Mixed EW-QCD corrections to the Higgs decay into bottom quarks

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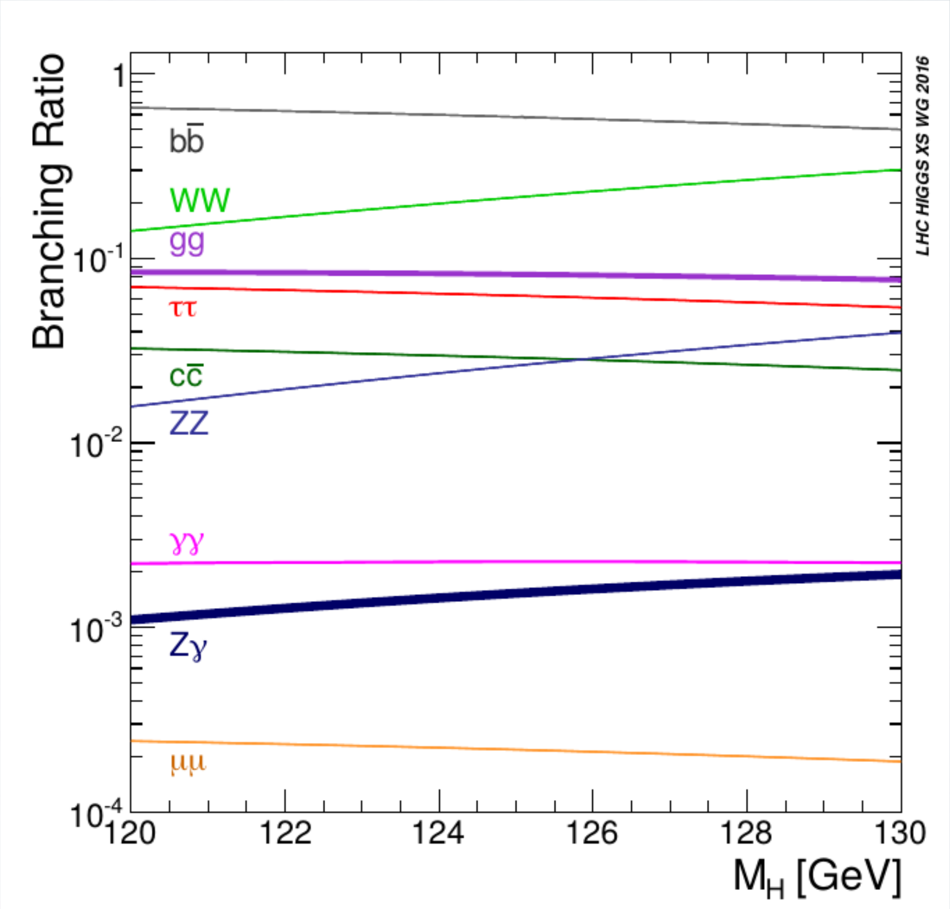
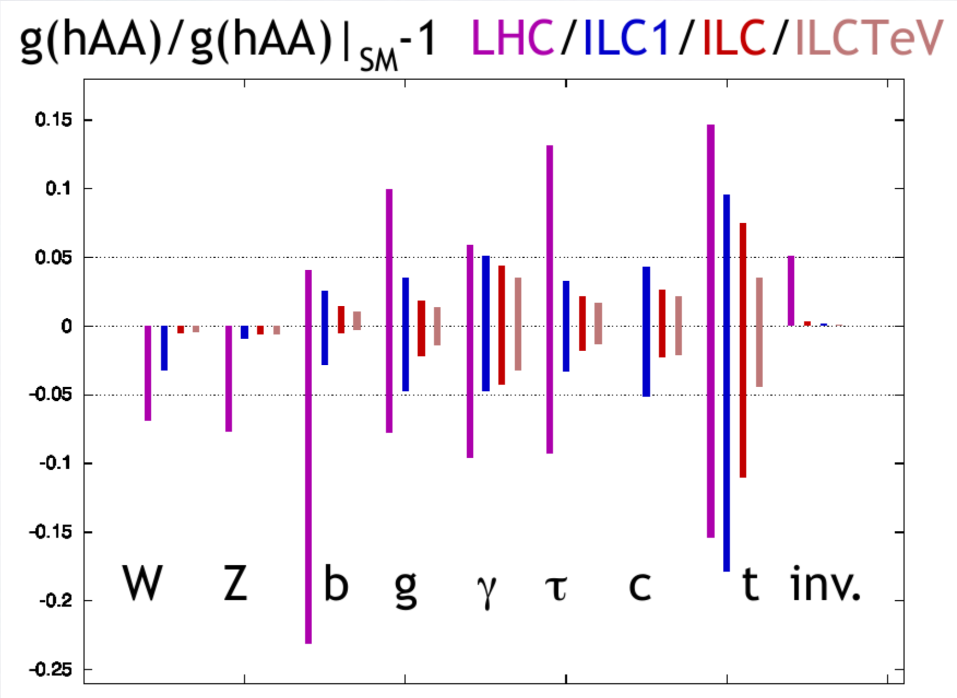
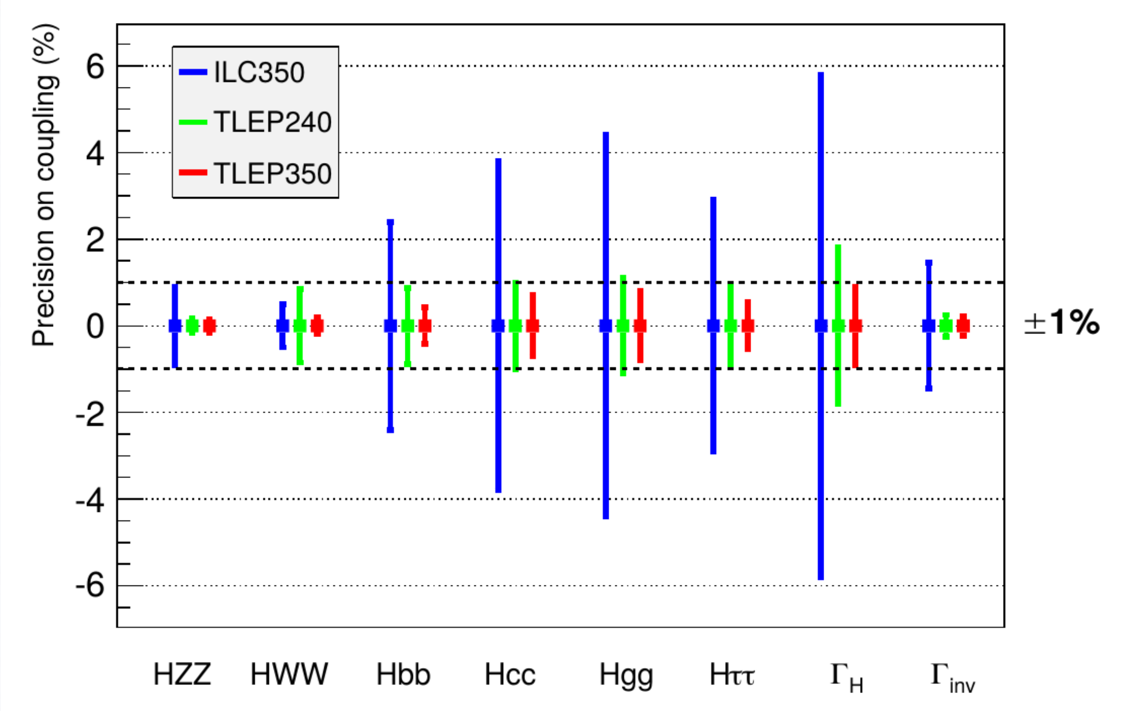
HIGGS DECAY TO BOTTOM QUARKS

