PAUL SCHERRER INSTITUT PSI

Second order QCD corrections to the g + g → H + H four-point amplitude

Pulak Banerjee

Paul Scherrer Institut

based on the work : Two-loop massless QCD corrections to the g + g → H + H four-point amplitude

with Sophia Borowka, Prasanna K Dhani, Thomas Gehrmann and V. Ravindran

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Motivation and goal

- Era of precision measurement: W mass, Trilinear and quartic gauge boson coupling…

- Prospects at luminosity upgraded hadron colliders

    1. Precise measurements of couplings,
    2. Study rare decays which can probe new physics,
    3. Study processes with two massive final states

HH production $\xrightarrow{3000\,\mathrm{fb}^{-1}}$ **~120 K of HH** $\longrightarrow$ trilinear coupling

- Theoretical challenge : precisely compute all such contributing processes.

- In HH production, we compute one such important process, that contributes at NNLO in perturbative QCD.

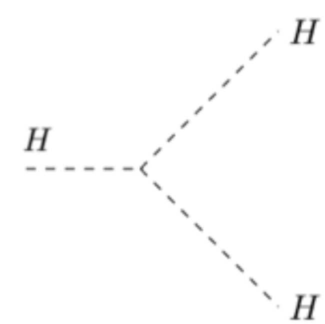
Plan

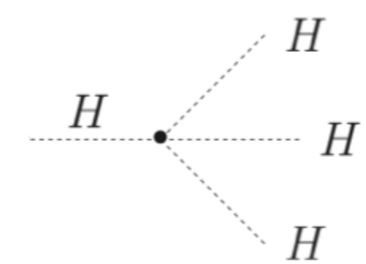
- Introduction
- HH @ LHC
- HH status
- Computation of Class B diagrams @NNLO
- UV renormalization and operator mixing
- IR factorization
- Numerical evaluation
- Conclusion

Introduction

- First two runs of the LHC : Large amounts of high precision data…
- The Higgs mass is precisely known; coupling strengths, spin are already determined.
- The strength of self coupling is not known till now: crucial to understand EWSB
- Higgs potential: $$V(H) = \frac{1}{2}M_H^2 H^2 + \lambda\, v H^3 + \frac{1}{4}\lambda' H^4$$

  with $\lambda = \lambda' = M_H^2/(2v^2)$

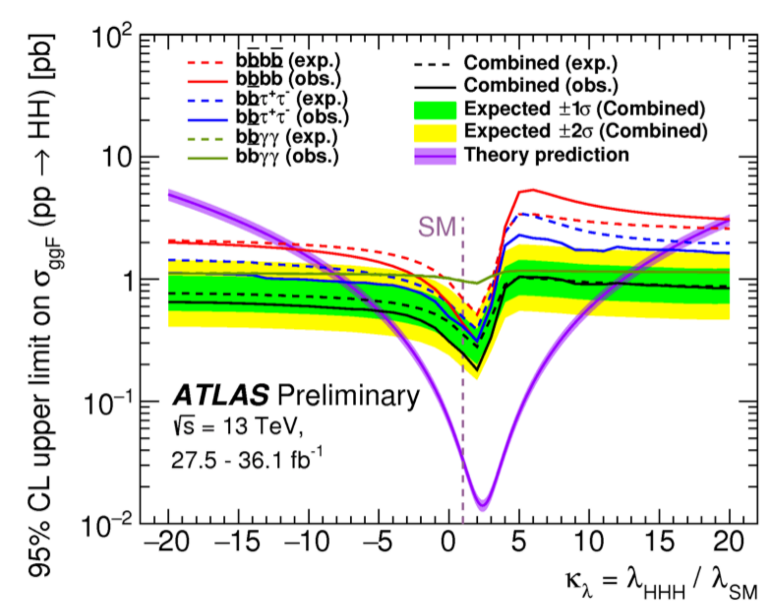




- For many BSM scenarios, the trilinear and quartic couplings deviate from the SM predictions. It is thus important to precisely measure these couplings.

κ_λ between -5.0 and 12.0

at a 95% confidence level.



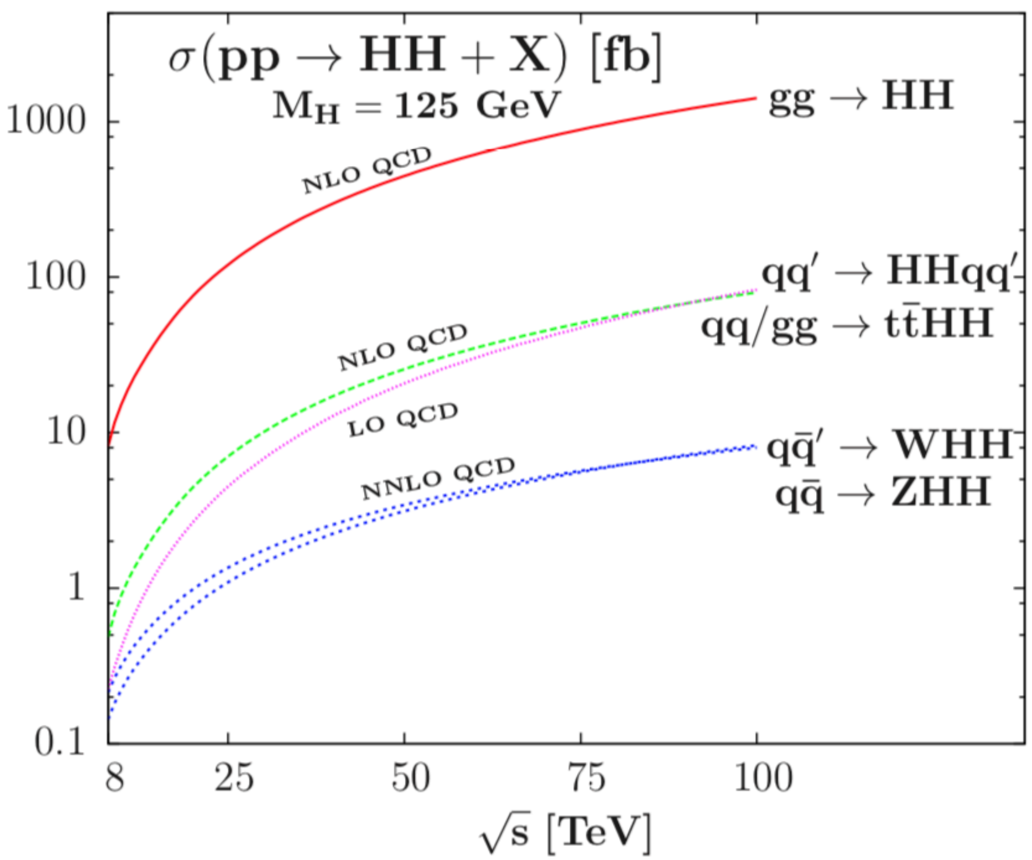
[Atlas]

