Mixed EW-QCD corrections to the Higgs decay into bottom quarks

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In collaboration with Matthew Schiavi, Ciaran Williams
HIGGS DECAY TO BOTTOM QUARKS

- $H \rightarrow bb$ has largest BR
- Effects every on-shell Higgs measurement via the width
- Will be measured to sub percentage at future Lepton Colliders
\[ \Gamma_{H \rightarrow bb} = y_b^2 A_b + \alpha_s y_b^2 B_b + \alpha_s^2 (y_b^2 C_b + y_b y_t C_{bt}) + \alpha_s^3 (y_b^2 D_b + y_b y_t D_{bt}) \\
+ \alpha (y_b^2 E_b + y_b y_t E_{bt}) \\
+ \alpha \alpha_s (y_b^2 F_b + y_b y_t F_{bt}) + \ldots \]
THEORETICAL PREDICTION

\[ \Gamma_{H \to bb} = y_b^2 A_b + \alpha_s y_b^2 B_b + \alpha_s^2 \left( y_b^2 C_b + y_b y_t C_{bt} \right) + \alpha_s^3 \left( y_b^2 D_b + y_b y_t D_{bt} \right) \]
\[ + \alpha \left( y_b^2 E_b + y_b y_t E_{bt} \right) \]
\[ + \alpha \alpha_s \left( y_b^2 F_b + y_b y_t F_{bt} \right) \]
\[ + \ldots \]

Strong Corrections

- Up to N4LO inclusively
  - Baikov, Chetyrkin, Kühn; Herzog, Ruijl, Ueda, Vermaseren, Vogt

- Up to N3LO differentially \((y_b^2\) piece)
  - Mondini, Schiavi, Williams
Theoretical Prediction

\[ \Gamma_{H \rightarrow bb} = y_b^2 A_b + \alpha_s y_b^2 B_b + \alpha_s^2 \left( y_b^2 C_b + y_b y_t C_{bt} \right) + \alpha_s^3 \left( y_b^2 D_b + y_b y_t D_{bt} \right) + \alpha \left( y_b^2 E_b + y_b y_t E_{bt} \right) + \alpha \alpha_s \left( y_b^2 F_b + y_b y_t F_{bt} \right) + \ldots \]

Electroweak Corrections

- NLO inclusively

Dabelstein, Hollik; Mihalia, Schmidt, Steinhauser
THEORETICAL PREDICTION

\[ \Gamma_{H \to bb} = y_b^2 A_b + \alpha_s y_b^2 B_b + \alpha_s^2 \left( y_b^2 C_b + y_b y_t C_{bt} \right) + \alpha_s^3 \left( y_b^2 D_b + y_b y_t D_{bt} \right) + \alpha \left( y_b^2 E_b + y_b y_t E_{bt} \right) + \alpha \alpha_s \left( y_b^2 F_b + y_b y_t F_{bt} \right) + \ldots \]

Mixed QCD-EW Corrections

- NNLO inclusively with asymptotic expansion  
  Mihalia, Schmidt, Steinhauser

- Master integrals for 2-loop amplitudes  
  Aglietti, Bonciani;  
  Aglietti, Bonciani, Degrassi, Vicini;  
  Chaubey, Weinzierl
LOOP AMPLITUDE

- Amplitude given by Feynman diagrams

\[ A = \sum_i a_i I_i \] • A
LOOP AMPLITUDE

- Amplitude given by Feynman diagrams

\[ A = \sum_i a_i I_i \]
LOOP AMPLITUDE

- Amplitude given by Feynman diagrams

\[ A = \sum_i a_i I_i \]

- Project onto basis

\[ A = \sum_i c_i f_i \]

- Integration-by-parts identities
- Integrand Reduction
- General Unitarity
- Numerical Unitarity

Tkachov; Chetyrkin, Tkachov
Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt; Mastrolia, Ossola; Zhang; Mastrolia, Mirabella, Ossola, Peraro
Bern, Dixon, Dunbar, Kosower; Cachazo, Svrcek, Witten;
Britto, Cachazo, Feng
Ita; Abreu, Febres Cordero, Ita, Jaquier, Page
LOOP AMPLITUDE

➢ Amplitude given by Feynman diagrams

\[ A = \sum_i a_i I_i \]

➢ Project onto basis

\[ A = \sum_i c_i f_i \]

- Integration-by-parts identities

- Integrand Reduction

- General Unitarity

- Numerical Unitarity

➢ Calculation of master integrals

- Feynman parameter

- Mellin-Barnes

- Differential equations

- Difference equation

Tkachov; Chetyrkin, Tkachov

Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt; Mastrolia, Ossola; Zhang; Mastrolia, Mirabella, Ossola, Peraro

