Recent developments in qT subtraction: EW corrections and power suppressed contributions

Luca Buonocore

University of Zurich, University of Napoli,

INFN-sezione di Napoli

in collaboration with M. Grazzini and F. Tramontano

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University of Zurich[UZH]

Outline

- Motivations
- Drell-Yan NLO EW corrections with q_{T}-subtraction
- Power suppressed terms
- Conclusions

Motivations

- Increasing precision of experimental measurements at LHC and future colliders requires the **inclusion of EW corrections** in the theoretical predictions. Different tools available for the full NLO EW corrections (towards automation) [Les Houches 2017, arXiv:1803.07977]

- As for the subtraction of infrared divergences, local subtractions as CS and FKS are the standard, but limited to NLO.

- So far, q_{T}-subtraction successfully employed for the computation of QCD corrections up to (N)NNLO. Its extension to EW correction **might be relevant to develop a subtraction scheme for NNLO EW and NNLO QCDxEW** (pure mixed corrections)

- Drell-Yan massive dilepton production represents the ideal process to start with for its phenomenological relevance as benchmark process and for the structure of singularities (analogous to the heavy quark production in QCD)

Motivations

- Renewed interest in power suppressed terms for SCET factorization and applications to slicing methods (NJettines, q_T)

- Next-to-Leading Power (NLP) contribution for **inclusive color-singlet** production in qT-subtraction at NLO **are known to be quadratic**
  [M. *Grazzini, S. Kallweit, S. Pozzorini, D. Rathlev, M. Wiesemann, JHEP* 1608 (2016) 140]
  and it has been **explicitly computed** by two different groups
  [M. *A. Ebert, I. Moult, I. W. Stewart, F. J. Tackmann. G. Vita. H.X. Zhu, JHEP* 1904 (2019) 123],
  [L. *Cieri, C. Oleari, M. Rocco, arXiv*:1906.09044]

- NLP for processes including **radiation off massive final state** show a different behavior (**linear**) as in the case of heavy quark production
  [S. *Catani,S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli, arXiv*:1901.04005]

- The QED implementation provides the perfect environment to study analytically the NLP and to complete the analysis at NLO.

DY NLO EW corrections with q_T subtraction

q_T-subtraction scheme

Master Formula (**subtraction**) for color singlet production

$$d\sigma_{(\mathrm{N})\mathrm{NLO}} = \mathcal{H}^{F}_{(\mathrm{N})\mathrm{NLO}} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma^{F+\mathrm{jets}}_{(\mathrm{N})\mathrm{LO}} - d\sigma^{\mathrm{CT}}\right]$$

hard-virtual function auxiliary cross section

The counterterm is provided by **transverse momentum resummation**

$$d\sigma^{\mathrm{CT}} = d\sigma_{\mathrm{LO}} \otimes \Sigma^{F}\left(\frac{q_T}{M}\right) d^2 q_T$$

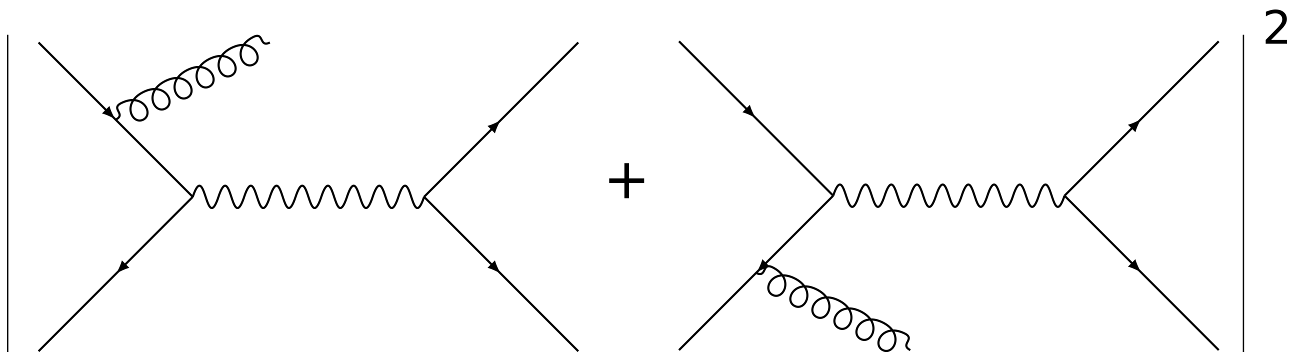
The counterterm is **non-local.** The actual implementation relies on the **introduction of a technical cut** $r_{\mathrm{cut}}=q_{\mathrm{T}}^{\mathrm{cut}}/M$ separating the two regions $q_T \neq 0$ and $q_T=0$ (**slicing**)

At NLO: all the IR divergence are contained in the small q_T limit

At NNLO, the slicing simplifies the structure of the singularities:

- $q_T \neq 0$, where the structure of the divergence is as NLO for the process F + jet
- $q_T=0$, where the genuine NNLO singularities occur (double not resolved emissions)

Structure of counterterm at NLO - Color singlet



$$\Sigma^{F(1;2)}_{c\bar{c}\to ab}(z)\tilde{I}_2\left(\frac{q_T}{M}\right) + \Sigma^{F(1;1)}_{c\bar{c}\to ab}(z)\tilde{I}_1\left(\frac{q_T}{M}\right)$$

$\ln^2 r_{\rm cut} \sim \dfrac{1}{\epsilon^2}$ $\qquad$ $\ln r_{\rm cut} \sim \dfrac{1}{\epsilon}$

In Mellin space

$$\Sigma^{F(1;2)}_{c\bar{c}\to ab,N} = -\frac{1}{2}A^{(1)}_c\delta_{ca}\delta_{\bar{c}b}$$

soft-collinear, proportional to the charges C_F, C_A

$$\Sigma^{F(1;1)}_{c\bar{c}\to ab,N} = -[\delta_{ca}\delta_{\bar{c}b}B^{(1)}_c + \delta_{ca}\gamma^{(1)}_{\bar{c}b,N} + \delta_{\bar{c}b}\gamma^{(1)}_{ca,N}]$$

soft

$$-\frac{3}{2}C_F, -\frac{1}{6}(11C_A - 2n_f)$$

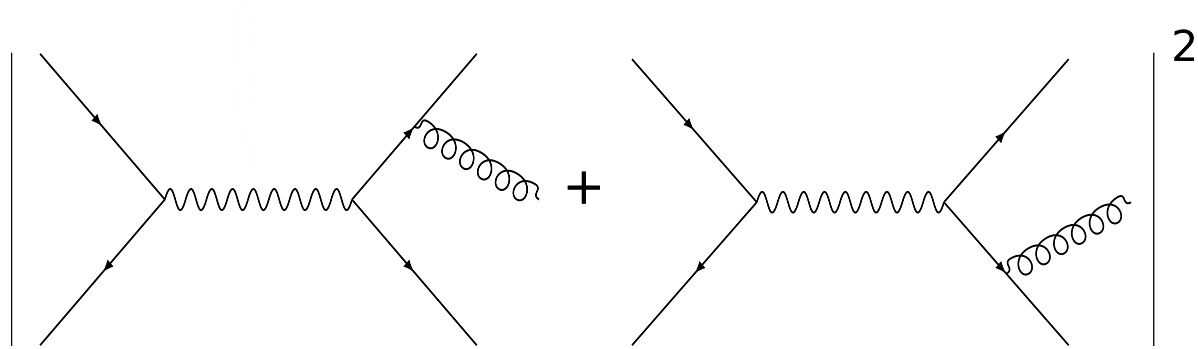
collinear

convolution with AP kernels

$$\int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z}\right) P_{ca}(z)$$

Structure of counterterm at NLO - heavy quarks

Recently q_T-subtraction extended to treat massive colorful final state ($t\bar{t}$ pair) up to NNLO [*S. Catani,S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli, arXiv*:1901.04005]



New contributions to the 'single pole part' with **non-trivial color structure**

$$\Sigma^{Q\overline{Q}}_{c\bar{c}\to ab,N} = \Sigma^{F(1;1)}_{c\bar{c}\to ab,N} - \delta_{ca}\delta_{\bar{c}b}\frac{< \tilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\overline{Q}}|(\Gamma_t^{c\bar{c}(1)} + \Gamma_t^{c\bar{c}(1)\dagger})|\tilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\overline{Q}} >}{|\tilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\overline{Q}}|^2}$$

$$\Gamma_t^{(1)} = -\frac{1}{4}\Bigg\{(\mathbf{T}_3^2+\mathbf{T}_4^2)(1-i\pi) + \sum_{i=1,2;j=3,4}\mathbf{T}_i\cdot\mathbf{T}_j \ln\frac{(2p_i\cdot p_j)^2}{M^2m^2}$$
$$+ 2\mathbf{T}_3\cdot\mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v}+1\right)\right]\Bigg\}$$

EW q_T-subtraction: Abelianization procedure

Caveat: only photons in the real emission process (full EW for the virtual process)

→ the subtraction sees only the QED part

Main idea: there is no need to compute it from scratch. Recycle QCD computation!