- **H -> bb has largest BR**
- **Effects every on-shell Higgs measurement via the width**
- **Will be measured to sub percentage at future Lepton Colliders**







THEORETICAL PREDICTION

$$\mathbf{\Gamma_{H\to bb} = y_b^2 A_b} + \alpha_\mathbf{s} \mathbf{y_b^2 B_b} + \alpha_\mathbf{s}^\mathbf{2} \left(\mathbf{y_b^2 C_b + y_b y_t C_{bt}}\right) + \alpha_\mathbf{s}^\mathbf{3} \left(\mathbf{y_b^2 D_b + y_b y_t D_{bt}}\right)$$
$$+\alpha \left(\mathbf{y_b^2 E_b + y_b y_t E_{bt}}\right)$$
$$+\alpha\alpha_\mathbf{s} \left(\mathbf{y_b^2 F_b + y_b y_t F_{bt}}\right)$$
$$+\dots$$

THEORETICAL PREDICTION

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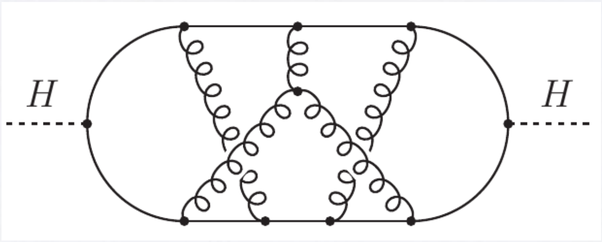
$$+\ldots$$

Strong Corrections

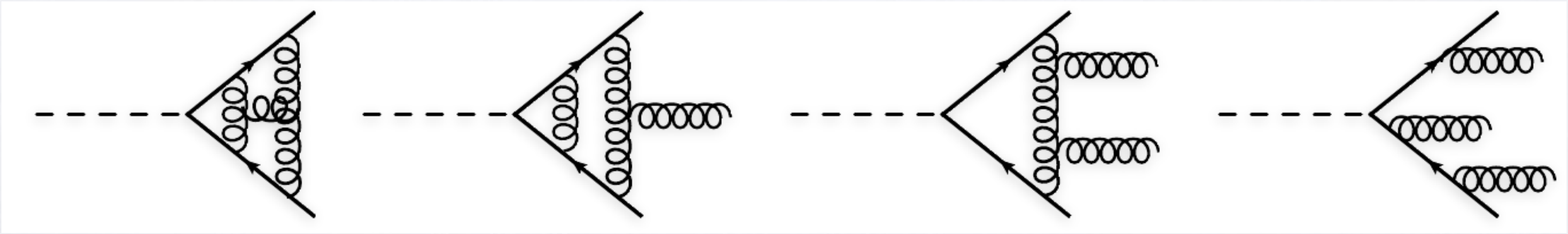
- **Up to N4LO inclusively**

Baikov, Chetyrkin, Kühn;
Herzog, Ruijl, Ueda, Vermaseren, Vogt



- **Up to N3LO differentially (y_b^2 piece)**

Mondini, Schiavi, Williams



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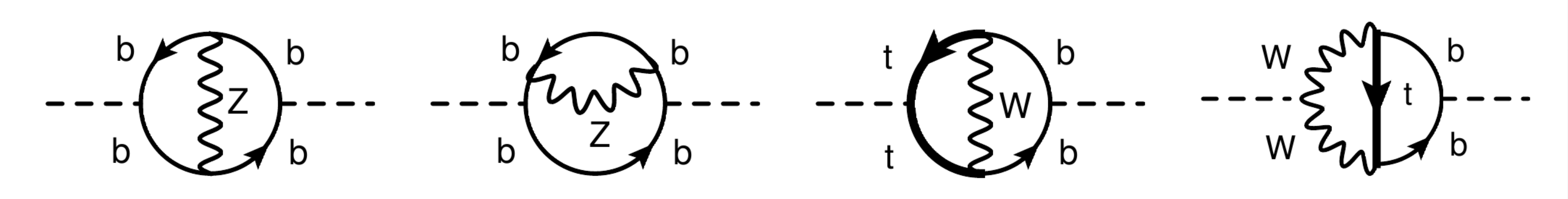
Electroweak Corrections