Bern, Dixon, Dunbar, Kosower; Cachazo, Svrcek, Witten; Britto, Cachazo, Feng

Ita; Abreu, Febres Cordero, Ita, Jaquier, Page

Smirnov; Tausk; Czakon; Smirnov, Smirnov

Kotikov; Remiddi; Gehrmann, Remiddi

Laporta; Lee, Smirnov, Smirnov
Methodology
## REVERSE UNITARITY VS OPTICAL THEOREM VS AMPLITUDE SQUARE

<table>
<thead>
<tr>
<th></th>
<th>Optical Theorem</th>
<th>Reverse Unitarity</th>
<th>Amplitude Square</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Master Integrals</strong></td>
<td>Low</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td><strong>V and R pieces separate</strong></td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Analytic Inclusive result</strong></td>
<td>yes (After PolyLog algebra)</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td><strong>Differential</strong></td>
<td>yes via PtB</td>
<td>yes via PtB</td>
<td>yes</td>
</tr>
</tbody>
</table>
OVERVIEW

➢ Generate Diagrams with QGraf

\[ M_{H \to bb} = \sum_i a_i D_i \]  

Nogueira

➢ Apply HVBM scheme for \( \gamma_5 \)

\[ \gamma_5 = \frac{i}{4!} \epsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \quad \{ \gamma_5, \gamma_\mu \} = 0, \quad [\gamma_5, \gamma_\mu] \]

\[ I^d[\hat{k}_1 \cdot \hat{k}_2] = -\frac{2e}{v_1^2} I^d[(k_1 \cdot v_\perp)(k_2 \cdot v_\perp)] \]

\[ M_{H \to bb} = \sum_i c_i I_i \]  

't Hooft, Veltman; Breitenlohner, Maison

Chetyrkin, Tkachov; Maierhöfer, Usovitsch, Uwer

➢ Integration-by-parts IDs with Kira

\[ \partial_x \vec{I} = A_x \vec{I} \]  

Kotikov; Gehrmann, Remiddi

➢ Differential Equations

➢ UV renormalization:
  • On-Shell scheme
  • MS for \( m_b \)

\[ M_{H \to bb}^T = Z_{y_b} M_{H \to bb} \]

➢ Combine real and virtual amplitudes

\[ \Gamma_{H \to bb} = \int d\Phi_2 M_{H \to bb}^{r,V} + \int d\Phi_3 M_{H \to bb}^{r,R} \]
INTEGRATION-BY-PARTS IDENTITIES

➢ Generated from Stokes Theorem

\[
\int \prod_{i=1}^{L} d^{d}k_{i} \frac{\partial}{\partial k_{\mu,i}} \left( \frac{q_{j}^{\mu}}{D_{1}^{\alpha_{1}} \ldots D_{N}^{\alpha_{N}}} \right) = 0 \quad \leftrightarrow \quad A\vec{I} = 0
\]

➢ Rank of As null space gives number of master integrals

➢ Limiting factors
  • Algebra in Gaussian Elimination
    - Finite Field Method
    - Intersection theory
  • Solving unnecessary Equations
    - Generate IBPs without higher powers
    - IBPs on the cut

➢ Implemented in Public Codes
  • Reduze
  • Fire
  • Air
  • Kira
  • Azurite
  • FiniteFlow

von Manteuffel, Schabinger; Maierhoefer, Usovitsch, Uwer; Peraro
Mastrolia, Mizera; Frellesvig, Gasparotto, Laporta, Mandal, Mastrolia, Mattiazzi, Mizera

Larsen, Zhang

Studerus, von Manteuffel
Smirnov

Anastasiou, Lazopolus

Maierhoefer, Usovitsch, Uwer

Georgoudis, Larsen, Zhang

Peraro
DIFFERENTIAL EQUATIONS

- Derivative in space spanned by MI
  \[ \partial_x \vec{f} = A_x \vec{f} \]

- $A_x$ inhabits properties of IBP
  - Block triangular
  - Rational in $x$ and $\varepsilon = (4-d)/2$

**Bottom up Approach**
- Solve each block separately
- Previously solved integrals appear as inhomogeneous part

**Matrix Approach**
- Conjecture: There is a basis such that:
  \[ \partial_x \vec{g} = \varepsilon \tilde{A}_x \vec{g} \]
- Makes integration simple
- But: Finding basis is difficult
FINDING CANONICAL DIFFERENTIAL EQUATIONS