Abelianization (*see Fabre talk*)

[D. *de Florian*, G. *F.R. Sborlini*, G. *Rodrigo*, *Eur.Phys.J.* C76 (2016) *no.5*, 282]

At NLO, the list of replacement is straightforward:

- DY-like : $C_A \to 0$

  $C_F \to e_f^2$ non trivial flavour structure

- FSR soft radiation: $\mathbf{T}_i^2 \to e_i^2 \mathbf{I}$

  $\mathbf{T}_i \cdot \mathbf{T}_j \to e_i e_j \mathbf{I}$ trivial color structure

Cross-checks: it reproduces the analytical structure in the eikonal approximation and we test the numerical convergence of the (real-counterterm) integral

EW corrections to the Drell-Yan Process

Relevant literature

- *Baur, Wackeroth, et al., PRD* 65 (2002) 033007, *PRD* 70 (2004) 073015
- *Dittmaier, Kramer, PRD* 65 (2002) 073007
- *Jadach, Płaczek, EPJC* 29 325 (2003), *D. Bardin et al., Acta Phys. Polon. B*40 (2009) 75
- *Carloni Calame et al., PRD* 69 (2004) 037301, *JHEP* 0612 (2006) 016, *JHEP* 0710 (2007) 109
- *Arbuzov et al., EPJC* 46, 407 (2006), *EPJC* 54 (2008) 451
- *Dittmaier, Huber, JHEP* 1001 (2010) 060

not meant to be exhaustive!

Tools

- Z/WGRAD, NLO EW to CC and NC DY
- SANC, NLO EW to CC and NC DY
- WINHAC, NLO EW + multiple photon to CC DY
- HORACE, NLO EW + matched multiple photon emission to CC and NC DY
- RADY, NLO EW + MSSM to NC DY

VALIDATION NLO EW for a massive lepton

Benchmark setup similar to [S. Dittmaier, M. Huber, JHEP 1001 (2010)]:

Physical Parameters (G_m-scheme):

- G_F = 1.16637 x10^{-5} GeV^{-2} α_0 = 1/137.03599911
- $M_{W,OS}$ = 80.403 GeV $M_{Z,OS}$ = 91.1876 GeV M_H = 115 GeV
- $G_{W,OS}$ = 2.141 GeV $G_{Z,OS}$ = 2.4952 GeV
- **m_l = 10 GeV**

Fiducial cuts:

- $M_{ll} > 50$ GeV $p_{T,l\pm} > 25$ GeV $|y_\pm| < 2.5$
- *no lepton-photon recombination*

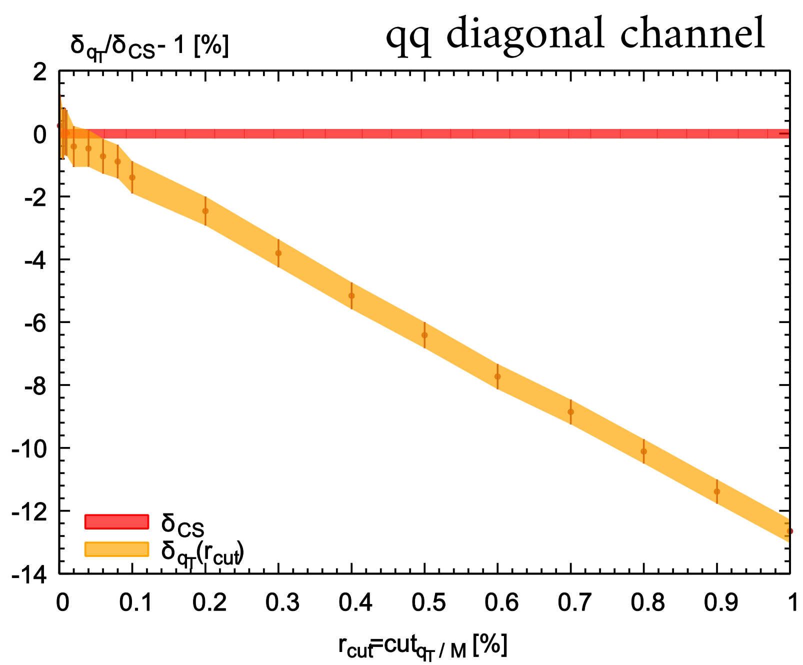
Preliminary

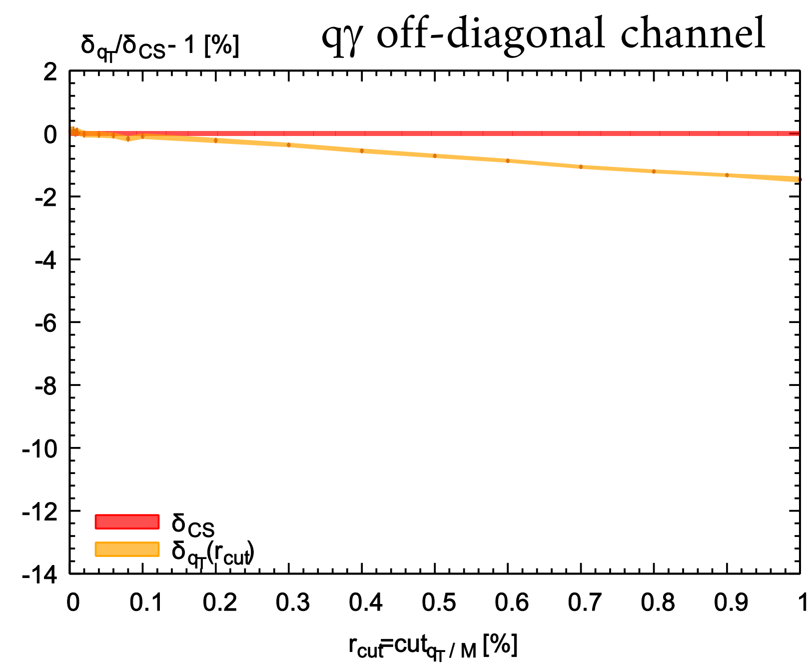
	q_T + GoSam	CS+RECOLA	SANC
$\sigma_{LO}^{q\bar{q}}$ (pb)	683.53 ± 0.03		683.26 ± 0.01
$\Delta\sigma_{q\bar{q}}$ (pb)	$-5.920 \pm 0.034^*$	-5.919 ± 0.008	-5.932 ± 0.020
$\sigma_{LO}^{\gamma\gamma}$ (pb)	1.2333 ± 0.0004		–
$\Delta\sigma_{q\gamma}$ (pb)	-0.6694 ± 0.0008	-0.6690 ± 0.0005	–

* uncertainty dominated by the real-ct contribution and extrapolation at r_{cut} =0

VALIDATION NLO EW for a massive lepton

Dependence of the NLO corrections on the r_{cut} regulator:





The q_T prediction has been obtained with a **linear extrapolation** (qq channel).

