$H \rightarrow b\bar{b}$ at N3LO accuracy

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RM, Matthew Schiavi, Ciaran Williams, JHEP 1906 (2019) 079
RM and Ciaran Williams, JHEP 1906 (2019) 120
Introduction

From Higgs observation in LHC Run I to precision Higgs measurements in Run II:

- Access different production and decay mechanisms
- Precisely probe couplings to other SM particles
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- Access different production and decay mechanisms
- Precisely probe couplings to other SM particles
Introduction

At future colliders, couplings such as $Hbb$ will be measured at the sub-percent level.

The increasing experimental precision mandates a similar increase in the precision of the corresponding theoretical predictions.

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**from M. Mangano’s Pheno 2019 talk**

<table>
<thead>
<tr>
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<th>HL-LHC $\delta \Gamma_H / \Gamma_H$ (%)</th>
<th>FCC-ee $\delta g_{HZZ} / g_{HZZ}$ (%)</th>
<th>FCC-hh $\delta g_{HWW} / g_{HWW}$ (%)</th>
<th>FCC-hh $\delta g_{Hbb} / g_{Hbb}$ (%)</th>
<th>FCC-hh $\delta g_{Hcc} / g_{Hcc}$ (%)</th>
<th>FCC-hh $\delta g_{Hgg} / g_{Hgg}$ (%)</th>
<th>FCC-hh $\delta g_{Hgg} / g_{Hgg}$ (2.5 (gg-&gt;H))</th>
<th>FCC-hh $\delta g_{Hcc} / g_{Hcc}$ (%)</th>
<th>FCC-hh $\delta g_{Hgg} / g_{Hgg}$ (%)</th>
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<td>$\delta g_{Hgg} / g_{Hgg}$ (%)</td>
<td>2.5 (gg-&gt;H)</td>
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<td>3.9</td>
<td>0.4 (*)</td>
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<td>tbd</td>
<td>3.9</td>
<td>0.4 (*)</td>
<td>tbd</td>
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<td>3.4</td>
<td>~10 (indirect)</td>
<td>0.95 (*)</td>
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<td>~10 (indirect)</td>
<td>0.95 (*)</td>
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<tr>
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<td>$\delta g_{Hcc} / g_{Hcc}$ (%)</td>
<td>9.8</td>
<td>~10 (indirect)</td>
<td>0.9 (*)</td>
<td>tbd</td>
<td>tbd</td>
<td>~10 (indirect)</td>
<td>0.9 (*)</td>
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<td>$\delta g_{Hgg} / g_{Hgg}$ (%)</td>
<td>50</td>
<td>~44 (indirect)</td>
<td>6.5</td>
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<td>tbd</td>
<td>~44 (indirect)</td>
<td>6.5</td>
<td>tbd</td>
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</table>

- pp at 14 TeV
  - 3 ab$^{-1}$
- pp at 100 TeV
- $e^+e^-$ at 91, 160, 240, 365 GeV
- ep at 3.5 TeV
Overview of the calculation

\[ \Gamma_{H \to b\bar{b}} = \Gamma_{H \to b\bar{b}}^{\text{LO}} + \Delta\Gamma_{H \to b\bar{b}}^{\text{NLO}} + \Delta\Gamma_{H \to b\bar{b}}^{\text{NNLO}} + \Delta\Gamma_{H \to b\bar{b}}^{\text{N3LO}} + \Delta\Gamma_{H \to b\bar{b}}^{\text{N4LO}} + \ldots \]

Inclusively known up to:

- **N4LO QCD** [Baikov, Chetyrkin, Kuhn hep-ph/0511063]
- **NLO EW** [Dabelstein, Hollik (1992); Kataev hep-ph/9708292]
- **Mixed QCDxEW** [Kataev hep-ph/9708292; Mihaila, Schmidt, Steinhauser 1509.02294] (also QCDxEW master integrals for Htt coupling [Chaubey, Weinzierl 1904.00382])

