

The Gravitational Potential of Two Point Masses at Five Loops

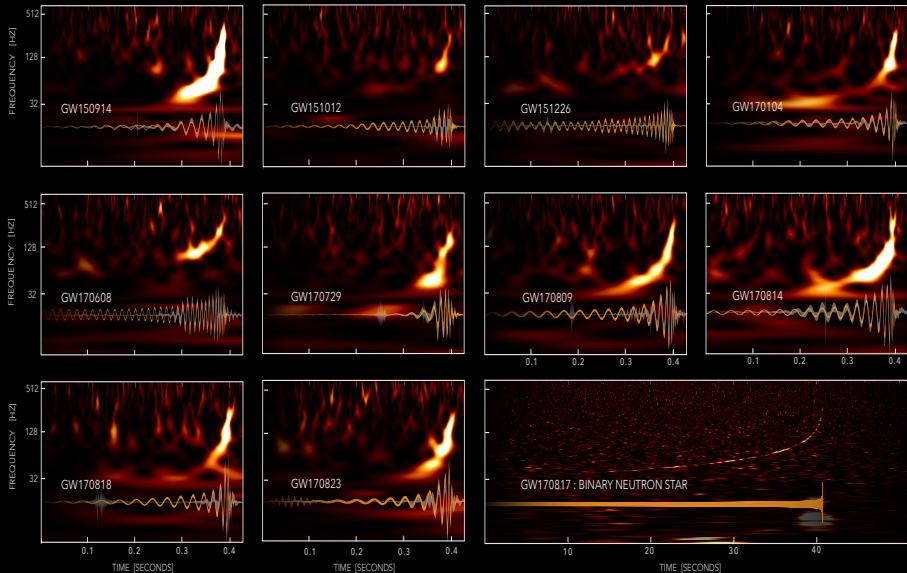
Andreas Maier



Avignon, 11 September 2019

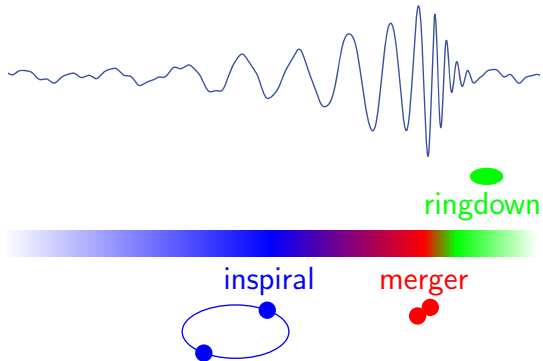
J. Blümlein, A. Maier, P. Marquard, arXiv:1902.11180

GRAVITATIONAL-WAVE TRANSIENT CATALOG-1



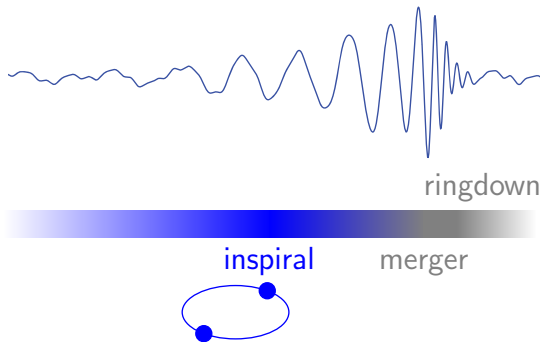
Gravitational waves

[LIGO Scientific Collaboration and Virgo Collaboration 2016]



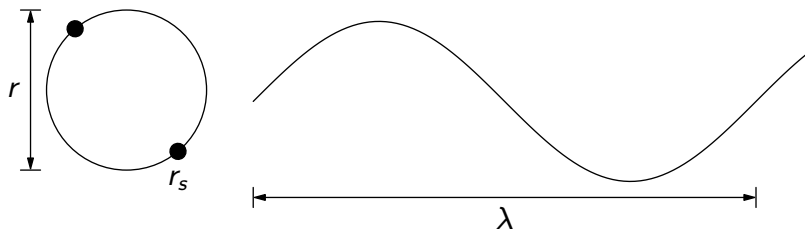
Gravitational waves

[LIGO Scientific Collaboration and Virgo Collaboration 2016]