- **NLO inclusively**

Dabelstein, Hollik;
Mihalia, Schmidt, Steinhauser



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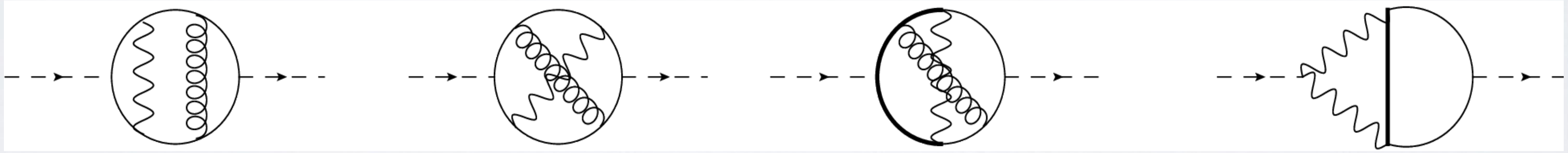
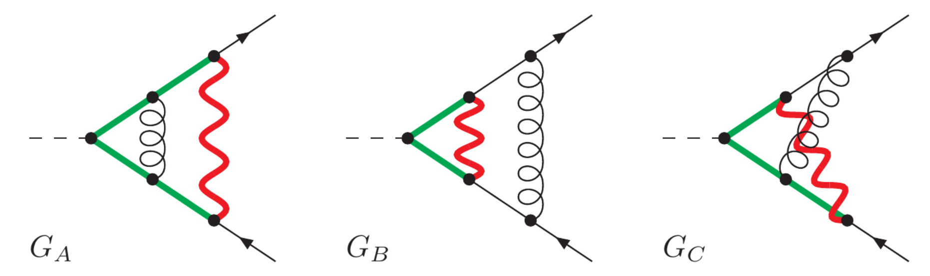
Mixed QCD-EW Corrections

- **NNLO inclusively with asymptotic expansion** — Mihalia, Schmidt, Steinhauser



- **Master integrals for 2-loop amplitudes** — Aglietti, Bonciani; Aglietti, Bonciani, Degrassi, Vicini; Chaubey, Weinzierl



LOOP AMPLITUDE

- **Amplitude given by Feynman diagrams**

$$A = \sum_i a_i I_i$$

- A

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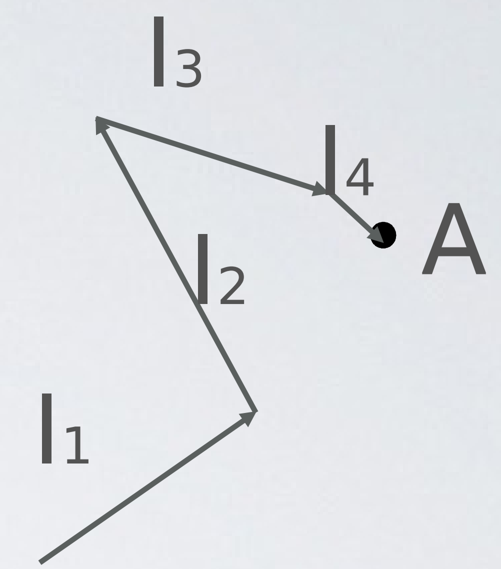

LOOP AMPLITUDE

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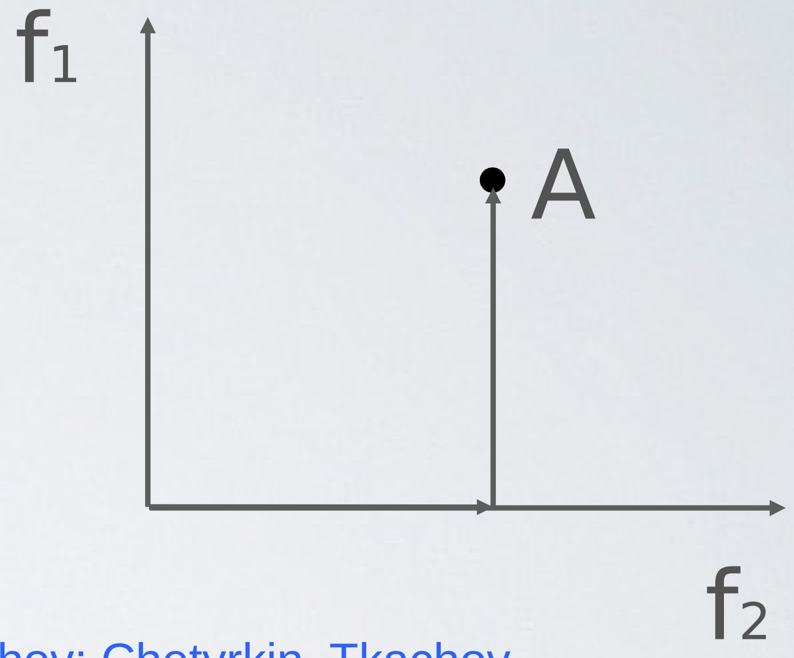
$$A = \sum_i a_i I_i$$