Differentially:

- **NNLO QCD** [Anastasiou, Herzog, Lazopoulos 1110.2368; Del Duca, Duhr, Somogyi, Tramontano, Trócsányi 1501.07226; Bernreuther, Cheng, Si 1805.06658]
- **Interfaced to VH production at NNLO QCD** [Ferrera, Somogyi, Tramontano 1705.10304; Caola, Luisoni, Melnikov, Röntsch 1712.06954; Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 1907.05836]  

**Aim:** provide fully-differential predictions at N3LO QCD accuracy
Overview of the calculation

- Treat the bottom quark as massless
- Focus on $y_b^2$ terms

\[
\Gamma^{N3LO}_{H\rightarrow bb} = y_b^2 A_b + \alpha_s y_b^2 B_b \\
+ \alpha_s^2 (y_b^2 C_b + y_b y_t C_{bt}) \\
+ \alpha_s^3 (y_b^2 D_b + y_b y_t D_{bt} + y_t^2 D_t)
\]

\[y_b y_t C_{bt}\]

\[y_t^2 D_t\]

+ $\mathcal{O}(\alpha_s)$ corrections

Ongoing work to include neglected terms (as well as EW and QCDxEW)

[Primo, Sasso, Somogyi, Tramontano 1812.07811]

talk by U. Schubert
Overview of the calculation

Differential N3LO coefficient:

\[
\frac{d \Delta \Gamma^{N3LO}_{H \rightarrow b\bar{b}}}{d \mathcal{O}_m} = \int d\Gamma_{H \rightarrow b\bar{b}}^{VVV} F^m_2(\Phi_2) d\Phi_2 \\
+ \int d\Gamma_{H \rightarrow b\bar{b}}^{RVV} F^m_3(\Phi_3) d\Phi_3 \\
+ \int d\Gamma_{H \rightarrow b\bar{b}}^{RRV} F^m_4(\Phi_4) d\Phi_4 \\
+ \int d\Gamma_{H \rightarrow b\bar{b}}^{RRR} F^m_5(\Phi_5) d\Phi_5
\]
Overview of the calculation

Differential N3LO coefficient:

\[
\frac{d \Delta \Gamma_{H \to b \bar{b}}^{\text{N3LO}}}{d \mathcal{O}_m} = \int d\Gamma_{H \to b \bar{b}}^{VVV} F_2^m(\Phi_2) d\Phi_2 \\
+ \int d\Gamma_{H \to b \bar{b}}^{RVV} F_3^m(\Phi_3) d\Phi_3 \\
+ \int d\Gamma_{H \to b \bar{b}}^{RRV} F_4^m(\Phi_4) d\Phi_4 \\
+ \int d\Gamma_{H \to b \bar{b}}^{RRR} F_5^m(\Phi_5) d\Phi_5
\]

triple-virtual (3 loops, 2 partons)
Overview of the calculation

Differential N3LO coefficient:

\[
\frac{d \Delta \Gamma^{N3LO}}{d \mathcal{O}_m}_{H \rightarrow b \bar{b}} = \int d\Gamma_{H \rightarrow b \bar{b}}^{VVV} F_2^m(\Phi_2) d\Phi_2 + \int d\Gamma_{H \rightarrow b \bar{b}}^{RVV} F_3^m(\Phi_3) d\Phi_3 + \int d\Gamma_{H \rightarrow b \bar{b}}^{RRV} F_4^m(\Phi_4) d\Phi_4 + \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR} F_5^m(\Phi_5) d\Phi_5
\]

triple-virtual (3 loops, 2 partons)
real double-virtual (2 loops, 3 partons)
Overview of the calculation