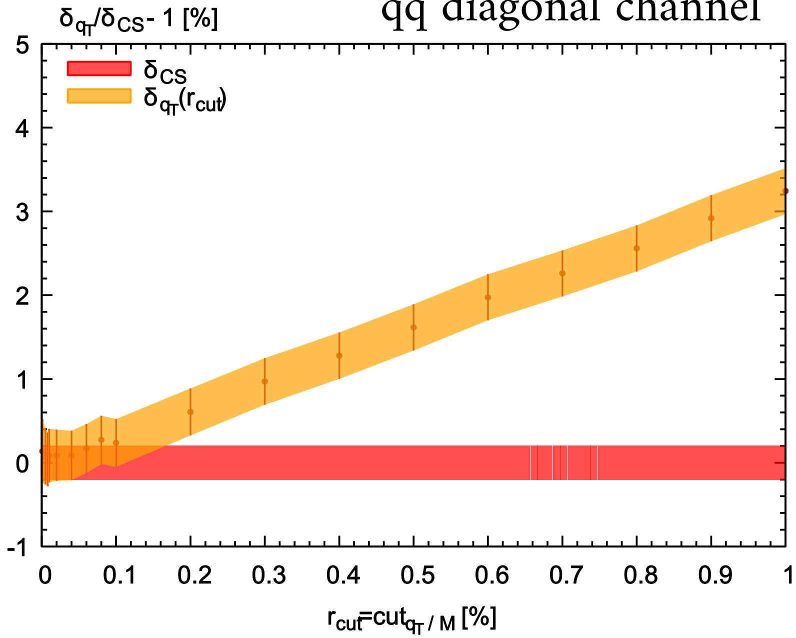
Remark: Sizeable r_{cut}-dependence also in the qγ channel. **Symmetrical cuts** on the p_T of the leptons **worsen** the r_{cut} dependence already for color singlet production (no final state radiation)

[M. Grazzini, S. Kallweit, M. Wiesemann, *Eur.Phys.J.C*78,537(2018)]

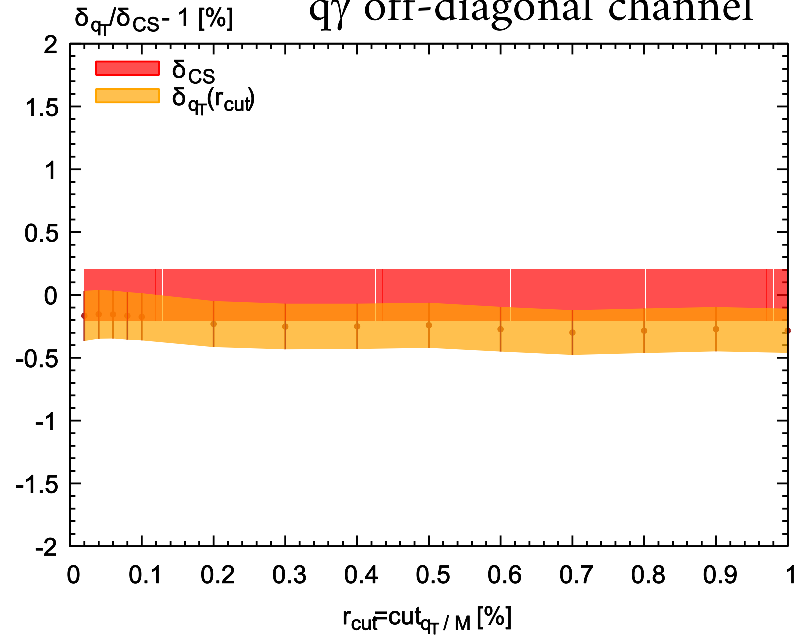
VALIDATION NLO EW for a massive lepton

Dependence of the NLO corrections on the r_{cut} regulator for the **inclusive case**

qq diagonal channel



qγ off-diagonal channel



- **Flat dependence in the qγ off-diagonal channel**, as it occurs in color singlet [M. Grazzini, S. Kallweit, M. Wiesemann, *Eur.Phys.J.C*78,537(2018)]
- Distinct **linear behavior** in the **qq diagonal channel** as in heavy quark production, **genuine effect** of the emission off massive final state

Power Suppressed Terms

Open questions

- Assess the analytical structure of the leading power suppressed term for FSR off massive final state for the inclusive total cross section.

- Shed light on the different behavior color singlet/colorful finale state

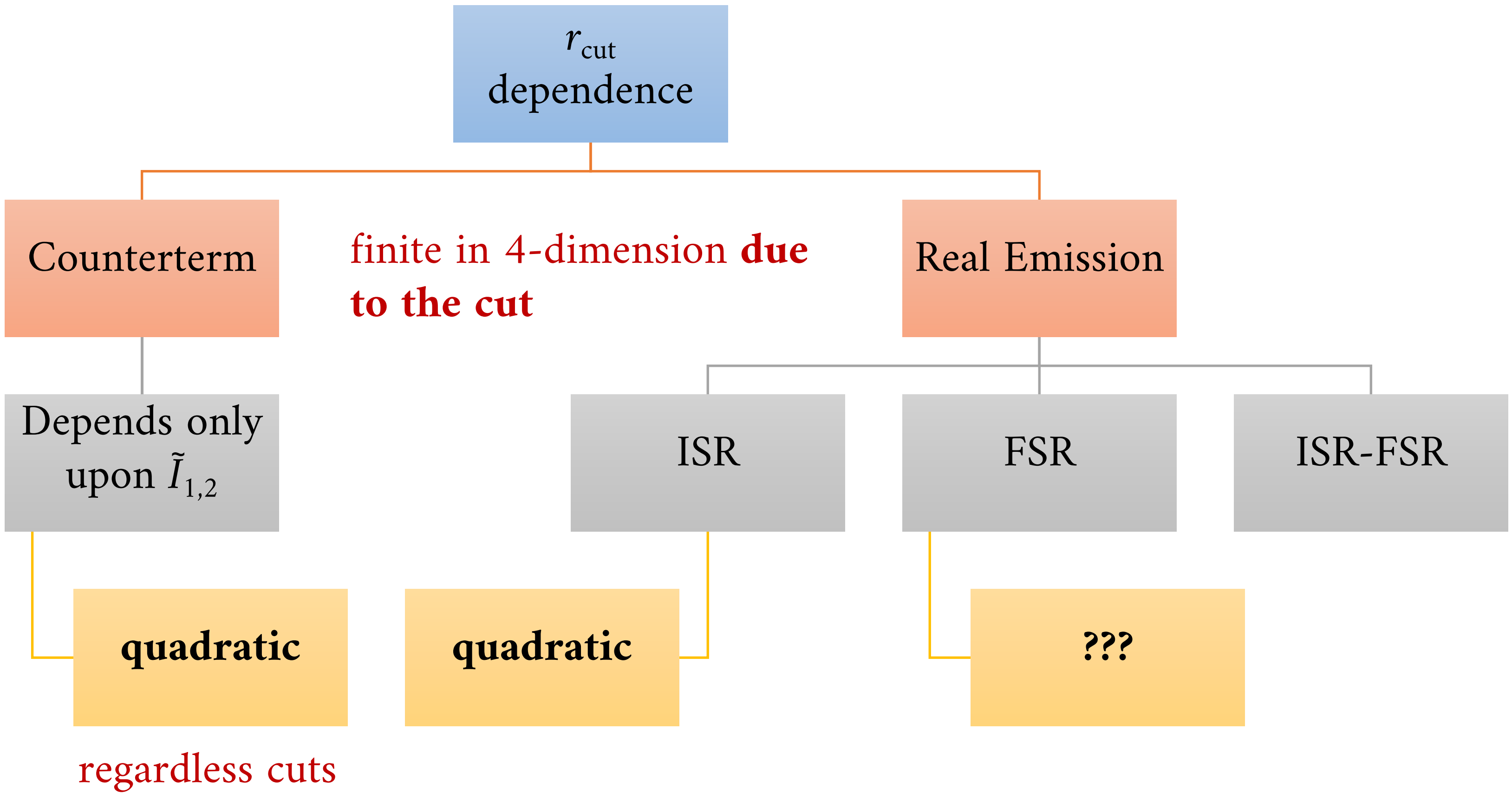
- Do subleading power suppressed terms show up some universal properties? Can they be reproduced in the soft limit?

