Threshold factorization of the Drell-Yan process at NLP

Sebastian Jaskiewicz (TU München)

Technische Universität München

RADCOR 2019 - Intl. Symposium on Radiative Corrections
9-13 September 2019
Avignon, France

Threshold factorization of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Sebastian Jaskiewicz and Leonardo Vernazza

In preparation

Leading-logarithmic threshold resummation of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

JHEP, 2019(3):43 arXiv:1809.10631

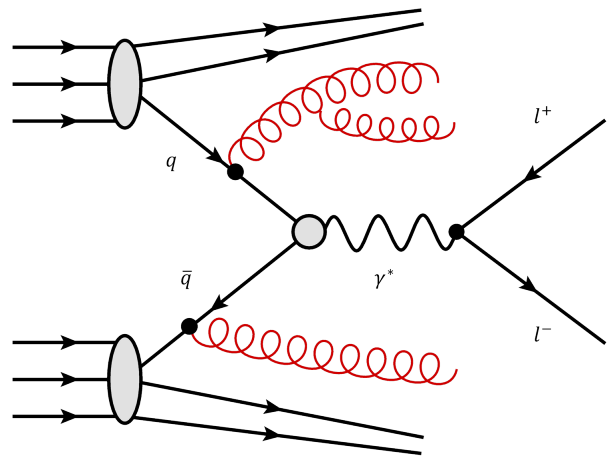
Outline

- Motivations - a short overview and introduction to SCET formalism
  $\rightarrow$ see also talk by Martin Beneke
- The Drell-Yan process - review of factorization at leading power within the position space SCET framework.
- The Drell-Yan process - new features at next-to-leading power
  - Emergence of collinear functions
  - Generalized soft functions
- Factorization formula at next-to-leading power - starting point for resummation
  - For resummation of leading logarithms $\rightarrow$ see talk by Robert Szafron
  - Issues beyond leading logarithmic accuracy

Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$



Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

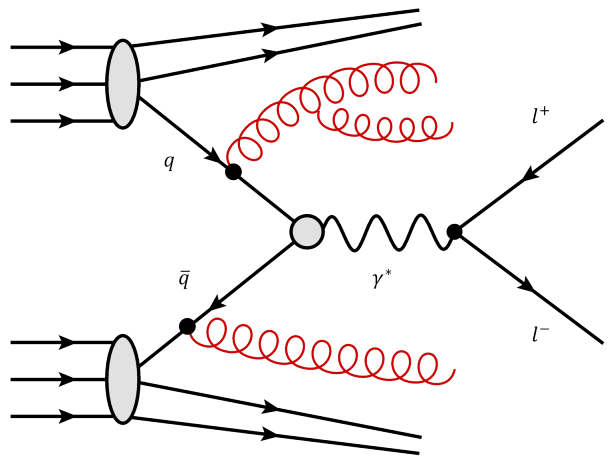
$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

- Leading power (LP) logarithms

Threshold Drell-Yan ($\mathrm{N^3LL}$) [T. Becher, M. Neubert, G. Xu, 0710.0680]

Thrust distribution in e^+e^- collisions ($\mathrm{N^3LL}$)

[T. Becher, M. Schwartz, 0803.0342]



Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

- Leading power (LP) logarithms

  Threshold Drell-Yan ($\mathrm{N^3LL}$) [T. Becher, M. Neubert, G. Xu, 0710.0680]

  Thrust distribution in e^+e^- collisions ($\mathrm{N^3LL}$)

  [T. Becher, M. Schwartz, 0803.0342]

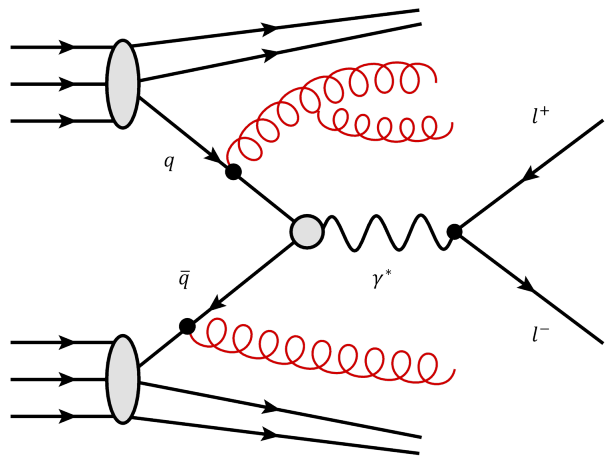
- Next-to-leading power (NLP) logarithms

  High-Energy Limit of QCD beyond Sudakov Approximation [T. Liu, A. Penin, 1709.01092]

  Subleading power resummed thrust spectrum for $H \to gg$ [I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

  Drell-Yan production at threshold [M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

  Diagrammatic resummation of threshold effects [N. Bahjat-Abbas, D. Bonocore, J. Sinninghe Damste, E. Laenen, L. Magnea, L. Vernazza, C. White, 1905.13710]



Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

- Leading power (LP) logarithms

  Threshold Drell-Yan ($\mathrm{N^3LL}$) [T. Becher, M. Neubert, G. Xu, 0710.0680]

  Thrust distribution in e^+e^- collisions ($\mathrm{N^3LL}$) [T. Becher, M. Schwartz, 0803.0342]

- Next-to-leading power (NLP) logarithms

  High-Energy Limit of QCD beyond Sudakov Approximation [T. Liu, A. Penin, 1709.01092]

  Subleading power resummed thrust spectrum for $H \to gg$ [I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

  Drell-Yan production at threshold [M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

  Diagrammatic resummation of threshold effects [N. Bahjat-Abbas, D. Bonocore, J. Sinninghe Damste, E. Laenen, L. Magnea, L. Vernazza, C. White, 1905.13710]

More on NLP:

- Violation of the Kluberg-Stern-Zuber theorem in SCET [M. Beneke, M. Garny, R. Szafron, J. Wang, 1907.05463]
- Next-to-leading power corrections to $V+1$ jet production in N-jettiness subtraction [R. Boughezal, A. Isgró , F. Petriello , 1907.12213]
- Next-to-leading power threshold effects for inclusive and exclusive processes with final state jets [M. van Beekveld, W. Beenakker, E. Laenen, C. D. White, 1905.08741]
- Regge Limit of Gauge Theory Amplitudes beyond Leading Power Approximation [A. Penin, 1909.02013]

Formalism of Soft Collinear Effective Theory (SCET)

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, 0206152]

$$\mathcal{L}_{\rm SCET} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\rm soft}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of a small parameter λ:

$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\rm LP} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, 0206152]

$$\mathcal{L}_{\rm SCET} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\rm soft}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of a small parameter λ:

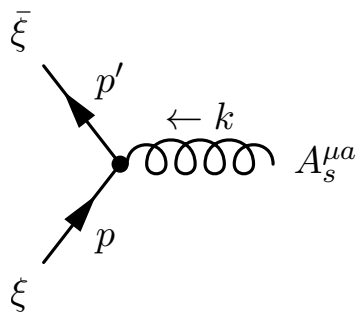
$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\rm LP} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Separate collinear sectors interact only through soft gluon interactions. Focusing in LP term:

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + g\, n_- A_s(x_-) + i\not{D}_{\perp c} \frac{1}{in_+ D_c}\, i\not{D}_{\perp c} \right) \frac{\not{n}_+}{2}\, \xi + \mathcal{L}_{c,\rm YM}^{(0)}$$

with $in_- D_c = in_- \partial + g\, n_- A_c(x)$, $x_-^\mu = (n_+ x)\dfrac{n_-^\mu}{2}$.

The soft interaction with each collinear field at LP is given by the standard eikonal vertex



$$ig_s t^a \frac{\not{n}_+}{2} n_{-\mu} \qquad \mathcal{O}(\lambda^0)$$

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, 0206152]

$$\mathcal{L}_{\rm SCET} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\rm soft}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of a small parameter λ:

$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\rm LP} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Separate collinear sectors interact only through soft gluon interactions. Focusing in LP term:

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + g\, n_- A_s(x_-) + i\not{D}_{\perp c} \frac{1}{in_+ D_c}\, i\not{D}_{\perp c} \right) \frac{\not{n}_+}{2}\, \xi + \mathcal{L}_{c,\rm YM}^{(0)}$$

with $in_- D_c = in_- \partial + g\, n_- A_c(x)$, $x_-^\mu = (n_+ x)\dfrac{n_-^\mu}{2}$.

The decoupling transformation, $\chi_c^{(0)} = Y_+^\dagger(0)\chi_c$, separates soft and collinear sectors at LP

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)} \frac{\not{n}_+}{2} \left(n_- \mathcal{A}_c + n_- \partial \right) \chi^{(0)}(x)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

SCET formalism: Lagrangian

The structure of the SCET Lagrangian beyond LP is more intricate

[M. Beneke, Th. Feldmann, 0211358]

$$\mathcal{L}_c^{(1)} = \bar{\xi}\left(x_\perp^\mu\, n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}_{\rm YM}^{(1)} + \left(\bar{q}\, W_c^\dagger\, i\not{D}_\perp\,\xi + \text{h.c.}\right)$$

$$\begin{aligned}\mathcal{L}_\xi^{(2)} = {}& \frac{1}{2}\,\bar{\xi}\left((n_-x)\, n_+^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu W_c \big[D_{\rm us}^\rho, g F_{\mu\nu}^{\rm us}\big] W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \\ & + \frac{1}{2}\,\bar{\xi}\left(i\not{D}_{\perp c}\,\frac{1}{in_+D_c}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger + x_\perp^\mu\gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\,\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi\end{aligned}$$

SCET formalism: Lagrangian

The structure of the SCET Lagrangian beyond LP is more intricate

[M. Beneke, Th. Feldmann, 0211358]

$$\mathcal{L}_c^{(1)} = \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}_{\rm YM}^{(1)} + \left(\bar{q}\, W_c^\dagger\, i\not{D}_\perp\,\xi + \text{h.c.}\right)$$

$$\begin{aligned}\mathcal{L}_\xi^{(2)} = {}& \frac{1}{2}\,\bar{\xi}\left((n_-x)\, n_+^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu W_c \big[D_{\rm us}^\rho, g F_{\mu\nu}^{\rm us}\big] W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \\ & + \frac{1}{2}\,\bar{\xi}\left(i\not{D}_{\perp c}\,\frac{1}{in_+D_c}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger + x_\perp^\mu\gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\,\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi\end{aligned}$$

- Importantly, there are no purely collinear interactions at subleading powers. In each vertex there is at least one soft field.
- coordinate space arguments appear in the Lagrangian due to multipole expansion of the soft modes:
  $\phi_s(x)\phi_c(x) = (\phi_s(x_-) + \underbrace{x_\perp\cdot\partial_\perp\phi_s(x_-)}_{\mathcal{O}(\lambda)} + \dots)\phi_c(x)$

SCET formalism: N-jet operator basis

Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i}\; C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger\, \xi_i \qquad\qquad \mathcal{A}_{i\perp}^{\mu} \equiv W_i^\dagger [iD_{\perp i}^{\mu}\, W_i]$$

by acting on these with derivatives $i\partial_{\perp i}^{\mu} \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current.

SCET formalism: N-jet operator basis

Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i}\; C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger\, \xi_i \qquad\qquad \mathcal{A}_{i\perp}^\mu \equiv W_i^\dagger [iD_{\perp i}^\mu\, W_i]$$

by acting on these with derivatives $i\partial_{\perp i}^\mu \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current

We adopt the notation: J_i^{An}, J_i^{Bn}, J_i^{Cn}, J_i^{Tn} where:

- $A, B, C...$ refers to number of fields in a given collinear direction
- n is the power of λ suppression (relative to $A0$) in a given sector.

at $\mathcal{O}(\lambda^2)$ for example we can construct J_i^{A2}, J_i^{B2}, J_i^{C2}, J_i^{T2} respectively:

$$i\partial_{\perp i}^\mu i\partial_{\perp i}^\nu \chi_i, \qquad \chi_i(t_{i_1}) i\partial_{\perp i}^\nu \mathcal{A}_{i\perp}^\mu(t_{i_2}), \qquad \chi_i(t_{i_1})\chi_i(t_{i_2})\mathcal{A}_{i\perp}^\mu(t_{i_3}), \qquad i\int d^4z\, \mathbf{T}\left[\chi_i(t_{i_1})\mathcal{L}^{(2)}(z)\right]$$

And a $\mathcal{O}(\lambda^2)$ 2-jet operator could be

$$J_1^{A2} J_2^{A0}, \qquad\qquad J_1^{A1} J_2^{B1}, \; ...$$

Drell-Yan process

The Drell-Yan process at threshold

$$A(p_A)B(p_B) \to \mathrm{DY}(Q) + X$$

$$z = \frac{Q^2}{\hat{s}} \to 1 \qquad \lambda = \sqrt{(1-z)}$$

$$\begin{aligned} p_c &= (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda) \\ p_{\bar{c}} &= (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda) \\ p_s &= (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2) \end{aligned}$$

$$Q^2\lambda^2 = Q^2(1-z) \gg \Lambda^2$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t})\, J_\mu^{A0A0}(t,\bar{t})$$

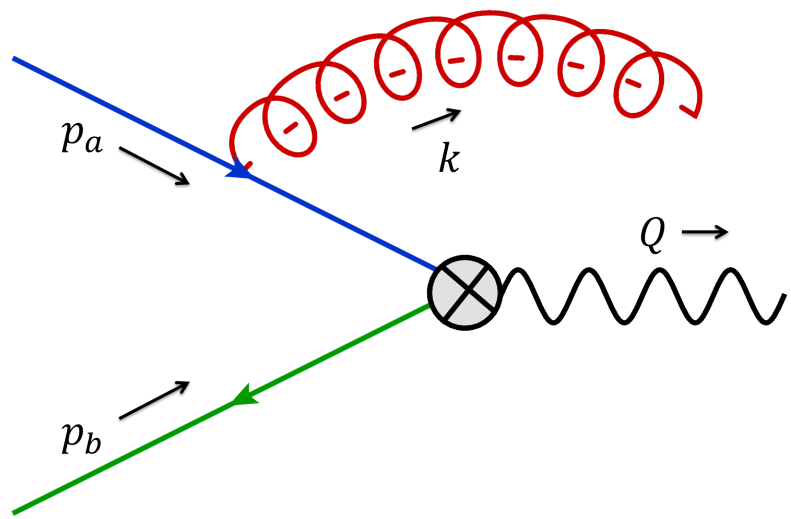
$$J_\rho^{A0A0}(t,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$

After decoupling, leading power current becomes:

$$J_\rho^{A0}(t,\bar{t}) = \bar{\chi}_{\bar{c}}^{(0)}(\bar{t}n_-)Y_-^\dagger(0)\gamma_{\perp\rho}Y_+(0)\chi_c^{(0)}(tn_+)$$

and states factorize:

$$\langle X| = \langle X_{\bar{c}}^{\mathrm{PDF}}|\, \langle X_c^{\mathrm{PDF}}|\, \langle X_s|$$



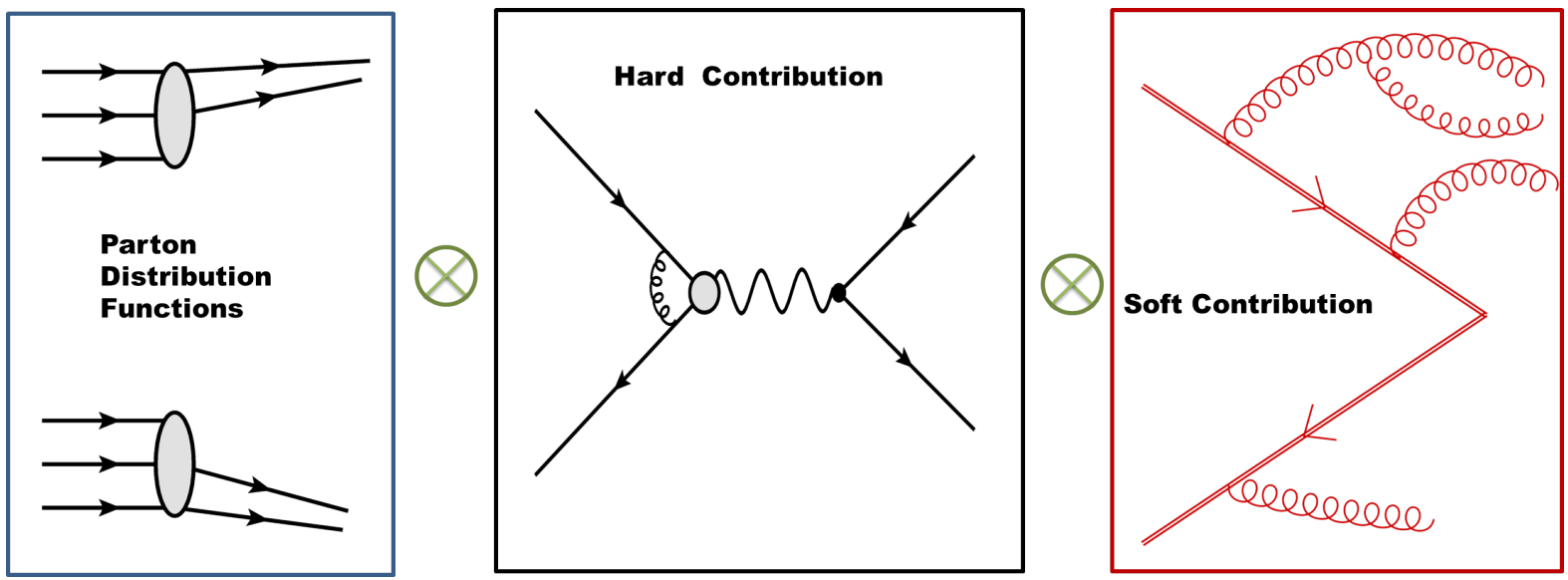
The Drell-Yan proccess - the leading power result