Differential N3LO coefficient:

\[
\frac{d \Delta \Gamma^{\text{N3LO}}_{H \rightarrow bb}}{d \mathcal{O}_m} = \int d\Gamma^{VVV}_{H \rightarrow bb} F_m^m(\Phi_2) d\Phi_2 \\
+ \int d\Gamma^{RVV}_{H \rightarrow bb} F_m^m(\Phi_3) d\Phi_3 \\
+ \int d\Gamma^{RRV}_{H \rightarrow bb} F_m^m(\Phi_4) d\Phi_4 \\
+ \int d\Gamma^{RRR}_{H \rightarrow bb} F_m^m(\Phi_5) d\Phi_5
\]

- triple-virtual (3 loops, 2 partons)
- real double-virtual (2 loops, 3 partons)
- double-real virtual (1 loop, 4 partons)
Overview of the calculation

Differential N3LO coefficient:

\[
\frac{d \Delta \Gamma^{N3LO}_{H \to b\bar{b}}}{d \mathcal{O}_m} = \int d\Gamma^{VVV}_{H \to b\bar{b}} F_2^m(\Phi_2) d\Phi_2 \\
+ \int d\Gamma^{RVV}_{H \to b\bar{b}} F_3^m(\Phi_3) d\Phi_3 \\
+ \int d\Gamma^{RRV}_{H \to b\bar{b}} F_4^m(\Phi_4) d\Phi_4 \\
+ \int d\Gamma^{RRR}_{H \to b\bar{b}} F_5^m(\Phi_5) d\Phi_5
\]

- triple-virtual (3 loops, 2 partons)
- real double-virtual (2 loops, 3 partons)
- double-real virtual (1 loop, 4 partons)
- triple-real (0 loops, 5 partons)
Overview of the calculation

\[
\frac{d \Delta \Gamma^{N3LO}_{H \rightarrow bb}}{d O_m} = \int d\Gamma_{H \rightarrow bb}^{VVV} F^m_2(\Phi_2) d\Phi_2 \\
+ \int d\Gamma_{H \rightarrow bb}^{RVV} F^m_3(\Phi_3) d\Phi_3 \\
+ \int d\Gamma_{H \rightarrow bb}^{RRV} F^m_4(\Phi_4) d\Phi_4 \\
+ \int d\Gamma_{H \rightarrow bb}^{RRR} F^m_5(\Phi_5) d\Phi_5
\]

\(F^m_i(\Phi_i)\) uses a jet-clustering algorithm to define an \(m\)-jet observable from \(i\) final-state partons

Each contribution contains soft and collinear IR divergences that cancel upon combination into a suitably-inclusive observable
Projection-to-Born method

We use the Projection-to-Born (P2B) method to deal with the IR divergences [Cacciari, Dreyer, Karlberg, Salam, Zanderighi 1506.02660]

*Main idea:* construct local counter-terms for the matrix elements projected onto a LO (Born) phase space.
Projection-to-Born method

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*Main idea*: construct local counter-terms for the matrix elements projected onto a LO (Born) phase space.

Example with $i=5$ partons clustered into $m=2$ jets:
Projection-to-Born method

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Main idea: construct local counter-terms for the matrix elements projected
onto a LO (Born) phase space.

Example with $i=5$ partons clustered into $m=2$ jets:

Generated event with $|\mathcal{M}|^2$

$$F_5^2(\Phi_5)$$
We use the Projection-to-Born (P2B) method to deal with the IR divergences [Cacciari, Dreyer, Karlberg, Salam, Zanderighi 1506.02660]

**Main idea**: construct local counter-terms for the matrix elements projected onto a LO (Born) phase space.

Example with $i=5$ partons clustered into $m=2$ jets:

\[ |M|^2 \times \text{cluster} - |M|^2 \times F_5^2(\Phi_5) - |M|^2 \times F_2^2(\Phi_B) \]
Projection-to-Born method

\[ |\mathcal{M}|^2 \times \left( F_5^2(\Phi_5) - F_2^2(\Phi_B) \right) \]

The IR divergences cancel exactly when the full phase space matches the Born-projected phase space.

This is the triple-unresolved region.

