Quark and Gluon Jet functions at 3-loops in QCD

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In collaboration with V. Ravindran & P. Banerjee

Plan

- Definition
- Contribution in predicting several observables
- Available results in the literature
- DIS soft plus jet function and its relation to SCET jet function
- Our result & Summary

Definition of Jet Function

- The parton jet function measures the probability that a parton field produces a jet of particles with momentum p from the vacuum.

- Mathematically, it is given by the vacuum matrix element of two gluon fields

Gluon field in the light-cone gauge $(n.\mathcal{A} = 0)$

$$\int d^d x\, e^{ipx} \langle 0|\mathcal{A}^a_\mu(x)\mathcal{A}^b_\nu(0)|0\rangle = \sum_X (2\pi)^d \delta^{(d)}(p - p_X)\langle 0|\mathcal{A}^a_\mu(0)|X\rangle\langle X|\mathcal{A}^b_\nu(0)|0\rangle$$

$$\equiv g_s^2 \theta(p^0)\delta^{ab}\left(-g_{\mu\nu} + \frac{n_\mu p_\nu + p_\mu n_\nu}{n.p}\right) J^g(p^2)$$

Gluon jet function

Definition

- The gluon field in the light-cone gauge is related to the field in a general gauge given by

$$\mathcal{A}^{\mu}(x) = \mathcal{A}^{a\mu}(x)t_a = W^{\dagger}(x)\left[iD^{\mu}W(x)\right]$$

Gluon field in a general gauge

Where light-like ($n.n = 0$) Wilson line given by

$$W(x) = \mathbf{P}\exp\left(ig_s\int_{-\infty}^{0} ds\, n.A(x+sn)\right)$$

Path ordering

Definition

* Xsection differential probing jet invariant mass takes the factorised form [Bauer et al. (arXiv: 0005275, 0011336, 0107001,0109045,0202088), Beneke et al. (0206152)]

Hard function　　Jet function　　Jet invariant mass

$$\frac{d\sigma}{d\tau} = H(Q) \times [B_a \otimes B_b \otimes J_{i_1} \otimes \ldots \otimes J_{i_N} \otimes S](\tau)$$

Hard scale　　Beam function　　Soft function　　$\frac{\tau}{Q} << 1$

* This factorisation is at LO in τ/Q and to all orders in α_s

$$A(\tau) \otimes B(\tau) \equiv \int d\tau' A(\tau - \tau') B(\tau')$$

Application

* A universal ingredient in SCET framework involving final state jets.

* Hence it appears in any jet process at hadron as well as e^+e^- colliders.

* Inclusive observables with soft-gluon resummation

  * Jet invariant mass   * Thrust distribution   * C-parameter

  [Catani et al. (Nucl.Phys. B407 (1993) 3-42), Chien et al. (1005.1644), Becher et al. (0803.0342), Abbate et al. (1006.3080, 1204.5746), Hoang et al. (1411.6633, 1501.04111)]

* Above mentioned observables are used for precise determination of QCD coupling from e^+e^- data.

* Another motivation: a major component of N-jettiness subtraction method. [Gaunt et al. (1505.04794), Boughezal et al. (1504.02131)]

Known Results

- One-loop and two-loop quark jet functions are known for some time. [Bauer et al. (0312109), Bosch et al. (0402094), Becher et al. (0603140)]

- Similarly, gluon jet function is also known up to two-loop order. [Becher et al. (0911.0681, 1008.1936)]

- The result of three-loop quark jet function has appeared more recently. [Bruser et al. (1804.09722)]

- All these results have been obtained through direct computation from formal SCET definition.

Goal

- QCD result $\xrightarrow{\text{Extraction}}$ Quark & Gluon jet function (3-loop)

- Extraction: Relating Soft+Jet function of DIS to SCET Jet function.

- Quark jet function: New independent calculation (Checked with the recent result from direct computation).

- Gluon jet function: New result from our work.