- **Project onto basis**

$$A = \sum_i c_i f_i$$



- Integration-by-parts identities — Tkachov; Chetyrkin, Tkachov
- Integrand Reduction — Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt; Mastrolia, Ossola; Zhang; Mastrolia, Mirabella, Ossola, Peraro
- General Unitarity — Bern, Dixon, Dunbar, Kosower; Cachazo, Svrcek, Witten; Britto, Cachazo, Feng
- Numerical Unitarity — Ita; Abreu, Febres Cordero, Ita, Jaquier, Page

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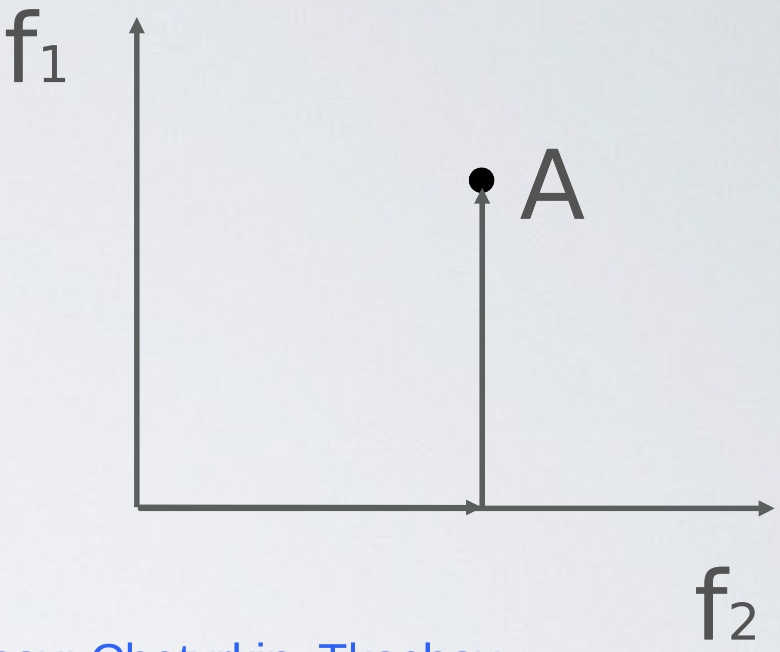

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- **Calculation of master integrals**

- Feynman parameter
- Mellin-Barnes — Smirnov; Tausk; Czakon; Smirnov, Smirnov
- Differential equations — Kotikov; Remiddi; Gehrmann, Remiddi
- Difference equation — Laporta; Lee, Smirnov, Smirnov

Methodology

REVERSE UNITARITY VS OPTICAL THEOREM VS AMPLITUDE SQUARE

	Optical Theorem	Reverse Unitarity	Amplitude Square
Number of Master Integrals	Low	High	Low
V and R pieces seperate	no	yes	yes
Analytic Inclusive result	yes (After PolyLog algebra)	yes	no
Differential	yes via PtB	yes via PtB	yes

OVERVIEW

- **Generate Diagrams with QGraf** $M_{H\to bb} = \sum_i a_i D_i$ Nogueira

- **Apply HVBM scheme for γ_5** 't Hooft, Veltman; Breitenlohner, Maison

$$\gamma_5 = \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \qquad \{\gamma_5, \tilde{\gamma_\mu}\} = 0, \quad [\gamma_5, \hat{\gamma_\mu}] \qquad I^d[\hat{k_1}\cdot\hat{k_2}] = -\frac{2\epsilon}{v_\perp^2} I^d[(k_1\cdot v_\perp)(k_2\cdot v_\perp)]$$

- **Integration-by-parts IDs with Kira** $M_{H\to bb} = \sum_i c_i I_i$ Chetyrkin, Tkachov; Maierhöfer, Usovitsch, Uwer

- **Differential Equations** $\partial_x \vec{I} = A_x \vec{I}$ Kotikov; Gehrmann, Remiddi

- **UV renormalization:**
  - On-Shell scheme
  - $\overline{\text{MS}}$ for m_b

$$M^r_{H\to bb} = Z_{y_b} M_{H\to bb}$$

- **Combine real and virtual amplitudes**

$$\Gamma_{H\to bb} = \int d\Phi_2 M^{r,V}_{H\to bb} + \int d\Phi_3 M^{r,R}_{H\to bb}$$