Born phase space in the Higgs rest frame:

\[ \Phi_B = \{p_1, p_2\} \]
\[ p_1 = \frac{m_H}{2}(1, \mathbf{n}_j) \]
\[ p_2 = \frac{m_H}{2}(1, -\mathbf{n}_j) \]

with \( \mathbf{n}_j \) the direction of the leading jet.
Projection-to-Born method

To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

\[
\frac{d \Delta \Gamma^{N3LO}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} = + \int d\Gamma_{H \rightarrow b \bar{b}}^{RVV} [F_{3m}(\Phi_3) - F_{2m}(\Phi_B)] d\Phi_3 \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR} [F_{4m}(\Phi_4) - F_{2m}(\Phi_B)] d\Phi_4 \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR} [F_{5m}(\Phi_5) - F_{2m}(\Phi_B)] d\Phi_5 \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{VVV} F_{2m}(\Phi_B) d\Phi_2 + \int d\Gamma_{H \rightarrow b \bar{b}}^{RVV} F_{2m}(\Phi_B) d\Phi_3 \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR} F_{2m}(\Phi_B) d\Phi_4 + \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR} F_{2m}(\Phi_B) d\Phi_5
\]
To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

\[
\frac{d \Delta \Gamma^{N3LO}}{d \mathcal{O}_m} = \int \frac{d \Gamma^{RVV}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} \left[ F_3^m(\Phi_3) - F_2^m(\Phi_B) \right] d\Phi_3 \\
+ \int \frac{d \Gamma^{RRV}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} \left[ F_4^m(\Phi_4) - F_2^m(\Phi_B) \right] d\Phi_4 \\
+ \int \frac{d \Gamma^{RRR}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} \left[ F_5^m(\Phi_5) - F_2^m(\Phi_B) \right] d\Phi_5 \\
+ \int \frac{d \Gamma^{VVV}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} F_2^m(\Phi_B) d\Phi_2 + \int \frac{d \Gamma^{RVV}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} F_2^m(\Phi_B) d\Phi_3 \\
+ \int \frac{d \Gamma^{RRR}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} F_2^m(\Phi_B) d\Phi_4 + \int \frac{d \Gamma^{RRR}_{H \rightarrow b \bar{b}}}{d \mathcal{O}_m} F_2^m(\Phi_B) d\Phi_5
\]

\[
\frac{d \Delta \Gamma^{N3LO, incl}}{d \mathcal{O}_m} = \int \Delta \Gamma^{N3LO}_{H \rightarrow b \bar{b}} F_2^m(\Phi_B) d\Phi_B
\]

**Ingredient 1:** Inclusive N3LO $H \rightarrow b \bar{b}$ width as a function of the Born kinematics
Projection-to-Born method

To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

\[
\frac{d \Delta \Gamma_{H \rightarrow b \bar{b}}^{N3LO}}{d \mathcal{O}_m} = + \int d\Gamma_{H \rightarrow b \bar{b}}^{RVV}\left[ F_{3}^{m}(\Phi_{3}) - F_{2}^{m}(\Phi_{B}) \right] d\Phi_{3} \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRV}\left[ F_{4}^{m}(\Phi_{4}) - F_{2}^{m}(\Phi_{B}) \right] d\Phi_{4} \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR}\left[ F_{5}^{m}(\Phi_{5}) - F_{2}^{m}(\Phi_{B}) \right] d\Phi_{5} \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{VVV}\Phi_{2}^{m}(\Phi_{B}) d\Phi_{2} + \int d\Gamma_{H \rightarrow b \bar{b}}^{RVV}\Phi_{2}^{m}(\Phi_{B}) d\Phi_{3} \\
+ \int d\Gamma_{H \rightarrow b \bar{b}}^{RRV}\Phi_{2}^{m}(\Phi_{B}) d\Phi_{4} + \int d\Gamma_{H \rightarrow b \bar{b}}^{RRR}\Phi_{2}^{m}(\Phi_{B}) d\Phi_{5}
\]