Usefulness

- The three-loop results contribute to the resummation for observables probing the invariant mass of final state jets at N3LL accuracy.

- The perspective of extending N-jettiness formalism, which has been applied successfully to several NNLO processes with final state jets to N3LO.

Why DIS?

- This is the simplest hadronic process consisting of only one jet i.e. one jet function.

- Due to the above reason, it is relatively easy to compare between QCD and SCET factorisation for this process.

- Theoretical predictions up to three-loop are known which constitute main input for our work. [Vermaseren et al. (0504242), Soar et al. (0912.0369)]

Inclusive Xsection

- Inclusive Xsection for the scattering of a lepton with a hadron in DIS is given by

$x = \frac{Q^2}{2P.q}$ Renorm. scale $z = \frac{Q^2}{2p.q}$ In. state col. fac. scale

$$\sigma^I(x, Q^2) = \sigma^I_B(\mu_R^2) \sum_{a=q,\overline{q},g} \int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z}, \mu_F^2\right) \Delta^I_a(a_s, z, Q^2, \mu_R^2, \mu_F^2)$$

Total MT squared Born Xsection PDF UV and IR finite CF

Inclusive Xsection

- UV renormalised and IR safe coefficient function can be factored in the following way:

Remaining hard part

$$\Delta_a^I(a_s, z, Q^2, \mu_F^2, \mu_R^2) = \Delta_a^{I,\mathrm{hard}}(a_s, z, Q^2, \mu_F^2, \mu_R^2) + \Delta^{I,\mathrm{SV}}(a_s, z, Q^2, \mu_F^2, \mu_R^2)$$

Soft+Virtual (Contributions coming from soft gluons)

$$\delta(1-z) \quad \& \quad \mathcal{D}_i(z) = \left[\frac{\log^i(1-z)}{1-z}\right]_+$$

Soft plus Virtual Xsection

- SV part of the coefficient function can be shown to factorise in the following way: [Ravindran (0512249, 0603041)]

$$\Delta^{I,\mathrm{SV}} = \left(Z^I\right)^2 |\hat{\mathcal{F}}^I|^2 \delta(1-z) \otimes \mathcal{C} e^{2\Phi^I_{\mathrm{SJ}}} \otimes \Gamma^{-1}_{II}$$