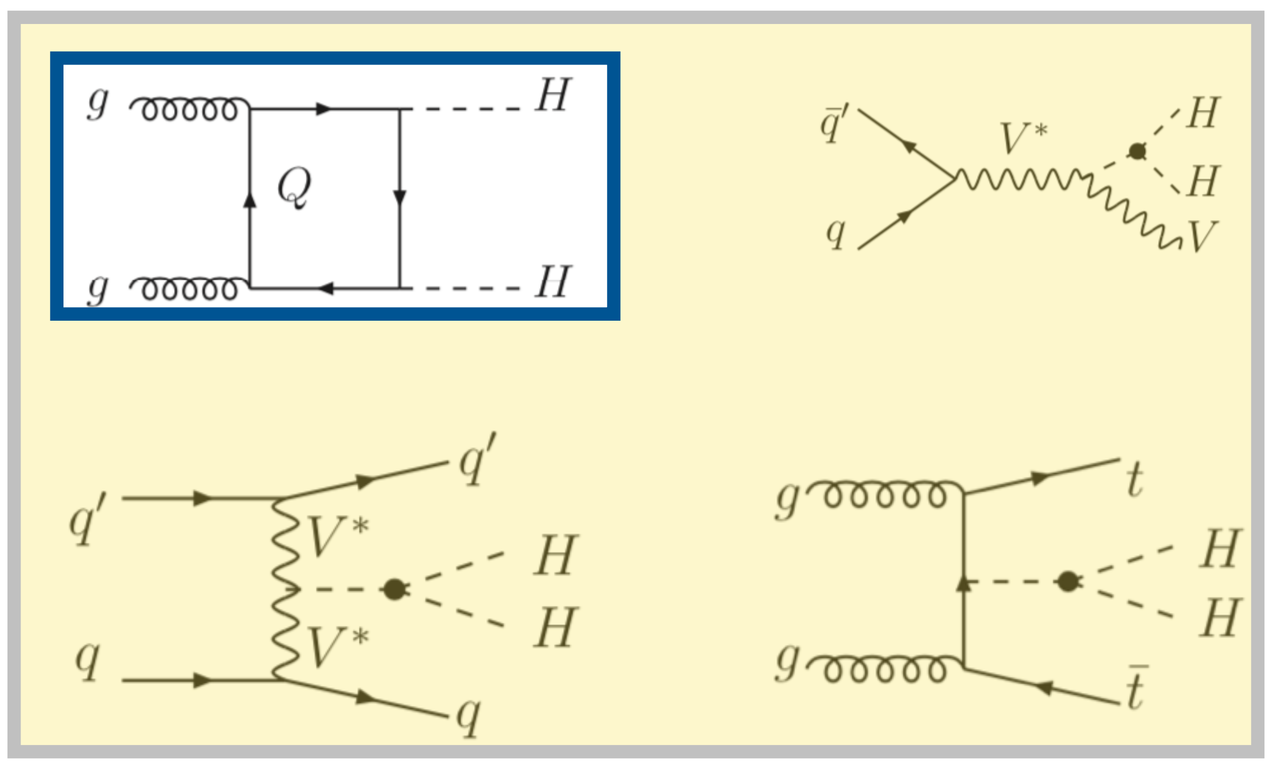
Introduction

- At the LHC measuring the quartic coupling via triple Higgs production is impossible, Xsecn ~ $\mathcal{O}(\mathrm{ab})$.

- Although very challenging, the trilinear coupling can be measured in hadron colliders with high luminosities, through Higgs pair production Xsecn~ $\mathcal{O}(\mathrm{fb})$

[Boudjema, Chopin] [Djouadi, Killian, Mühlleitner, Zerwas] [Barger et. al.] [Osland, Pandita] [Asakawa, Harada, Kanemura]





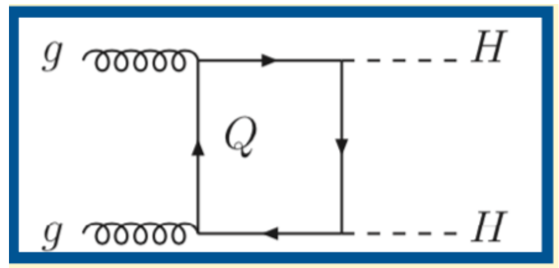
- Channels of production

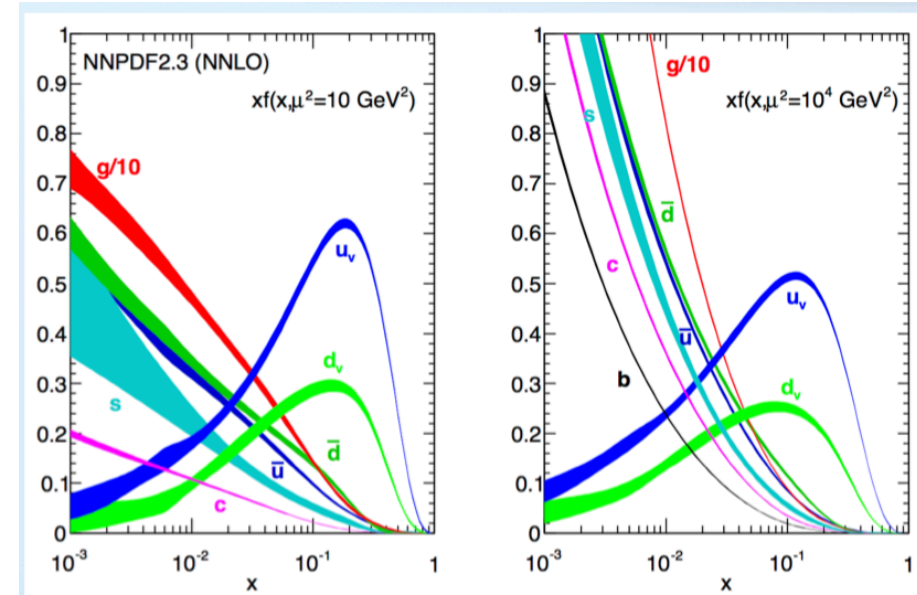
- For all processes Xsecn ~1000 times smaller than single Higgs [Baglio et. al.]