Compact binary systems

Power counting

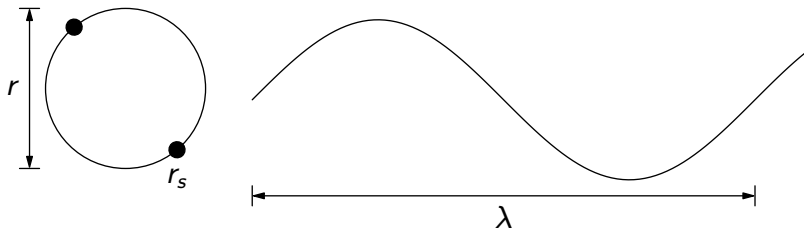


- Masses comparable: $m \equiv m_1 \sim m_2$
Generalisation to different masses straightforward
- Nonrelativistic system: $v \ll 1$
- Virial theorem: $mv^2 \sim \frac{Gm^2}{r}$

Post-Newtonian (PN) expansion:
Combined expansion in $v \sim \sqrt{Gm/r} \ll 1$

Post-Newtonian expansion

Scales

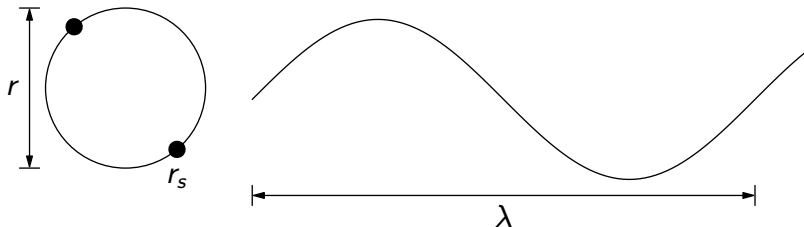


- $\omega \approx \frac{2v}{r} \Rightarrow \lambda \sim \frac{r}{v}$

- $r_s = 2GM \Rightarrow r_s \sim rv^2$

Post-Newtonian expansion

Scales at LIGO/VIRGO



- $10 \text{ km} \lesssim \lambda \lesssim 10\,000 \text{ km}$
- $10 \text{ km} \lesssim r_s \lesssim 100 \text{ km}$
- Inspiral: $0.1 \lesssim v \lesssim 0.5$, $100 \text{ km} \lesssim r \lesssim 1000 \text{ km}$

	black holes	neutron stars
masses	$\sim 10\text{--}50 m_\odot$	$\sim 1 m_\odot$
radiated energy	$\sim 1\text{--}5 m_\odot$	$\geq 0.04 m_\odot$
redshift	$\sim 0.1\text{--}0.5$	~ 0.01

Post-Newtonian expansion

Theory status

Conservative dynamics, no spin: complete results up to 4PN (v^8)

- ADM Hamiltonian formalism [Damour, Jaranowski, Schäfer 2016]
- Fokker Lagrangian in harmonic coordinates
[Bernard, Blanchet, Bohé, Faye, Marchant, Marsat 2017]
- Non-relativistic effective field theory
[Foffa, Mastrolia, Porto, Rothstein, Sturani, Sturm 2017–2019]

Method in this talk:
non-relativistic effective field theory [Goldberger, Rothstein 2004]

General Relativity

- Here: point-like objects
 - No spin
 - No finite-size effects (neutron stars: 5PN, black holes: 6PN)
- Harmonic gauge fixing: $\partial_\mu(\sqrt{-g}g^{\mu\nu} - \eta^{\mu\nu}) = 0$
 $g = \det(g^{\mu\nu})$
- Dimensional regularisation: $d = 3 - 2\epsilon$

$$S_{\text{GR}} = S_{\text{EH}} + S_{\text{GF}} + S_{\text{pp}}$$

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{-g} R$$

$$S_{\text{GF}} = -\frac{1}{32\pi G} \int d^{d+1}x \sqrt{-g} \Gamma_\mu \Gamma^\mu$$

$$S_{\text{pp}} = -\sum_i m_i \int d\tau_i = -\sum_i m_i \int dt \sqrt{-g_{\mu\nu} \frac{\partial x_i^\mu}{\partial t} \frac{\partial x_i^\nu}{\partial t}}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$\Gamma^\mu = g^{\alpha\beta} \Gamma_{\alpha\beta}^\mu$$

Non-relativistic effective theory

[Goldberger, Rothstein 2004]

Similar to non-relativistic QCD

[Caswell, Lepage 1985; Pineda, Soto 1997; Luke, Manohar, Rothstein 2000; ...]