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\rm LP}(z)$$

where

[G. P. Korchemsky *et al.*, 1993]
[T. Becher *et al.*, 0710.0680, S. Moch *et al.*, 0508265]

$$\hat{\sigma}^{\rm LP}(z) = |C(Q^2)|^2 \; Q \; S_{\rm DY}(Q(1-z))$$



The Drell-Yan proccess - the leading power result

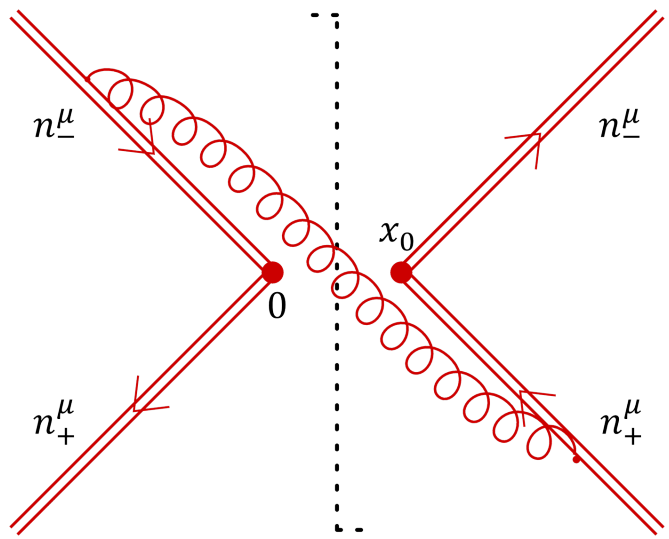
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_cQ^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\,\hat{\sigma}_{ab}^{\rm LP}(z)$$

where

[G. P. Korchemsky *et al.*, 1993]
[T. Becher *et al.*, 0710.0680, S. Moch *et al.*, 0508265]

$$\hat{\sigma}^{\rm LP}(z) = |C(Q^2)|^2\; Q\; S_{\rm DY}(Q(1-z))$$

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\,\frac{1}{N_c}\,{\rm Tr}\,\langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\,\mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$



Factorization formula at NLP

First a schematic formula:

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\rm NLP}(z)$$

The $\hat{\sigma}_{ab}^{\rm NLP}(z)$ is given by

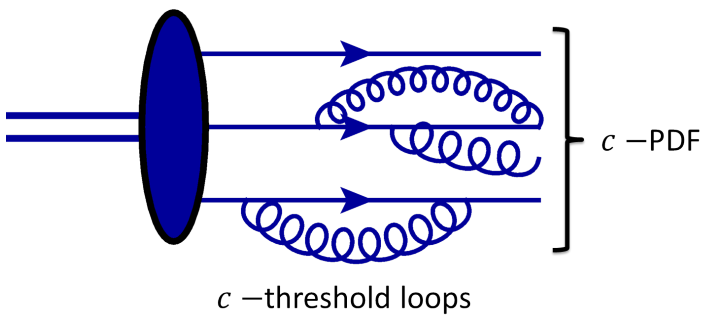
$$\hat{\sigma}^{\rm NLP} = \sum_{\rm terms} [C \otimes J \otimes \bar{J}]^2 \otimes S$$

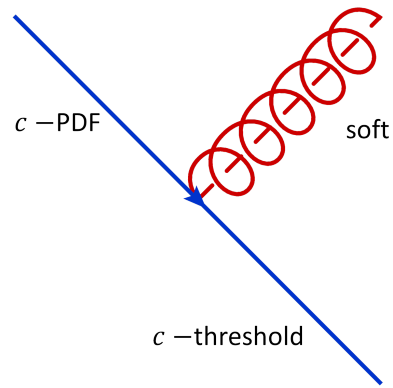
- C is the hard Wilson matching coefficient
- S is the generalized soft function
- J is the collinear function

Let us now motivate the emergence of this structure at next-to-leading power.

Collinear functions at LP and NLP

- The collinear function at LP is unity because of decoupling transformation. The threshold collinear modes can trivially be identified with c-PDF modes, $\chi_c \to \chi_c^{\rm PDF}$.

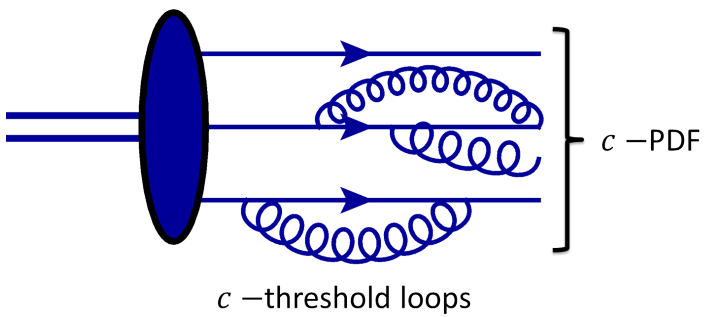


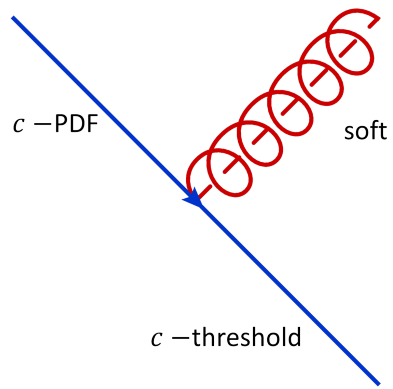


$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^{\mu} = i\int d^4z\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(z)\right]$$

Collinear functions at LP and NLP

- The collinear function at LP is unity because of decoupling transformation. The threshold collinear modes can trivially be identified with c-PDF modes, $\chi_c \to \chi_c^{\rm PDF}$.
- This is no longer true at NLP. Consider an example of subleading SCET Lagrangian: $\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar{\chi}^{(0)}_c z^\mu_\perp z^\rho_\perp \left[i\partial_\rho i n_- \partial \mathcal{B}^+_\mu(z_-)\right] \frac{\not{n}_+}{2}\chi^{(0)}_c$, $\mathcal{B}^\mu_\pm = Y^\dagger_\pm \left[iD^\mu_s Y_\pm\right]$
  [M. Beneke and Th. Feldmann, 0211358]
- Crucially, an insertion of a piece of a subleading Lagrangian comes with an integral over its position, $\int d^4z$, which induces a collinear scale.





$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^\mu = i\int d^4z\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(z)\right]$$

Collinear functions at NLP

- PDF collinear modes can be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

Collinear functions at NLP

- PDF collinear modes can be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

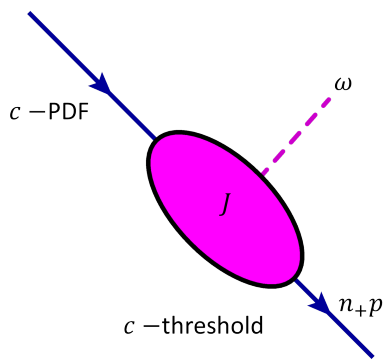
- Hence we define the matching equation which gives a SCET definition of what is known as the "radiative jet function"
  [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C.D. White, 1503.05156]
  see also [D. Bonocore, E. Laenen, L. Magnea, L. Vernazza, C.D. White, 1610.06842]

- $$i \int d^4z \, \mathbf{T}\Big[\chi_{c,\gamma f}\,(tn_+)\;\mathcal{L}^{(2)}(z)\Big]$$
  $$= 2\pi \sum_i \int du \int \frac{d(n_+z)}{2}\; \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t, u; \frac{n_+z}{2}\right) \chi^{\mathrm{PDF}}_{c,\beta b}(un_+)\, \mathfrak{s}_{i;\mu,d}(z_-)$$

$$\mathfrak{s}_i(z_-) \in \left\{ \frac{i\partial^\mu_\perp}{in_-\partial} \mathcal{B}^+_{\mu\perp}(z_-)\,,\; \frac{i\partial_{[\mu\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]}(z_-)\,,\; \frac{1}{(in_-\partial)} \big[\mathcal{B}^+_{\mu\perp}(z_-), \mathcal{B}^+_{\nu\perp}(z_-)\big], \ldots \right\}$$

[M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

[M. Beneke, A. Broggio, SJ, L. Vernazza, to appear soon]



Collinear functions at NLP

- PDF collinear modes can be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

- Hence we define the matching equation which gives a SCET definition of what is known as the "radiative jet function"
  [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C.D. White, 1503.05156]
  see also [D. Bonocore, E. Laenen, L. Magnea, L. Vernazza, C.D. White, 1610.06842]

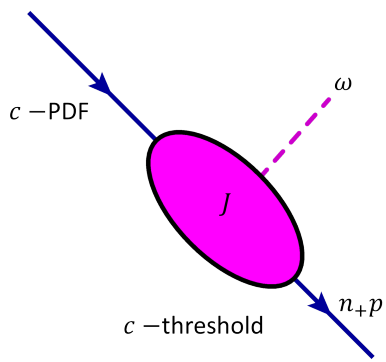
-

$$i \int d^4z\, \mathbf{T}\Big[\chi_{c,\gamma f}\,(tn_+)\;\mathcal{L}^{(2)}(z)\Big]$$
$$= 2\pi \sum_i \int du \int \frac{d(n_+z)}{2}\; \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t, u; \frac{n_+z}{2}\right) \chi^{\mathrm{PDF}}_{c,\beta b}(un_+)\, \mathfrak{s}_{i;\mu,d}(z_-)$$

$$\mathfrak{s}_i(z_-) \in \left\{ \frac{i\partial^\mu_\perp}{in_-\partial} \mathcal{B}^+_{\mu_\perp}(z_-)\,,\; \frac{i\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]}(z_-)\,,\; \frac{1}{(in_-\partial)} \big[\mathcal{B}^+_{\mu_\perp}(z_-), \mathcal{B}^+_{\nu_\perp}(z_-)\big], \ldots \right\}$$

[M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

[M. Beneke, A. Broggio, SJ, L. Vernazza, to appear soon]



Equation of motion:

$$n_+\mathcal{B}^+(z_-) = -2\frac{i\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_-) - 2\frac{[\mathcal{B}^\mu_\perp, [in_-\partial\mathcal{B}_{\mu_\perp}]]}{(in_-\partial)^2} + \ldots$$

Collinear functions at NLP

- PDF collinear modes can be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

- Hence we define the matching equation which gives a SCET definition of what is known as the "radiative jet function"
  [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C.D. White, 1503.05156]
  see also [D. Bonocore, E. Laenen, L. Magnea, L. Vernazza, C.D. White, 1610.06842]

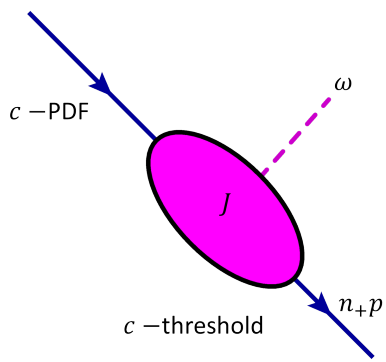
-

$$i \int d^4z\, \mathbf{T}\Big[\chi_{c,\gamma f}\,(tn_+)\;\mathcal{L}^{(2)}(z)\Big]$$
$$= 2\pi \sum_i \int du \int \frac{d(n_+z)}{2}\; \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t, u; \frac{n_+z}{2}\right) \chi^{\mathrm{PDF}}_{c,\beta b}(un_+)\, \mathfrak{s}_{i;\mu,d}(z_-)$$

$$\mathfrak{s}_i(z_-) \in \left\{ \frac{i\partial^\mu_\perp}{in_-\partial} \mathcal{B}^+_{\mu_\perp}(z_-)\,,\; \frac{i\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]}(z_-)\,,\; \frac{1}{(in_-\partial)} \big[\mathcal{B}^+_{\mu_\perp}(z_-), \mathcal{B}^+_{\nu_\perp}(z_-)\big], \ldots \right\}$$

[M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

[M. Beneke, A. Broggio, SJ, L. Vernazza, to appear soon]



A similar definition appeared recently in
[I. Moult, I.W. Stewart, G. Vita, 1905.07411]

General collinear functions

- The discussed construction is actually general at subleading powers, not only next-to-leading power
- There can be many Lagrangian insertions at various positions each with its own ω_i conjugate to the large component of threshold collinear momentum

We can separate the Lagrangian insertions

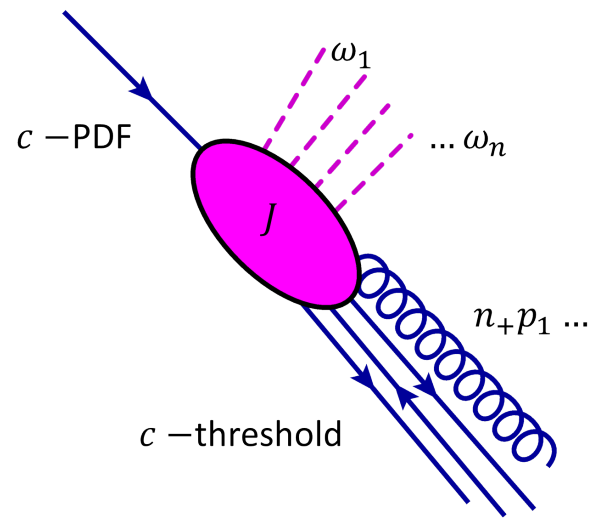
$$\mathcal{L}_V^{(n)}(z) = \mathcal{L}_c^{(n)}(z) \otimes \mathcal{L}_s^{(n)}(z_-)$$

$$i^n \left(\prod_{j=1}^{n} \int d^4 z_j \right)$$

$$\times \mathbf{T} \left[\chi_c(t_1 n_+) \chi_c(t_2 n_+) ... \times \mathcal{L}^{(n)}(z_1) \times ... \times \mathcal{L}^{(m)}(z_n) \right]$$

$$= 2\pi \sum_i \int du \left(\prod_{j=1}^{n} \int dz_{j-} \right) \tilde{J}_i\,(t_1, t_2 ..., u; z_{1-}, ..., z_{n-})$$

$$\times \chi_c^{\rm PDF}(u n_+)\ \mathfrak{s}_i(z_{1-}, ..., z_{n-})$$



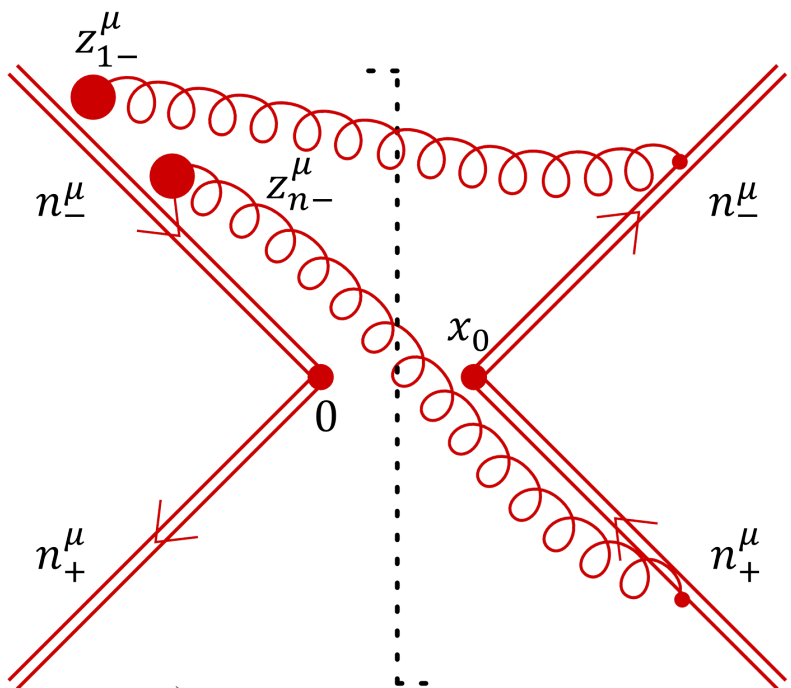
Generalized soft functions

The necessary presence of the time-ordered products at NLP gives rise to the concept of generalized soft functions with explicit gauge fields. Schematically we have

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(n_+ z_j)}{4\pi}\, e^{-i\omega_j (n_+ z_j)/2} \right)$$
$$\times \,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\times \mathcal{L}_s^{(n)}(z_{1-})\times\ldots\times\mathcal{L}_s^{(n)}(z_{n-})\right]|0\rangle$$

$\mathcal{L}_s^{(n)}(z_{j-})$ contains $\mathcal{B}^+_{\perp\nu}(z_{j-})$ fields, not only Wilson lines. For more details on generalized soft functions see talk by Robert Szafron.