HH @ LHC

- **Gluon flux is high at the energies of LHC…**

dominant channel





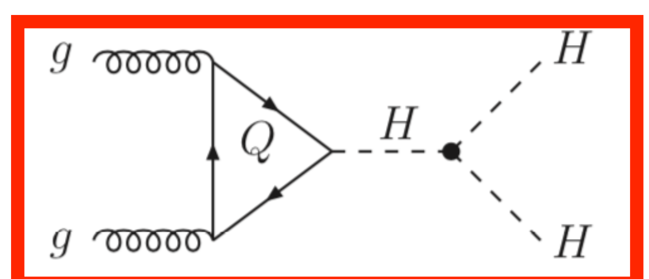
Q are mainly t quark in SM, b < 1% @ LO

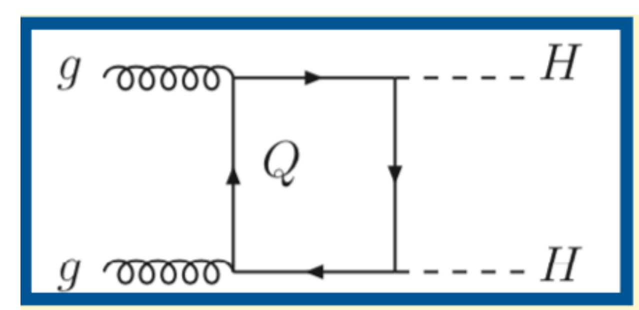
- **Although in SM the sub leading channels have much less Xsection compared to gluon fusion, nevertheless in MSSM, HH production through bB is enhanced.**

see Ravindran's talk

- **At HL-LHC, HH production should be observed.**

- **Sensitivity to** λ_{HHH}



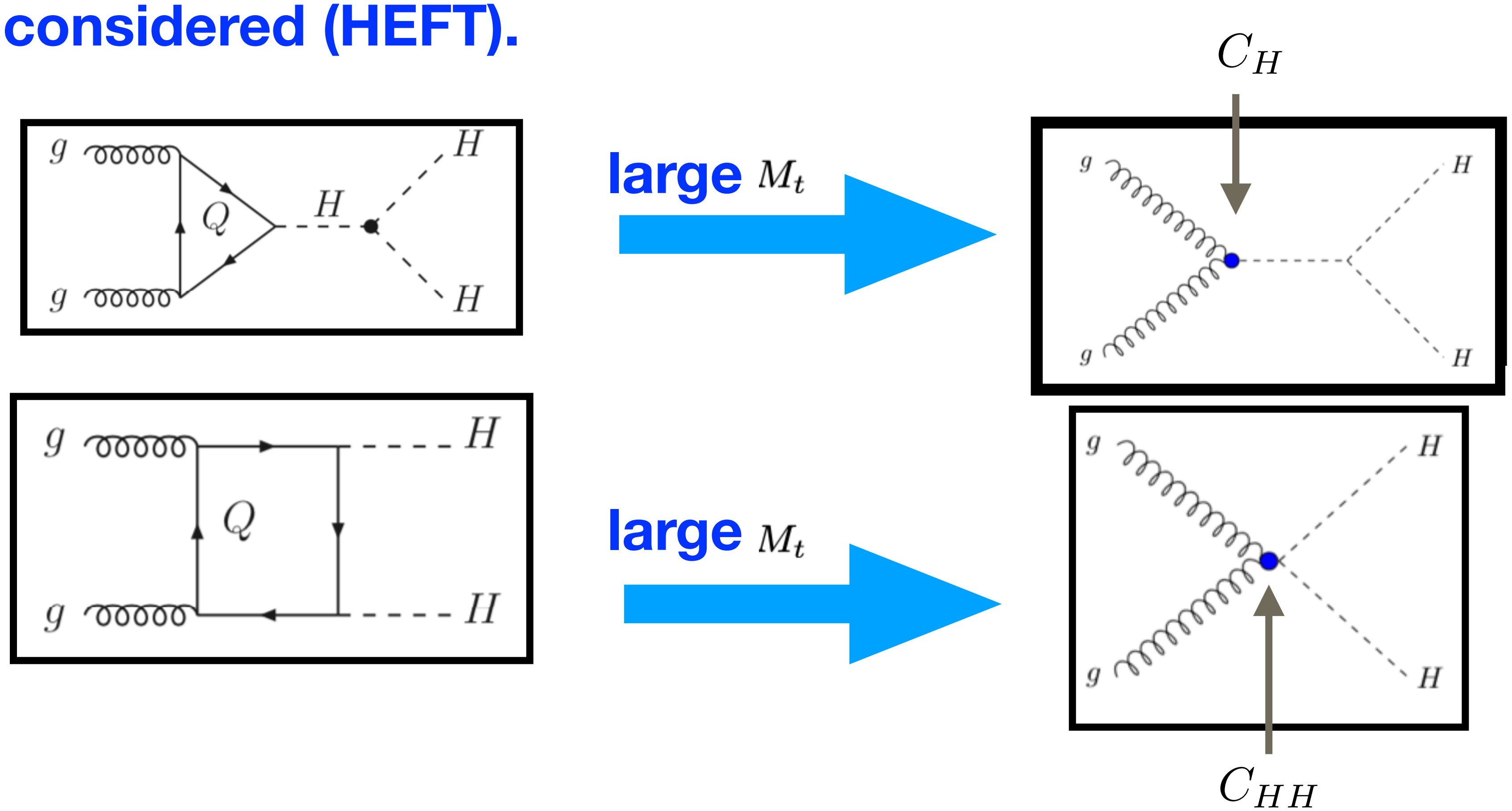


This one is sensitive

- **To determine** λ_{HHH} **contributions from different channels are needed.**

HH @ LHC

- **Computing QCD corrections for these channels with full top quark mass at higher orders (HO) in perturbation theory is really difficult !**
- **To compute HO corrections, effective theory approach, where large M_t limit is considered (HEFT).**



$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\left(C_H(a_s)\frac{\phi}{v} - C_{HH}(a_s)\frac{\phi^2}{v^2}\right) G_{\mu\nu}G^{\mu\nu}$$