Full theory:

General relativity

$$S_{\text{GR}} = S_{\text{EH}} + S_{\text{GF}} + S_{pp} \quad \longrightarrow$$

potential gravitons:

$$k_0 \sim \frac{v}{r}, \vec{k} \sim \frac{1}{r}$$

radiation gravitons:

$$k_0 \sim \frac{v}{r}, \vec{k} \sim \frac{v}{r}$$

Effective theory:

NRGR

$$S_{\text{NRGR}} = \int dt \frac{1}{2} m_i v_i^2 + \frac{G m_1 m_2}{r} + \dots$$

classical potentials

radiation gravitons

Potential matching

Expansion of action

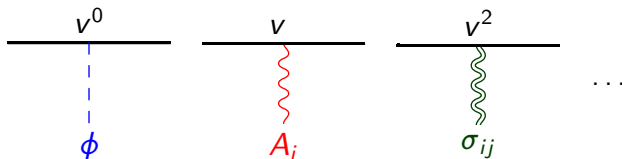
Expand S_{GR} in $v \sim \sqrt{Gm/r} \ll 1$, e.g.

$$S_{\text{pp}} = - \sum_i m_i \int dt \sqrt{-g_{\mu\nu} \frac{\partial x_i^\mu}{\partial t} \frac{\partial x_i^\nu}{\partial t}} = - \sum_i m_i \int dt \sqrt{-g_{00}} + \mathcal{O}(v_i)$$

Coupling to **spatial components** of metric **suppressed**

Temporal Kaluza-Klein decomposition [Kol, Smolkin 2010]

$$g^{\mu\nu} = e^{2\phi} \begin{pmatrix} -1 & & A_j \\ A_i & e^{-2\frac{d-1}{d-2}\phi} (\delta_{ij} + \sigma_{ij}) & -A_i A_j \end{pmatrix}$$



Potential matching

Diagrammatic expansion

Equate amplitude in effective and full theory:

$$\begin{aligned} & \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ | \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \frac{1}{2!} \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ | \text{---} | \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \frac{1}{3!} \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ | \text{---} | \text{---} | \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \dots \\ = & \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ | \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \text{---} \overline{\text{---}} \left[\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \end{array} \right] \text{---} \overline{\text{---}} + \dots \end{aligned}$$

All momenta potential, $p_0 \sim \frac{v}{r} \ll p_i \sim \frac{1}{r}$

\hookrightarrow expand propagators:

$$\frac{1}{\vec{p}^2 - p_0^2} = \frac{1}{\vec{p}^2} + \frac{p_0^2}{\vec{p}^4} + \mathcal{O}(v^4)$$

Potential matching

Diagrammatic expansion

$$V = i \log \left(1 + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \color{red}{\text{wavy}} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \color{blue}{\text{triangle}} \\ \text{---} \end{array} \right. \\ \left. + \begin{array}{c} \text{---} \\ | | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \color{blue}{\text{X}} \\ \text{---} \end{array} + \dots \right) \\ = i \left(\begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \underbrace{\begin{array}{c} \text{---} \\ \color{red}{\text{wavy}} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \color{blue}{\text{triangle}} \\ \text{---} \end{array}}_{\text{1PN}} + \dots \right)$$

Potential matching

Known results

Confirmation of previous results:

- 1PN: [Goldberger, Rothstein 2004]
- 2PN: [Gilmore, Ross 2008]
- 3PN: [Foffa, Sturani 2011; Blümlein, Maier, Marquard 20XX]
- 4PN:
 - “static” contribution $v = 0$:
[Foffa, Mastrolia, Sturani, Sturm 2016; Damour, Jaranowski 2017]
 - $v \neq 0$: [Foffa, Sturani 2019; Foffa, Porto, Rothstein, Sturani 2019]