Ingredient 2: Differential NNLO $H \rightarrow b\bar{b}j$ width and its Born projection
Differential NNLO $H\rightarrow b\bar{b}j$ width

$$\frac{d \Delta \Gamma_{H\rightarrow b\bar{b}j}^{\text{NNLO}}}{d \mathcal{O}_m} = \int d\Gamma_{H\rightarrow b\bar{b}j}^{VV} F_3^m(\Phi_3) d\Phi_3$$

$$+ \int d\Gamma_{H\rightarrow b\bar{b}j}^{RV} F_4^m(\Phi_4) d\Phi_4$$

$$+ \int d\Gamma_{H\rightarrow b\bar{b}j}^{RR} F_5^m(\Phi_5) d\Phi_5$$
Differential NNLO $H \to b\bar{b}j$ width

\[ \frac{d \Delta \Gamma_{H \to b\bar{b}j}^{\text{NNLO}}}{d \mathcal{O}_m} = \int d\Gamma_{H \to b\bar{b}j}^{VV} F_3^m(\Phi_3) d\Phi_3 \]
\[ + \int d\Gamma_{H \to b\bar{b}j}^{RV} F_4^m(\Phi_4) d\Phi_4 \]
\[ + \int d\Gamma_{H \to b\bar{b}j}^{RR} F_5^m(\Phi_5) d\Phi_5 \]

two-loop amplitudes for $H \to b\bar{b}g$
Differential NNLO $H \to b\bar{b}j$ width

\[
\frac{d\Delta\Gamma^{\text{NNLO}}_{H \to b\bar{b}j}}{d\mathcal{O}_m} = \int d\Gamma_{H \to b\bar{b}j}^{VV} F_m^3(\Phi_3) d\Phi_3 \\
+ \int d\Gamma_{H \to b\bar{b}j}^{RV} F_m^4(\Phi_4) d\Phi_4 \\
+ \int d\Gamma_{H \to b\bar{b}j}^{RR} F_m^5(\Phi_5) d\Phi_5
\]

two-loop amplitudes for $H \to bbg$

one-loop amplitudes for $H \to bbgg$ and $H \to bbqq$
Differential NNLO $H \to b \bar{b} j$ width

$$\frac{d \Delta \Gamma_{H \to b \bar{b} j}^{\text{NNLO}}}{d \mathcal{O}_m} = \int d\Gamma_{H \to b \bar{b} j}^{VV} F_3^m(\Phi_3) d\Phi_3$$

$$+ \int d\Gamma_{H \to b \bar{b} j}^{RV} F_4^m(\Phi_4) d\Phi_4$$

$$+ \int d\Gamma_{H \to b \bar{b} j}^{RR} F_5^m(\Phi_5) d\Phi_5$$

two-loop amplitudes for $H \to b \bar{b}g$

one-loop amplitudes for $H \to b \bar{b}gg$ and $H \to b \bar{b}qqg$

tree-level amplitudes for $H \to b \bar{b}ggg$ and $H \to b \bar{b}qqg$
Differential NNLO $H \to b\bar{b}j$ width

Two-loop $H \to b\bar{b}g$ amplitudes calculated using the MIs from [Gehrmann, Remiddi hep-ph/0008287 and hep-ph/0101124]

Checks:
- IR poles against the known IR structure [Catani hep-ph/9802439]
- Finite part against an independent calculation [Ahmed, Mahakhud, Mathews, Rana, Ravindran 1405.2324]
- Two-loop soft/collinear-gluon limits

One-loop $H \to 4$ partons amplitudes calculated analytically using generalized unitarity for helicity amplitudes [Bern, Dixon, Dunbar, Kosower hep-ph/9403226]

Tree-level $H \to 5$ partons amplitudes calculated using BCFW recursion relations [Britto, Cachazo, Feng, Witten hep-th/0501052]
N-jettiness slicing