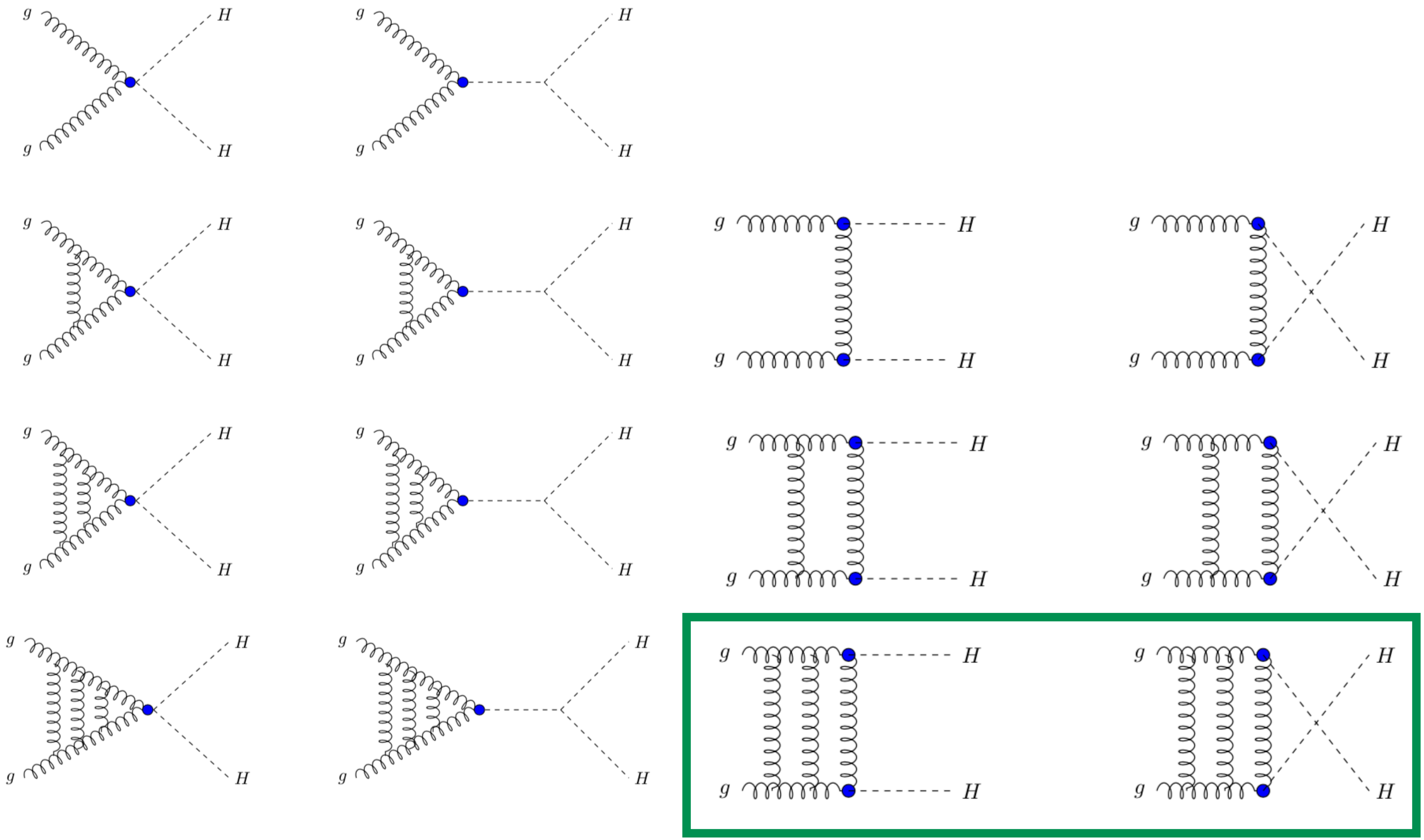
- **For production of two CP odd pseudoscalar Higgs via gluon channel…**

  see Matthew's talk

- **We work in effective theory and compute QCD corrections for HH production for certain class of diagrams, relevant for N3LO Xsecn.**

WHAT WE COMPUTE...

- **Diagrams : Expand the Lagrangian in terms of a_s**



- **LO**
- **NLO**
- **NNLO**
- **NNNLO**

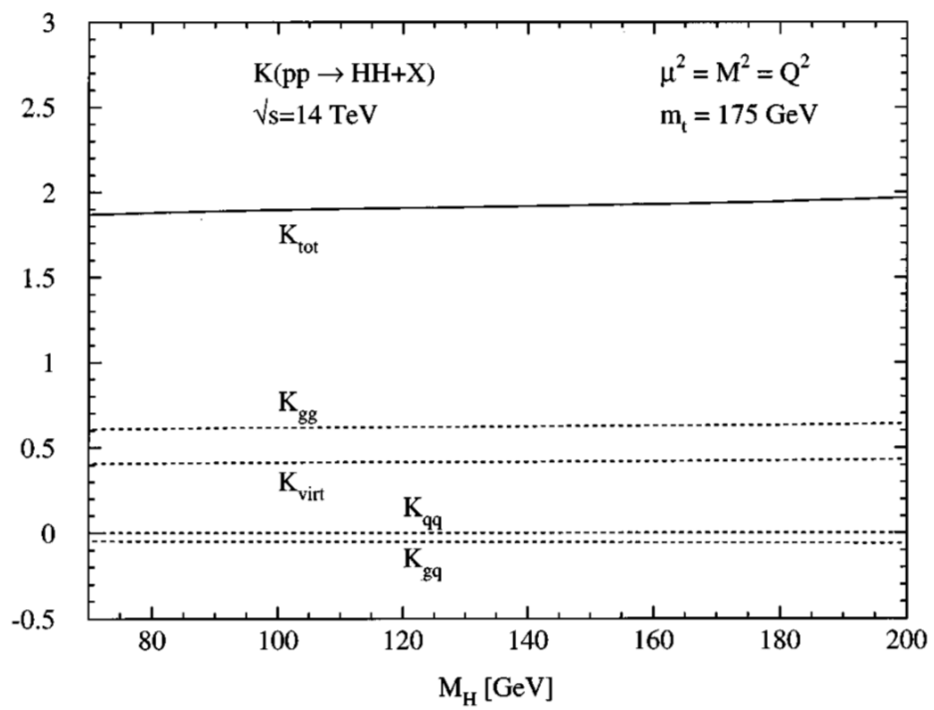
- **Class A**
- **Class B**

OUR COMPUTATION !