- Is there a systematic way to compute them beyond inclusive observables?

Overview

We focus on **pure QED case (laboratory process)**:

- ISR, FSR and their interference are gauge invariant subsets and can be treated separately



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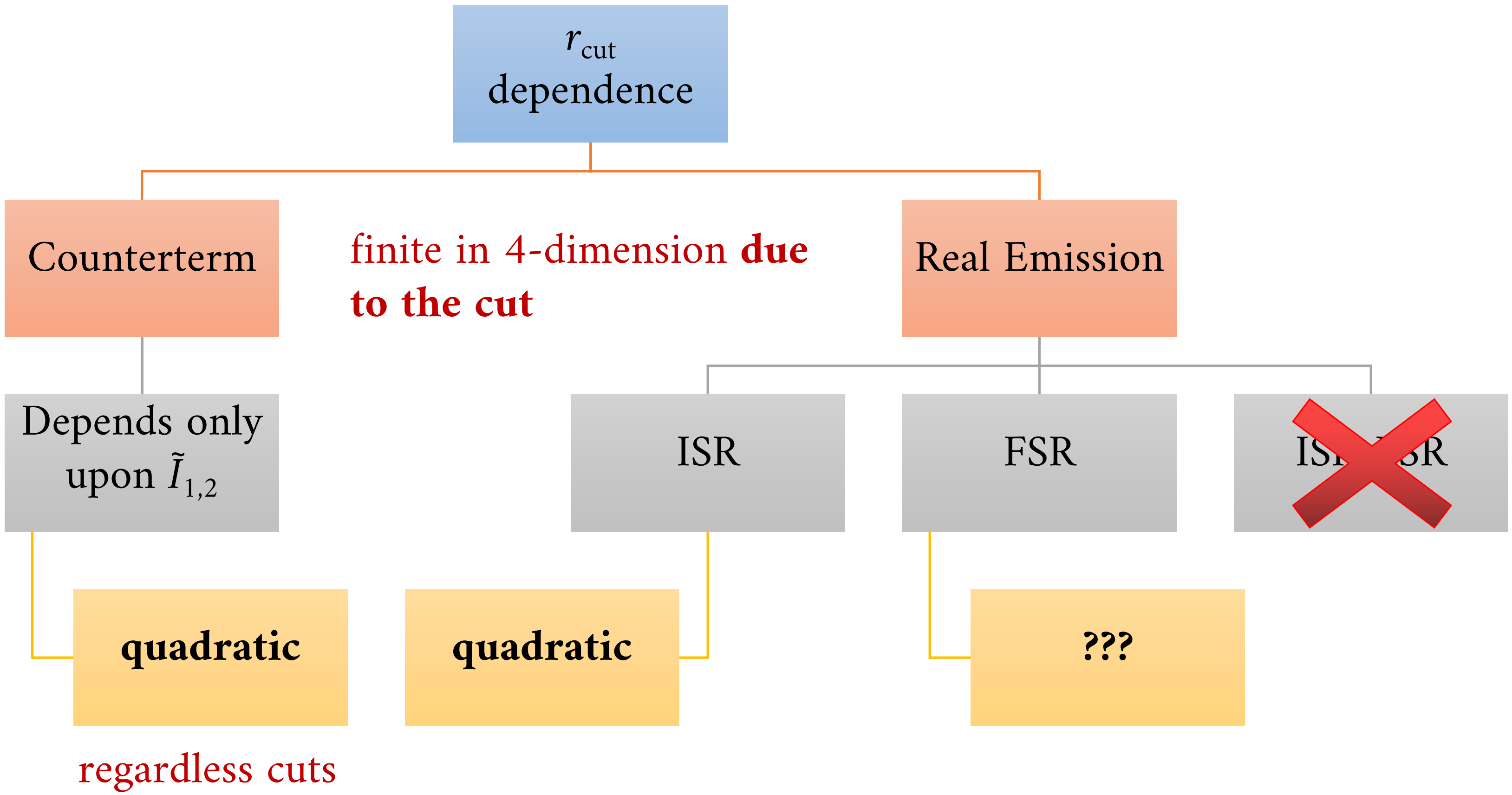

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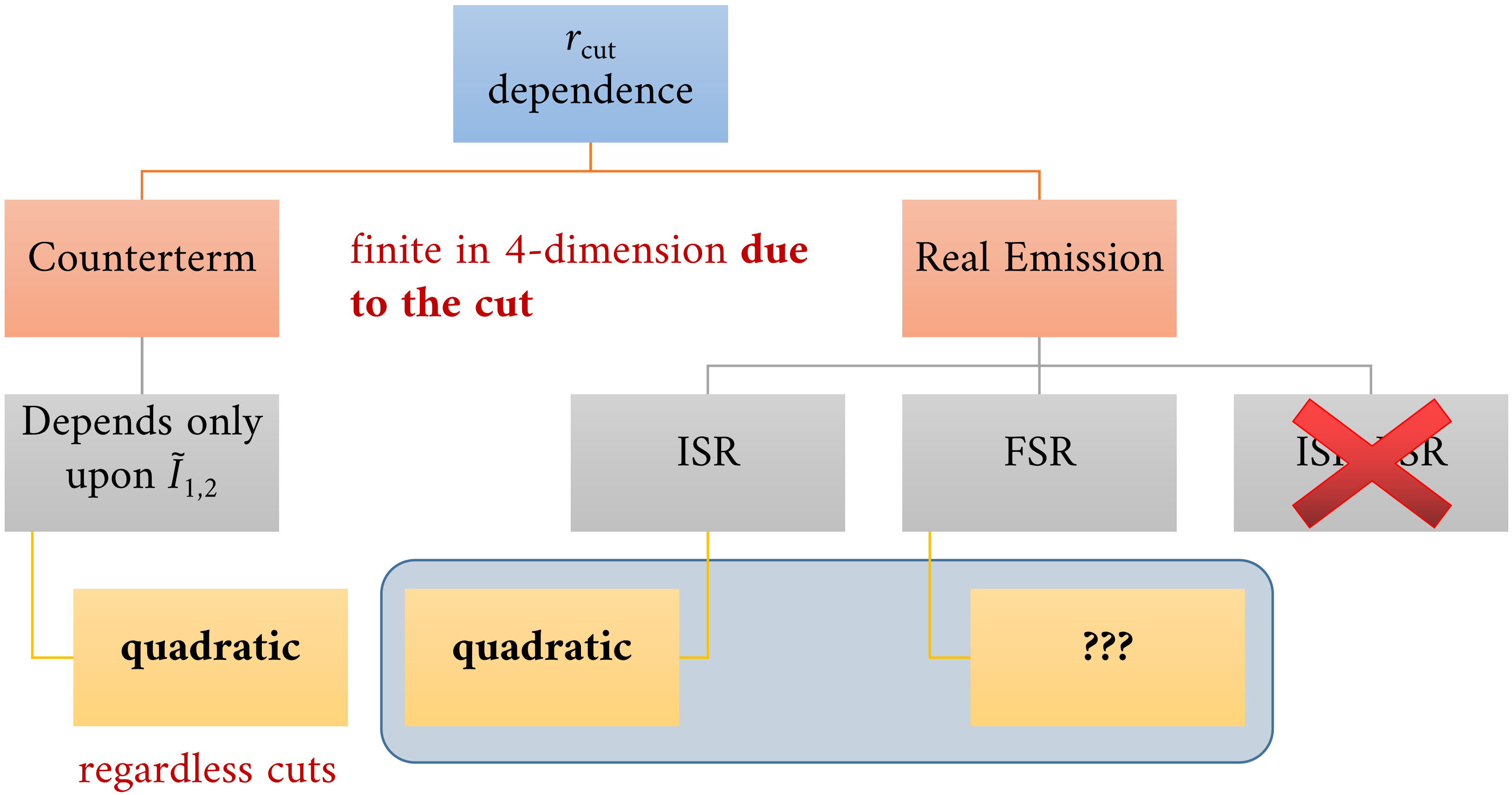