We regulate the IR divergences present in our NNLO $H \to b\bar{b}j$ calculation by using **N-jettiness slicing** [Boughezal, Focke, Liu, Petriello 1504.02131; Gaunt, Stahlhofen, Tackmann, Walsh 1505.04794]. For a parton-level event we define the 3-jettiness variable [Stewart, Tackmann, Waalewijn 1004.2489]:

$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$

- The index $j$ runs over the $m$ partons in the phase space
- The momenta $q_i$ are the momenta of the three most energetic jets
- $Q_i = 2E_i$ with $E_i$ the energy of the $i$-th jet.
N-jettiness slicing

\[ H \rightarrow b\bar{b}j \text{ at NNLO} \]

\[ \tau_3 = \sum_{j=1, m} \min_{i=1, 2, 3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} \approx 0 \]

Doubly-unresolved region

All radiation is either soft or collinear
N-jettiness slicing

\[ H \rightarrow b\bar{b}j \text{ at NNLO} \]

\[
\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} > 0
\]

Singly-unresolved region
At least one parton is resolved
N-jettiness slicing

Introduce a variable $\tau_3^{\text{cut}}$ that separates the phase space into two regions:
N-jettiness slicing

Introduce a variable $\tau_3^{\text{cut}}$ that separates the phase space into two regions:

- The region $\tau_3 < \tau_3^{\text{cut}}$ contains all of the \textit{doubly-unresolved} regions of phase space and here the decay width is approximated using this factorization theorem from SCET [Stewart, Tackmann, Waalewijn 0910.0467]:

$$\Gamma_{H \to b\bar{b}j} (\tau_3 < \tau_3^{\text{cut}}) \approx \int \prod_{i=1}^{3} J_i \otimes S \otimes \mathcal{H} + \mathcal{O}(\tau_3^{\text{cut}})$$

\textbf{Jet functions} \hspace{1cm} \textbf{Soft function} \hspace{1cm} \textbf{Hard function}

[Becher, Neubert hep-ph/0603140] \hspace{1cm} [Boughezal, Liu, Petriello 1504.02540; Campbell, Ellis, RM, Williams 1711.09984] \hspace{1cm} (finite part of the two-loop amplitudes)
N-jettiness slicing

Introduce a variable $\tau_3^{\text{cut}}$ that separates the phase space into two regions:

- The region $\tau_3 < \tau_3^{\text{cut}}$ contains all of the *doubly-unresolved* regions of phase space and here the decay width is approximated using this factorization theorem from SCET [Stewart, Tackmann, Waalewijn 0910.0467]:

\[
\Gamma_{H \to b \bar{b} j j} (\tau_3 < \tau_3^{\text{cut}}) \approx \int \prod_{i=1}^{3} \mathcal{J}_i \otimes S \otimes \mathcal{H} + \mathcal{O}(\tau_3^{\text{cut}})
\]

Jet functions


Soft function

[Boughezal, Liu, Petriello 1504.02540; Campbell, Ellis, RM, Williams 1711.09984]

Hard function

(finite part of the two-loop amplitudes)

- The region $\tau_3 > \tau_3^{\text{cut}}$ contains the *singly-unresolved* and *fully-resolved* regions. It is the NLO calculation of $H \to b \bar{b} j j$. In our case we regulate the IR divergences using Catani-Seymour dipoles [hep-ph/9605323].
Results

We have implemented our NNLO $H \rightarrow b\bar{b}j$ calculation into a parton-level MC code based on MCFM [Campbell, Ellis et al].

We use the **Durham jet algorithm**. Starting at the parton level, for every pair of partons $(i,j)$:

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{Q^2}$$

If $y_{ij} < y_{\text{cut}}$ the pairs are combined into a new object with momentum $p_i + p_j$. The algorithm repeats until no further clusterings are possible and the remaining objects are classified as jets.

We present results in the Higgs *rest frame*. 
Validation of the $H \to b\bar{b}j$ NNLO N-jettiness calculation

Dependence of the NNLO $H \to 3j$ coefficient on the unphysical parameter $\tau_3^{\text{cut}}$ for three clustering options.

