding-energy of quasi-circular orbit.
- "μ" terms have no classical effects, by dimension shifting.
- Apparent collinear divergence due to expansion in small $q \ll m, s$. Small-q limit does not commute with small-m regge limit of [Amati, Ciafaloni, Veneziano, '90].

PREDICTIONS FOR BINDING ENERGY

Comparision with state-of-the-art numerical relativity ("truth"), PN, and EOB approximations. [Antonelli *et al.*, 2019]



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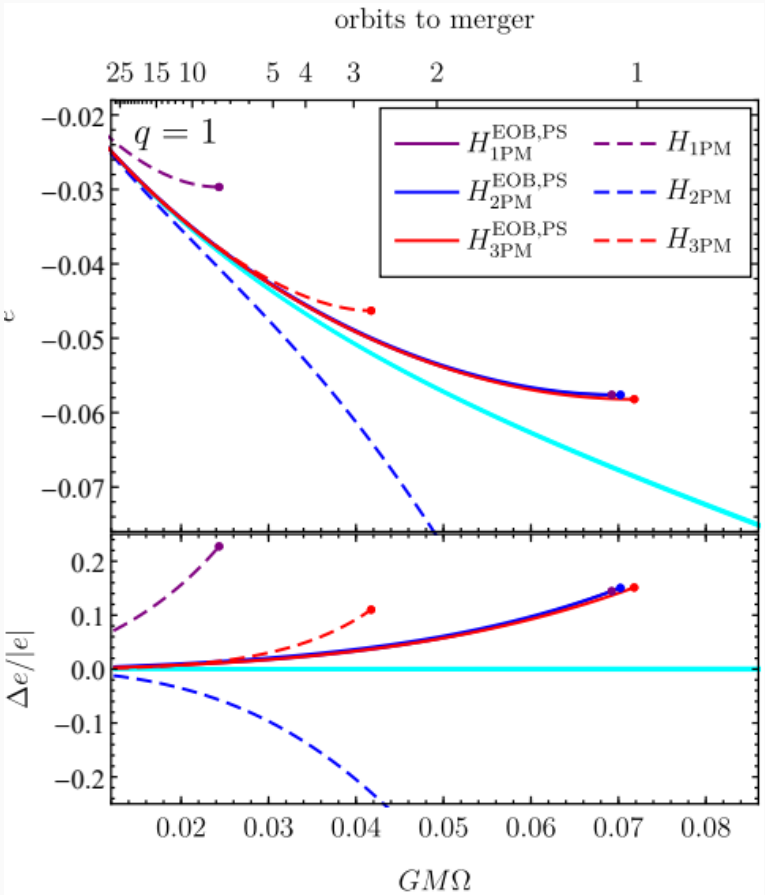

- Clear improvement over lower PM orders.
- Not reaching accuracy of 4PN.
- Hyperbolic orbit comparison would be more illuminating.

FUTURE OUTLOOK

- **Higher orders** within reach. Double copy and EFT integration methods expected to scale well.

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- **Tail effect**, i.e. conservative radiation reaction, to be treated by ultrasoft modes in EFT.
- **Relativistic integration** to be further explored to bypass velocity resummation $\Longrightarrow$ Canonical differential equations for integrals in soft expansion.

FUTURE OUTLOOK

- **Higher orders** within reach. Double copy and EFT integration methods expected to scale well.
- **Tail effect**, i.e. conservative radiation reaction, to be treated by ultrasoft modes in EFT.
- **Relativistic integration** to be further explored to bypass velocity resummation $\Longrightarrow$ Canonical differential equations for integrals in soft expansion.
- **Spin, finite-size effects** in PM expansion [Bini, Damour, '17; Vines, '17, Bini, Damour, '18; Guevera, Ochirov, Vines, '18; Vines, Steinhoff, Buonanno, '18; Chung, Huang, Kim, Lee, '18; Maybee O'Connell, Vines, '19; Guevera, Ochirov, Vines, '19]

Thank you!