Real emission

Integration

- Deal with the $\boldsymbol{r}_{\mathbf{cut}}$ **constraints**
- Handle **3-body kinematics**

Expansion

- Taylor expansion unfeasible
- **Deal with distributions**

Real emission

Integration

- Deal with the $\boldsymbol{r}_{\mathbf{cut}}$ **constraints**
- Handle **3-body kinematics**

Expansion

- Taylor expansion unfeasible
- **Deal with distributions**

Technical, but it relies on standard mathematical methods (plus distribution, Mellins transform)

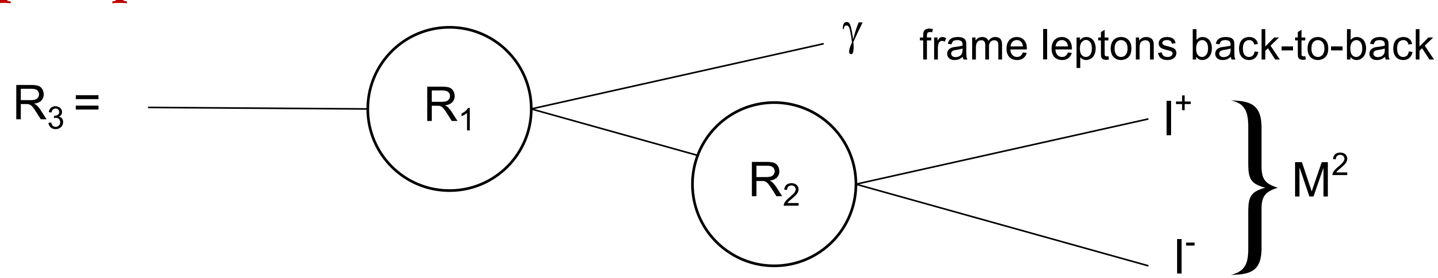
Real emission

Integration

- Deal with the r_{cut} **constraints**
- Handle **3-body kinematics**

- Taylor expansion

Phase Space parametrisation



$$= \frac{1}{16}\frac{1}{(2\pi)^4}\int dM^2 dq_T^2 \frac{1}{\sqrt{(s-M^2)^2-4q_T^2 s}}\sqrt{1-\frac{4m^2}{M^2}}\int d\Omega|\mathcal{M}|^2$$

- q_T appears explicitly among the integration variables. It allows **a simplified treatment of the cut** in the integration.

Real emission: master formula

The dependence of the partonic real emission partonic cross section on the $r_{\rm cut}$ regulator can be recast in the following master formula

$$\boxed{\frac{d\sigma}{dr_{\rm cut}^2} = -\frac{1}{32}\frac{1}{(2\pi)^4}\int_{z_{\rm min}}^{z_{\rm max}} \frac{z\,dz}{\sqrt{(1-z)^2 - 4zr_{\rm cut}^2}}\sqrt{1-\frac{4m^2}{zs}}\int d\Omega|\mathcal{M}|^2}$$

in terms of the energy fraction z

$$z = \frac{M^2}{s}, \qquad z_{\rm min} = \frac{4m^2}{s}, \qquad z_{\rm max} = 1 - 2r_{\rm cut}\sqrt{1+r_{\rm cut}^2} + 2r_{\rm cut}^2$$

Important remarks

- $r_{\rm cut}$ dependence is contained in the **jacobian square root factor** (**quadratic**) and in the **upper integration limit** $z_{\rm max}$
- the square root factor **vanishes** in $z_{\rm max}$ (**non trivial interplay**)
- in the small-$r_{\rm cut}$ limit: $z_{\rm max}=1-2r_{\rm cut}+O\left(r^2{}_{\rm cut}\right)$ **linear dependence**

Integration

First step: integration over the angle between the two leptons

We use **partial fractioning** to recast the expression as a combination of known angular integrals (**relevant for heavy-quark production**)
[*W. Beenakker, H. Kuijf, W. L. van Neerven, Phys.Rev.* D40 (1989) 54-82]

$$\boxed{\frac{d\sigma^{\mathrm{FS}}}{dr_{\mathrm{cut}}^2} = \frac{4\alpha^3 e_q^2}{3s} \int_{z_{\min}}^{z_{\max}} dz \left[\frac{K_1(z; m^2/s)}{(1-z)^2\sqrt{(1-z)^2 - 4zr_{\mathrm{cut}}^2}} + \frac{K_2(z; m^2/s) r_{\mathrm{cut}}^2}{(1-z)^4\sqrt{(1-z)^2 - 4zr_{\mathrm{cut}}^2}} \right]}$$

$$\boxed{\frac{d\sigma^{\mathrm{IS}}}{dr_{\mathrm{cut}}^2} = -\frac{4\alpha^3 e_q^4}{9s} \int_{z_{\min}}^{z_{\max}} dz \left[\frac{K_3(z; m^2/s)}{r_{\mathrm{cut}}^2\sqrt{(1-z)^2 - 4zr_{\mathrm{cut}}^2}} + \frac{K_4(z; m^2/s)}{\sqrt{(1-z)^2 - 4zr_{\mathrm{cut}}^2}} \right]}$$

- The coefficient functions *Ks* are regular at $z=1$
- FS distributions are more singular in $z=1$. They pick the linear dependence in $z_{\max}$
- In IS, the interplay between the square root factor and $z_{\max}$ leads to quadratic dependence

Results

Born cross section

$$\sigma_0(s) = \frac{2\pi}{9s}\alpha^2 e_q^2 \beta(3-\beta^2) \qquad \beta = \sqrt{1-\frac{4m^2}{s}}$$

pure linear NLP (no logs!)

$$\begin{aligned}\sigma^{\rm FS}(s;r_{\rm cut}) - \sigma_{\rm lim} &= \sigma_0(s)\frac{\alpha}{2\pi}\left\{\left[2-\frac{(1+\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta}\right]\log\left(r_{\rm cut}^2\right)\right.\\ &\left.-\frac{3\pi}{8}\left[\frac{6(5-\beta^2)}{3-\beta^2}+\frac{(-47+8\beta^2+3\beta^4)}{\beta(3-\beta^2)}\log\frac{1+\beta}{1-\beta}\right]r_{\rm cut}\right\} + O(r_{\rm cut}^2)\\ &\equiv \sigma^{\rm LP}(r_{\rm cut}) + \sigma^{\rm NLP}(r_{\rm cut}) + O(r_{\rm cut}^2)\end{aligned}$$