Asymptotic behavior is established in each region.

$y_{\text{cut}} = 0.0001$ corresponds to imposing a very weak jet cut.
P2B with N-jettiness slicing

\[
\frac{d \Delta \Gamma^{N3LO}_{H \to bb}}{d \mathcal{O}_m} = \frac{d \Delta \Gamma^{N3LO, incl}_{H \to bb}}{d \mathcal{O}^B_m} \\
+ \int d\Gamma^{RVV}_{H \to bb} [F_3^m(\Phi_3) - F_2^m(\Phi_B)] d\Phi_3 \\
+ \int d\Gamma^{RRV}_{H \to bb} [F_4^m(\Phi_4) - F_2^m(\Phi_B)] d\Phi_4 \\
+ \int d\Gamma^{RRR}_{H \to bb} [F_5^m(\Phi_5) - F_2^m(\Phi_B)] d\Phi_5
\]

Problem when \( m=2 \): how to define 3-jettiness for 2-jet events?

Differential NNLO \( H \to bb \) calculation using N-jettiness slicing
Focus on triple-real contribution as an example:

\[
\int d\Gamma_{H \rightarrow b\bar{b}}^{RRR} \left[ F_5^m (\Phi_5) - F_2^m (\Phi_B) \right] d\Phi_5
\]

\(F_5^m (\Phi_5)\) picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):
P2B with N-jettiness slicing

Focus on triple-real contribution as an example:

\[
\int d\Gamma_{H \rightarrow b\bar{b}}^{RRR} \left[ F_5^m(\Phi_5) - F_2^m(\Phi_B) \right] d\Phi_5
\]

\( F_5^m(\Phi_5) \) picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):

a) events with 3 or more jets:
   straightforward to compute 3-jettiness
P2B with N-jettiness slicing

Focus on triple-real contribution as an example:

\[ \int d\Gamma_{H\rightarrow b\bar{b}}^{RRR} [F_5^m(\Phi_5) - F_2^m(\Phi_B)] d\Phi_5 \]

\( F_5^m(\Phi_5) \) picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):

b) events with 2 jets: reverse last step of clustering to obtain exactly 3 sub-jets. Then apply 3-(sub)jettiness slicing.
Validation of the P2B+SCET method at NNLO

We introduce the transverse momentum and pseudo-rapidity of the leading jet with respect to a fictitious beam axis to fully test the IR cancellations.
Validation at N3LO

Dependence of the 2-jet N3LO coefficient on the 3-(sub)jettiness slicing parameter $\tau_3^{\text{cut}}$

- The change in the N3LO coefficient in this region is about 1%.

Use $\tau_3^{\text{cut}} = 0.02$ GeV for predictions.
The observed pattern is similar to the results obtained for $e^+e^- \rightarrow \text{jets}$ at the same order [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 0802.0813; Weinzierl 0807.3241]
The size of the corrections is observable-dependent. The scale dependence is considerably reduced as higher-order terms are included.
Can broadly observe three regions:

1) LO boundary: all phase spaces contribute, good convergence of the series and small residual scale dependence

2) “Bulk”: only phase spaces with 3+ partons contribute, NNLO-like calculation

3) “Tail”: only phase spaces with 4+ partons contribute, NLO-like calculation
• We entered an era of precision Higgs physics at the LHC.

• Precise theoretical predictions for Higgs observables are needed to successfully compare theory and experiment.

• We computed the $H \rightarrow b\bar{b}$ decay at N3LO accuracy focusing on the contribution in which the Higgs boson couples directly to massless bottom quarks.

• Using the Projection-to-Born method + N-jettiness slicing, we produced differential distributions and jet rates in the Higgs rest frame.