➢ Via Magnus theorem
  • From linear DEQ
    \[ A(x, \epsilon) = A_0(x) + \epsilon A_1(x) \]
  • Magnus Theorem provides basis change
    \[ B(x) = e^{\Omega[A_0](x)} \quad g'(x, \epsilon) = B(x, \epsilon) f'(x, \epsilon) \]
  • Applied to many examples:
    2-loop: Higgs+Jet, mixed QCD-EW corrections to DY, Muon-Electron scattering, qq → tt
    3-loop: Ladder topology Higgs+Jet

➢ Many more strategies
  • Unit leading singularity
  • Rational Ansatz for basis change
  • Reduction to fuchsian form and Eigenvalue normalisation
  • Factorisation of Picard-Fuchs operator

Ageri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, US

Henn
Gehrmann, von Manteuffel, Tancredi, Weihs

Lee; Lee, Smirnov

Adams, Chaubery, Weinzierl
SOLVING CANONICAL DIFFERENTIAL EQUATIONS

Canonical form

\[ \partial_x g(x, \epsilon) = \epsilon \tilde{A}_x(x) \tilde{g}(x, \epsilon) \]
\[ d\tilde{g}(x, \epsilon) = \epsilon \sum_i M_i d\log(\eta_i) \tilde{g}(x, \epsilon) \]

- Kinematic Dependence encoded in \( \eta \)
- \( \eta \) forms the alphabet

Solution given by

\[ g(x, \epsilon) = \left[ 1 + \sum_{i=1}^{\infty} \int_{\gamma} dA \ldots dA \right] \tilde{g}(x_0, \epsilon) \]

<table>
<thead>
<tr>
<th>rational ( \eta )s</th>
<th>rational ( \eta )s</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Chen iterated Integrals</td>
<td>• Generalised Polylogarithms</td>
</tr>
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</table>

\[ C(\tilde{\eta}_n; x) = \int_{\gamma} d\log(\eta_1) \ldots d\log(\eta_n) \]

\[ G(\tilde{0}_n; x) = \frac{1}{n!} \log(x)^n \]
\[ G(\tilde{w}_n; x) = \int_0^x \frac{dt}{t - w_1} G(\tilde{w}_{n-1}; t) \]
BOUNDARY CONDITIONS

➢ Solution given by

\[ \tilde{g}(x, \epsilon) = \left[ 1 + \sum_{i=1}^{\infty} \int_{\gamma} dA \ldots dA \right] \tilde{g}(x_0, \epsilon) \]

➢ Two general ways to fix the boundary

<table>
<thead>
<tr>
<th>Known limits</th>
<th>Pseudo-thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Taking the limit x to x₀</td>
<td>• Solution has unphysical divergences</td>
</tr>
<tr>
<td>• Fix boundary constant by matching the solution to known function</td>
<td>• Demanding absence of unphysical divergences gives relations between boundary constant</td>
</tr>
</tbody>
</table>

• Leftover constants must be provided
Status of Calculation
WEAK CORRECTION

✓ Generated the squared amplitude
✓ Analytic PS integration via reverse unitarity
✓ Computed the MI using DEQ
✓ Unrenormalized expression cross checked against independent amplitude calculation
✓ Counterterms taken from literature

⇒ All poles cancel

✓ Real Contributions are finite and experimentally distinguishable

⇒ Not included here

✗ Check against existing result is in progress
MIXED QCD-EW CORRECTION

- Only Z-contributions considered so far

**Virtual Contribution**
- ✔ Generated Amplitude
- ✔ Applied IBPs
- ✔ Plugged in MI
- ❗ UV Renormalization in progress

**Real Contribution**
- ✔ Generated Amplitude
- ✔ Applied IBPs
- ✔ Plugged in MI
- ❗ UV Renormalization in progress

"NLO-like"

IR pole cancellation

**To-Do:**
- UV renormalization
- W-contributions
- IR pole cancellation
- Check against existing results
Conclusions
CONCLUSIONS

➢ H to bb most important decay channel

➢ Future (lepton) colliders can probe bottom yukawa coupling to subpercent level

➢ Inclusively: N4LO QCD, NLO EW and the approximated mixed QCD-EW are known

➢ Differentially: up to N3LO QCD is known

➢ Presented progress on differential NLO EW and mixed QCD-EW calculation

OUTLOOK

➢ Finish NLO EW and exact mixed QCD-EW calculation

➢ Check approximation of mixed QCD-EW result by computing inclusive width

➢ Combine with N3LO QCD corrections in coherent code
Thank you for your attention