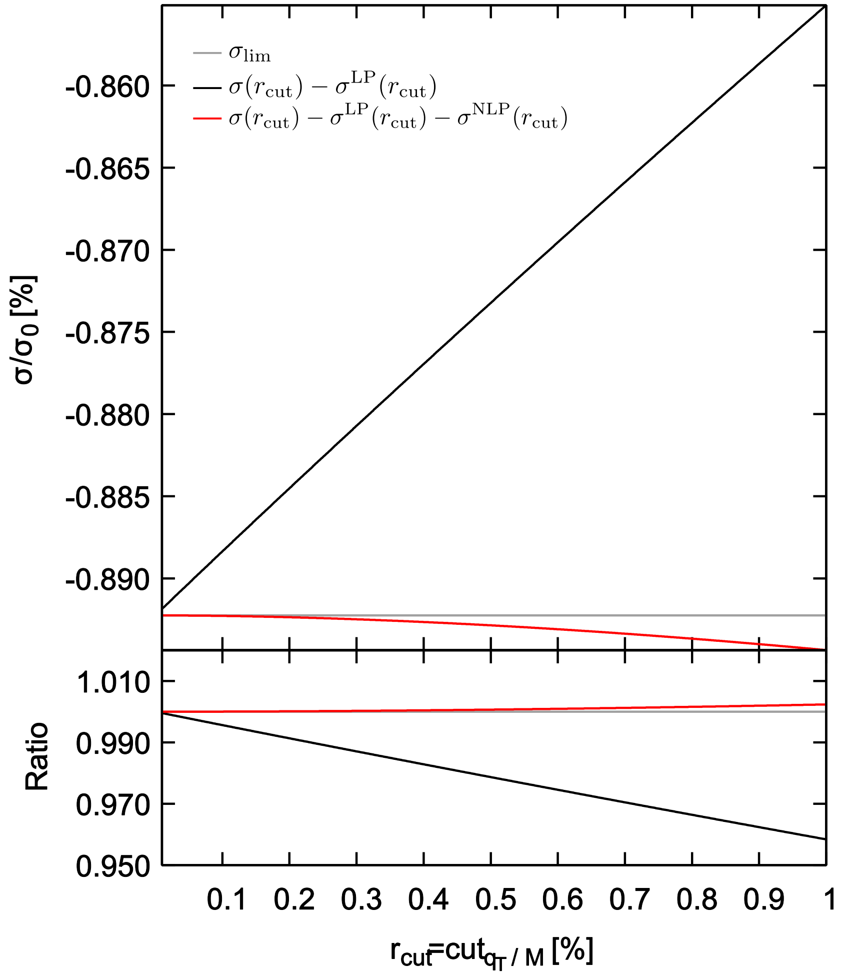
$$\begin{aligned}\sigma^{\rm IS}(s;r_{\rm cut}) - \sigma_{\rm lim} &= \sigma_0(s)\frac{\alpha}{2\pi}e_q^2\left\{\ln^2 r_{\rm cut}^2 - 4\left(2\ln 2-\frac{4}{3}-\ln\frac{1-\beta^2}{\beta^2}-\frac{1}{\beta(3-\beta^2)}\ln\frac{1+\beta}{1-\beta}\right)\ln r_{\rm cut}^2\right.\\ &\left.-\frac{3}{2}\frac{(1+\beta^2)(1-\beta^2)^2}{\beta^4(3-\beta^2)}r_{\rm cut}^2\ln r_{\rm cut}^2 - \frac{3}{2}\frac{(1+\beta^2)(1-\beta^2)^2}{\beta^4(3-\beta^2)}\left(1-4\ln 2+2\ln\frac{1-\beta^2}{\beta^2}\right)r_{\rm cut}^2\right\} + \ldots\ldots\\ &\equiv \sigma^{\rm LP}(r_{\rm cut}) + \sigma^{\rm NLP}(r_{\rm cut}) + \ldots\ldots\end{aligned}$$

- **Check**: as byproduct we re-derive the result for the production of a color-singlet system of fixed mass [L. *Cieri, C. Oleari, M. Rocco, arXiv*:1906.09044]
- **Remark:** Our partonic result is a smooth function of β, at variance with what happens for the on-mass shell color singlet production

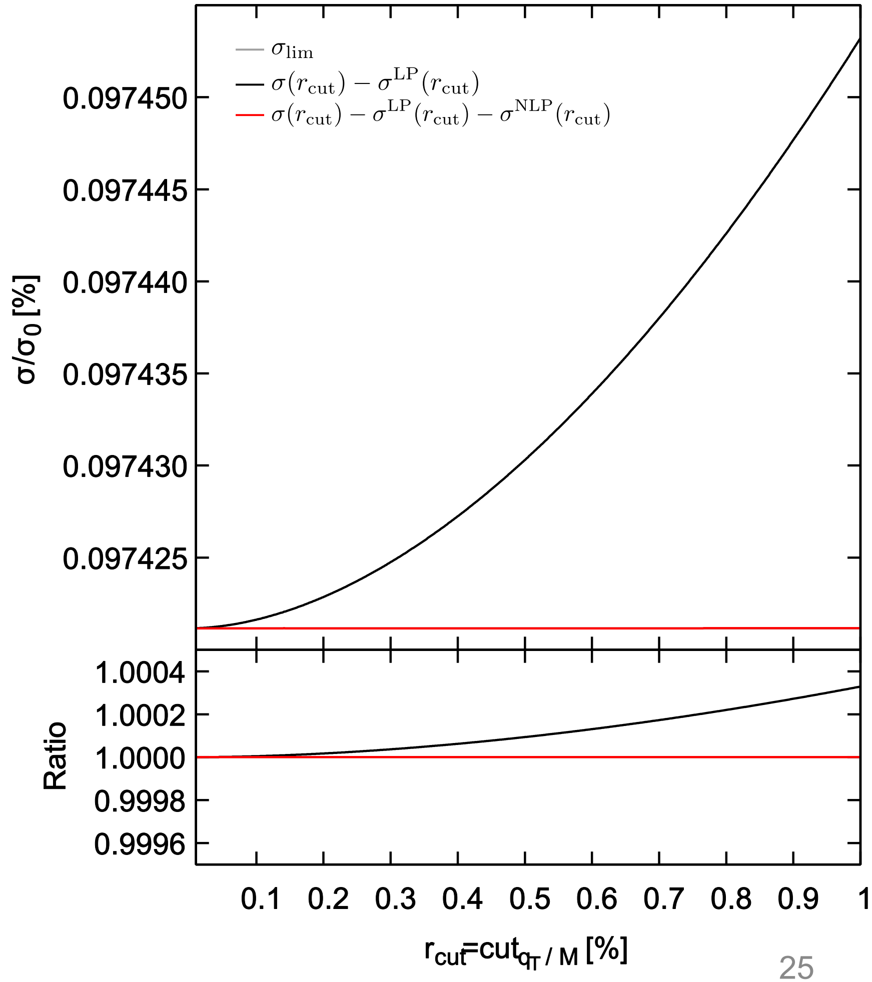
Validation: numerical checks

Dependence of the **real emission partonic cross section** on the r_{cut} regulator

FSR case



ISR case



Open questions

- Assess the analytical structure of the leading power suppressed term for FSR off massive final state for the inclusive total cross section. ✓ **pure linear NLP**

- Shed light on the different behavior color singlet/colorful finale state ✓

- Do subleading power suppressed terms show up some universal properties? Can they be reproduced in the soft limit?

- Is there a systematic way to compute them beyond inclusive observables?

FSR - Soft Power Counting

A **power counting argument** establishes the correspondence between the **soft expansion** and the power corrections in the $r_{\rm cut}$ regulator

$$d\sigma^{\rm FS} = (d\Phi_R^{(0)} + d\Phi_R^{(1)} + \dots) \times (|\mathcal{M}|^{2(0)} + |\mathcal{M}|^{2(1)} + \dots)$$

Matrix Element

Phase Space

	Soft	Next-to-Soft
Soft	LP	**NLP**
Next-to-Soft	**NLP**	higher orders

- NLP comes from **finite contributions** of the real emission cross sections which are cut away integrating up to $r_{\rm cut}$.
- **Physical understanding**: the correspondence between small-$q_{\rm T}$ region and singular configuration is not one-to-one

Going beyond inclusive predictions - Preliminary

NLP can be integrated in the **unresolved** region in a full differential fashion!

$$d\sigma^{l^+l^-}_{NLO} = \mathcal{H}^{l^+l^-}_{NLO} \otimes d\sigma^{l^+l^-}_{LO} + \left[d\sigma^{l^+l^-+\gamma}_{LO} - d\sigma^{l^+l^-,\, CT}_{NLO} \right]_{\frac{q_T}{M} > r_{\mathrm{cut}}} + \boxed{\left[d\sigma^{l^+l^-+\gamma,\, FS}_{LO} \right]^{NLP}_{\frac{q_T}{M} < r_{\mathrm{cut}}}}$$

Numerical Scheme

Ingredients

- **LP local counterterm** (universal behavior)
- **local mapping:** massive FKS [L. *Buonocore, P. Nason, F. Tramontano, Eur. Phys. J.* C78 (2018) *no.2,* 151]