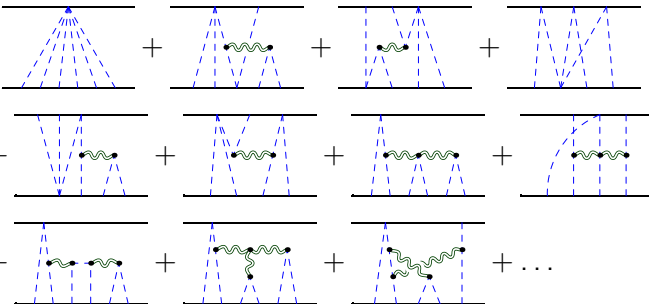
New:

- **5PN static contribution:**

[Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla 27 Feb 2019; Blümlein, Maier, Marquard 28 Feb 2019]

Potential matching

Static 5PN calculation

$$-iV_{5\text{PN}}^S =$$


The diagram illustrates the static 5PN calculation as a sum of Feynman diagrams. The diagrams are arranged in three rows and four columns, with a plus sign and an ellipsis following the last diagram in the third row. Each diagram consists of two horizontal black lines representing the worldlines of two particles. Blue dashed lines represent graviton exchanges between the particles. Green wavy lines represent matter field exchanges. The diagrams show various topologies of graviton and matter field exchanges, including tree-level, one-loop, and two-loop configurations. The first row shows tree-level diagrams with one, two, and three graviton exchanges. The second row shows one-loop diagrams with one, two, and three graviton exchanges and one matter field exchange. The third row shows two-loop diagrams with one, two, and three graviton exchanges and one matter field exchange. The ellipsis indicates that there are more diagrams in the series.

Potential matching

Feynman rules

$$\text{--- } p \text{ ---} = -\frac{i}{2c_d \vec{p}^2}$$

$$\begin{array}{c} i_1 i_2 \\ \text{~~~~~} \\ p \\ \text{~~~~~} \\ j_1 j_2 \end{array} = -\frac{i}{2\vec{p}^2} (\delta_{i_1 j_1} \delta_{i_2 j_2} + \delta_{i_1 j_2} \delta_{i_2 j_1} + (2 - c_d) \delta_{i_1 i_2} \delta_{j_1 j_2})$$

$$\begin{array}{c} m_i \\ \text{---} \\ \text{---} \\ n \end{array} = -i \frac{m_i}{m_{\text{Pl}}^n}$$

$$\begin{array}{c} p_1 \\ \text{---} \\ \text{---} \\ p_2 \end{array} \begin{array}{c} i_1 i_2 \\ \text{~~~~~} \\ \text{---} \\ \text{---} \end{array} = i \frac{c_d}{2m_{\text{Pl}}} (V_{\phi\phi\sigma}^{i_1 i_2} + V_{\phi\phi\sigma}^{i_2 i_1})$$

$$V_{\phi\phi\sigma}^{i_1 i_2} = \vec{p}_1 \cdot \vec{p}_2 \delta^{i_1 i_2} - 2p_1^{i_1} p_2^{i_2}$$

$$\begin{array}{c} p_1 \quad j_1 j_2 \\ \text{---} \\ \text{---} \\ p_2 \quad i_1 i_2 \end{array} = i \frac{c_d}{16m_{\text{Pl}}^2} (V_{\phi\phi\sigma\sigma}^{i_1 i_2 j_1 j_2} + V_{\phi\phi\sigma\sigma}^{i_2 i_1 j_1 j_2} + V_{\phi\phi\sigma\sigma}^{i_1 i_2 j_2 j_1} + V_{\phi\phi\sigma\sigma}^{i_2 i_1 j_2 j_1})$$

$$V_{\phi\phi\sigma\sigma}^{i_1 i_2 j_1 j_2} = \vec{p}_1 \cdot \vec{p}_2 (\delta^{i_1 i_2} \delta^{j_1 j_2} - 2\delta^{i_1 j_1} \delta^{i_2 j_2}) - 2(p_1^{i_1} p_2^{i_2} \delta^{j_1 j_2} + p_1^{j_1} p_2^{j_2} \delta^{i_1 i_2}) + 8\delta^{i_1 j_1} p_1^{i_2} p_2^{j_2}$$