HH status

- **LO: Computed with full top quark mass dependence** [Glover, van der Bij]

$\mathrm{NLO}_{\mathrm{HEFT}}$ **: In HEFT approach, scale variation is ~ 20% at NLO ; K ~ 2** [Dawson, Dittmaier, Spira]



- **Top mass effects in real radiation ~ -10%**

  [Frederix et al.] [Maltoni et. al.]

- $1/m_t$ **expansion effect on Xsecn ~** $\pm 10\%$

  [Grigo, Hoff, Steinhauser, Melnikov]

- **Expansion around** p_T and m_H [Bonciani, Degrassi, Giardino, Gröber]

- **NLO: Full top quark mass dependence**

  A. **Xsection is ~14% less than born improved HEFT(** $\sqrt{s} = 13\,\mathrm{TeV}$ **)**

  B. **NLO top mass effects ~ -15%**

  C. **Top mass scheme uncertainties** $\lesssim$ **30%**

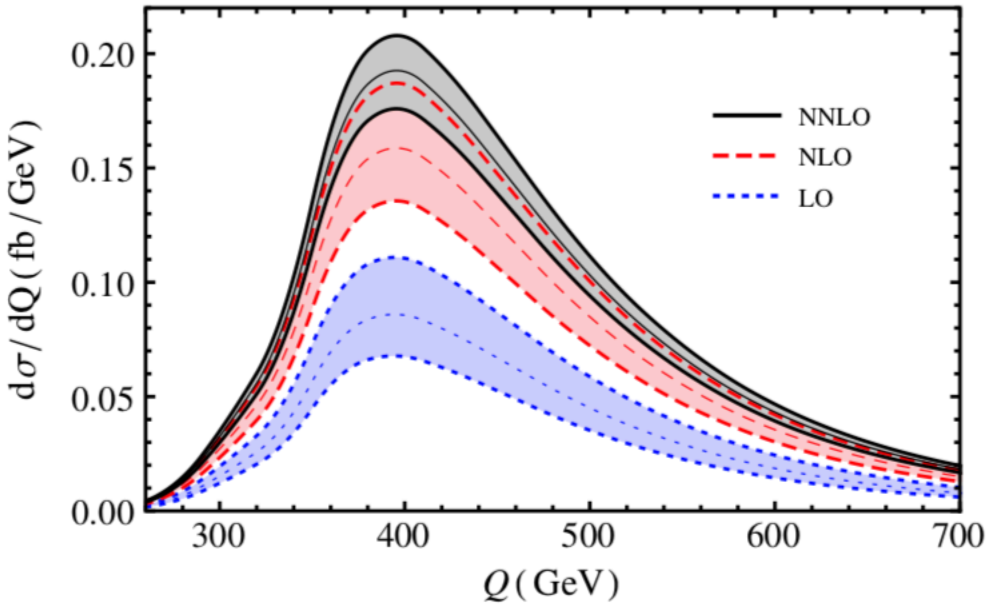
[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher]

HH status

- At NLO, $K \lesssim 2$: was seen also for single Higgs production.
- Beyond NLO ?? Exact result is difficult to compute. In HEFT :
- Computation of NNLO-SV contribution for HH pair:

$$K^{\mathrm{SV}}_{\mathrm{NNLO}} = 2.37$$
$$K_{\mathrm{NLO}} = 1.92$$

[de Florian, Mazzitelli]



Xsectn is ~20% more compared to NLO

Better convergence compared to NLO (~8%)

[de Florian, Mazzitelli]

- At NNLO (SV) the top mass effects lead to an uncertainty of $\pm 5\%$.

[Grigo, Hoff, Steinhauser]

- Soft gluon resummation at NNLL [Shao, Li, Li, Wang] [de Florian, Mazzitelli]
- Differential distribution [Q.Li, Q.-S. Yan, X. Zhao] [Maierhofer, Papaefstathiou]

HH status

- Re-weighting approach: the NNLO results can be combined with exact NLO top quark mass dependent result. [Grazzini et. al.]

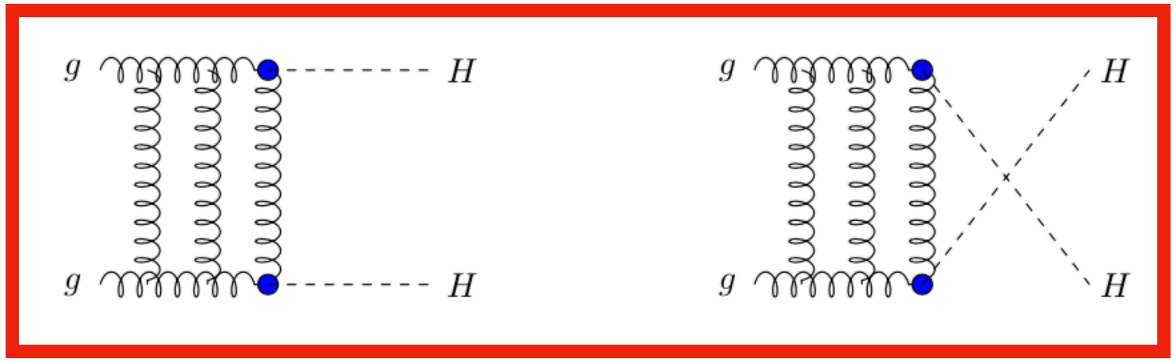
- Beyond NNLO? Although the class A diagrams are known to 3 loop, class B are known only up to 1 loop.

- To precisely extract λ_{HHH}, the two loop corrections of class B is necessary.

- The class A diagrams @N3LO are already known. [Baikov et. al.] [Gehrmann et. al.]

- Matching coefficients. [Grigo, Melnikov, Steinhauser] [Gerlach, Herren, Steinhauser] [Spira]

- In this work, we calculate the last missing piece required for a full N3LO SV calculation of HH production in the HEFT.