[M. Beneke , F. Campanario, T. Mannel, B.D. Pecjak, 0411395]



Derivation of the factorization formula at NLP

Factorization formula at NLP

First step in derivation is to extend the matching equation of the DY to SCET current up to NLP accuracy:

$$\bar{\psi}\gamma_\rho\psi(0) = \sum_{m_1,m_2} \int \{dt_k\}\,\{d\bar{t}_{\bar{k}}\}\,\widetilde{C}^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)\;J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)$$

$$J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right) = J_{\bar{c}}^{\,m_1}\left(\{\bar{t}_{\bar{k}}\}\right)\,\Gamma_\rho^{m_1,m_2}\,J_c^{\,m_2}\left(\{t_k\}\right)$$

Contrast with LP where the current is simply given by:

$$J_\rho^{A0A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$

Factorization formula at NLP

First step in derivation is to extend the matching equation of the DY to SCET current up to NLP accuracy:

$$\bar{\psi}\gamma_\rho\psi(0) = \sum_{m_1,m_2} \int \{dt_k\}\,\{d\bar{t}_{\bar{k}}\}\,\widetilde{C}^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)\;J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)$$

$$J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right) = J_{\bar{c}}^{\,m_1}\left(\{\bar{t}_{\bar{k}}\}\right)\,\Gamma_\rho^{m_1,m_2}\,J_c^{\,m_2}\left(\{t_k\}\right)$$

Contrast with LP where the current is simply given by:

$$J_\rho^{A0A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$

Now must consider all possible sources of power suppression. In the presented formalism, this means including power suppressed currents, A1, B1, A2, B2, C2 and T1, T2 with all possible Lagrangian insertions for each direction. For example:

$$\begin{aligned} J_\rho^{A0A2}(t,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}i\partial_\perp^\mu i\partial_{\perp\mu}\chi_c(tn_+) \\ J_\rho^{A0B2}(t_1,t_2,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\mathcal{A}_{c\perp}^\mu(t_2n_+)i\partial_\mu\chi_c(t_1n_+) \end{aligned}$$

Factorization formula at NLP

First step in derivation is to extend the matching equation of the DY to SCET current up to NLP accuracy:

$$\bar{\psi}\gamma_\rho\psi(0) = \sum_{m_1,m_2} \int \{dt_k\}\,\{d\bar{t}_{\bar{k}}\}\,\widetilde{C}^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)\;J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)$$

$$J_\rho^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right) = J_{\bar{c}}^{\,m_1}\left(\{\bar{t}_{\bar{k}}\}\right)\,\Gamma_\rho^{m_1,m_2}\,J_c^{\,m_2}\left(\{t_k\}\right)$$

Note that currents without time-ordered product operators with a Lagrangian insertion can be discarded: lead to scaleless integrals just as at LP!

$$J_c^{T2}(t) = i\int d^4z\,\mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}^{(2)}(z)\right]$$

Now must consider all possible sources of power suppression. In the presented formalism, this means including power suppressed currents, ~~A1~~, B1, ~~A2~~, B2, ~~C2~~ and T1, T2 with all possible Lagrangian insertions for each direction. For example:

$$J_\rho^{A0A2}(t,\bar{t}\,) \;=\; \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}i\partial_\perp^\mu i\partial_{\perp\mu}\chi_c(tn_+)$$

$$J_\rho^{A0B2}(t_1,t_2,\bar{t}\,) \;=\; \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\mathcal{A}_{c\perp}^\mu(t_2n_+)i\partial_\mu\chi_c(t_1n_+)$$

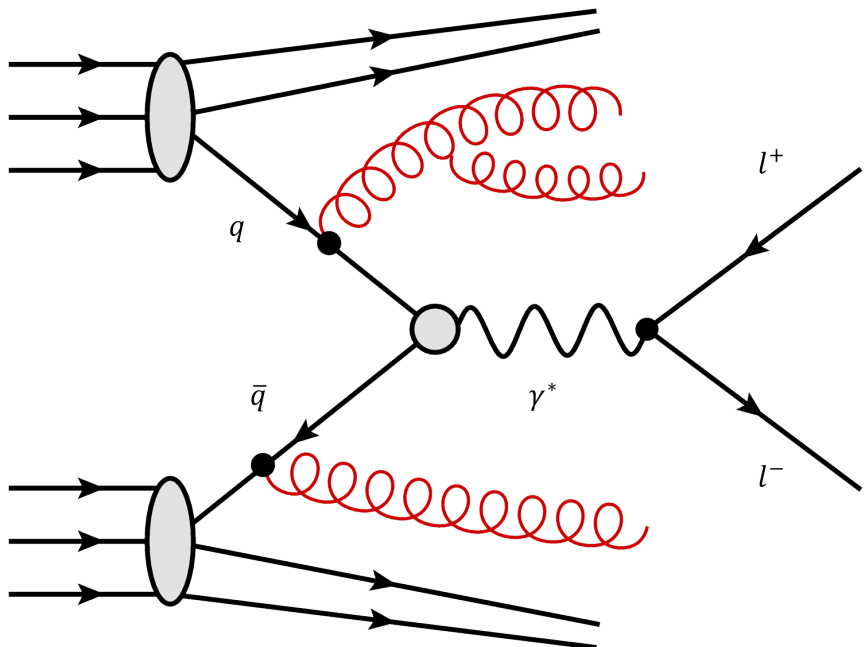
Factorization formula at NLP

Cross section is given by a combination of the lepton and hadronic tensor

$$d\sigma = \frac{4\pi\alpha_{\rm EM}^2}{3sq^2}\frac{d^4q}{(2\pi)^4}\left(-g^{\mu\rho}W_{\mu\rho}\right)$$

where the hadronic tensor is given by

$$\begin{aligned} g^{\mu\rho}W_{\mu\rho} &= \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle \\ &= \sum_X \langle A(p_A)B(p_B)|J_\rho^\dagger(0)|X\rangle\langle X|J^\rho(0)|A(p_A)B(p_B)\rangle \\ &\qquad \times(2\pi)^4\delta\left(p_A+p_B-q-p_{X_s}-p_{X_c^{\rm PDF}}-p_{X_{\bar{c}}^{\rm PDF}}\right) \end{aligned}$$



Factorization formula at NLP

Cross section is given by a combination of the lepton and hadronic tensor

$$d\sigma = \frac{4\pi\alpha_{\rm EM}^2}{3sq^2}\frac{d^4q}{(2\pi)^4}\left(-g^{\mu\rho}W_{\mu\rho}\right)$$

where the hadronic tensor is given by

$$\begin{aligned} g^{\mu\rho}W_{\mu\rho} &= \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle \\ &= \sum_X \langle A(p_A)B(p_B)|J_\rho^\dagger(0)|X\rangle\langle X|J^\rho(0)|A(p_A)B(p_B)\rangle \\ &\quad \times(2\pi)^4\delta\left(p_A+p_B-q-p_{X_s}-p_{X_c^{\rm PDF}}-p_{X_{\bar c}^{\rm PDF}}\right) \end{aligned}$$

The recipe for derivation of the factoritation formula

- Consider the matrix element, with only soft radiation allowed due to threhold kinematics
- Consider all possible insertions of subleading power Lagrangian and perform the second matching of threshold collinear fields to PDF collinear fileds. Here we introduce the new objects: collinear functions

$$i\int d^4z\,\mathbf{T}\left[\chi_{c,\gamma f}\,(tn_+)\;\mathcal{L}^{(2)}(z)\right] = 2\pi\sum_i\int du\int\frac{d(n_+z)}{2}\,\tilde J_{i;\gamma\beta,\mu,fbd}\left(t,u;\frac{n_+z}{2}\right)\chi^{\rm PDF}_{c,\beta b}(un_+)\,\mathfrak{s}_{i;\mu,d}(z_-)$$

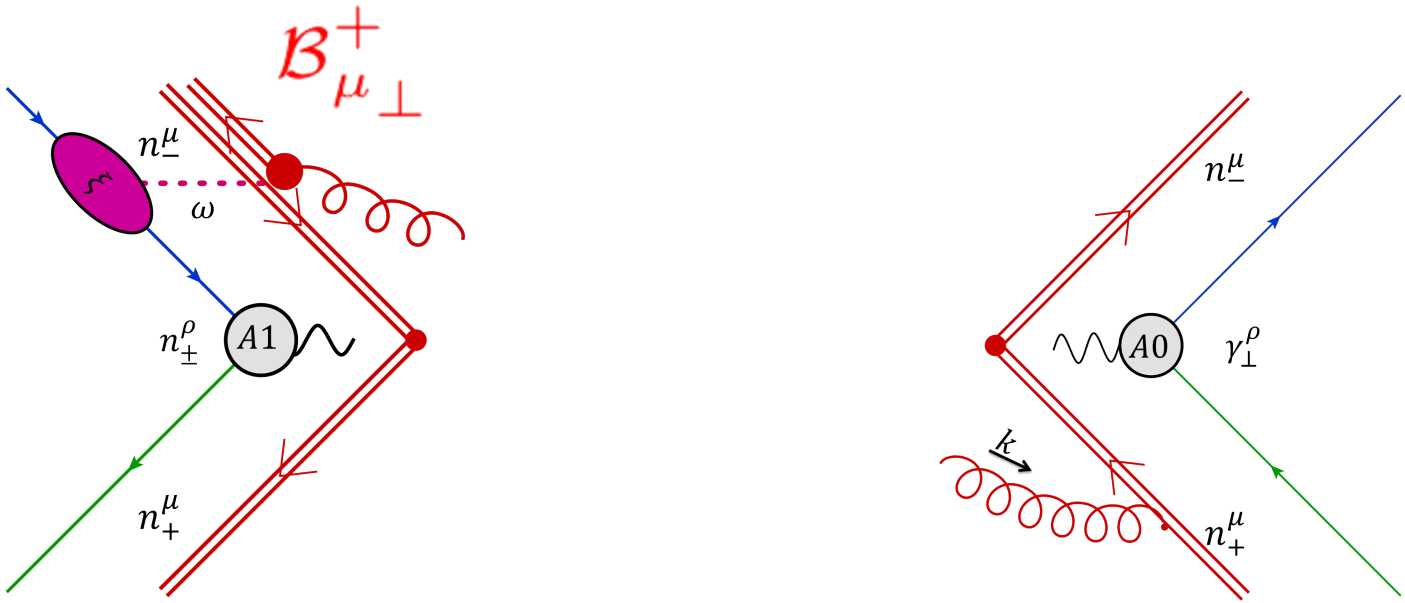
- Usual steps follow: square amplitude, sum over intermediate states $\to$ standard PDFs

Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho} W_{\mu\rho} \quad = \quad \int d^4x e^{-iq\cdot x} \langle A(p_A) B(p_B) | J^{\dagger\,\rho}(x) J_\rho(0) | A(p_A) B(p_B) \rangle$$

Consider : $\quad J_c^{T2}(t) = i \int d^4z\, \mathbf{T}\left[J_c^{A1}(t)\, \mathcal{L}^{(1)}(z) \right]$



$$J_\rho^{A0,A1}(t,\bar{t}) \quad = \quad \bar{\chi}_{\bar{c}}(\bar{t}\, n_-)\, n_{+\rho}\; i\not{\partial}_\perp \chi_c(t\, n_+)$$

$$J_\rho^{A0,B1}(t_1,t_2,\bar{t}) \quad = \quad \bar{\chi}_{\bar{c}}(\bar{t}\, n_-)\, n_{\pm\rho}\; \not{\mathcal{A}}_{\perp c}(t_2\, n_+) \chi_c(t_1\, n_+)$$

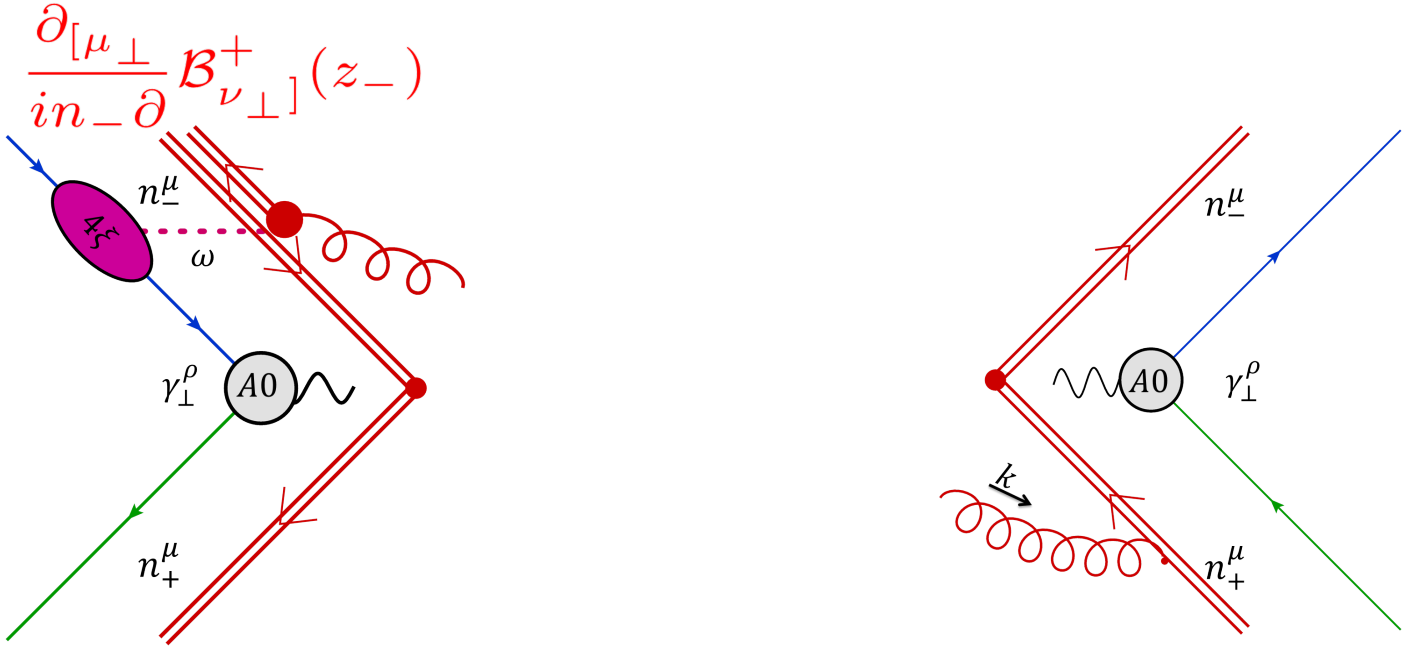
Contributions from power suppressed currents can start contributing at NNLP!
Only the LP J^{A0A0} and insertions of the Lagrangian needed up to NLP.

Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho}W_{\mu\rho} \quad = \quad \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

Consider : $\quad J_c^{T2}(t) = i\int d^4z\,\mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}^{(2)}(z)\right]$



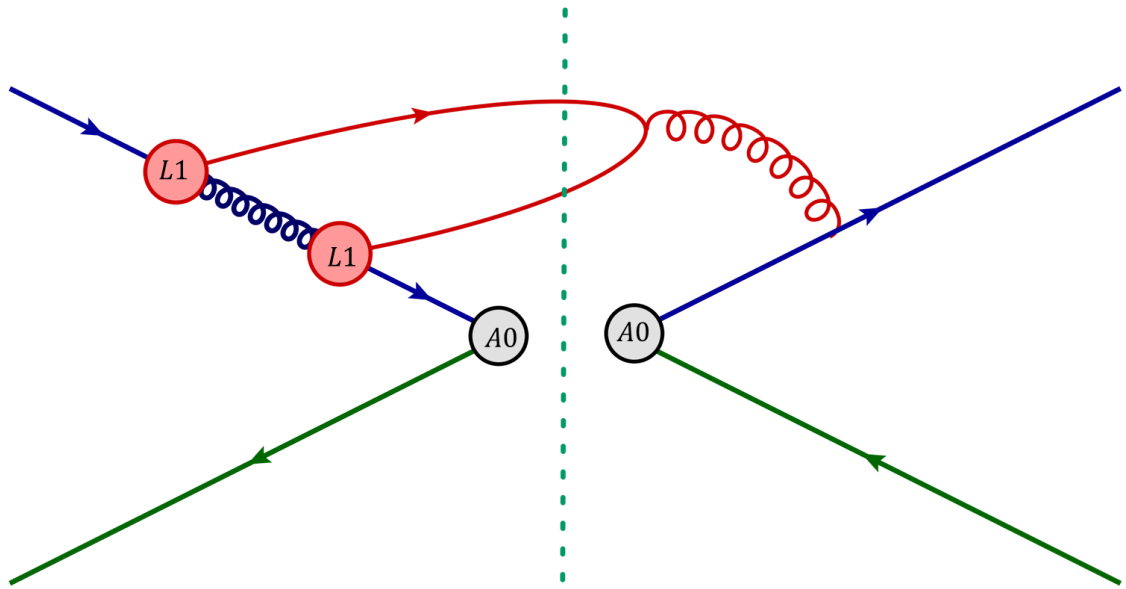
$$\widetilde{S}_{4\xi;\mu\nu}(x^0;\omega) \;=\; \int dz_-\, e^{-i\omega\, z_-}\,\frac{1}{N_c}\,\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger Y_-\right](x^0)\mathbf{T}\left(\left[Y_-^\dagger Y_+\right](0)\,\frac{i\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^+_{\nu_\perp]}(z_-)\right)|0\rangle$$

Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho}W_{\mu\rho} \quad = \quad \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

Consider : $J_c^{T2}(t) = i^2 \int d^4z_1\, d^4z_2\, \mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}^{(1)}_{q\xi}(z_1)\mathcal{L}^{(1)}_{q\xi}(z_2)\right]$



$$\mathcal{L}^{(1)}_{\xi q}(z) \quad = \quad \bar{q}_+(z_-)\mathcal{A}_{c\perp}(z)\chi_c(z) + \text{h.c.}$$

Power corrections due to soft quark contributions studied in B-physics [M. Beneke, F. Campanario, T. Mannel, B. Pecjak, 0411395]

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^\rho_\perp\right]_{\beta\gamma}\int d(n_+p)\;C^{\,A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(\,x_a\,n_+p_A,\,x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

For comparison, LP result is:

$$\Delta_{\,\rm LP}(z) = |C(Q^2)|^2\;Q\;S_{\rm DY}(Q(1-z))$$

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^\rho_\perp\right]_{\beta\gamma}\int d(n_+p)\;C^{\,A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(x_a\,n_+p_A,\, x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_{\,i}\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\;e^{-i\omega_j z_{j-}}\;\times\;\frac{1}{N_c}\,\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y^\dagger_+\,Y_-\right](x)\right)\mathbf{T}\left(\left[Y^\dagger_-\,Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \;\in\; \left\{\frac{i\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu\perp}(z_{1-}),\,\frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu}{}_\perp(z_{1-}),\left[in_-\partial\mathcal{B}^+_{\mu\perp}(z_{1-})\right]\right],\right.$$

$$\left.\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu\perp}(z_{1-}),\mathcal{B}^+_{\nu\perp}(z_{1-})\right],\,\frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu\perp}(z_{1-})\mathcal{B}^+_{\nu\perp}(z_{2-}),\,\frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\right\}$$

For comparison, LP result is:

$$\Delta_{\rm LP}(z) = |C(Q^2)|^2\;Q\;S_{\rm DY}(Q(1-z))$$

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^\rho_\perp\right]_{\beta\gamma}\int d(n_+p)\;C^{\,A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(x_a\,n_+p_A,\, x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_{\,i}\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\;e^{-i\omega_j z_{j-}}\;\times\;\frac{1}{N_c}\,\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y^\dagger_+\,Y_-\right](x)\right)\mathbf{T}\left(\left[Y^\dagger_-\,Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \;\in\; \left\{\frac{i\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu\perp}(z_{1-}),\,\frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu}{}_\perp(z_{1-}),\left[in_-\partial\mathcal{B}^+_{\mu\perp}(z_{1-})\right]\right],\right.$$

$$\left.\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu\perp}(z_{1-}),\mathcal{B}^+_{\nu\perp}(z_{1-})\right],\,\frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu\perp}(z_{1-})\mathcal{B}^+_{\nu\perp}(z_{2-}),\,\frac{1}{(in_-\partial)^2}\,q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\right\}$$

At NNLO accuracy there are three contributions:

- Collinear: 1-loop collinear and NLO soft functions
- Hard: 1-loop hard and NLO soft functions
- Soft: NNLO soft functions

Only one of the soft building blocks starts with a single gluon emission.

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^\rho_\perp\right]_{\beta\gamma}\int d(n_+p)\;C^{\,A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(x_a\,n_+p_A,\, x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

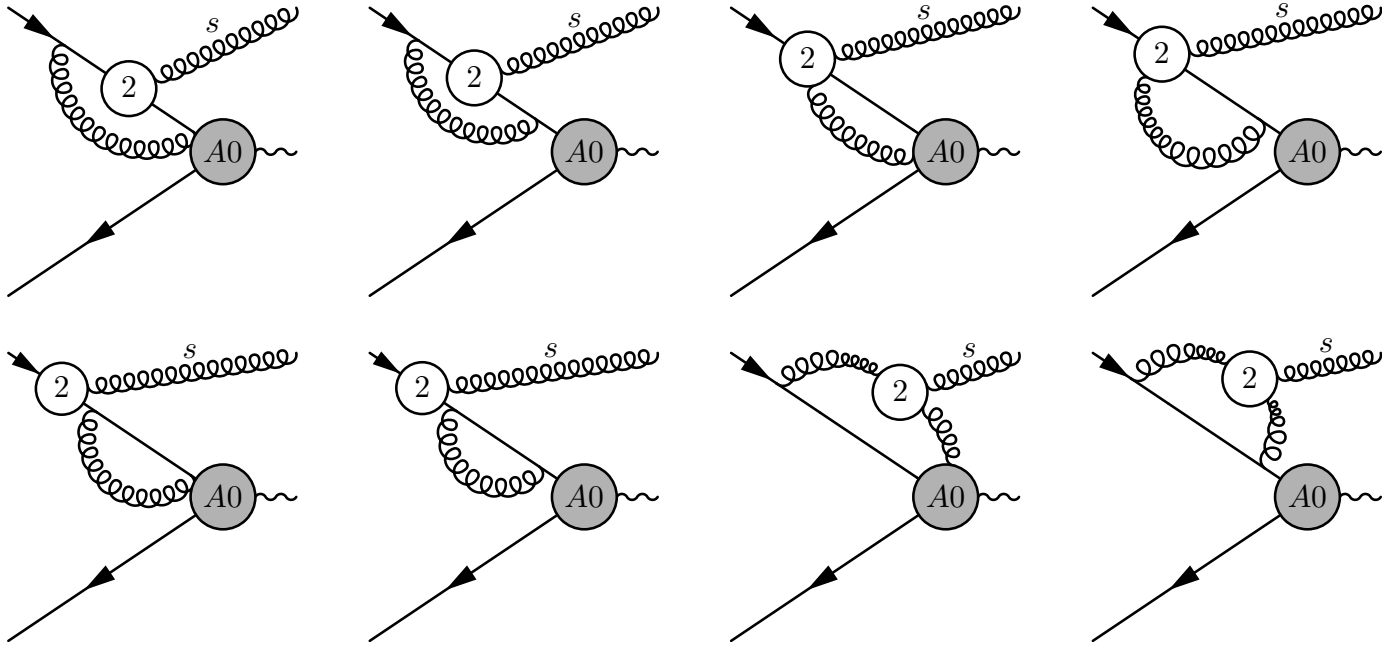
$$\Delta^{dyn\,(2)}_{\rm NLP-coll}(z) \quad = \quad 4\,Q\int d\omega\; J^{(1)}_{1,2}\,(x_a\,n_+p_A;\omega)\,S^{(1)}_1(\Omega;\omega)$$

At NNLO accuracy there are three contributions:

- Collinear: 1-loop collinear and NLO soft functions
- Hard: 1-loop hard and NLO soft functions
- Soft: NNLO soft functions

Only one of the soft building blocks starts with a single gluon emission.

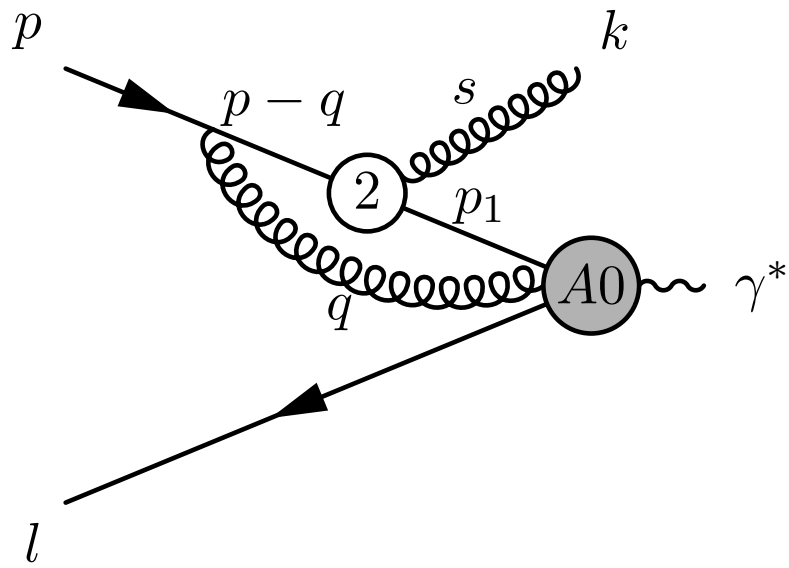
1-loop collinear function calculation



$$\langle g(k)_K|\mathcal{J}^{1g}_{\gamma f}(0)|q(p_A)_q\rangle = \int dt\, dn_+p_1\, e^{itn_+p_1} \int \frac{dn_+p_a}{2\pi} du\, e^{i\, n_+p_a\, u} \int \frac{d\omega}{2\pi}\, dz_-\, e^{-i\,\omega\, z_-}$$

$$\times \int \frac{dn_+p}{2\pi}\, e^{-i\, n_+p\, t} \Big(J^A_{1,2;\gamma\beta,fb}\,(n_+p, n_+p_a; \omega)\, \langle 0|\chi^{\rm PDF}_{c,\beta b}(un_+)|q(p_A)_q\rangle\, \langle g(k)_K|\mathfrak{s}_{1;A}(z_-)\,|0\rangle \Big)$$

1-loop collinear function calculation



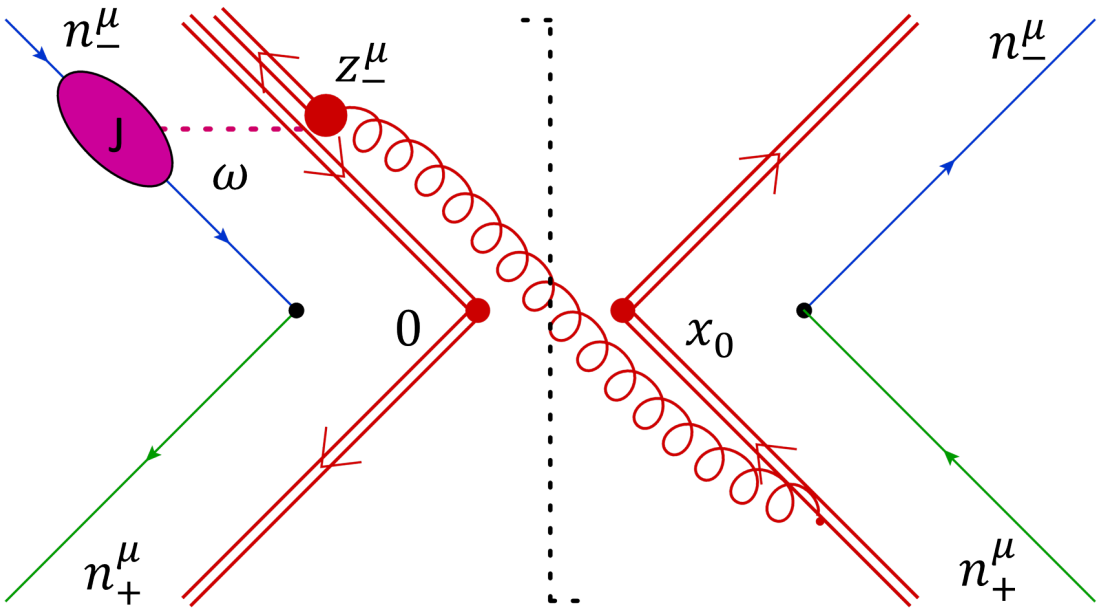
$$I_{\text{fig}} = \frac{g_s\,\alpha_s}{4\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\mathbf{T}^b}{(n_+p)}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\left\{2\left(\frac{(n_+k)}{(n_-k)}n_-^\nu - n_+^\nu\right)\right.$$

$$\left. + \left(\frac{k_\perp^2\, n_-^\nu}{(n_-k)^2} - \frac{k_\perp^\nu}{(n_-k)}\right)\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} + 2 + \zeta_2\right) + \frac{\left[\gamma_\perp^\nu\,,\, \not{k}_\perp\right]}{(n_-k)}\left(-\frac{1}{\epsilon^2} + \zeta_2\right)\right\} u_c(p)\epsilon^*_{\nu b}(k) + \mathcal{O}(\epsilon)$$

Final result

After combination with leptonic part and stripping off the PDFs

$$\Delta^{dyn\,(2)}_{\text{NLP}-\text{coll}}(z) \quad = \quad 4\,Q \int d\omega\, J^{(1)}_{1,2}\,(x_a\, n_+ p_A; \omega)\, S^{(1)}_1(\Omega; \omega)$$



$$\mathfrak{s} = \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_-)$$

Final result

After combination with leptonic part and stripping off the PDFs

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-coll}}(z) \quad = \quad 4\,Q \int d\omega\, J^{(1)}_{1,2}\,(x_a\, n_+p_A;\omega)\, S^{(1)}_1(\Omega;\omega)$$

$$J_{1,2}(x_a n_+p_A;\omega) \quad = \quad -\frac{1}{x_a(n_+p_A)} - 2\frac{\partial}{x_a\partial(n_+p_A)}$$
$$+\frac{\alpha}{4\pi}\frac{1}{x_a(n_+p_A)}\left[\frac{\omega(x_a n_+p_A)}{\mu^2}\right]^{-\epsilon}\left(C_A\left(5+\mathcal{O}(\epsilon^1)\right) - C_F\left(\frac{4}{\epsilon}+5+\mathcal{O}(\epsilon^1)\right)\right)$$

Comments on the final result

$$\int d\omega \, J^{(1)}_{1,2}(x_a n_+ p_A;\omega)\, \widetilde{S}^{(1)}_1(x,\omega)$$

$$J_{1,2}(x_a n_+ p_A;\omega) = -\frac{1}{x_a(n_+p_A)} - 2\frac{\partial}{x_a \partial(n_+p_A)} + \frac{\alpha}{4\pi}\frac{1}{x_a(n_+p_A)}\left[\frac{\omega(x_a n_+ p_A)}{\mu^2}\right]^{-\epsilon}\left(C_A\left(5+\mathcal{O}(\epsilon^1)\right) - C_F\left(\frac{4}{\epsilon}+5+\mathcal{O}(\epsilon^1)\right)\right)$$

$$S_1(\Omega,\omega) = \frac{\alpha\, C_F}{2\pi}\frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\frac{1}{\omega^{1+\epsilon}}\frac{1}{(\Omega-\omega)^{\epsilon}}\theta(\omega)\theta(\Omega-\omega) + \mathcal{O}(\alpha^2)$$

Comments on the final result

$$\int d\omega \, J^{(1)}_{1,2}(x_a n_+ p_A; \omega) \, \widetilde{S}^{(1)}_1(x, \omega)$$

The factorization formula is valid for unrenormalized objects. Performing the convolution in d - dimensions reproduces fixed NNLO result:

$$\Delta^{(2)}_{\mathrm{NLP-coll}} = \frac{\alpha_s^2}{(4\pi)^2}\left(C_A C_F \left(\frac{20}{\epsilon} - 60\log(1-z) + 8 + \mathcal{O}(\epsilon^1) \right) \right.$$
$$\left. + C_F^2 \left(\frac{-16}{\epsilon^2} - \frac{20}{\epsilon} + \frac{48}{\epsilon}\log(1-z) + 60\log(1-z) - 72\log^2(1-z) + \mathcal{O}(\epsilon^1) \right) \right)$$

after we set the scale to hard. In agreement with equation (4.22) of 1503.05156

Comments on the final result

$$\int d\omega \, J_{1,2}^{(1)}(x_a n_+ p_A; \omega) \, \widetilde{S}_1^{(1)}(x, \omega)$$

For resummation, we treat the two objects independently, and expand in ϵ prior to performing the final convolution. However, there is a problem! At two loops:

$$J_{1,2}^{(1)}(x_a n_+ p_A; \omega) \sim \alpha \log(\omega)$$

and

$$S_1(\Omega, \omega) \sim \alpha \, \delta(\omega) + \mathcal{O}(\alpha^2)$$

For LL resummation, only tree level collinear function is needed, since the soft function begins at one loop due to the explicit field insertions. Beyond this not yet understood.

Summary

- Introduction of the collinear functions at amplitude level and explicit computation within SCET framework.
- Presented general factorization formula for DY threshold production at next-to-leading power, checked its validity up to fixed NNLO and identified a problem for resummation at NLL
- For leading logarithmic resummation - see slides from Robert Szafron's talk

Thank you

Back up slides

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s(x + sn_\mp)\right]$$

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s(x+sn_\mp)\right]$$

The LP quark Lagrangian is

$$\mathcal{L}_{\rm LP} = \bar\chi\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\,i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and <u>after</u> the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar\chi^{(0)}\frac{\not{n}_+}{2}(\,n_-\mathcal{A}_c + n_-\partial)\;\chi^{(0)}(x)$$

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s(x+sn_\mp)\right]$$

The LP quark Lagrangian is

$$\mathcal{L}_{\rm LP} = \bar\chi\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\,i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and <u>after</u> the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar\chi^{(0)}\frac{\not{n}_+}{2}(\,n_-\mathcal{A}_c + n_-\partial)\;\chi^{(0)}(x)$$

From now on we use decoupled fields. Leading power current becomes

$$J_\rho^{A0}(t,\bar t\,) = \bar\chi_{\bar c}^{(0)}(\bar t n_-)Y_-^\dagger(0)\gamma_{\perp\rho}Y_+(0)\chi_c^{(0)}(tn_+)$$

Matching to quark current at NLP

N-jet operators are built out of following relevant building blocks.