Potential matching

Feynman rules

$$\begin{array}{c} i_1 i_2 \\ p_1 \\ p_2 \\ j_1 j_2 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} k_1 k_2 \end{array} = \frac{i}{32m_{\text{Pl}}} (\tilde{V}_{\sigma\sigma\sigma}^{i_1 i_2 j_1 j_2, k_1 k_2} + \tilde{V}_{\sigma\sigma\sigma}^{i_2 i_1 j_1 j_2, k_1 k_2})$$

$$\tilde{V}_{\sigma\sigma\sigma}^{i_1 i_2 j_1 j_2, k_1 k_2} = V_{\sigma\sigma\sigma}^{i_1 i_2 j_1 j_2, k_1 k_2} + V_{\sigma\sigma\sigma}^{i_1 i_2 j_2 j_1, k_1 k_2} + V_{\sigma\sigma\sigma}^{i_1 i_2 j_1 j_2, k_2 k_1} + V_{\sigma\sigma\sigma}^{i_1 i_2 j_2 j_1, k_2 k_1}$$

$$\begin{aligned}
 V_{\sigma\sigma\sigma}^{i_1 i_2 j_1 j_2, k_1 k_2} = & (\vec{p}_1^2 + \vec{p}_1 \cdot \vec{p}_2 + \vec{p}_2^2) \left(-\delta^{j_1 j_2} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) \right. \\
 & \left. + 2 \left[\delta^{i_1 j_1} \left(4\delta^{i_2 k_1} \delta^{j_2 k_2} - \delta^{i_2 j_2} \delta^{k_1 k_2} \right) - \delta^{i_1 i_2} \delta^{j_1 k_1} \delta^{j_2 k_2} \right] \right) \\
 & + 2 \left\{ 4 \left(p_1^{k_2} p_2^{i_2} - p_1^{i_2} p_2^{k_2} \right) \delta^{i_1 j_1} \delta^{j_2 k_1} \right. \\
 & + 2 \left[\left(p_1^{i_1} + p_2^{i_1} \right) p_2^{i_2} \delta^{j_1 k_1} \delta^{j_2 k_2} - p_1^{k_1} p_2^{k_2} \delta^{i_1 j_1} \delta^{i_2 j_2} \right] \\
 & + \delta^{j_1 j_2} \left[p_1^{k_1} p_2^{k_2} \delta^{i_1 i_2} + 2 \left(p_1^{k_2} p_2^{i_2} - p_1^{i_2} p_2^{k_2} \right) \delta^{i_1 k_1} - \left(p_1^{i_1} + p_2^{i_1} \right) p_2^{i_2} \delta^{k_1 k_2} \right] \\
 & + p_2^{j_2} \left(4p_1^{i_2} \delta^{i_1 k_1} \delta^{j_1 k_2} + p_1^{j_1} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) \right. \\
 & \left. + 2 \left[\delta^{i_1 j_1} \left(p_1^{i_2} \delta^{k_1 k_2} - 2p_1^{k_2} \delta^{i_2 k_1} \right) - p_1^{k_2} \delta^{i_1 i_2} \delta^{j_1 k_1} \right] \right) \\
 & + p_1^{j_2} \left(p_1^{j_1} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) - 4p_2^{i_2} \delta^{i_1 k_1} \delta^{j_1 k_2} \right. \\
 & \left. \left. + 2 \left[p_2^{k_2} \delta^{i_1 i_2} \delta^{j_1 k_1} + \delta^{i_1 j_1} \left(2p_2^{k_2} \delta^{i_2 k_1} - p_2^{i_2} \delta^{k_1 k_2} \right) \right] \right) \right\}
 \end{aligned}$$

