Two-loop mixed EW-QCD amplitudes for Drell-Yan lepton pair production

Matthias Heller
in collaboration with: Andreas von Manteuffel, Robert Schabinger

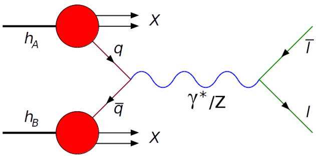
10.09.2019

The Drell-Yan process

Drell-Yan process at LHC

neutral and charged current:

$$pp \to l^+ l^- + X,\ l = e, \mu$$
$$pp \to l^\pm \nu_l + X,\ l = e, \mu$$



Drell-Yan process is important for:

- Determining masses of Z- and W-Boson
- Search for new gauge bosons
- Determining forward-backward asymmetry (measuring sinθ_W)
- Determining Parton distribution function
- Calibration of luminosity
  $L = N/\sigma$

Expanding the Amplitude

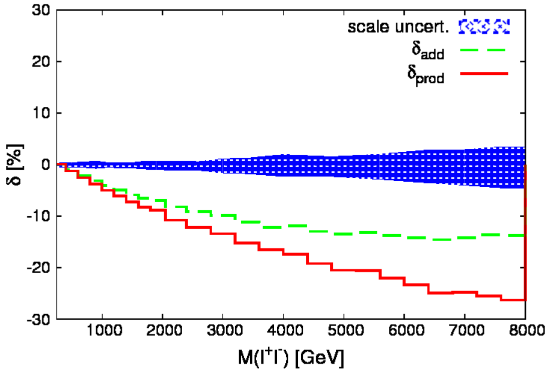
For cross section we need squared amplitude after summation over colors and spins:

$$\sum_{spin}\sum_{color}|\mathcal{M}|^2$$
$$= \left(\frac{\alpha}{\pi}\right)^2 \left|\mathcal{M}^{(1,0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{M}^{(2,0)} + \left(\frac{\alpha_s}{\pi}\right)\mathcal{M}^{(1,1)} + \left(\frac{\alpha}{\pi}\right)\left(\frac{\alpha_s}{\pi}\right)\mathcal{M}^{(2,1)}\right|^2$$
$$= \left(\frac{\alpha}{\pi}\right)^2 \Bigg(\left|\mathcal{M}^{(1,0)}\right|^2 + \left(\frac{\alpha}{\pi}\right)\cdot 2\cdot \mathrm{Re}\left[\mathcal{M}^{*(1,0)}\mathcal{M}^{(2,0)}\right] +$$
$$\left(\frac{\alpha_s}{\pi}\right)\cdot 2\cdot \mathrm{Re}\left[\mathcal{M}^{*(1,0)}\mathcal{M}^{(1,1)}\right] + \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\cdot$$
$$\cdot 2\cdot\left(\mathrm{Re}\left[\mathcal{M}^{*(1,1)}\mathcal{M}^{(2,0)}\right] + \mathrm{Re}\left[\mathcal{M}^{*(1,0)}\mathcal{M}^{(2,1)}\right]\right)\Bigg)$$

Pertubative corrections

- QCD corrections NNLO [Hamberg, van Neerven, Matsuura '91, Harlander Kilgore '02, Anastasiou, Dixon, Melnikov, Petriello '03, Melnikov, Petriello '06]
- QED on-shell NNLO [Berends, van Neerven, Burgers '88, Blümlein, De Freitas, Raab, Schönwald '19, talk by Kay Schönwald]
- EW NLO [Baur, Brein, Holllik, Schappacher, Wackeroth '01, Dittmaier, Krämer '01, Baur, Wackeroth '04]
- Mixed QED-QCD [Kilgore Sturm, '11, talk by Ignacio Fabre]
- Mixed EW-QCD on-shell [Dittmaier, Huss, Schwinn '14,'15, talk by Narayan Rana]

Pertubative corrections

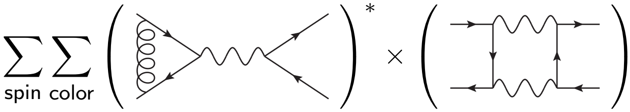


[Campbell, Wackeroth, Zhou '16]