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

(A1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\not{\partial}_\perp]\chi_c(tn_+),\ \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho(-i)\overleftarrow{\not{\partial}}_\perp]\chi_c(tn_+)$$

(B1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2n_+)]\chi_c(t_1n_+),\ \bar{\chi}_{\bar{c}}(\bar{t}_1n_-)[n_\pm^\rho \not{\mathcal{A}}_{\bar{c}\perp}(\bar{t}_2n_-)]\,\chi_c(tn_+)$$

With the the scaling

$$[n_\pm^\rho i\not{\partial}_\perp]\,\chi_c(tn_+)\sim\lambda$$
$$[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2n_+)]\,\chi_c(tn_+)\sim\lambda$$

relative to LP.

Time-ordered products

$$\left(J^{Tm}_{W,V}(s,t)\right)^{\mu} = i\int d^4x\, \mathbf{T}\left[J^{\mu}_{W}(s,t)\,\mathcal{L}^{(n)}_{V}(x)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

The NLP soft-collinear SCET quark-gluon interaction Lagrangian written in terms of building blocks $\mathcal{B}^{\mu}_{\pm} = Y^{\dagger}_{\pm}\left[iD^{\mu}_{s}Y_{\pm}\right]$ and $q^{\pm}(x) = Y^{\dagger}_{\pm}(x)\,q_s(x)$ is

[M. Beneke, F. Campanario, T. Mannel, B.D. Pecjak , 0411395]

$$\begin{aligned}
\mathcal{L}^{(1)}_{\xi} &= \bar{\chi}_c i x^{\mu}_{\perp}\left[in_-\partial \mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{1\xi} &= \frac{1}{2}\bar{\chi}_c in_- x n^{\mu}_{+}\left[in_-\partial \mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{2\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp}x^{\rho}_{\perp}\left[i\partial_{\rho} in_-\partial \mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{3\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp}x^{\rho}_{\perp}\left[\mathcal{B}^{+}_{\rho}(x_-),\, in_-\partial \mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{4\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[i\partial_{\nu}\mathcal{B}^{+}_{\mu}(x_-) - i\partial_{\mu}\mathcal{B}^{+}_{\nu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(2)}_{5\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[\mathcal{B}^{+}_{\nu}(x_-),\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(1)}_{\xi q} &= \bar{q}_{+}(x_-)\mathcal{A}_{c\perp}\chi_c + \text{h.c.}
\end{aligned}$$

Definition of PDFs

$$\mathcal{A}_{\perp\mu} = Y_+^\dagger W_c^\dagger \left[i\, D_c\, W_c \right] Y_+$$

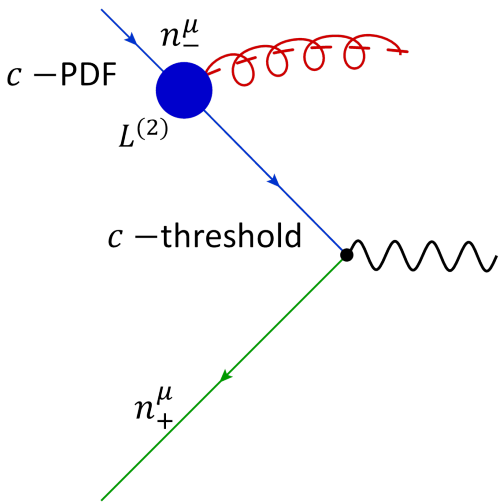
$$\langle A(p_A)|\bar{\chi}_{c,\alpha a}(x + u' n_+)\chi_{c,\beta b}(u n_+)|A(p_A)\rangle = \frac{\delta_{ba}}{N_c} \left(\frac{\not{n}_-}{4} \right)_{\beta\alpha} n_+ p_A \times \int_0^1 dx_a\, f_{a/A}(x_a)\, e^{i(x + u' n_+ - u\, n_+)\cdot x_a p_A}$$

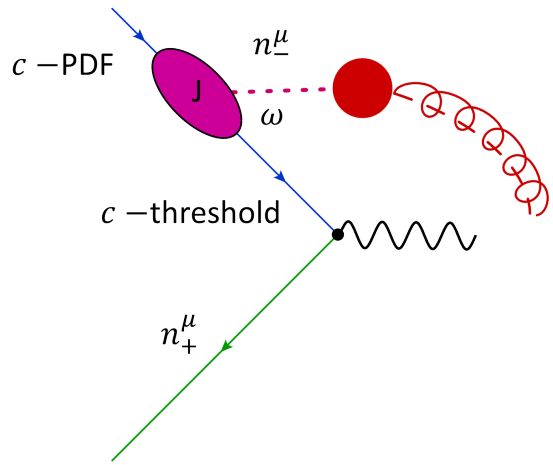
Collinear functions

Threshold collinear fields are matched to collinear-PDF fields

$$\int dt\, e^{i(n_+p)t}\, i \int d^4z\, e^{i\omega(n_+z)/2}\, \mathbf{T}\left[\chi_c(tn_+) \times \mathcal{L}_c^{(n)}(z)\right]$$

$$= \int d(n_+p') \int dt\, e^{i(n_+p')t}\, J(n_+p, n_+p'; \omega)\, \chi_c^{\rm PDF}(tn_+)$$





Soft functions

We introduce the soft operator

$$\widetilde{\mathcal{S}}_{2\xi}\left(x, z_{-}\right)=\bar{\mathbf{T}}\left[Y_{+}^{\dagger}(x) Y_{-}(x)\right] \mathbf{T}\left[Y_{-}^{\dagger}(0) Y_{+}(0) \frac{i \partial_{\perp}^{\nu}}{i n_{-} \partial} \mathcal{B}_{\perp \nu}^{+}\left(z_{-}\right)\right]$$

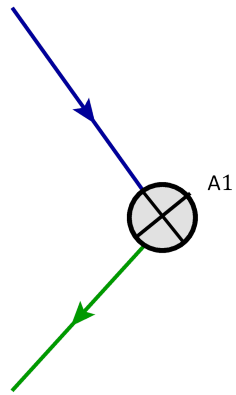
and the Fourier transform of its (colour-traced) vacuum matrix element

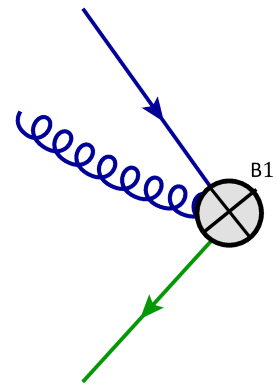
$$S_{2\xi}(\Omega, \omega)=\int \frac{dx^{0}}{4\pi} \int \frac{d(n_{+}z)}{4\pi} e^{i x^{0} \Omega / 2-i \omega\left(n_{+} z\right) / 2} \frac{1}{N_{c}} \operatorname{Tr}\left\langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^{0}, z_{-})| 0\right\rangle$$

Possible contributing structures

First we check whether subleading power contributions start at order λ.

- Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_+^\rho i\partial\!\!\!/_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_+^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$

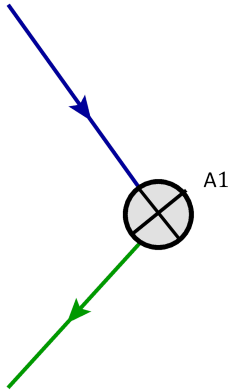


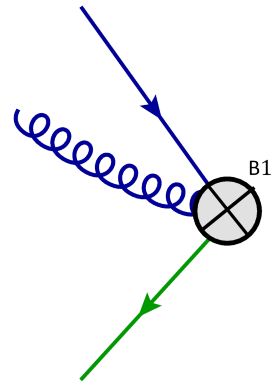


Possible contributing structures

First we check whether subleading power contributions start at order λ.

- Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_+^\rho i\partial\!\!\!/_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_+^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$

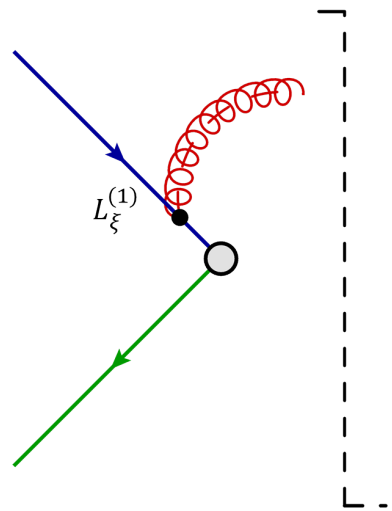




- Another possibility is a single power suppressed time-ordered product of the form $\left(J^{T1}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x\,\mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x)\right]$

Only one possibility

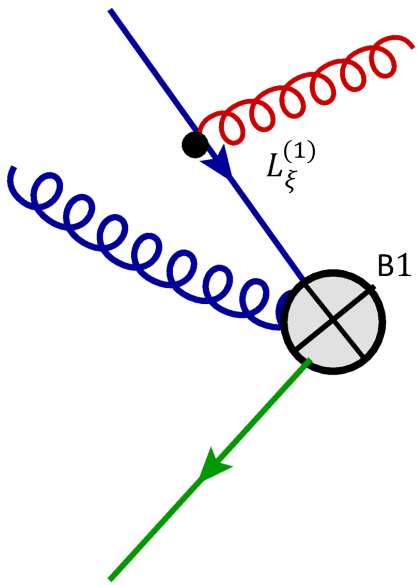
$$\mathcal{L}^{(1)}_\xi \quad = \quad \bar{\chi}_c i x^\mu_\perp \left[in_-\partial \mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c$$

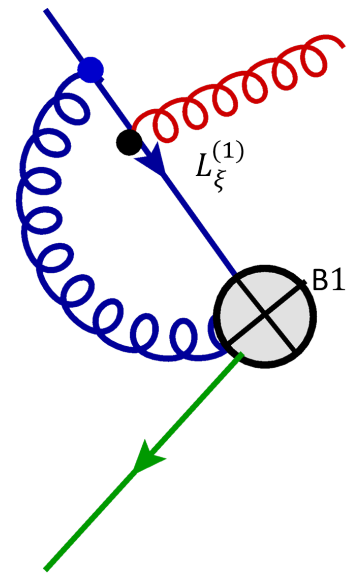


Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- B1-type current, $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\perp^\rho \mathcal{A}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$, with Lagrangian insertion $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[in_- \partial \mathcal{B}_\mu^+\right] \dfrac{\not{n}_+}{2}\chi_c$

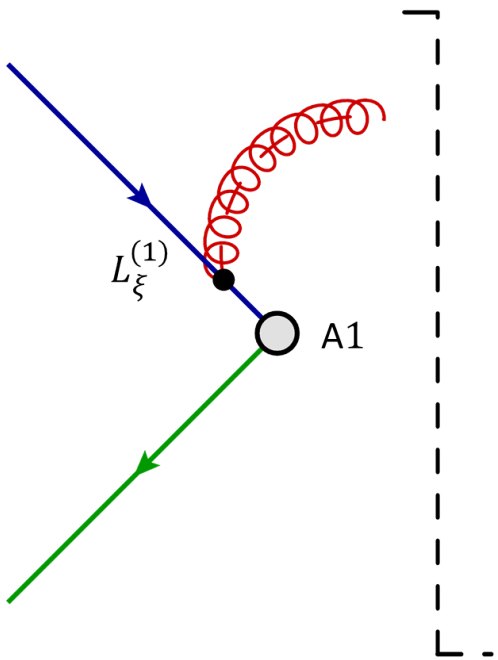




Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- A1-type current with $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[in_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$ insertion



Feynman rule for emission of a soft gluon from $\mathcal{B}_\mu^+$ is

$$g\,T^A \left[-\frac{k_\perp^\mu n_{-\nu}}{(n_- k)} + g_\perp^{\mu\nu} \right] \epsilon_\nu^*\, e^{+ik\cdot z_-}$$

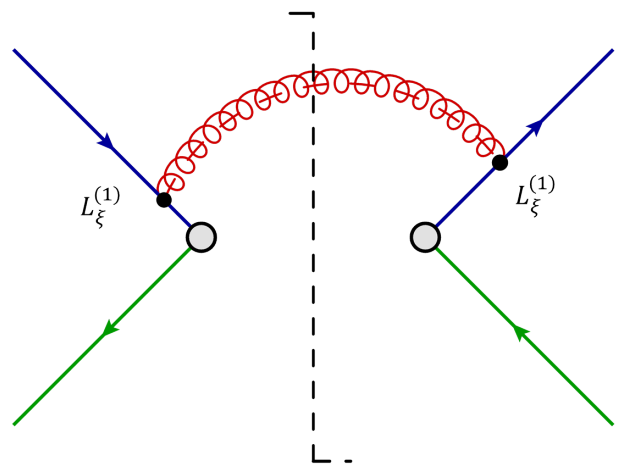
Considering the Lagrangian insertions

Previous arguments allow us also to drop following possible contributions

$$\left(J^{T1}_{A0,\xi}(s,t)\right)^{\mu} = i\int d^4x_1\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi}(x_1)\right]$$

$$\left(\bar{J}^{T1}_{A0,\xi}(\bar{s},\bar{t}\,)\right)^{\mu} = (-i)\int d^4x_2\,\mathbf{T}\left[\bar{J}^{\mu}_{A0}(\bar{s},\bar{t}\,)\,\mathcal{L}^{(1)}_{\xi}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi} \quad = \quad \bar{\chi}_c i x^{\mu}_{\perp}\left[in_{-}\partial\mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_{+}}{2}\chi_c$$

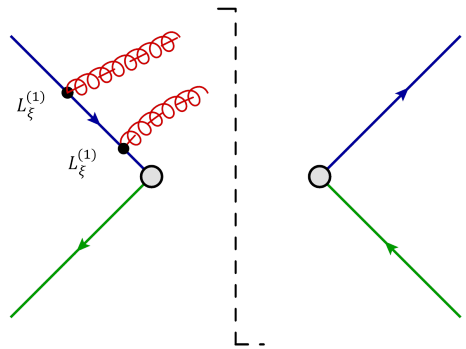


Considering the Lagrangian insertions

The following contributions start at $\mathcal{O}(\alpha^2)$

$$\left(J^{T2}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x_1 i\int d^4x_2\,\mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x_1)\,\mathcal{L}^{(1)}_\xi(x_2)\right]$$

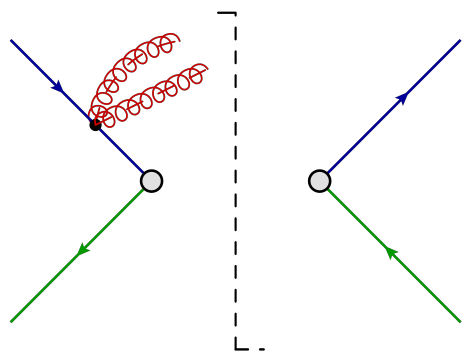
$$\mathcal{L}^{(1)}_\xi = \bar\chi_c i x^\mu_\perp \left[in_-\partial \mathcal{B}^+_\mu\right]\frac{\not n_+}{2}\chi_c$$



$$\left(J^{T2}_{A0,V}(s,t)\right)^\mu = i\int d^4x\,\mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(2)}_V(x)\right]$$

$$\mathcal{L}^{(2)}_{3\xi} = \frac{1}{2}\bar\chi_c x^\mu_\perp x^\rho_\perp \left[\mathcal{B}^+_\rho,\, in_-\partial\mathcal{B}^+_\mu\right]\frac{\not n_+}{2}\chi_c$$

$$\mathcal{L}^{(2)}_{5\xi} = \frac{1}{2}\bar\chi_c\left(i\not\partial_\perp + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial}ix^\mu_\perp\gamma^\nu_\perp \times\left[\mathcal{B}^+_\nu,\mathcal{B}^+_\mu\right]\frac{\not n_+}{2}\chi_c + \text{h.c.}$$

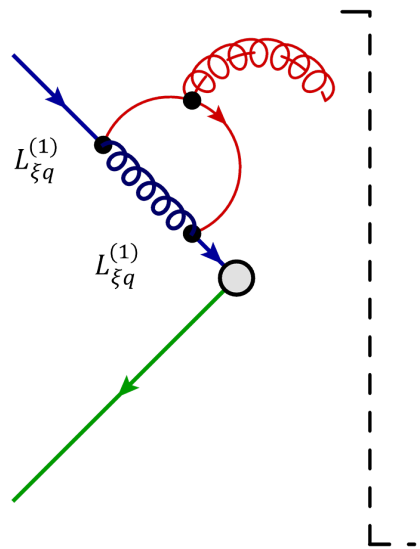


Considering the Lagrangian insertions

It is also possible to construct diagrams containing soft quarks

$$\left(J^{T2}_{A0,\xi q}(s,t)\right)^{\mu} = i\int d^4x_1 i\int d^4x_2\, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi q}(x_1)\,\mathcal{L}^{(1)}_{\xi q}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} \quad = \quad \bar{q}_+ \mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$