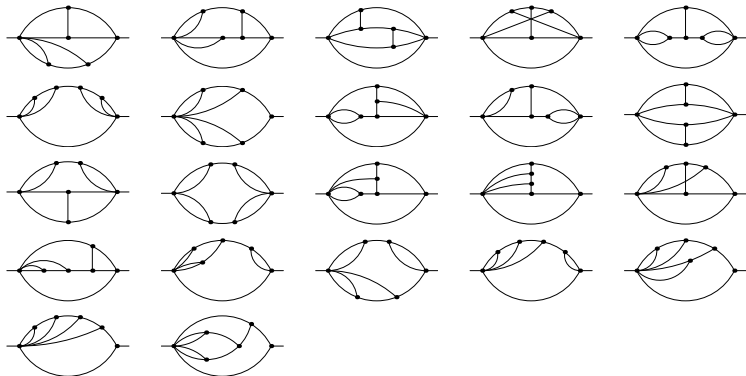
$$c_d = 2 \frac{d-1}{d-2}, \quad m_{\text{Pl}} = \sqrt{32\pi G}$$

Potential matching

Diagram families

Massless propagators:

$$P_f(q) = \int \frac{d^d l_1}{\pi^{d/2}} \cdots \frac{d^d l_5}{\pi^{d/2}} \frac{\mathcal{N}(q, l_1, \dots, l_5)}{\vec{p}_1^{2a_1} \cdots \vec{p}_{10}^{2a_{10}}}$$



Potential matching

Master integrals

Reduction to master integrals (crusher): [Chetyrkin, Tkachov 1981; Laporta 2000]

$$V_{5\text{PN}}^S = c_0 \text{ (diagram 1) } + c_1 \text{ (diagram 2) } + c_2 \text{ (diagram 3) } \\ + c_3 \text{ (diagram 4) } + \mathcal{O}(\epsilon)$$

c_j : Laurent series in $\epsilon = \frac{3-d}{2}$, polynomials in m_1, m_2, r^{-1}, G^{-1}

Master integrals factorise into $\begin{matrix} a \\ \text{---} \\ q \\ \text{---} \\ b \end{matrix}$ and known

[Lee, Mingulov 2015]

Potential matching

Result

$$V_N^S = -\frac{G}{r} m_1 m_2$$

$$V_{1PN}^S = \frac{G^2}{2r^2} m_1 m_2 (m_1 + m_2)$$

$$V_{2PN}^S = -\frac{G^3}{r^3} m_1 m_2 \left[\frac{1}{2} (m_1^2 + m_2^2) + 3m_1 m_2 \right]$$

$$V_{3PN}^S = \frac{G^4}{r^4} m_1 m_2 \left[\frac{3}{8} (m_1^3 + m_2^3) + 6m_1 m_2 (m_1 + m_2) \right]$$

$$V_{4PN}^S = -\frac{G^5}{r^5} m_1 m_2 \left[\frac{3}{8} (m_1^4 + m_2^4) + \frac{31}{3} m_1 m_2 (m_1^2 + m_2^2) + \frac{141}{4} m_1^2 m_2^2 \right]$$

$$V_{5PN}^S = \frac{G^6}{r^6} m_1 m_2 \left[\frac{5}{16} (m_1^5 + m_2^5) + \frac{91}{6} m_1 m_2 (m_1^3 + m_2^3) + \frac{653}{6} m_1^2 m_2^2 (m_1 + m_2) \right]$$

Potential matching

Velocity corrections

Full corrections include velocities and *higher time derivatives*:

$$\begin{aligned}\mathcal{L}_{2\text{PN}} = & + \frac{G^3}{r^3} m_1 m_2 \left[\frac{1}{2} (m_1^2 + m_2^2) + 3 m_1 m_2 \right] \\ & + G m_1 m_2 r \left[\frac{15}{8} \vec{a}_1 \vec{a}_2 - \frac{1}{8} (\vec{a}_1 \vec{r})(\vec{a}_2 \vec{r}) \right] \\ & + (\text{terms depending on } \vec{v}_1, \vec{v}_2)\end{aligned}$$