• Our calculation could be used outside of the rest frame for LHC/FC applications.
Extra slides
Inclusive N3LO $H \to b\bar{b}$ width

Can be obtained through the *optical theorem* by computing the massless $\mathcal{O}(\alpha_s^3)$ four-loop correlator of the quark-scalar current [Chetyrkin hep-ph/9608318]

\[
\Delta \Gamma_{H \to b\bar{b}}^{N3LO} = \Gamma_{H \to b\bar{b}}^{LO} \left( \frac{\alpha_s}{\pi} \right)^3 \left[ s_3 + L \left( 2s_2 \beta_0 + s_1 \beta_1 + 2s_2 \gamma_m^0 + 2s_1 \gamma_m^1 + 2 \gamma_m^2 \right) \\
+ L^2 \left( s_1 \beta_0^2 + 3s_1 \beta_0 \gamma_m^0 + \beta_1 \gamma_m^0 + 2s_1 (\gamma_m^0)^2 + 2 \beta_0 \gamma_m^1 + 4 \gamma_m^0 \gamma_m^1 \right) \\
+ L^3 \left( \frac{2}{3} \beta_0^2 \gamma_m^0 + 2 \beta_0 (\gamma_m^0)^2 + \frac{4}{3} (\gamma_m^0)^3 \right) \right]
\]

\[L = \log \left( \mu^2 / m_H^2 \right)\]
Can broadly observe three regions:

1) At LO $m_j=0$. Must ensure that first bin be inclusive enough for IR cancellations. Large corrections

2) “Bulk”: phase spaces with 3+ partons contribute, NNLO-like calculation

3) “Tail”: phase spaces with 4+ partons contribute, NLO-like calculation
Two-loop amplitudes for $H \rightarrow b\bar{b}g$

**Soft-gluon limit:** $p_3 \rightarrow 0$ which means $y, z \rightarrow 0$ simultaneously

$$2 \text{Re} \left( M_{H \rightarrow b\bar{b}g}^{(2)} M_{H \rightarrow b\bar{b}g}^{(0)^*} \right) \rightarrow 2 \text{Re} \left( S^{(0)}(y, z) M_{H \rightarrow b\bar{b}}^{(2)} M_{H \rightarrow b\bar{b}}^{(0)^*} \right. $$

$$+ S^{(1)}(y, z) M_{H \rightarrow b\bar{b}}^{(1)} M_{H \rightarrow b\bar{b}}^{(0)^*} $$

$$\left. + S^{(2)}(y, z) M_{H \rightarrow b\bar{b}}^{(0)} M_{H \rightarrow b\bar{b}}^{(0)^*} \right)$$

$y = z = 10^{-10}$

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<th>Coefficient</th>
<th>Known limit</th>
<th>Our result</th>
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<tbody>
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<td>6.52342650793 $\cdot 10^7$</td>
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</table>
Two-loop amplitudes for $H \rightarrow b\bar{b}g$

**Collinear limit:** $t \rightarrow 0$ which means $y \rightarrow 0$ while $z$ is fixed

$$2 \text{Re} \left( \mathcal{M}_{H \rightarrow b\bar{b}g}^{(2)} \mathcal{M}_{H \rightarrow b\bar{b}g}^{(0)*} \right) \rightarrow 2 \text{Re} \left( C^{(0)}(y, z) \mathcal{M}_{H \rightarrow b\bar{b}}^{(2)} \mathcal{M}_{H \rightarrow b\bar{b}}^{(0)*} \right)$$

$$+ C^{(1)}(y, z) \mathcal{M}_{H \rightarrow b\bar{b}}^{(1)} \mathcal{M}_{H \rightarrow b\bar{b}}^{(0)*}$$

$$+ C^{(2)}(y, z) \mathcal{M}_{H \rightarrow b\bar{b}}^{(0)} \mathcal{M}_{H \rightarrow b\bar{b}}^{(0)*}$$

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<th>Coefficient</th>
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<th>Our result</th>
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<td>$\epsilon^0$</td>
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</tbody>
</table>

$y = 10^{-12}$

$z = 0.23$