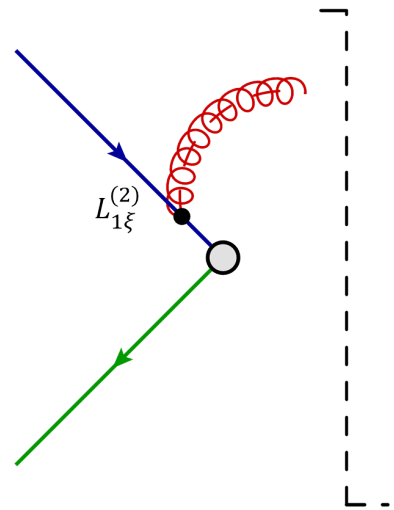


These contributions also start at $\mathcal{O}(\alpha^2)$

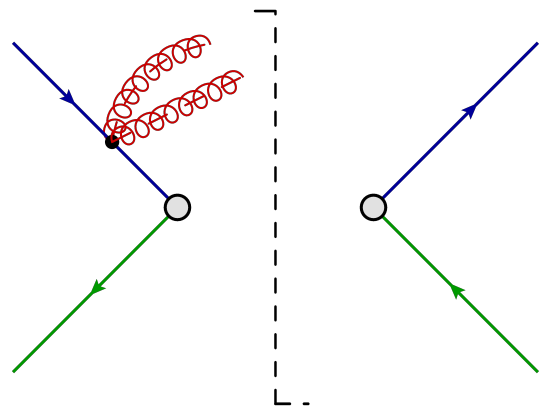
Considering the Lagrangian insertions

Two more possible contributions with following Lagrangian terms making up the time-ordered product

$$\mathcal{L}^{(2)}_{1\xi} = \frac{1}{2}\bar{\chi}_c i n_- x n_+^\mu \left[i n_- \partial \mathcal{B}^+_\mu \right] \frac{\not{n}_+}{2} \chi_c$$



$$\mathcal{L}^{(2)}_{4\xi} = \frac{1}{2}\bar{\chi}_c \left(i\not{\partial}_\perp + \not{\mathcal{A}}_{c\perp} \right) \frac{1}{i n_+ \partial} i x^\mu_\perp \gamma^\nu_\perp \times \left[i\partial_{\nu_\perp} \mathcal{B}^+_{\mu_\perp} - i\partial_{\mu_\perp} \mathcal{B}^+_{\nu_\perp} \right] \frac{\not{n}_+}{2} \chi_c + \text{h.c.}$$



Conclusion

We therefore find that for LL resummation at NLP in the quark-antiquark channel only the single time-ordered product contribution:

$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^{\mu} = i\int d^4x\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(x)\right]$$

To NLP LL accuracy the matching equation is then extended to

$$\bar{\psi}\gamma^{\mu}\psi(0) = \int dt\,d\bar{t}\,\widetilde{C}^{A0}(t,\bar{t}\,)\left[J^{\mu}_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^{\mu} + \;\bar{c}\text{-term}\right]$$

Again we consider

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle$$

A power suppressed amplitude

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\left[J^\mu_{A0}(t,\bar{t}\,) + \boxed{i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(x)\right]} + \ \bar{c}\text{-term}\right]$$

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\, d\bar{t}\, e^{it\, n_+p} e^{i\bar{t}n_-\bar{p}} C^{A0}(n_+p, n_-\bar{p})$$
$$\times\langle X|\mathbf{T}\Bigg[\underbrace{\bar{\chi}_{\bar{c}}(\bar{t}n_-)Y^\dagger_-(0)\gamma^\mu_\perp\, Y_+(0)\chi_c\,(tn_+)}_{J^\mu_{A0}(t,\bar{t}\,)}\, i\int d^4z\, \bar{\chi}_{c,e}\,(z)\,\frac{1}{2}z^\nu_\perp z^\rho_\perp$$
$$\times\left[\boxed{\left(\frac{in_-\partial_z}{in_-\partial_z}\right)}(in_-\partial_z) i\partial^\rho_\perp \mathcal{B}^+_{\perp\nu,ed}\,(z_-)\right]\frac{\not{n}_+}{2}\chi_{c,d}\,(z)\Bigg]|A(p_A)B(p_B)\rangle$$

A power suppressed amplitude

The states factorize as at leading power: $\langle X| = \langle X_{\bar{c}}^{\rm PDF}| \, \langle X_c^{\rm PDF}| \, \langle X_s|$ as they are eigenstates of the LP Lagrangian

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\, d\bar{t}\, e^{it\, n_+p} e^{i\bar{t} n_-\bar{p}} C^{A0}(n_+p, n_-\bar{p})$$

$$\times \langle X_{\bar{c}}^{\rm PDF}|\bar{\chi}_{\bar{c},\alpha a}(\bar{t}n_-)|B(p_B)\rangle \gamma^\mu_{\perp,\alpha\gamma}$$

$$\times i\int d^4z \langle X_c^{\rm PDF}|\frac{1}{2} z_\perp^\nu z_\perp^\rho (in_-\partial_z)^2\, \mathbf{T}\left[\chi_{c,\gamma f}\,(tn_+)\,\bar{\chi}_{c,e}\,(z)\,\frac{\not{n}_+}{2}\chi_{c,d}\,(z)\right]|A(p_A)\rangle$$

$$\times \langle X_s|\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\frac{i\partial_\perp^\rho}{in_-\partial_z}\mathcal{B}^+_{\perp\nu,ed}\,(z_-)\right)|0\rangle$$

Amplitude with collinear function

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int d(n_+p_a)d(n_-p_b)$$

$$\delta\left(n_-\bar{p}+(n_-p_b)\right)\, C^{A0}(n_+p,n_-\bar{p})$$

$$\times\int\frac{d\omega}{2\pi}\, J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+p,n_+p_a;\omega\right)\langle X^{\mathrm{PDF}}_{\bar{c}}|\hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle$$

$$\times\gamma^{\mu}_{\perp,\alpha\gamma}\langle X^{\mathrm{PDF}}_{c}|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$

$$\times\int\frac{dn_+z}{2}e^{-i\omega\frac{n_+z}{2}}\langle X_s|\mathbf{T}\left(\left[Y^{\dagger}_-(0)Y_+(0)\right]_{af}\frac{i\partial^{\rho}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\perp\nu,ed}\left(z_-\right)\right)|0\rangle$$

Computation of collinear function

The short-distance coefficient can be extracted by computing the partonic matrix element $\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle$. Running to collinear scale: only tree level collinear function is necessary.

Collinear function:

$$J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+q_a,(n_+q);\omega\right)=-\delta_{bd}\delta_{fe}\delta_{\beta\gamma}\delta\left((n_+q_a)-(n_+q)\right)\frac{g^{\nu\rho}_{\perp}}{(n_+q)}$$

LP + NLP amplitude

We are considering the matching up to NLP

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\left[J^\mu_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^\mu + \;\bar{c}\text{-term}\right]$$

For which we obtained

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{dn_+p_a}{2\pi}\frac{dn_-p_b}{2\pi}\, C^{A0}(n_+p_a, -n_-p_b)$$

$$\times\, \langle X_{\bar{c},\mathrm{PDF}}|\hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle\, \gamma^\mu_{\perp\alpha\beta}\, \langle X_{c,\mathrm{PDF}}|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$

$$\times\Bigg\{ \langle X_s|\mathbf{T}\Big[Y^\dagger_-(0)Y_+(0)\Big]_{ab}|0\rangle$$

$$+\frac{1}{2}\int\frac{d\omega}{4\pi}\, J^{(O)}_{2\xi}(n_+p_a;\omega)\int d(n_+z)\, e^{-i\omega(n_+z)/2}$$

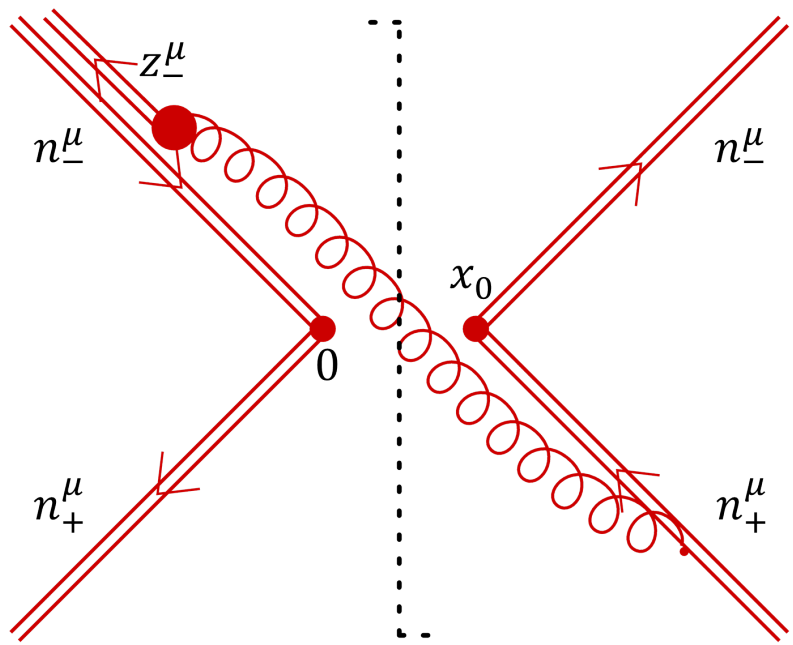
$$\times\langle X_s|\mathbf{T}\left(\Big[Y^\dagger_-(0)Y_+(0)\Big]_{af}\,\frac{i\partial^\nu_\perp}{in_-\partial_z}\mathcal{B}^+_{\perp\nu;fb}(z_-)\right)|0\rangle\Bigg\} + \bar{c}\text{-term}$$

Note that $J^{(\mathrm{O})}_{2\xi}\,(n_+p_a;\omega) = -\dfrac{2}{n_+p_a}$

Relevant soft function

The generalized soft function at cross section level here is

$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}$$
$$\times \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\,\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}_{\perp\nu}^+(z_-)\right]|0\rangle$$



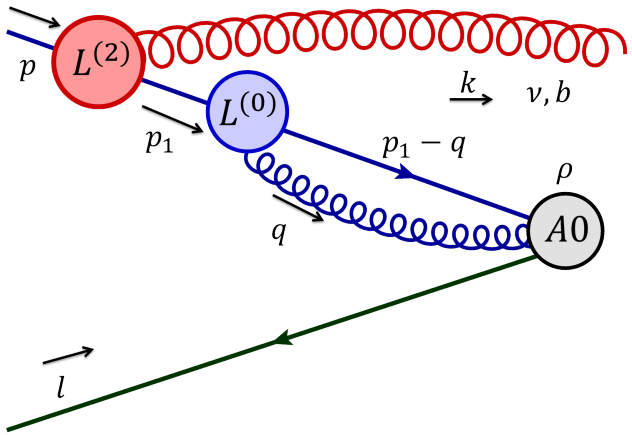
For details on renormalization of soft functions and resummation see Robert's talk.

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

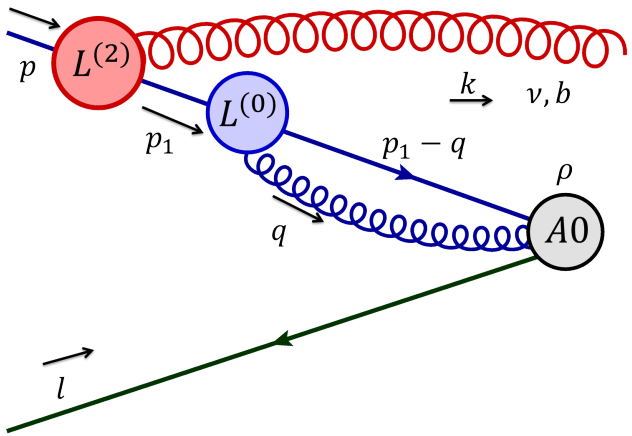
Power suppressed amplitude calculation

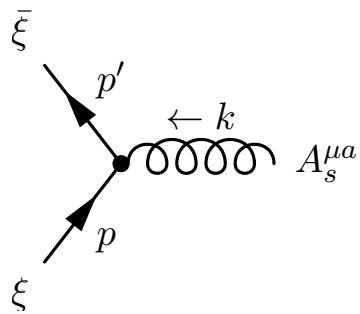
Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.





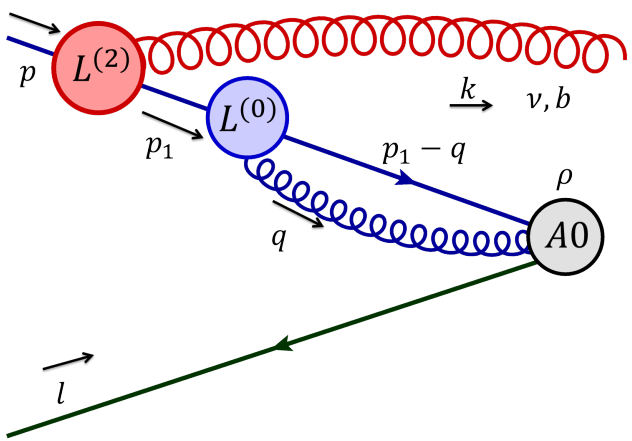
$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \dfrac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu \; + \; (k X_\perp) X_\perp^\rho n_-^\nu \; + \; X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]$$

 [M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

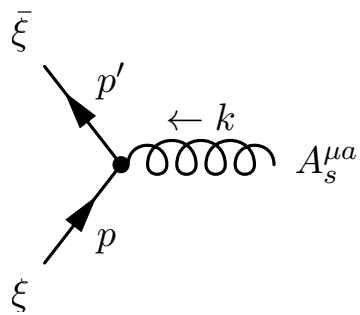
Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\mathcal{L}_\xi^{(2)} = \frac{1}{2}\bar{\chi}_c i\ (n_- x)\ n_+^\mu \left[in_- \partial \mathcal{B}_\mu^+(x_-) \right] \frac{\not{n}_+}{2} \chi_c + \ldots$$

[M. Beneke and Th. Feldmann, 0211358]

$$X^\alpha = -\frac{\partial}{\partial p_{1\alpha}} \left\{ (2\pi)^4 \delta^4 (p - k_+ - p_1) \right\}$$



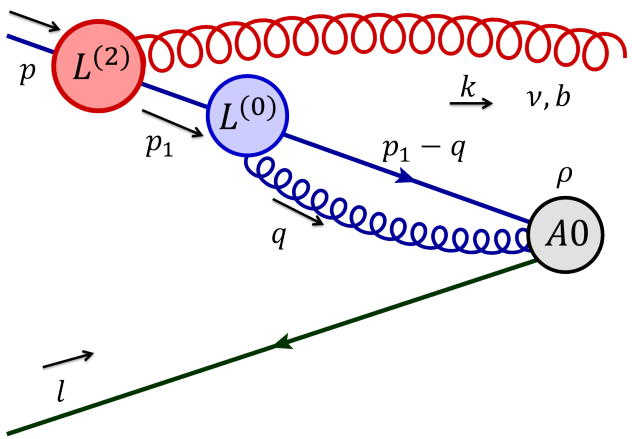
$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \dfrac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu \ + \ (k X_\perp) X_\perp^\rho n_-^\nu \ + \ X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]$$

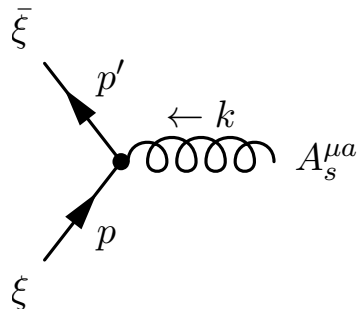
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\ \left((n_+k)n_{-\nu}-(n_-k)n_{+\nu}\right)\ \left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\ \left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)\ \left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\ \left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\ \left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$



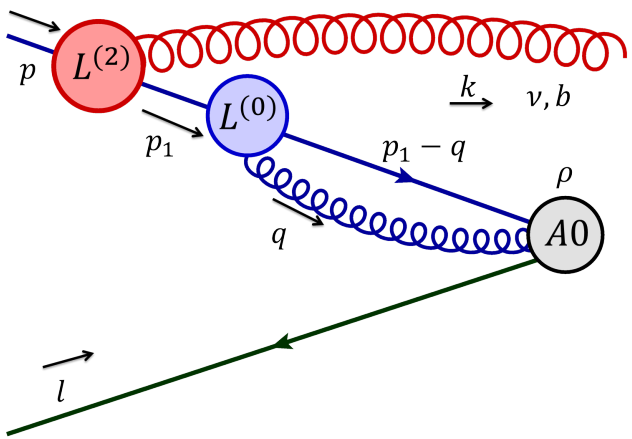
$$ig_s t^a\begin{cases}\dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\ (n_-X)n_+^\rho n_-^\nu\ +\ (kX_\perp)X_\perp^\rho n_-^\nu\ +\ X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\ \right]$$

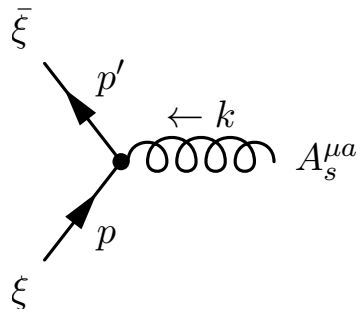
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\boxed{((n_+k)n_{-\nu}-(n_-k)n_{+\nu})}\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$