INTEGRATION-BY-PARTS IDENTITIES

- **Generated from Stokes Theorem**

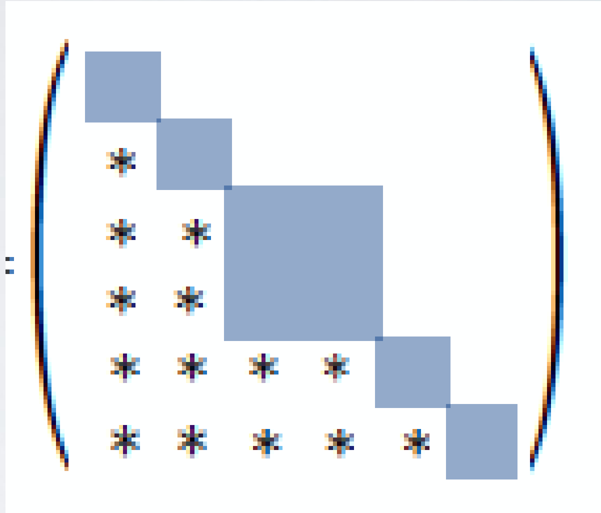
$$\int \prod_{i=1}^{L} d^d k_i \frac{\partial}{\partial k_{\mu,i}} \left(\frac{q_j^\mu}{D_1^{\alpha_1} \dots D_N^{\alpha_N}} \right) = 0 \qquad \leftrightarrow \qquad A\vec{I} = 0$$

- **Rank of As null space gives number of master integrals**
- **Limiting factors**
  - Algebra in Gaussian Elimination
    - Finite Field Method — von Manteuffel, Schabinger; Maierhoefer, Usovitsch, Uwer; Peraro
    - Intersection theory — Mastrolia, Mizera; Frellesvig, Gasparotto, Laporta, Mandal, Mastrolia, Mattiazzi, Mizera
  - Solving unnecessary Equations
    - Generate IBPs without higher powers — Larsen, Zhang
    - IBPs on the cut — Larsen, Zhang
- **Implemented in Public Codes**
  - Reduze — Studerus, von Manteuffel
  - Fire — Smirnov
  - Air — Anastasiou, Lazopolus
  - Kira — Maierhoefer, Usovitsch, Uwer
  - Azurite — Georgoudis, Larsen, Zhang
  - FiniteFlow — Peraro

DIFFERENTIAL EQUATIONS

- **Derivative in space spanned by MI**

$$\partial_x \vec{f} = A_x \vec{f}$$



- A_x **inhabits properties of IBP**
  - Block triangular
  - Rational in x and ε=(4-d)/2

Bottom up Approach	Matrix Approach
• Solve each block separately • Previously solved integrals appear as inhomogeneous part	• Conjecture: There is a basis such that: Henn $\partial_x \vec{g} = \epsilon \tilde{A}_x \vec{g}$ • Makes integration simple • But: Finding basis is difficult

FINDING CANONICAL DIFFERENTIAL EQUATIONS

- **Via Magnus theorem**

Ageri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, US

  - From linear DEQ

$$A(x,\epsilon) = A_0(x) + \epsilon A_1(x)$$

  - Magnus Theorem provides basis change

$$B(x) = e^{\Omega[A_0](x)} \qquad \vec{g}(x,\epsilon) = B(x,\epsilon)\vec{f}(x,\epsilon)$$

  - Applied to many examples:

    2-loop: Higgs+Jet, mixed QCD-EW corrections to DY, Muon-Electron scattering, qq→tt

    3-loop: Ladder topology Higgs+Jet

- **Many more strategies**
  - Unit leading singularity — Henn
  - Rational Ansatz for basis change — Gehrmann, von Manteuffel, Tancredi, Weihs
  - Reduction to fuchsian form and Eigenvalue normalisation — Lee; Lee, Smirnov
  - Factorisation of Picard-Fuchs operator — Adams, Chaubery, Weinzierl