PB, S Borowka, PK Dhani, T Gehrmann, V Ravindran

Computation of Class B diagrams @NNLO

- **Diagram generation by QGRAF: 2 @LO, 37@NLO, 865@NNLO**

[Nogueira]

- **Kinematics:** $s = m_{\mathrm{h}}^2 \frac{(1+x)^2}{x}, \quad t = -m_{\mathrm{h}}^2 y, \quad u = -m_{\mathrm{h}}^2 z.$ $\qquad s + t + u = 2m_{\mathrm{h}}^2$

- **Amplitude and projectors:** $\mathcal{M}_{ab}^{\mu\nu} = \delta_{ab}\left(\mathcal{T}_1^{\mu\nu}\, \mathcal{M}_1 + \mathcal{T}_2^{\mu\nu}\, \mathcal{M}_2\right)$ **a,b are colour indices**

$$\mathcal{T}_1^{\mu\nu} = g^{\mu\nu} - \frac{1}{p_1 \cdot p_2}\left(p_1^\nu p_2^\mu\right)$$

$$\mathcal{T}_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_1 \cdot p_2\, p_T^2}\left(m_{\mathrm{h}}^2\, p_2^\mu p_1^\nu - 2p_1 \cdot p_3\, p_2^\mu p_3^\nu - 2p_2 \cdot p_3\, p_3^\mu p_1^\nu + 2p_1 \cdot p_2\, p_3^\mu p_3^\nu\right)$$

$$p_T^2 = (tu - m_{\mathrm{h}}^4)/s.$$

[Glover, van der Bij]

- **Hence,**

$$\mathcal{M}_i = \frac{1}{N^2-1}\, P_i^{\mu\nu} \mathcal{M}_{\mu\nu}^{ab} \delta_{ab}.$$

and

$$P_1^{\mu\nu} = \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_1^{\mu\nu} - \frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_2^{\mu\nu},$$

$$P_2^{\mu\nu} = -\frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_1^{\mu\nu} + \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_2^{\mu\nu}.$$

with $d = 4 - 2\epsilon$

Transverse nature => No external ghosts required

Now $\mathcal{M}_i = \mathcal{M}_i^A + \mathcal{M}_i^B, \qquad i = 1, 2$

Computation of Class B diagrams @NNLO

- Complexity in computing the coefficients $\mathcal{M}_i$ becomes involved due to the tensorial nature of the amplitudes.

- These coefficients were calculated using in-house routines in FORM.

[Vermaseren]

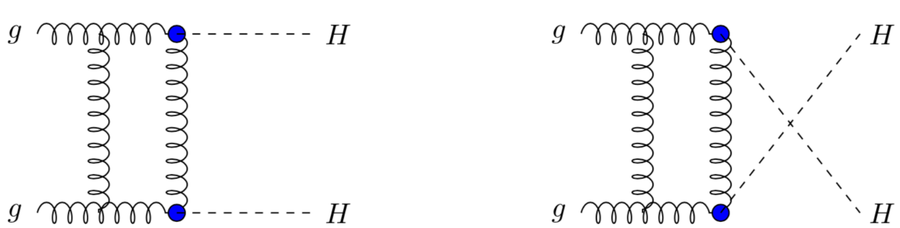
- At each stage, simplification was done to ensure the expressions remain compact.

$$\mathcal{M}_i = \mathcal{M}_i^A + \mathcal{M}_i^B, \qquad i = 1, 2$$

known

known up to 3 loops

NEW @ 2 loop !



[de Florian, Mazzitelli]

- Reduze 2 : Shift propagators to transform diagrams to different basis.

[von Manteuffel, Studerus]

Computation of Class B diagrams @NNLO

- Reduction of huge number of scalar Feynman integrals to Master Integrals; done independently in LiteRed and REDUZE 2.

  [Lee] [von Manteuffel, Studerus]

- 149 Master integrals.

- Integrals calculated for the process $q\bar{q} \to VV$

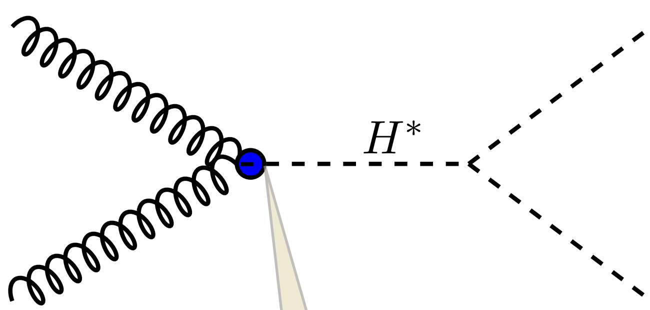
  [Gehrmann, von Manteuffel, Tancredi, Weihs] [Gehrmann, Tancredi, Weihs]

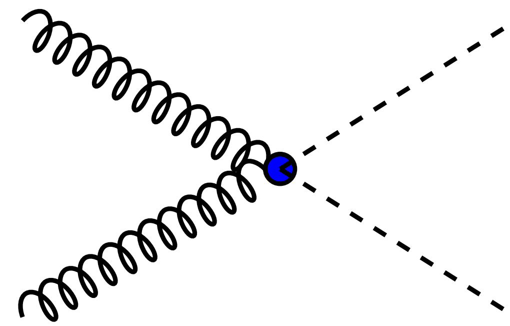
- Using the integrals, we compute the UV and IR divergent amplitudes.

UV renormalization and operator mixing: Known

- **Coupling constant renomalization:** $$\hat{a}_s \mu^{2\epsilon} S_\epsilon = a_s \mu_R^{2\epsilon} \left[1 - a_s \left(\frac{\beta_0}{\epsilon}\right) + a_s^2 \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right) + \mathcal{O}(a_s^3)\right]$$

- **Effective vertices**



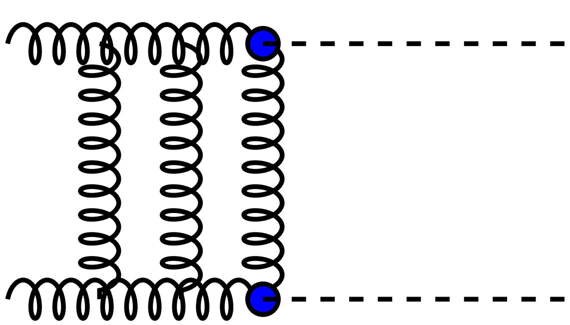


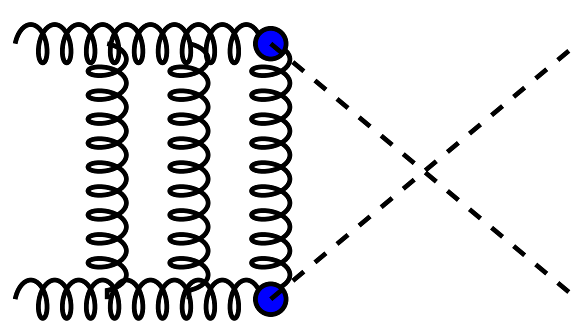
Effective vertex

Multiply overall renormalisation constant $Z_{\mathcal{O}}$

[Nielsen] [Spiridonov, Chetyrkin]