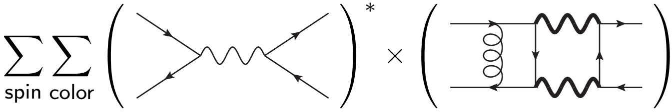
Next important step: mixed QCD-EW corrections for off-shell Z production

Corrections of order $\alpha_s \alpha$

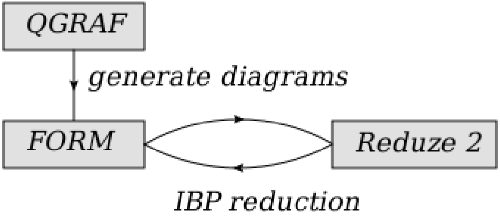
- Squared 1-Loop $\left(\text{Re}\left[\mathcal{M}^{*(0,1)}\mathcal{M}^{(1,0)}\right]\right)$



- 2-Loop with Tree $\left(\text{Re}\left[\mathcal{M}^{*(0,0)}\mathcal{M}^{(1,1)}\right]\right)$



Calculation setup



Software used:

- QGRAF [Nogueira '93]
- FORM 4.1 [Kuipers, Ueda, Vermaseren, Vollinga '12]
- Reduze 2 [Studerus, von Manteuffel '12]
- Mathematica

Outline

- Case of QED-QCD
- Extension to full EW sector
  - Calculation of master integrals
  - Treatment of γ_5
  - UV-Renormalization
  - IR structure

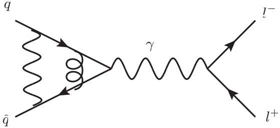
The case of QED-QCD

The case of QED-QCD

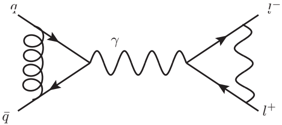
- consider first only virtual QEDxQCD contributions
- much easier, because
    - no masses
    - no γ_5
- results are known [Kilgore and Sturm, '11]
- full agreement with their result

Gauge invariant subsets

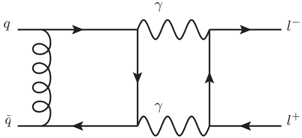
One can distinguish the contribution between 4 gauge invariant subsets:



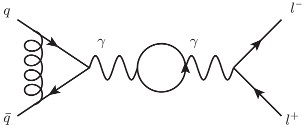
$(a) \propto Q_q^2$



$(b) \propto Q_l^2$



$(c) \propto Q_q Q_l$



$(d) \propto \sum Q_f^2$

IR structure

- IR poles would cancel among divergences arising from real correction
- not calculated yet
- use Soft and Jet functions [Kilgore and Sturm, '11] instead

$$\mathcal{J}^{(1,0)} = -\left(\frac{1}{2\epsilon^2} + \frac{3}{4\epsilon}\right)(Q_q^2 + Q_l^2) \qquad \mathcal{J}^{(0,1)} = -\left(\frac{1}{2\epsilon^2} + \frac{3}{4\epsilon}\right) C_F$$

$$\mathcal{J}^{(1,1)} = \left(\frac{1}{4\epsilon^4} + \frac{3}{4\epsilon^3} + \frac{9}{16\epsilon^2}\right) C_F(Q_q^2 + Q_l^2) - \frac{1}{2\epsilon}\left(\frac{3}{16} - \frac{3}{2}\zeta_2 + 3\zeta_3\right) C_F Q_q^2$$

$$\mathcal{S}^{(1,0)} = -\frac{1}{2\epsilon}\left[(Q_q^2 + Q_l^2)\ln\left(\frac{\mu^2}{-s}\right) + 2Q_q Q_l\left(\ln\left(\frac{\mu^2}{-t}\right) - \ln\left(\frac{\mu^2}{-u}\right)\right)\right]$$

$$\mathcal{S}^{(0,1)} = -\frac{1}{2\epsilon} C_F \ln\left(\frac{\mu^2}{-s}\right)$$

$$\mathcal{S}^{(1,1)} = \frac{1}{4\epsilon^2} C_F \ln\left(\frac{\mu^2}{-s}\right)\left[(Q_q^2 + Q_l^2)\ln\left(\frac{\mu^2}{-s}\right) + 2Q_q Q_l\left(\ln\left(\frac{\mu^2}{-t}\right) - \ln\left(\frac{\mu^2}{-u}\right)\right)\right]$$