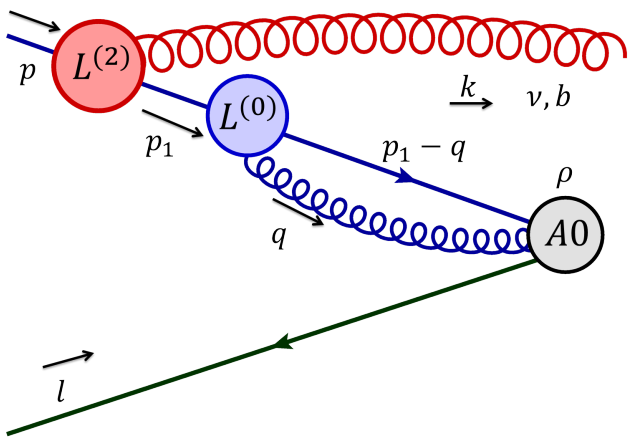
$$ig_s t^a\begin{cases}\dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu}+(kX_\perp)X_\perp^\rho n_-^\nu+X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\right]$$

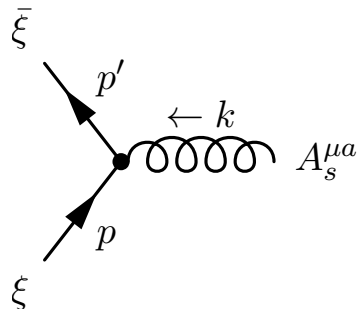
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\ ((n_+k)n_{-\nu}-(n_-k)n_{+\nu})\ \left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\ \left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\ \left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$



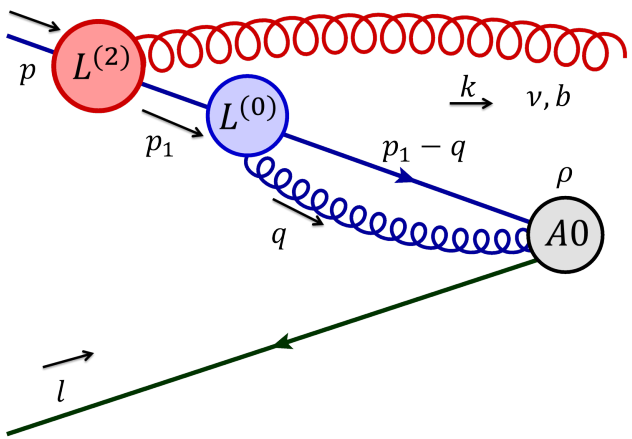
$$ig_s t^a\begin{cases}\dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\ (n_-X)n_+^\rho n_-^\nu\ +\boxed{(kX_\perp)X_\perp^\rho n_-^\nu}+\ X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\ \right]$$

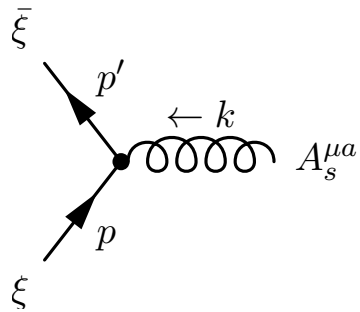
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\ ((n_+k)n_{-\nu}-(n_-k)n_{+\nu})\ \left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)\ \left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$



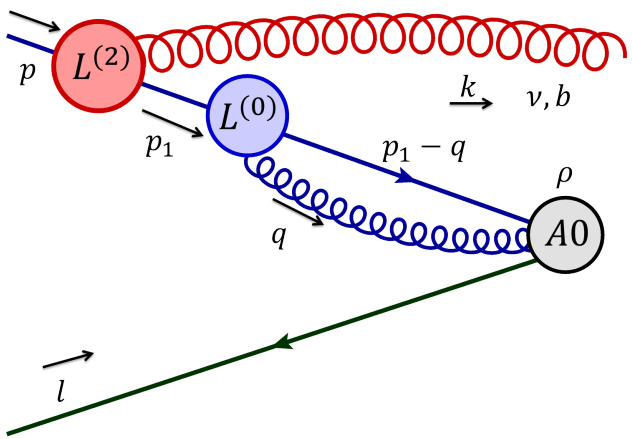
$$ig_s t^a\begin{cases}\dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\ (n_-X)n_+^\rho n_-^\nu\ +\ (kX_\perp)X_\perp^\rho n_-^\nu\ +\ \boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\boxed{((n_+k)n_{-\nu}-(n_-k)n_{+\nu})}\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$

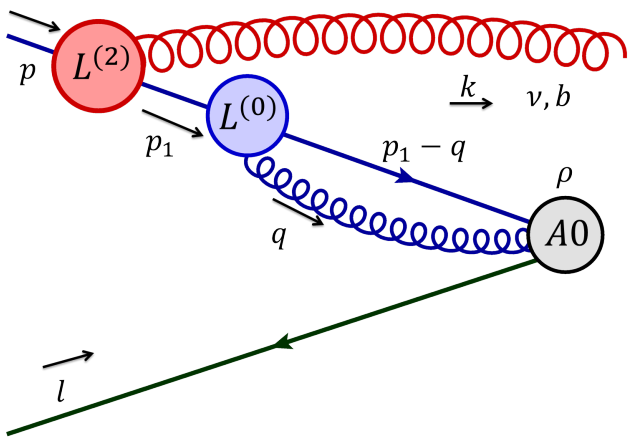
$$(n_+k)(n_-\epsilon^*) = 2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2}-k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu}+\boxed{(kX_\perp)X_\perp^\rho n_-^\nu}+\boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\boxed{\left(\frac{-2k_\perp^2}{n_-k}n_{-\nu}+2k_{\perp\nu}\right)}\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$

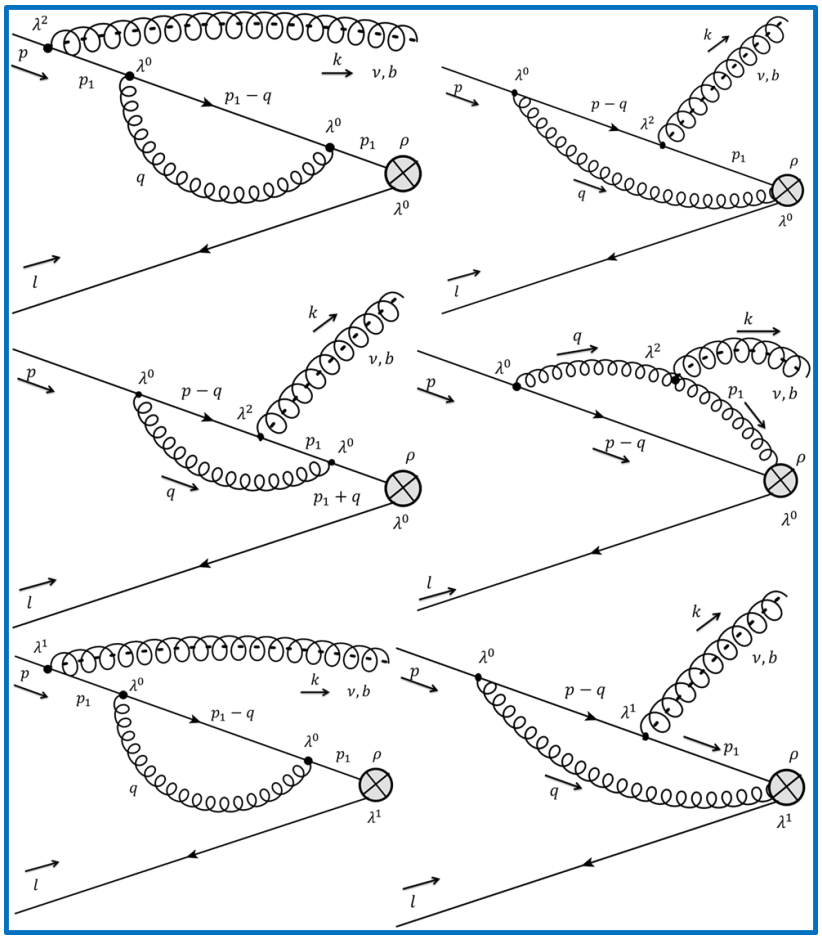
$$(n_+k)(n_-\epsilon^*)=2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2}-k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu}+\boxed{(kX_\perp)X_\perp^\rho n_-^\nu}+\boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Amplitude calculation: 1-real emission

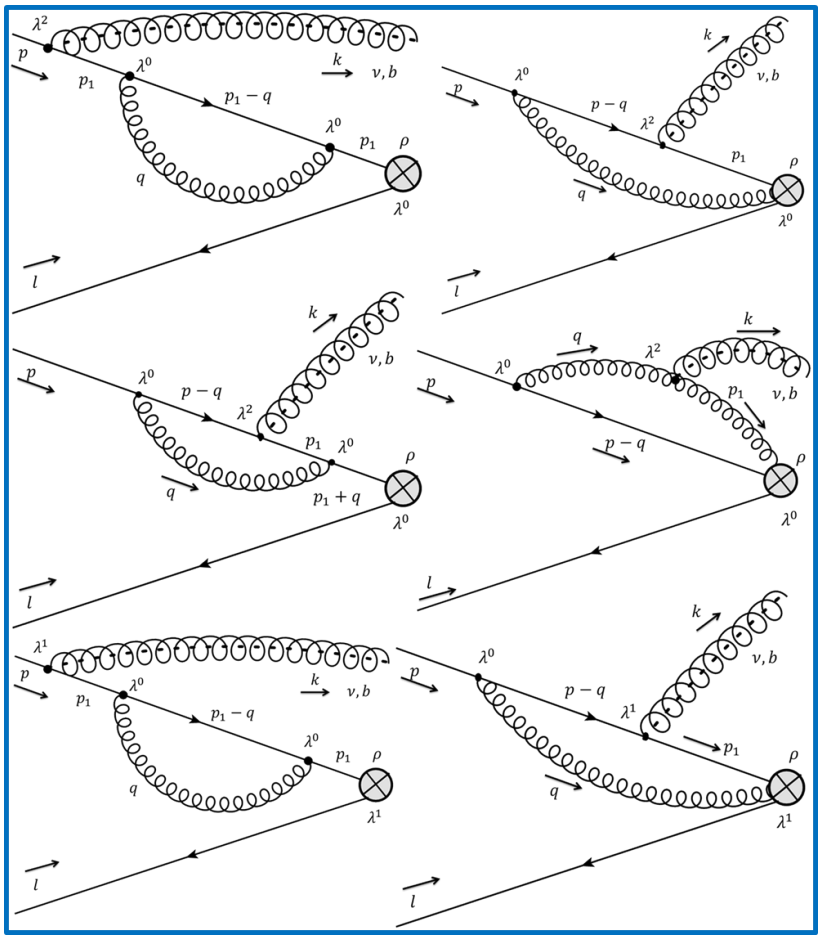
$$\mathcal{A} = C \otimes J_{2\xi} \otimes\langle X| \frac{\partial^{\mu}_{\perp}}{in_{-}\partial}\mathcal{B}^{+}_{\mu\perp} |0\rangle + C \otimes J_{4\xi} \otimes\langle X| \frac{\partial_{[\mu\perp}}{in_{-}\partial}\mathcal{B}^{+}_{\nu\perp]} |0\rangle + C \otimes J_{\xi} \otimes\langle X| \mathcal{B}^{+}_{\mu\perp} |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes \boxed{J_{2\xi}} \otimes \langle X| \frac{\partial_\perp^\mu}{in_-\partial} \mathcal{B}^+_{\mu_\perp} |0\rangle + C \otimes \boxed{J_{4\xi}} \otimes \langle X| \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]} |0\rangle + C \otimes \boxed{J_\xi} \otimes \langle X| \mathcal{B}^+_{\mu_\perp} |0\rangle$$

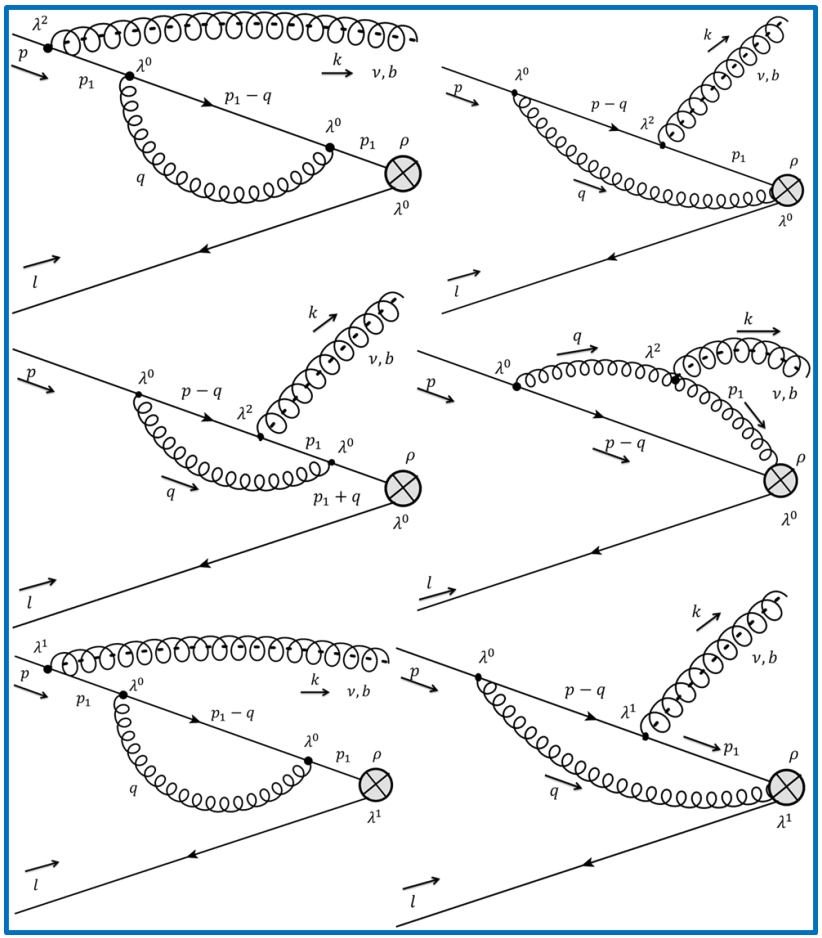


1-loop collinear $\otimes$ 1-real soft emission

Extract 1-loop collinear functions

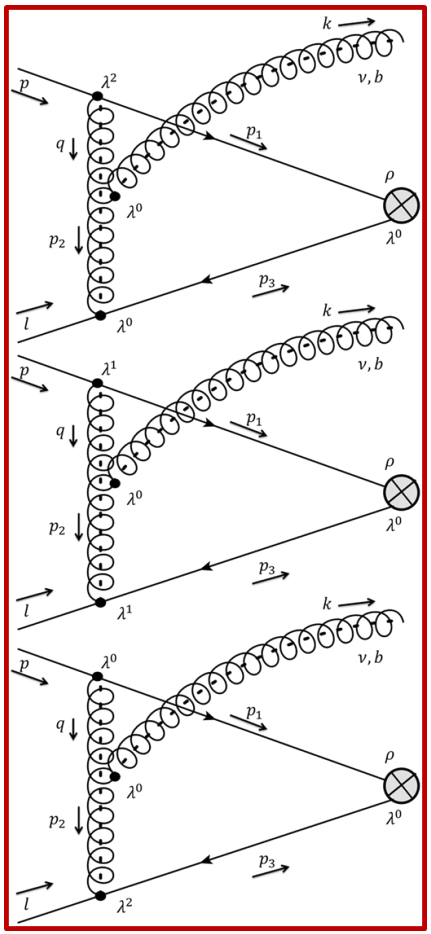
Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes J_{2\xi} \otimes\langle X| \boxed{\frac{\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu_\perp}} |0\rangle + C \otimes J_{4\xi} \otimes\langle X| \boxed{\frac{\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^+_{\nu_\perp]}} |0\rangle + C \otimes J_{\xi} \otimes\langle X| \boxed{\mathcal{B}^+_{\mu_\perp}} |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission

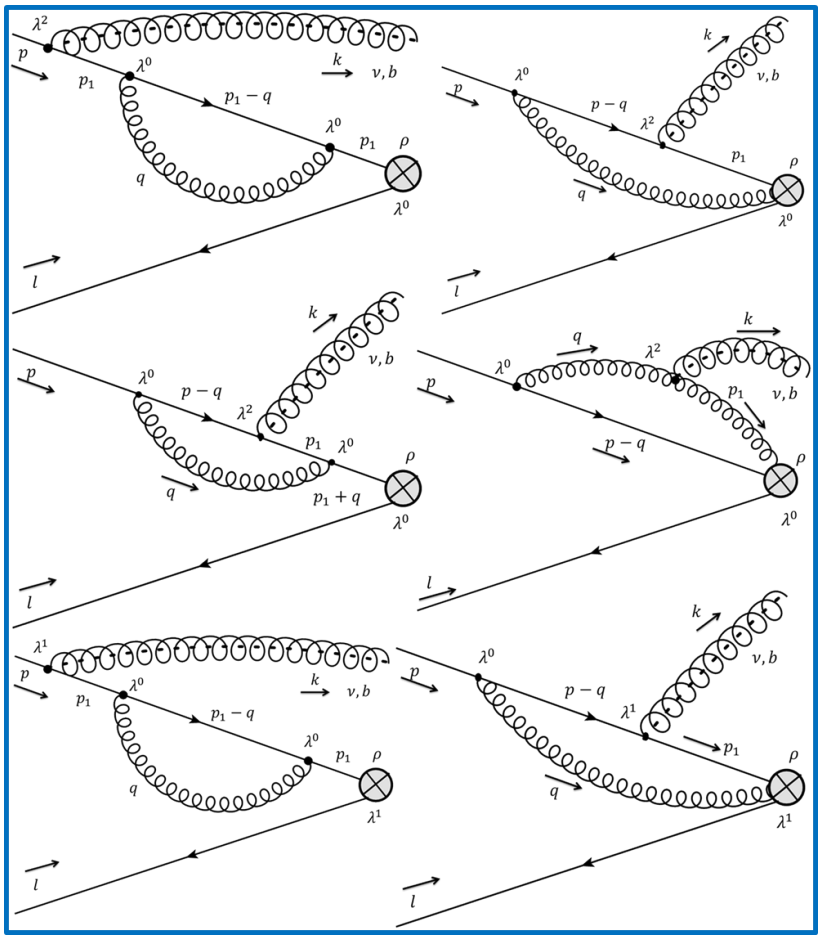
Extract 1-loop collinear functions



1-loop soft $\otimes$ 1-real soft emission

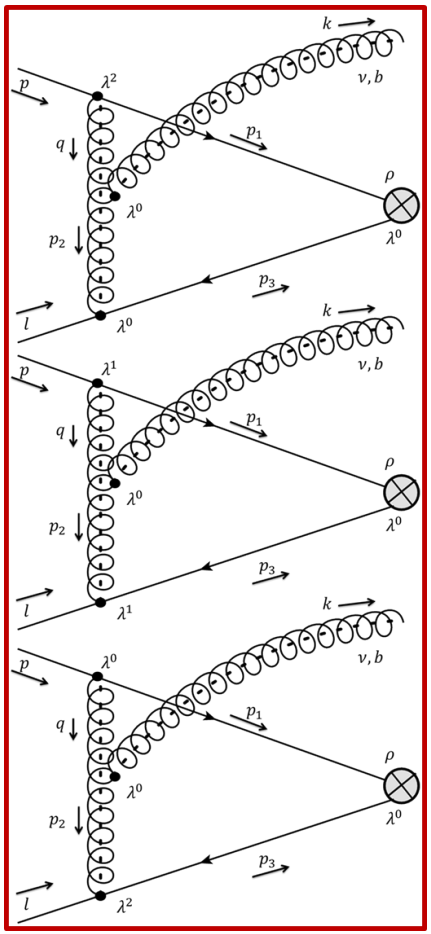
Amplitude calculation: 1-real emission

$$\mathcal{A} = \boxed{C} \otimes J_{2\xi} \otimes \langle X| \frac{\partial^{\mu}_{\perp}}{in_- \partial} \mathcal{B}^{+}_{\mu\perp} |0\rangle + \boxed{C} \otimes J_{4\xi} \otimes \langle X| \frac{\partial_{[\mu\perp}}{in_- \partial} \mathcal{B}^{+}_{\nu\perp]} |0\rangle + \boxed{C} \otimes J_{\xi} \otimes \langle X| \mathcal{B}^{+}_{\mu\perp} |0\rangle$$

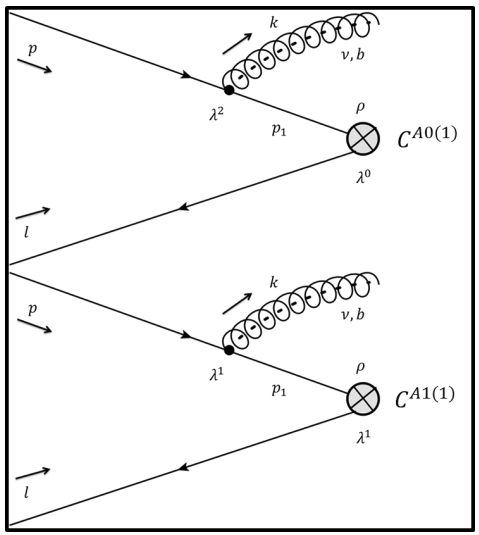


1-loop collinear $\otimes$ 1-real soft emission

Extract 1-loop collinear functions



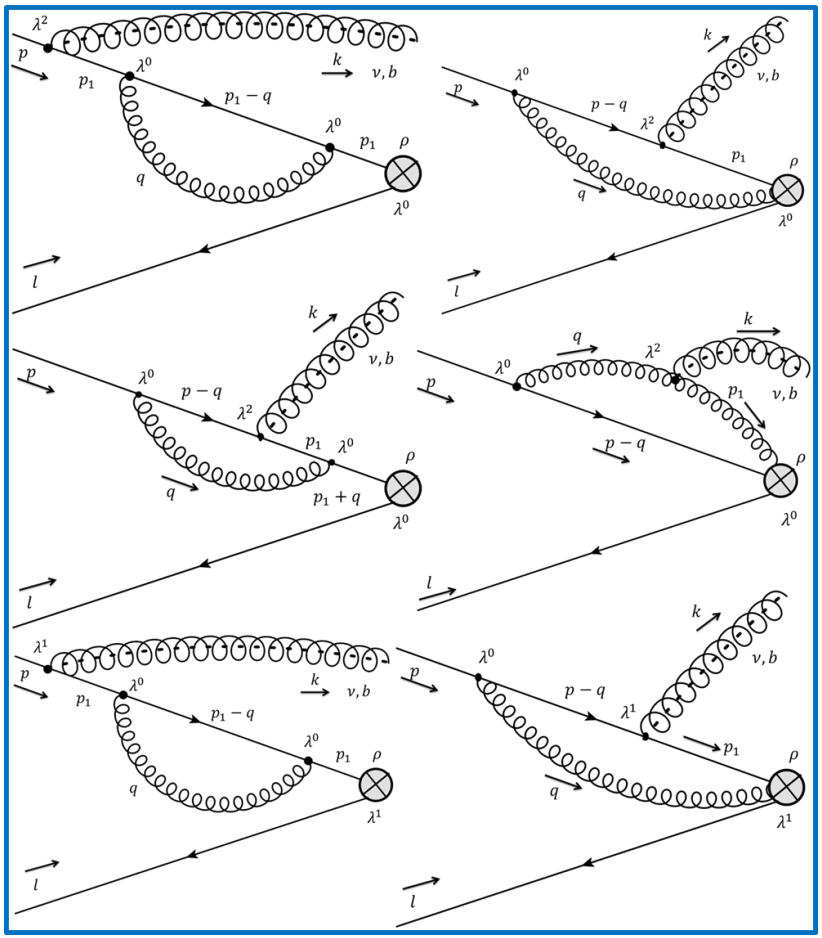
1-loop soft $\otimes$ 1-real soft emission



1-loop hard $\otimes$ 1-real soft emission

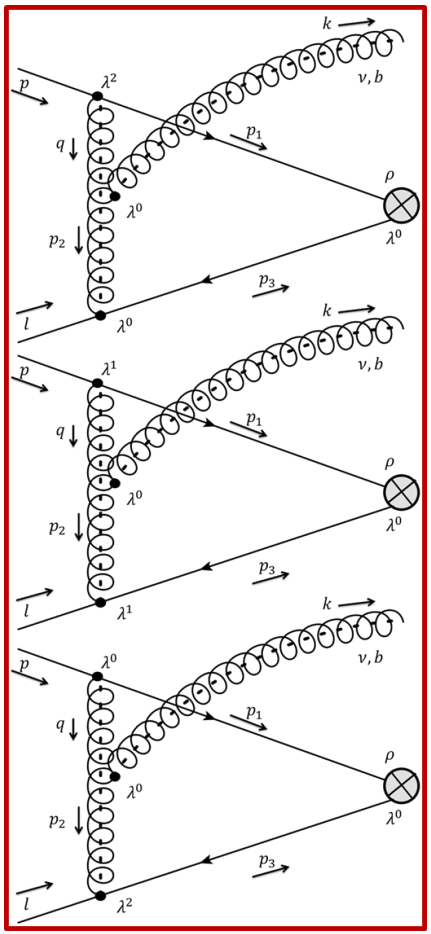
Amplitude calculation: 1-real emission

We find agreement with the method of regions expansion for the 1-real 1-virtual amplitude. For explicit results see next slides.

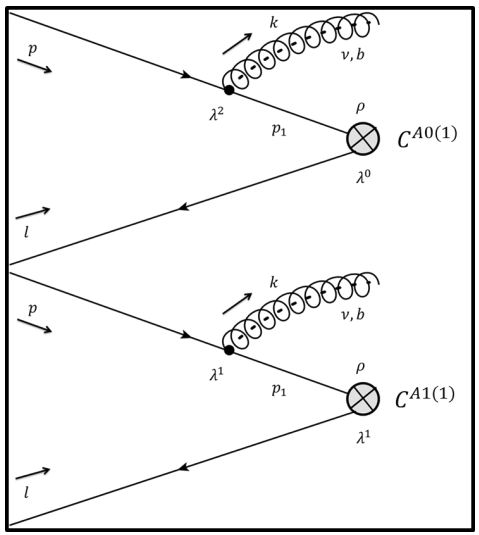


1-loop collinear $\otimes$ 1-real soft emission

Extract 1-loop collinear functions



1-loop soft $\otimes$ 1-real soft emission



1-loop hard $\otimes$ 1-real soft emission

Result for the power suppressed amplitude: C_F

$$
\begin{aligned}
i\,C_F\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)n_{-\nu}\left(\frac{3}{\epsilon}+2-\frac{3}{2}\zeta(2)\epsilon+\left(-\zeta(2)-\frac{21\zeta(3)}{3}-4\right)\epsilon^2\right)\\
&+(n_-k)n_{+\nu}\left(-\frac{1}{\epsilon}-3+(-6+\frac{1}{2}\zeta(2))\epsilon+\left(\frac{3}{2}\zeta(2)-12+\frac{7\zeta(3)}{3}\right)\epsilon^2\right)\\
&+k_{\perp\nu}\left(+\frac{2}{\epsilon}-1+(-6-\zeta(2))\epsilon+\left(+\frac{\zeta(2)}{2}-\frac{14\zeta(3)}{3}-16\right)\epsilon^2\right)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(+\frac{1}{2}+\epsilon+\left(-\frac{1}{4}\zeta(2)+2\right)\epsilon^2\right)\Bigg)
\end{aligned}
$$

$$
i\,C_F\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(+1+4\epsilon-\frac{1}{2}(\zeta(2)-20)\epsilon^2\right)
$$

$$
i\,C_F\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(-1-4\epsilon+\frac{1}{2}(\zeta(2)-20)\epsilon^2\right)
$$

Result for the power suppressed amplitude: C_A

$$
\begin{aligned}
& i\,C_A\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg((n_+k)n_{-\nu}\Bigg(-\frac{1}{2\,\epsilon^2}-\frac{3}{2\,\epsilon}+\frac{1}{4}(\zeta(2)-18)\\
& +\frac{1}{12}(9\zeta(2)+14\zeta(3)-48)\epsilon+\frac{1}{32}(72\zeta(2)+112\zeta(3)+47\zeta(4)-288)\epsilon^2\Bigg)\\
& +(n_-k)n_{+\nu}\Bigg(-\frac{1}{2\epsilon^2}-\frac{3}{2\epsilon}+\frac{1}{4}(\zeta(2)+2)+\frac{1}{12}(9\zeta(2)+14\zeta(3)-24)\epsilon\\
& -\frac{1}{32}(8\zeta(2)-112\zeta(3)-47\zeta(4)+32)\epsilon^2\Bigg)\\
& +k_{\perp\nu}\Bigg(-\frac{1}{\epsilon^2}-\frac{3}{\epsilon}+\frac{1}{2}(\zeta(2)-8)\\
& +\left(\frac{3\zeta(2)}{2}+\frac{7\zeta(3)}{3}-6\right)\epsilon+\left(2\zeta(2)+7\zeta(3)+\frac{47\zeta(4)}{16}-10\right)\epsilon^2\Bigg)\\
& +[\not{k}_\perp,\gamma_{\perp\nu}]\left(\frac{1}{4}\left(-2-4\epsilon+(\zeta(2)-8)\epsilon^2\right)\right)\Bigg)
\end{aligned}
$$

$$
i\,C_A\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(+\frac{1}{\epsilon}+2-\frac{1}{2}(\zeta(2)-6)\epsilon+\left(-\zeta(2)-\frac{7\zeta(3)}{3}+5\right)\epsilon^2\right)
$$

$$
i\,C_A\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(-\frac{1}{\epsilon}-2+\frac{1}{2}(\zeta(2)-6)\epsilon+\left(\zeta(2)+\frac{7\zeta(3)}{3}-5\right)\epsilon^2\right)
$$

Results for power suppressed amplitude: Soft

$$
\begin{aligned}
i\,C_A\gamma_{\perp\rho}\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)\,n_{-\nu}\left(+\frac{1}{\epsilon^2}+\frac{\zeta(2)}{2}-\frac{7}{3}\zeta(3)\epsilon-\frac{1}{16}39\zeta(4)\epsilon^2\right)\\
&+(n_-k)\,n_{+\nu}\left(0\right)\\
&+k_{\perp\nu}\left(\frac{1}{\epsilon^2}+\frac{1}{2}\zeta(2)-\frac{7}{3}\zeta(3)\epsilon-\frac{39}{16}\zeta(4)\epsilon^2\right)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(+\frac{1}{2\,\epsilon^2}+\frac{1}{4}\zeta(2)-\frac{7}{6}\zeta(3)\epsilon-\frac{1}{32}39\zeta(4)\epsilon^2\right)\Bigg)\\
&g\,\frac{C_A}{2}t^b\frac{1}{(n_+p)}n_+^\rho\frac{i\alpha\,e^{\epsilon\gamma_E}}{(4\pi)}\frac{1}{\epsilon^2}\frac{\Gamma[1-\epsilon]^3}{\Gamma[2-2\epsilon]}\,\Gamma[1+\epsilon]^2\\
\times&\left[-(1-2\epsilon)\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}+2\gamma_{\perp\nu}+\frac{2\not{k}_\perp k_{\perp\nu}}{(n_-k)(n_+k)}+\frac{\not{k}_\perp n_{+\nu}}{(n_+k)}\,(1-2\epsilon)\right]
\end{aligned}
$$

Results for power suppressed amplitude: soft $\times$ hard

$$
\begin{aligned}
iC_F\,\gamma_\perp^\rho\,\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)n_{-\nu}\left(\frac{2}{\epsilon^2}+\frac{1}{\epsilon}+5-\frac{1}{6}\pi^2+\mathcal{O}(\epsilon)\right)\\
&+(n_-k)n_{+\nu}\left(+\frac{2}{\epsilon}+3+\mathcal{O}(\epsilon)\right)\\
&+k_{\perp\nu}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\frac{\pi^2}{6}+8+\mathcal{O}(\epsilon)\right)\\
&+\left[\not{k}_\perp,\gamma_{\perp\nu}\right]\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}-\frac{\pi^2}{12}+4+\mathcal{O}(\epsilon)\right)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&i\,g t^b\;n_+^\rho C_F\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\frac{\pi^2}{6}+\mathcal{O}(\epsilon)\right)\\
&\qquad\frac{1}{(n_+p)(n_-k)}\left(\not{k}_\perp n_{-\nu}-(n_-k)\gamma_{\perp\nu}\right)
\end{aligned}
$$

NLP factorization formula

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\begin{aligned}
\hat{\sigma}(z) \quad = \quad & \sum_{\text{terms}} \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i' \, D(-\hat{s}; \omega_i, \bar{\omega}_i) D^*(-\hat{s}; \omega_i', \bar{\omega}_i') \\
& \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2 + \vec{q}^{\,2}}} \, \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x} \\
& \times \widetilde{S}(x; \omega_i, \bar{\omega}_i, \omega_i', \bar{\omega}_i')
\end{aligned}$$

and

$$\begin{aligned}
D(-\hat{s}; \omega_i, \bar{\omega}_i) \quad = \quad & \int d(n_+ p_i) d(n_- \bar{p}_i) \, C(n_+ p_i, n_- \bar{p}_i) \\
& \times J(n_+ p_i, x_a n_+ p_A; \omega_i) \, \bar{J}(n_- \bar{p}_i, -x_b n_- p_B; \bar{\omega}_i)
\end{aligned}$$

$J_{1,2}$ Collinear function

$$J_{1;\gamma\beta}\left(n_+p, x_a\, n_+p_A; \omega\right) \quad = \quad \delta_{\gamma\beta}\left[J_{1,1}\left(x_a n_+p_A; \omega\right)\frac{\partial}{\partial(n_+p)}\delta(n_+p - x_a\, n_+p_A)\right.$$
$$\left. + J_{1,2}\left(x_a n_+p_A; \omega\right)\delta(n_+p - x_a\, n_+p_A)\right].$$

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-hard}}(z) = 4\left(-H^{(1)}(Q^2) + \epsilon\, H^{(1)}(Q^2)\right)\int d\omega\, S_1^{(1)}\left(\Omega;\, \omega\,\right)$$

Thank you