[Kataev, Krasnikov, Pivovarov]



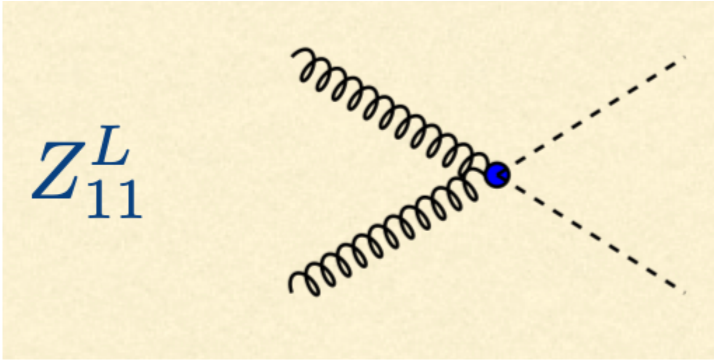


Multiply $Z_{\mathcal{O}}^2$

Subtleties with operator renormalisation

- However at two loop, even after following the above procedure, there are $1/\epsilon$ divergences proportional to β_1

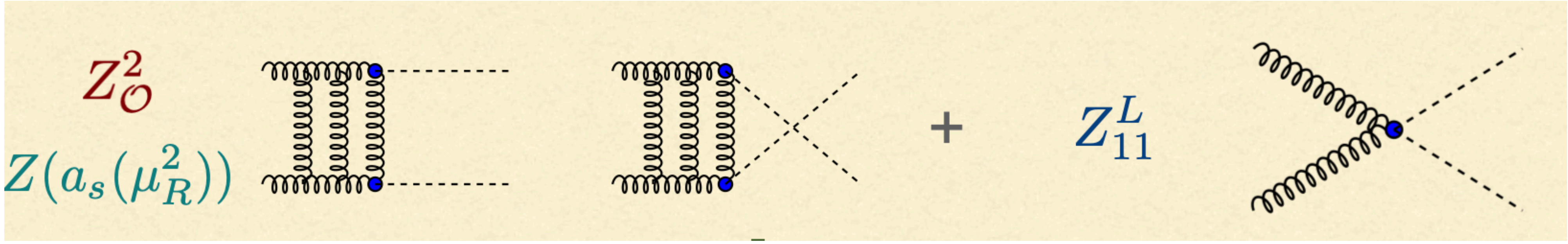
- Due to presence of two $G_{\mu\nu}G^{\mu\nu}\phi$ operators in class B, at 2 loop we need a new counterterm :



$$\boxed{Z_{11}^L = a_s^2 \frac{2\beta_1}{\epsilon} + \mathcal{O}(a_s^3)}$$

Zoller

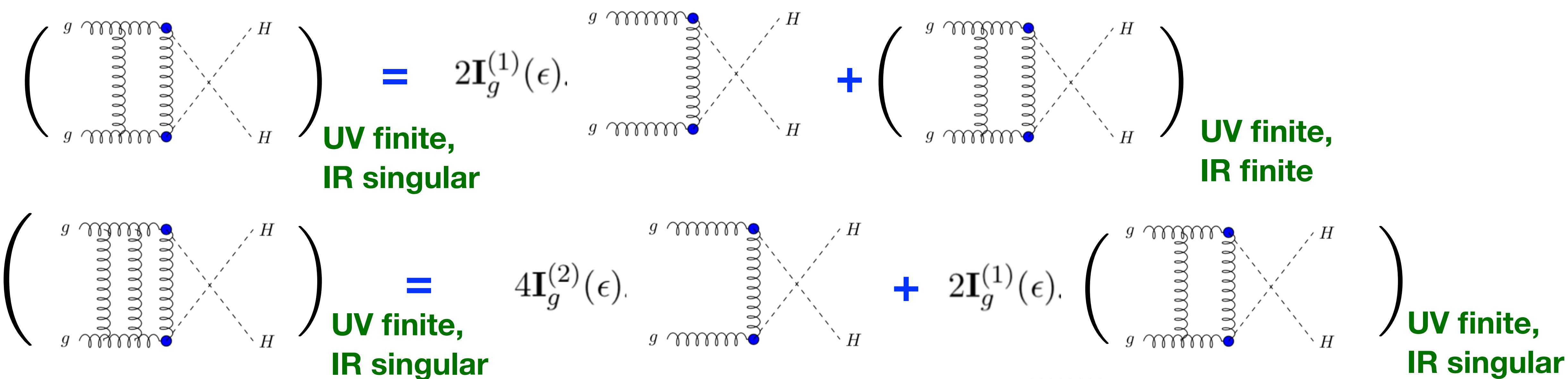
Thus

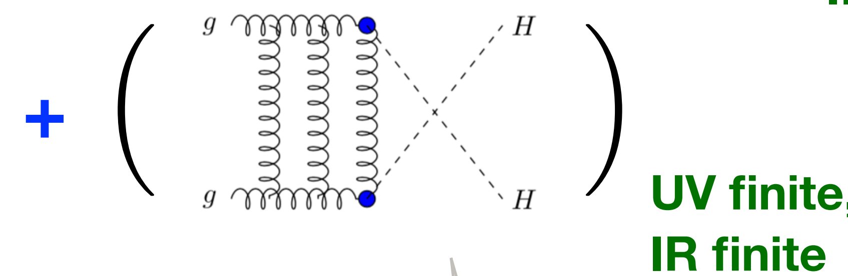


UV finite

IR factorization

- **UV finite and IR divergent projected amplitudes:** [Catani]





$$\mathbf{I}_g^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)}\left(\frac{C_A}{\epsilon^2}+\frac{\beta_0}{2\epsilon}\right)\left(-\frac{\mu_R^2}{s}\right)^{\epsilon},$$

$$\mathbf{I}_g^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}_g^{(1)}(\epsilon)\left[\mathbf{I}_g^{(1)}(\epsilon)+\frac{\beta_0}{\epsilon}\right] + \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left[\frac{\beta_0}{2\epsilon}+K\right]\mathbf{I}_g^{(1)}(2\epsilon) + 2\mathbf{H}_g^{(2)}(\epsilon).$$

$$K = \left(\frac{67}{18}-\zeta_2\right)C_A - \frac{10}{9}T_F n_f,$$

$$\mathbf{H}_g^{(2)}(\epsilon) = -\left(-\frac{\mu_R^2}{s}\right)^{2\epsilon}\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)} \times \frac{1}{2\epsilon}\left\{C_A^2\left(-\frac{5}{24}-\frac{11}{48}\zeta_2-\frac{1}{4}\zeta_3\right)+C_A n_f\left(\frac{29}{54}+\frac{1}{24}\zeta_2\right)-\frac{1}{4}C_F n_f-\frac{5}{54}n_f^2\right\}$$