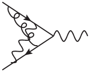
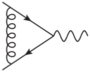
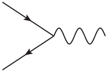
Expansion of the Hard function

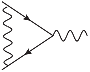
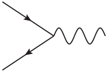
One can expand the hard scattering function in terms of Jet and Soft functions:

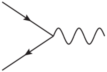
$$
\begin{aligned}
\mathcal{M} = \mathcal{H}^{(1,0)} &+ \left(\frac{\alpha}{\pi}\right)\left[\mathcal{J}^{(1,0)}\mathcal{H}^{(1,0)} + \mathcal{S}^{(1,0)}\mathcal{H}^{(1,0)} + \mathcal{H}^{(2,0)}\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)\left[\mathcal{J}^{(0,1)}\mathcal{H}^{(1,0)} + \mathcal{S}^{(0,1)}\mathcal{H}^{(1,0)} + \mathcal{H}^{(1,1)}\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\Bigg[\left(\mathcal{J}^{(1,1)} + \mathcal{J}^{(0,1)}\mathcal{J}^{(1,0)} + \mathcal{J}^{(1,0)}\mathcal{J}^{(0,1)} + \mathcal{S}^{(1,1)}\right)\mathcal{H}^{(1,0)} \\
&\qquad + \left(\mathcal{J}^{(1,0)} + \mathcal{S}^{(1,0)}\right)\mathcal{H}^{(1,1)} + \left(\mathcal{J}^{(0,1)} + \mathcal{S}^{(0,1)}\right)\mathcal{H}^{(2,0)} \\
&\qquad + \mathcal{H}^{(2,1)}\Bigg]
\end{aligned}
$$

IR structure diagrammatically

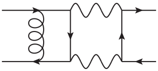
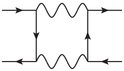
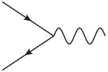
Vertex diagrams:

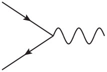
 $- \left(\mathcal{J}^{(1,0)}_{Q_q^2} + \mathcal{S}^{(1,0)}_{Q_q^2} \right) \Bigg($  $- \left(\mathcal{J}^{(0,1)}_{C_F} + \mathcal{S}^{(0,1)}_{C_F} \right)$  $\Bigg)$

$- \left(\mathcal{J}^{(0,1)}_{C_F} + \mathcal{S}^{(0,1)}_{C_F} \right) \Bigg($  $- \left(\mathcal{J}^{(1,0)}_{Q_q^2} + \mathcal{S}^{(1,0)}_{Q_q^2} \right)$  $\Bigg)$

$- \left(\mathcal{J}^{(1,1)}_{C_F Q_q^2} + \mathcal{S}^{(1,1)}_{C_F Q_q^2} \right)$  $=$ finite

Box diagrams:

 $- \left(\mathcal{J}^{(0,1)}_{C_F} + \mathcal{S}^{(0,1)}_{C_F} \right) \Bigg($  $- \mathcal{S}^{(1,0)}_{Q_l Q_q}$  $\Bigg)$

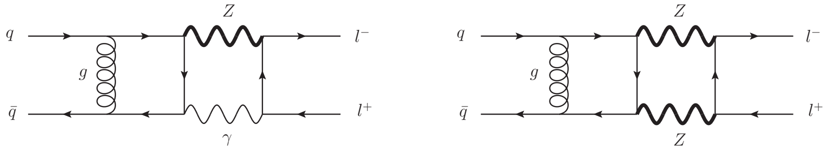
$- \mathcal{S}^{(1,1)}_{C_F Q_l Q_q}$  $=$ finite