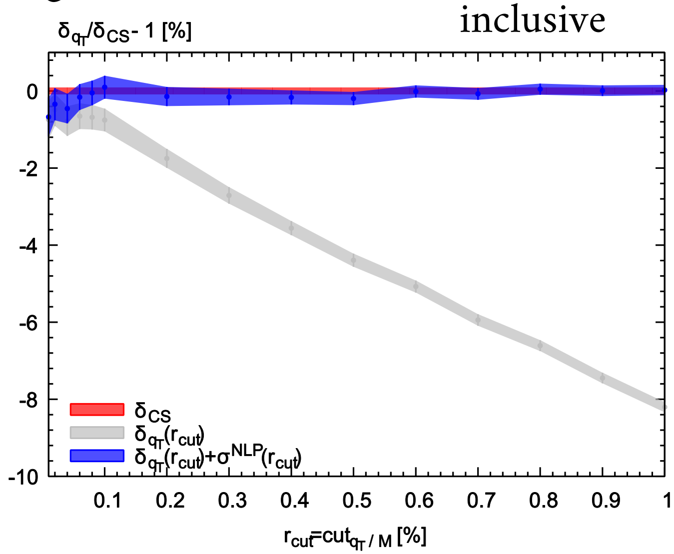
Integration of NLP in the unresolved region

- $$\sigma^{\mathrm{NLP}} = \int \left[d\sigma^{l^+l^-+\gamma,\, FS}_{LO} - d\sigma^{CT,LP}_{NLO} \right]_{\frac{q_T}{M} < r_{\mathrm{cut}}}$$

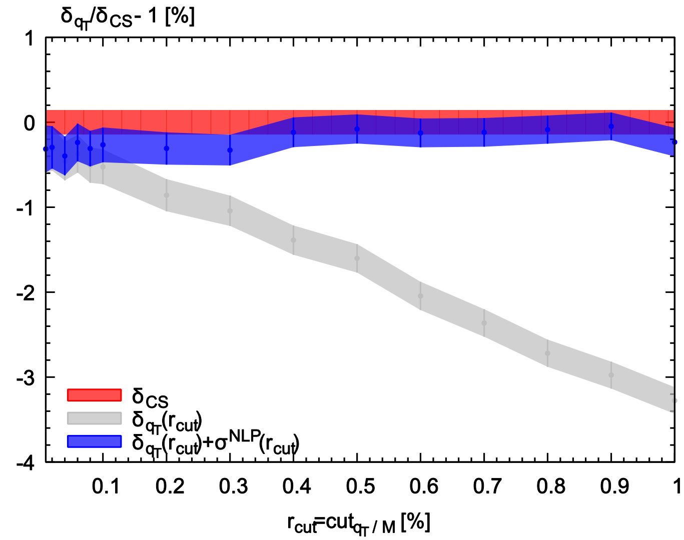
Going beyond inclusive predictions - Preliminary

Pure QED case

inclusive

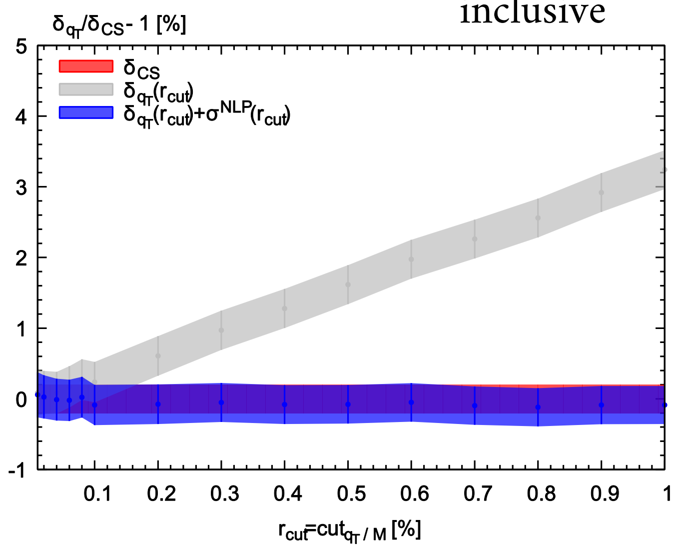


$|y_l| < 2.5, \quad p_{T,l^-} > 25\,\mathrm{GeV}, \quad p_{T,l^+} > 20\,\mathrm{GeV}$



Including Z resonance

inclusive



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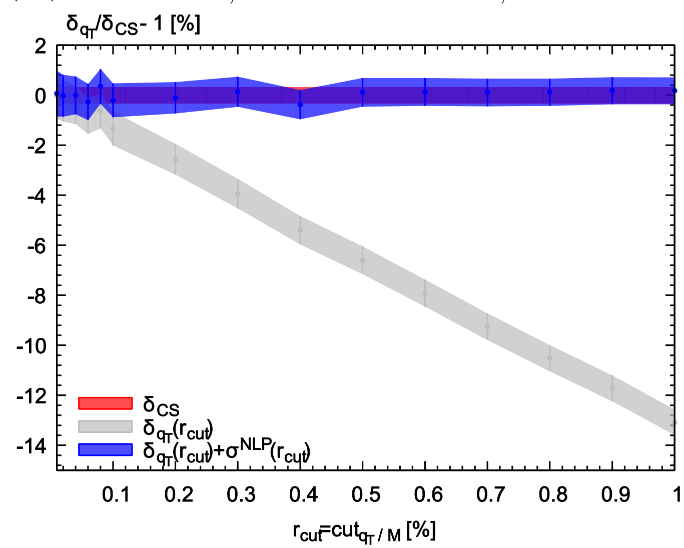


Open questions

- Assess the analytical structure of the leading power suppressed term for FSR off massive final state for the inclusive total cross section. ✓ **pure linear NLP**

- Shed light on the different behavior color singlet/colorful finale state ✓

- Do subleading power suppressed terms show up some universal properties? Can they be reproduced in the soft limit? ✗ **requires next-to-soft**

- Is there a systematic way to compute them beyond inclusive observables? ✓ **(at least) at NLO**

Conclusions and Outlook

1. We have obtained the **first implementation of NLO EW corrections** with **q_T-subtraction**

   - **Work in progress:** development of a subtraction scheme suitable for higher order corrections in EW and mixed QCDxEW

2. We have extended the analysis of NLP corrections for the qt-subtraction formula at NLO to the case of **radiation emitted of a massive final state**

   Outlook:

   - symmetrical cuts

   - extension to NNLO

BACKUP

q_T-counterterm contribution

We can consider **separately** the contributions of the q_T-counterterm and the real emission matrix element, which are **finite in 4-dimensions** due to the cut.

As for the **counterterm**, all the r_{cut}-dependence is contained in the $\tilde{I}_{1,2}$ functions (large logs from the resummation programs).

In the small r-limit, they behave as

$$\tilde{I}_1(r) = -\frac{1}{r^2} + \frac{b_0^2}{4}(1 - 2\ln r) + O(r^2) \qquad \tilde{I}_2(r) = \frac{4\ln r}{r^2} + \frac{b_0^2}{2}\left(-1 + 2\ln^2 r\right) + O(r^2)$$

They depend **quadratic ally** on r_{cut}, modulo logarithmic terms (it holds also at NNLO and higher orders).

- NLP corrections from the **counterterm are quadratic** (independently of the perturbative order).
- A **linear behavior** might come only from the **real emission**.