Can be eliminated using

- *Total time derivatives* $\delta\mathcal{L} \propto \frac{d}{dt} F(\vec{r}, \vec{v}_1, \vec{v}_2)$
- *Equations of motion* $\delta\mathcal{L} \propto \left(\vec{a}_1 + \frac{G m_2}{r^3} \vec{r} \right) \left(\vec{a}_2 - \frac{G m_1}{r^3} \vec{r} \right)$

$$\begin{aligned}\mathcal{L}_{2\text{PN}} = & - \frac{G^3}{r^3} m_1 m_2 \left[\frac{1}{4} (m_1^2 + m_2^2) + \frac{5}{4} m_1 m_2 \right] \\ & + (\text{terms depending on } \vec{v}_1, \vec{v}_2)\end{aligned}$$

Conclusions

- Inspiral phase of compact binary systems described well by *Post-Newtonian (PN) expansion* $v \sim \sqrt{Gm/r} \ll 1$
- Effective field theories and calculational methods from particle physics very effective for high PN orders
- Static gravitational potential now known at five loops (5PN)

Backup

Potential matching

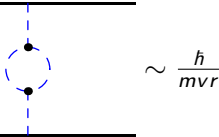
Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)



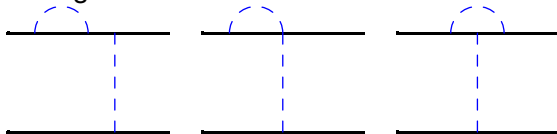
$$\sim \frac{\hbar}{mvr}$$

- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections



Absorbed into renormalisation of sources

- No source-reducible diagrams [Fischler 1977]

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Initially *time-ordered* diagrams:

$$\frac{\text{Diagram}}{\Theta(y^0 - x^0)} = \frac{1}{2} \left(\frac{\text{Diagram}}{\Theta(y^0 - x^0)} + \frac{\text{Diagram}}{\Theta(x^0 - y^0)} \right) = \frac{1}{2} \text{Diagram}$$

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Initially *time-ordered* diagrams:

The diagrammatic equation shows the sum of two time-ordered diagrams. The first diagram consists of two horizontal lines with two vertical dashed lines between them, each with a right-pointing arrow. The second diagram is similar but the two vertical dashed lines cross each other. This sum is equal to a single diagram with two horizontal lines and two vertical dashed lines, with a right-pointing arrow on the top horizontal line. This is further equal to $\frac{1}{2}$ times a diagram with two horizontal lines and two vertical dashed lines. Finally, this is equal to $\frac{1}{2}$ times the square of a diagram with two horizontal lines and one vertical dashed line.

$$\begin{array}{c} \text{---} \rightarrow \text{---} \\ | \quad | \\ \text{---} \rightarrow \text{---} \end{array} + \begin{array}{c} \text{---} \rightarrow \text{---} \\ \diagdown \quad \diagup \\ | \quad | \\ \diagup \quad \diagdown \\ \text{---} \rightarrow \text{---} \end{array} = \begin{array}{c} \text{---} \rightarrow \text{---} \\ | \quad | \\ \text{---} \rightarrow \text{---} \end{array} = \frac{1}{2} \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} = \frac{1}{2} \left(\begin{array}{c} \text{---} \text{---} \\ | \\ \text{---} \text{---} \end{array} \right)^2$$

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Initially *time-ordered* diagrams:

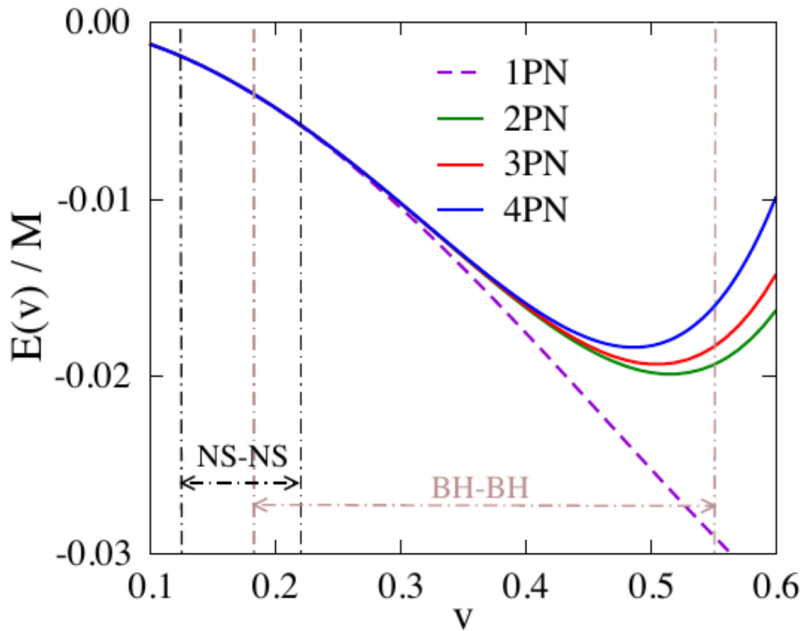
$$\begin{array}{c} \text{---} \rightarrow \text{---} \\ | \quad | \\ \text{---} \rightarrow \text{---} \end{array} + \begin{array}{c} \text{---} \rightarrow \text{---} \\ \diagdown \quad \diagup \\ \diagup \quad \diagdown \\ \text{---} \rightarrow \text{---} \end{array} = \begin{array}{c} \text{---} \rightarrow \text{---} \\ | \quad | \\ \text{---} \rightarrow \text{---} \end{array} = \frac{1}{2} \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} = \frac{1}{2} \left(\begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} \right)^2$$

$$-iV = \log \left(1 + \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} + \frac{1}{2} \left(\begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} \right)^2 + \dots \right) = \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \end{array} + \dots$$

Potential matching

Number of diagrams

	QGRAF	source irred	no source loops	no tadpoles	sym
N	1	1	1	1	1
1PN	2	2	2	2	1
2PN	19	19	19	15	5
3PN	360	276	258	122	8
4PN	10081	5407	4685	1815	50
5PN	332020	128080	101570	27582	154



Potential matching

Results for master integrals

$$\text{Diagram 1} = e^{5\epsilon\gamma_E} \frac{\Gamma(6 - \frac{5d}{2}) \Gamma^6(-1 + \frac{d}{2})}{\Gamma(-6 + 3d)}$$

$$\text{Diagram 2} = e^{5\epsilon\gamma_E} \frac{\Gamma(7 - \frac{5d}{2}) \Gamma(3 - d) \Gamma(2 - \frac{d}{2}) \Gamma^7(-1 + \frac{d}{2}) \Gamma(5 - 2d)}{\Gamma(5 - \frac{3}{2}d) \Gamma(-2 + d) \Gamma(-3 + \frac{3}{2}d) \Gamma(-7 + 3d)}$$

$$\text{Diagram 3} = e^{5\epsilon\gamma_E} \frac{\Gamma(7 - \frac{5d}{2}) \Gamma^2(3 - d) \Gamma^7(-1 + \frac{d}{2}) \Gamma(-6 + \frac{5d}{2})}{\Gamma(6 - 2d) \Gamma^2(-3 + \frac{3d}{2}) \Gamma(-7 + 3d)}$$

$$\text{Diagram 4} = 6\pi^{7/2} \left[\frac{2}{\epsilon} - 4 - 4 \ln(2) - (48 + 8 \ln(2) - 4 \ln^2(2) - 105\zeta_2) \epsilon + \mathcal{O}(\epsilon^2) \right]$$

$$\begin{aligned} V_{5PN}^S \stackrel{\epsilon \rightarrow 0}{=} & \frac{G^6}{r^6} (m_1 m_2) \pi^{-7/2} \left\{ \frac{15}{32} (m_1^5 + m_2^5) \left[\text{Diagram 1} \right]_{\epsilon^0} + \frac{91}{4} m_1 m_2 (m_1^3 + m_2^3) \left[\text{Diagram 1} \right]_{\epsilon^0} \right. \\ & + m_1^2 m_2^2 (m_1 + m_2) \left(\left[\frac{293}{4} \text{Diagram 1} - \frac{45}{16} \text{Diagram 2} + \frac{45}{32} \text{Diagram 3} \right]_{\epsilon^0} \right. \\ & \left. \left. + \left[\frac{519}{16} \text{Diagram 2} - \frac{627}{32} \text{Diagram 3} + 2 \text{Diagram 4} \right]_{\epsilon^{-1}} \right) \right\} \end{aligned}$$