Final Result

- finite result after IR subtraction: $2\text{Re}\left[\mathcal{H}^{*(2,0)}\mathcal{H}^{(1,1)} + \mathcal{H}^{*(1,0)}\mathcal{H}^{(2,1)}\right]$
- complete agreement with result of Kilgore and Sturm
- Functions in final result:
  - $\log\left(\frac{s}{\mu^2}\right)$ up to power 4
  - $\log\left(\frac{-u}{s}\right)$,$\log\left(\frac{-u}{s}\right)^2$,$\log\left(\frac{-u}{s}\right)^3$,$\text{Li}_2\left(\frac{-u}{s}\right)$,$\text{Li}_3\left(\frac{-u}{s}\right)$,$\text{Li}_4\left(\frac{-u}{s}\right)$
  - $\log\left(\frac{-t}{s}\right)$,$\log\left(\frac{-t}{s}\right)^2$,$\log\left(\frac{-t}{s}\right)^3$,$\text{Li}_2\left(\frac{-t}{s}\right)$,$\text{Li}_3\left(\frac{-t}{s}\right)$,$\text{Li}_4\left(\frac{-t}{s}\right)$

Extension to full EW sector

Extension to full EW sector



Now consider also contributions of EW sector

- now massive particles (Z and W boson)
- 3 new integral families
- γ_5 in Kreimer's scheme
- master integrals involve unrationalizable square root
- overlap of UV and IR divergences

Calculation of master integrals

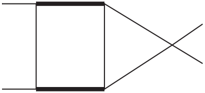
- we calculate all master integrals using method of differential equations
- canonical dlog-form [Kotikov '10, Henn '13, Remiddi, Tancredi '16, Adams, Weinzierl '18]:

$$\mathrm{d}\vec{m} = \epsilon\, \mathrm{dln}(l_a) A^{(a)} \vec{m}$$

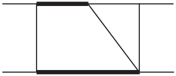
- for zero-mass [Smirnov '99] and one-mass integrals direct integration of DE in terms of generalized polylogarithms [Bonciani, Di Vita, Mastrolia, Schubert '16, von Manteuffel, Schabinger '17]
- two-mass integrals require algebraic letters with unrationalizable square [van Straten '14, Besier, Festi, Harrison, Naskrecki '19]

Construction of ansatz for algebraic letters

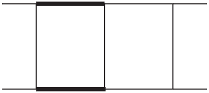
$\epsilon^4 s r_1$



$\epsilon^4 r_3$



$\epsilon^4 s r_2$



3 square roots:

$$r_1 = \sqrt{s(s-4m^2)},\ r_2 = \sqrt{-st(4m^2(t+m^2)-st)},\ r_3 = \sqrt{s(t^2(s-4m^2)+sm^2(m^2-2t))}$$

Only 2 can be rationalized [Bonciani, Di Vita, Mastrolia, Schubert '16]:

$$s = -m^2\frac{(1-w)^2}{w},\ t = -m^2\frac{w(1+z)^2}{z(1+w)^2}$$

$$r_1 = \frac{-m^2(1-w)(1+w)}{w},\ r_2 = \frac{-m^4(1-w)(1-z)(1+z)}{z(1+w)},\ r_3 = \frac{m^4(1-w)}{wz(1+w)}\sqrt{}$$

$$\sqrt{} = \sqrt{(1+w^2z^2)(w+z)^2+2wz(w-z)^2+4wz^2(1+w^2)}$$

Construction of ansatz for algebraic letters

- Method of [Duhr, Gangl, Rhodes '11]
- construct functions, that do not introduce new symbols
- e.g. $S(Li_2(a)) = -(1-a) \otimes a$ requires that both a and $1-a$ factor over alphabet
- issues with algebraic letters
  - no unique factorization, e.g. $\sqrt{x}$, $\sqrt{y}$, $(x-y) = (\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$
  - non-integer powers of letters, e.g. $\sqrt{l}$, $l^{1/4}, \ldots$
  - test factorization of letters numerically, e.g. $g = 1-a = c l_1^{a_1} l_2^{a_2} \ldots$
    $\to \ln(g) - \ln(c) - a_1 \ln(l_1) - a_2 \ln(l_2) - \ldots = 0$
  - very complicated relations
  - no obvious way to choose letters