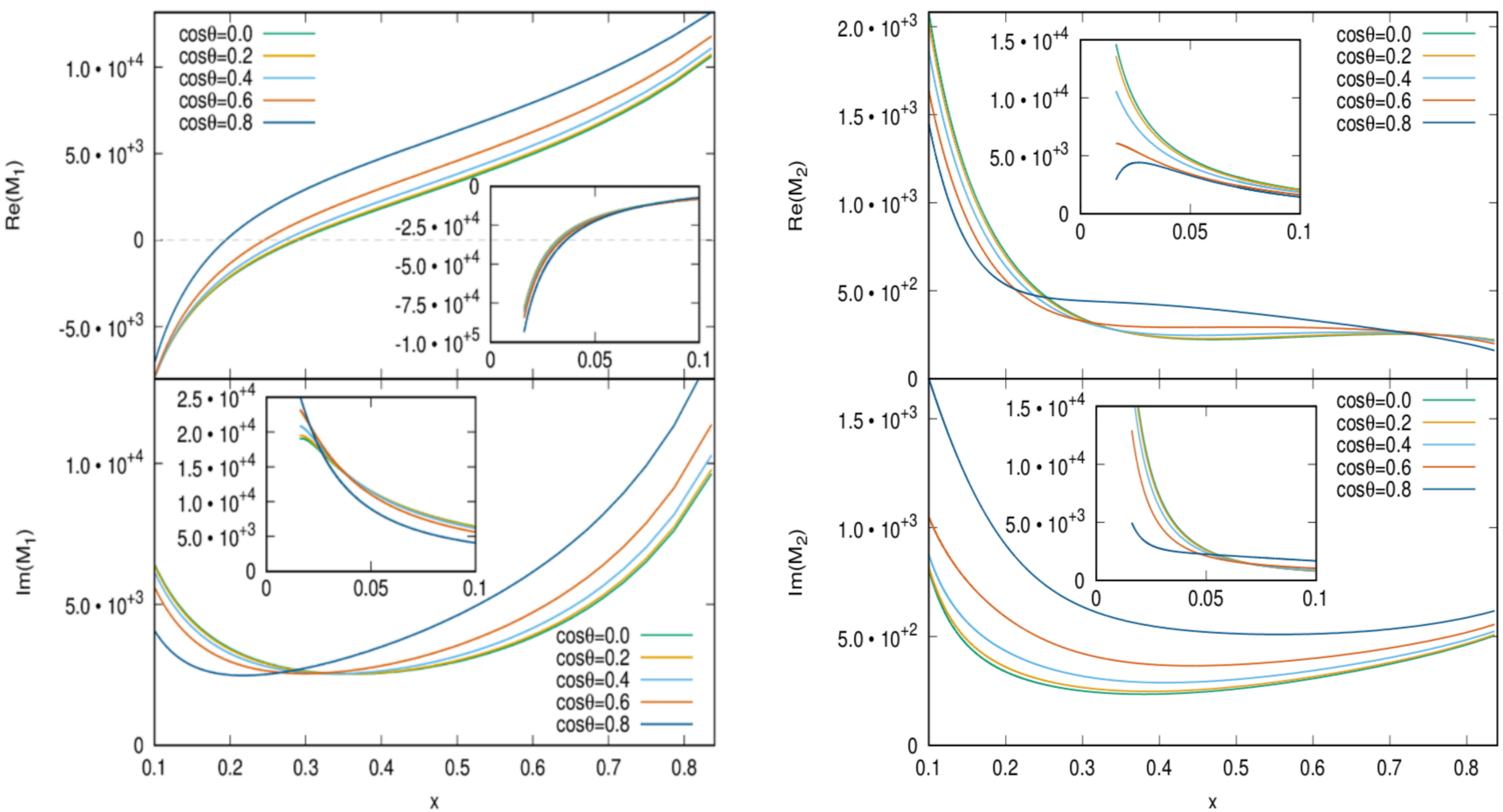
[Sterman, Tejeda-Yeomans] [Becher, Neubert] [Gardi, Magnea]

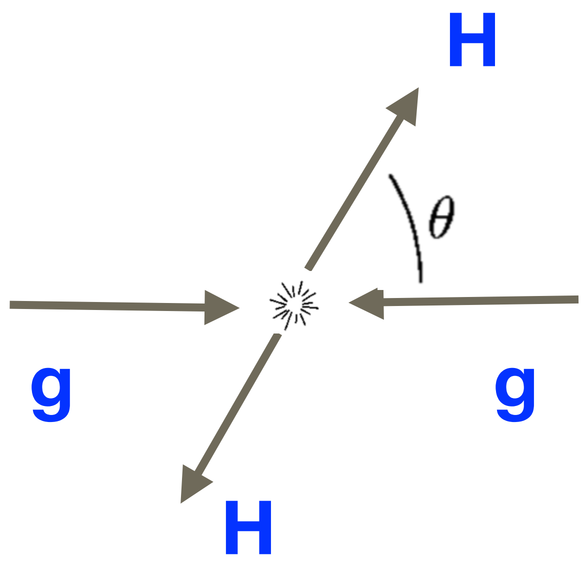
Plots: next slide !

- **Better numerical stability: finite amplitudes were extensively simplified using in-house routines.**

Numerical evaluation

- **For** $m_{\mathrm{h}} = 125\,\mathrm{GeV} \quad \mu_R^2 = m_{\mathrm{h}}^2/2,$





- **Bonosic amplitude =>** $|M_i\rangle_{\mathrm{Cos}(\theta)\to-\mathrm{Cos}(\theta)} = |M_i\rangle$

check on computation !

- **Amplitude shows stable behaviour.**

- **High precision needed for huge rational coefficients in expression.**

Conclusion

- We have computed a two loop amplitude in HEFT framework for HH pair production.

- This amplitude is essential for predicting the soft-virtual cross section at N3LO for di-Higgs production in the effective theory.

- Combine these amplitudes into fully differential calculation will require more work.

- Top quark mass effects in HH production for NNLO virtual corrections have been known recently.

[Davies, Steinhauser]

Thank You