SOLVING CANONICAL DIFFERENTIAL EQUATIONS

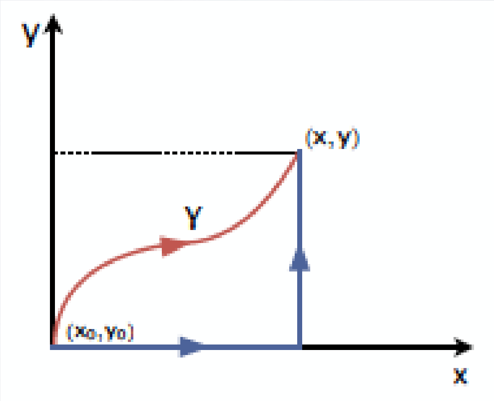
➢ Canonical form

$$\partial_x \vec{g}(x,\epsilon) = \epsilon \tilde{A}_x(x)\vec{g}(x,\epsilon) \qquad d\vec{g}(x,\epsilon) = \epsilon \sum_i M_i dlog(\eta_i)\vec{g}(x,\epsilon)$$

- Kinematic Dependence encoded in η
- *η* forms the alphabet

➢ Solution given by

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_\gamma dA \ldots dA\right] \vec{g}(x_0,\epsilon)$$



rational ηs	rational ηs
• Chen iterated Integrals	• Generalised Polylogarithms
$\mathbf{C(\tilde{\eta}_n;x) = \int_\gamma dlog(\eta_1)\ldots dlog(\eta_n)}$	$\mathbf{G(\tilde{0}_n;x) = \frac{1}{n!}log(x)^n}$ $\mathbf{G(\tilde{w}_n;x) = \int_0^x \frac{dt}{t-w_1} G(\tilde{w}_{n-1};t)}$

BOUNDARY CONDITIONS

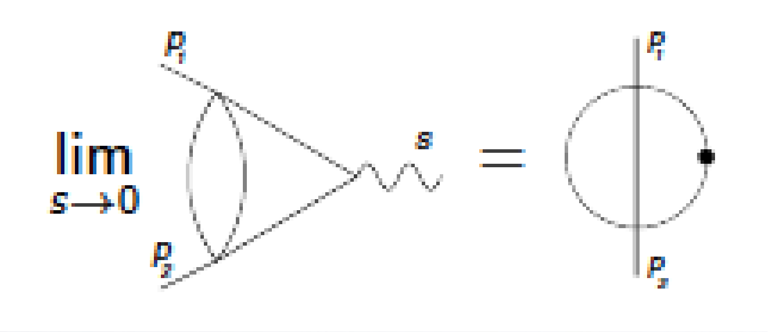
- **Solution given by**

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_{\gamma} dA \ldots dA\right] \vec{g}(x_0,\epsilon)$$

- **Two general ways to fix the boundary**

Known limits

- Taking the limit x to x_0
- Fix boundary constant by matching the solution to known function



Pseudo-thresholds

- Solution has unphysical divergences
- Demanding absence of unphysical divergences gives relations between boundary constant
- Leftover constants must be provided

Status of Calculation

WEAK CORRECTION

- ✔ **Generated the squared amplitude**
- ✔ **Analytic PS integration via reverse unitarity**
- ✔ **Computed the MI using DEQ**
- ✔ **Unrenormalized expression cross checked against independent amplitude calculation**
- ✔ **Counterterms taken from literature** Denner

  ➡ **All poles cancel**
- ✔ **Real Contributions are finite and experimentally distinguishable**

  ➡ **Not included here**
- ✗ **Check against existing result is in progress**

MIXED QCD-EW CORRECTION

- **Only Z-contributions considered so far**

Virtual Contribution

- ✓ Generated Amplitude
- ✓ Applied IBPs
- ✓ Plugged in MI
- ✗ UV Renormalization in progress

Real Contribution

- ✓ Generated Amplitude
- ✓ Applied IBPs
- ✓ Plugged in MI
- ✗ UV Renormalization in progress

"NLO-like"
IR pole cancellation

To-Do:

- UV renormalziation
- W-contributions
- IR pole cancellation
- Check against existing results

Conclusions

CONCLUSIONS

- H to bb most important decay channel
- Future (lepton) colliders can probe bottom yukawa coupling to subpercent level
- Inclusively: N4LO QCD, NLO EW and the approximated mixed QCD-EW are known
- Differentially: up to N3LO QCD is known
- Presented progress on differential NLO EW and mixed QCD-EW calculation

OUTLOOK

- Finish NLO EW and exact mixed QCD-EW calculation
- Check approximation of mixed QCD-EW result by computing inclusive width
- Combine with N3LO QCD corrections in coherent code

Thank you for your attention