Simplification of algebraic letters

- aim: simplify complicated letters (reduce degree)
- observation: if $l = x + y\sqrt{\ }$ is a letter, then $\bar{l} = x - y\sqrt{\ }$ factorizes over the alphabet
- $\sqrt{\ }$ itself is a letter
- consequence: $l\bar{l}$ factorizes over the rational part of the alphabet
- construction of better letters: make ansatz $l = x + y\sqrt{\ }$ and require that $l\bar{l}$ factorizes over rational part. Pick letters with lowest degree

Improved letters in our case

- rational part

$$\mathcal{L}_r = \{1-w, -w, 1+w, 1-w+w^2, 1-z, -z, 1+z \\ 1-wz, 1+w^2z, -z-w^2, z-w\}$$

- algebraic part

$$\mathcal{L}_a = \{r, -(1-w)(z-w)(1-wz)+r(1+w), -(1-w)\left(4wz+(w+z)(1+wz)\right) \\ -r(1+w), r^2-2wz^2(1-w)^2+r(w+z)(1+wz), \\ r^2(1-z)^2+2z^2(z+w^2)(1+w^2z)+r(1-z)(1+z)\left(2wz-(w+z)(1+wz)\right)\}$$

- simplified algebraic part

$$\tilde{\mathcal{L}}_a = \{r, \frac{1}{2}(2+z-w+wz(w+z)+r), \frac{1}{2}(2w^2+z-w+wz(w+z)+r), \\ \frac{1}{2}(-(w+z)(1-wz)+r), \frac{1}{2}(-(z-w)(1+wz)+r)\}$$

Success with new letters

- with new letters we were able to integrate DE in physical region of phase space
- observation: no roots of letters needed, much lower degree for arguments
- for weight 3: Li_3 and Li_{21}, for weight 4: Li_4, Li_{22} and Li_{31}
- simple integration constants:
  - weight 3: $\frac{3}{2}\zeta_3 - \frac{3}{2}\pi^3 i$
  - weight 4: $\frac{1335349}{32}\zeta_2^2 - 116\zeta_3\pi i$

The Problem with γ_5

- in dimensional regularization dimension d is a complex number
- anti commuting γ_5 and cyclic trace only well defined in $d = 4$

Using an anti commuting γ_5 and a cyclic trace, it can be shown that:

$$(d-4) \cdot \mathrm{tr}(\gamma_5 \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4}) = 0.$$

Solution of the problem - γ_5 schemes

Give up either anti commutativity of γ_5 or cyclicity of $\mathrm{tr}(\cdot)$:

- BM-scheme ['t Hooft, Veltmann, Breitenlohner, Maison '77]
  - $\{\gamma_5, \gamma_\mu\} \propto \mathcal{O}(\epsilon)$
  - spurious anomalies, which must be cancelled by finite counter terms
- Kreimer's scheme [Korner, Kreimer, Schilcher '92]
  - give up cyclicity of $\mathrm{tr}(\cdot)$
  - define new functional, called $\mathrm{Tr}(\cdot)$, that is only for $d = 4$ the ordinary trace

In this calculation, we used Kreimer's scheme.

Passarino-Veltman tensor reduction

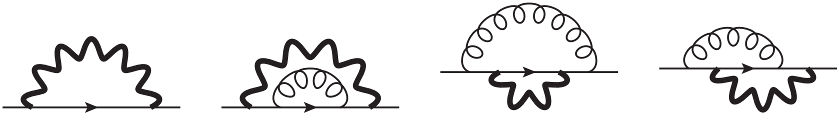
- because of γ_5, there arise 4 dimensional epsilon tensors in Fermion traces
- these must not be contracted with loop momentums, because integrals are calculated in d dimensions
- use Passarino-Veltman tensor reduction