Coefficient Functions

Coefficient functions $K_{1,2}$ occurring in the study of the FSR

$$K_1(z;m/s) = -\left[\frac{4m^2}{s}z^2 + z(1+z)^2\right]\sqrt{1-\frac{4m^2}{sz}} + z(1+z^2+\frac{4m^2}{s}z - \frac{8m^4}{s^2})\log\frac{1+\sqrt{1-\frac{4m^2}{sz}}}{1-\sqrt{1-\frac{4m^2}{sz}}}$$

$$K_2(z;m/s) = 2z^2\left\{\left[1+z(6+z)+\frac{4m^2}{s}z\right]\sqrt{1-\frac{4m^2}{sz}} - \left(1+z^2+\frac{4m^2}{s}(2+z)-\frac{8m^4}{s^2}\right)\log\frac{1+\sqrt{1-\frac{4m^2}{sz}}}{1-\sqrt{1-\frac{4m^2}{sz}}}\right\}$$

Coefficient functions $K_{3,4}$ occurring in the study of the ISR

$$K_3(z;m^2/s) = \sqrt{1-\frac{4m^2}{sz}}\left(z+\frac{2m^2}{s}\right)\frac{1+z^2}{z^2}$$

AP splitting

$$K_4(z;m^2/s) = K_3(z;m^2/s)\frac{-2z}{1+z^2}$$

Expansions

FSR

$$\frac{\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^2\sqrt{(1-z)^2-4zr_{\rm cut}^2}} = \frac{1}{4}\delta(1-z)\frac{1}{r_{\rm cut}^2} + \frac{\pi}{8}\left[\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\rm cut}} + \mathcal{O}(1)$$

contribution to linear NLP

$$\frac{\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^4\sqrt{(1-z)^2-4zr_{\rm cut}^2}} = \frac{1}{24}\delta(1-z)\frac{1}{r_{\rm cut}^2} + \frac{\pi}{64}\left[3\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\rm cut}} + \mathcal{O}(1)$$

Remark: up to the consider order, no dependence on the lower limit $z_{\min}$

ISR

$$\begin{aligned} T(z, r_{\rm cut}, a) &\equiv \frac{\Theta(z-a)\Theta(z_{\max}-z)}{\sqrt{(1-z)^2-4zr_{\rm cut}^2}} \\ &= T^{(0,1)}(z,a)\ln r_{\rm cut}^2 + T^{(0,0)}(z,a) \\ &+ T^{(2,1)}(z,a)r_{\rm cut}^2\ln r_{\rm cut}^2 + T^{(2,1)}(z,a)r_{\rm cut}^2 + \ldots\ldots. \end{aligned}$$

FSR - Hadronic cross section

Residual r_{cut} in the convolution with pdf **might be an extra source** of the linear LP at the level of the hadronic cross section:

extra r_{cut}-dependence

$$\sigma_H(S,\mu_F) = \int_{z_0=\frac{4m^2}{S}}^{z_{\max}(r_{\text{cut}})} \frac{dz_H}{z_H} L(z_H, z_0, \mu_F)\hat{\sigma}\left(\hat{s}=\frac{4m^2}{z_H}, r_{\text{cut}}\right)$$

collision energy

parton luminosity

Remark: $z_{\max}$ is the same as before!

Analyticity argument:

- the new contributions to the linear NLP **are proportional to the partonic cross section** σ (or its LP expansion) picked at z_H=1
- z_H=1 corresponds to the threshold for the production of massive final state
- the cross section vanishes at the threshold for analyticity.

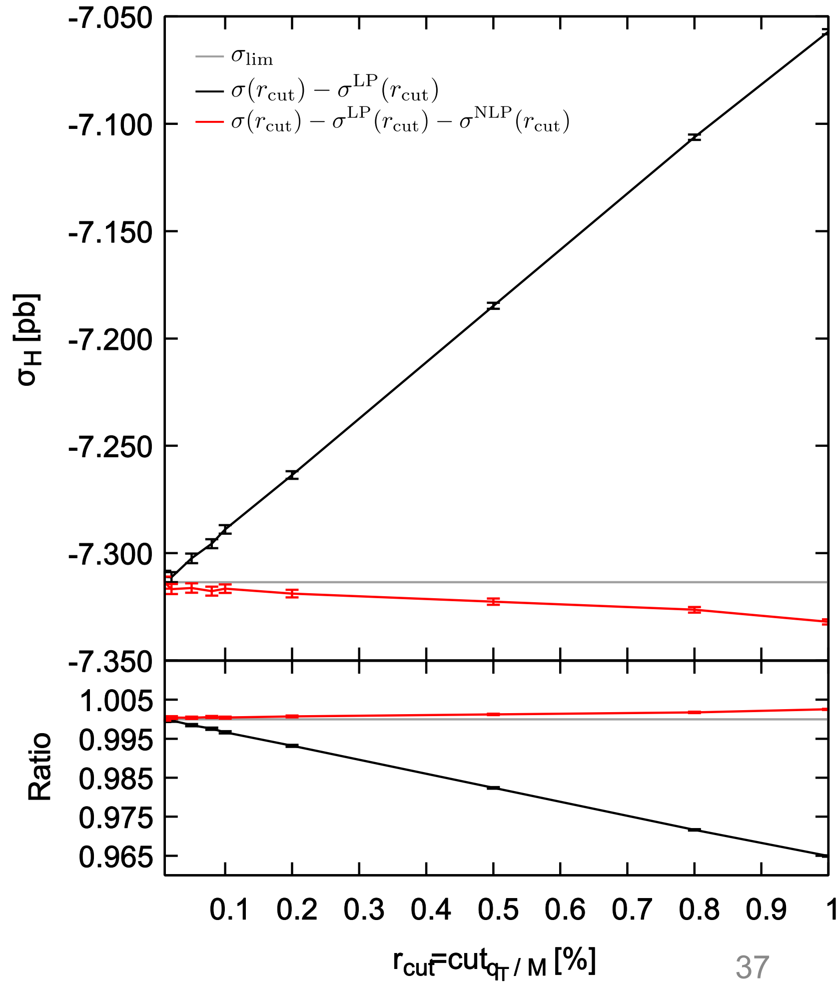
→ No extra contribution!

FSR - Hadronic cross section: numerical check

Residual r_{cut} in the convolution with pdf **might be an extra source** of the linear LP at the level of the hadronic cross section

We compute for different values of r_{cut}

- the real emission hadronic cross section
- the LP partonic contribution convoluted with the pdf (with $z_{max}=1$)
- the linear NLP partonic contribution convoluted with the pdf (with $z_{max}=1$)



The linear NLP **is entirely controlled by the result at the partonic level**

FSR - Soft Power Counting

$$\frac{\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^2\sqrt{(1-z)^2-4zr_{\text{cut}}^2}} = \frac{1}{4}\delta(1-z)\frac{1}{r_{\text{cut}}^2} + \frac{\pi}{8}\left[\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\text{cut}}} + \mathcal{O}(1)$$

$$\frac{\Theta(z_{\max}-z)\Theta(z-z_{\min})}{(1-z)^4\sqrt{(1-z)^2-4zr_{\text{cut}}^2}} = \frac{1}{24}\delta(1-z)\frac{1}{r_{\text{cut}}^2} + \frac{\pi}{64}\left[3\delta(1-z)+2\delta'(1-z)\right]\frac{1}{r_{\text{cut}}} + \mathcal{O}(1)$$

- At NLP, the δ'(1-z) distribution occurs implying **going beyond the strictly soft approximation**
- A **power counting argument** establishes the correspondence between the soft expansion and the power corrections in the r_{cut} regulator (in a full differential fashion):

  1. Eikonal Current J_{ij} and soft phase space of the radiation reproduce only LP

$$I_{ij}^{\text{soft}} \sim \int_{sr_{\text{cut}}^2}^{q_T^{\max}} dq_T^2 \int_0^\infty \frac{dk^+}{k^+}\int_0^{2\pi} d\phi J_{ij} = \int_{sr_{\text{cut}}^2}^{q_T} \frac{dq_T^2}{q_T^2}\int_0^\infty dy f(y,\Phi_B) \qquad y = \left(\frac{k^+}{q_T}\right)^2$$

  2. An extra energy power (next-to-soft) gives an extra qT factor (NLP)

$$E\gamma = \frac{1}{\sqrt{2}}\left(k^+ + \frac{q_T^2}{2k^+}\right) = q_T\frac{1+2y}{\sqrt{2y}}$$