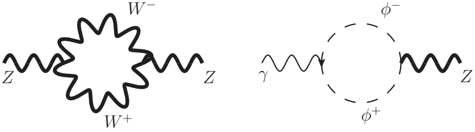
$$I^{\mu_1 \dots \mu_n} = \int d^d k_1 \dots \mathrm{d}^d k_m \frac{k_{i_1}^{\mu_1} \dots k_{i_n}^{\mu_n}}{D_1 \dots D_N} = \sum_j I_j \, p_{j_1}^{\mu_1} \dots g^{\mu_i \mu_{i+1}} \dots p_{j_n}^{\mu_n}$$

The right hand side can then be contracted with 4 dimensional epsilon tensors.
For R_ξ- gauge, up to rank 10 needed (finite field methods)

UV-Renormalization

- non-trivial wave function renormalization (use on-shell scheme)
- all other counter terms from Z, W and γ two-point functions
  $\rightarrow$ pure one-loop contribution
- Renormalization conditions can be chosen (on-shell, $\overline{MS}$, ...)

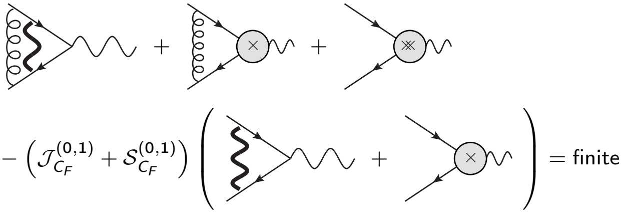




IR structure

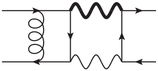
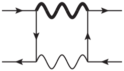
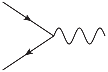
- infrared structure of unbroken Standard Model derived in [Kilgore '13]
- here: Standard Model after spontaneous symmetry breaking
- same Soft and Jet functions as in pure QED!

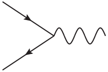
Z Vertex:



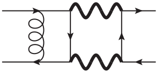
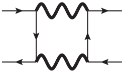
IR structure

ZA Box:

 $-\left(\mathcal{J}^{(0,1)}_{C_F}+\mathcal{S}^{(0,1)}_{C_F}\right)\Bigg($  $-\,\mathcal{S}^{(1,0)}_{Q_l Q_q}$  $\Bigg)$

$-\,\mathcal{S}^{(1,1)}_{C_F Q_l Q_q}$  $=$ finite

ZZ Box:

 $-\left(\mathcal{J}^{(0,1)}_{C_F}+\mathcal{S}^{(0,1)}_{C_F}\right)$  $=$ finite

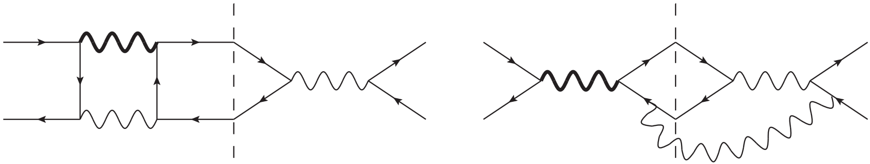
Summary and Outlook

- no new infrared structures in EW calculation compared to QED
- new method of solving DE with algebraic letters (try for other cases)
- C++ code for numerical evaluation of amplitude

Backup Slides

The Reading Point

- $\mathrm{Tr}(\cdot)$ is not cyclic $\rightarrow$ define reading prescription where to start reading the trace
- to restore standard Adler-Bell-Jackiw anomaly start at axial vertex (or symmetrize if there are more)
- Standard Model is free of anomalies $\rightarrow$ one can choose arbitrary starting point
- but: keep the same starting point for all diagrams!!



UV-Renormalization

Kilgore und Sturm gave renormalized result in $\overline{\text{MS}}$-scheme For α_s, renormalization is trivial:

$$\frac{\alpha_s^B}{\pi} = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon} \frac{\alpha_s}{\pi}$$

For QED, following Kilgore and Sturm, we have

$$\frac{\alpha^B}{\pi} = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon} \frac{\alpha}{\pi} \left(1 + \frac{\alpha}{\pi}\frac{1}{3\epsilon}\left(N_l Q_l^2 + N_c N_q Q_q^2\right) + \frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\frac{1}{8\epsilon} C_F N_c N_q Q_q^2\right)$$

Only fermion-loop diagrams contribute to β function