Mellin convolution

Additional UV renorm. for Higgs

Soft plus Jet function

Pure virtual contribution

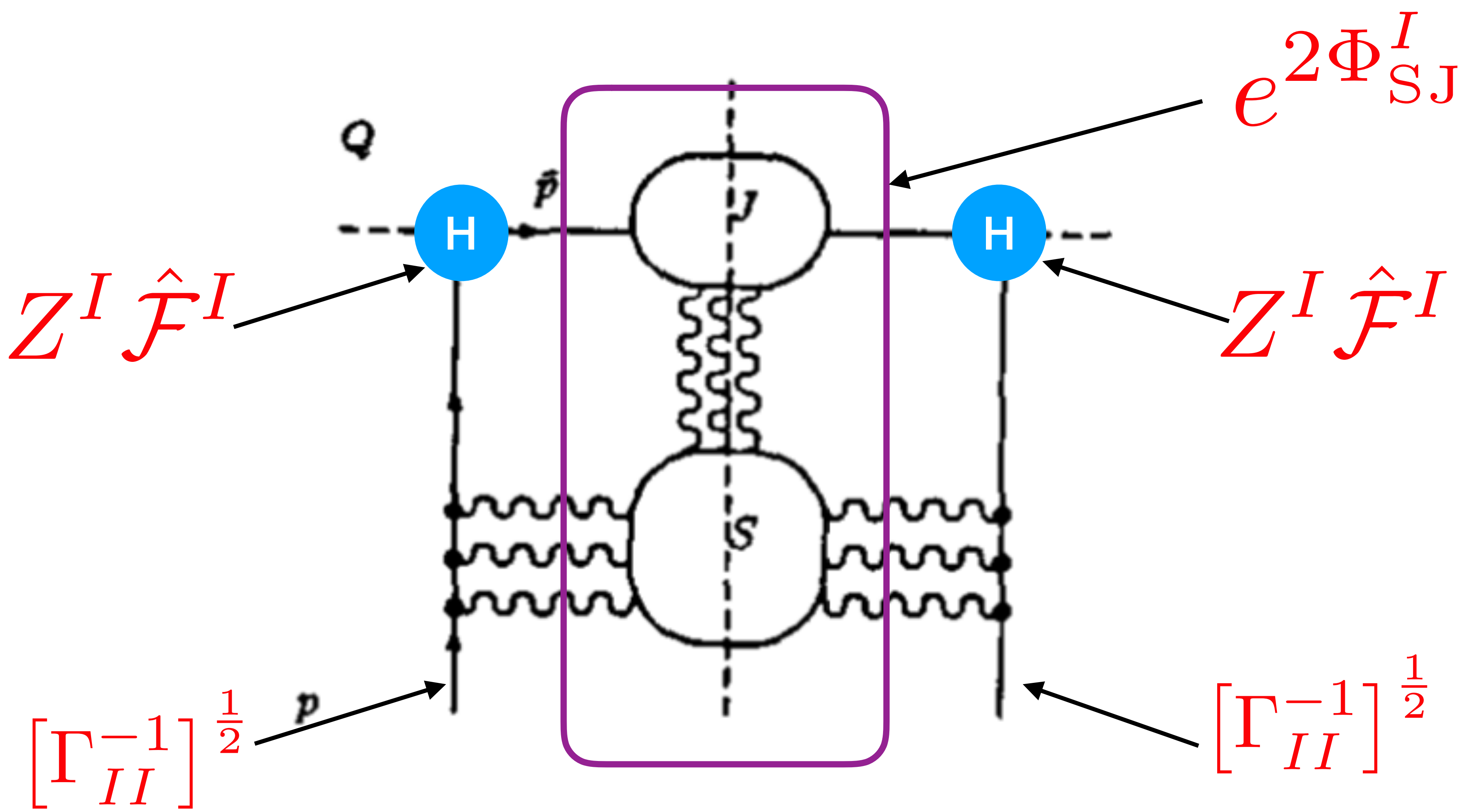
Convoluted exp.

Diagonal IR sub. kernel

Soft plus Virtual Xsection

[Catani et al. (1989)]

- The previous slide can be seen pictorially as



UV Renormalisation Constant

- It satisfies following RGE

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z^I \left(\hat{a}_s, \mu_R^2, \mu^2, \epsilon\right) = \sum_{i=1}^{\infty} a_s^i \left(\mu_R^2\right) \gamma_{i-1}^I$$

UV anomalous dimension

- For Higgs effective coupling, it is known to all orders in QCD $\beta-$function. [Spiridonov]

- In QCD, $\beta-$function is known up to five-loops.

[Herzog et al. (1701.01404)]

IR Kernels

* Obeys following RGE

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma\left(z, \mu_F^2, \epsilon\right) = \frac{1}{2} P\left(z, \mu_F^2\right) \otimes \Gamma\left(z, \mu_F^2, \epsilon\right)$$

DGLAP splitting functions

* Complete result available up to three-loop in literature.

[Moch, Vermaseren and Vogt (0403192, 0404111)]

* Partial result at four-loops. [Davies et al. (1610.07477), Moch et al. (1707.08315)]

Form Factor

[Sudakov, Mueller, Collins, Sen]

- It obeys Sudakov equation given by

Finite

$$Q^2 \frac{d}{dQ^2} \log \hat{\mathcal{F}}_I = \frac{1}{2} \left[K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon \right) + G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) \right]$$

Poles in regularisation parameter

- RG invariance implies

Cusp ano. dim.

$$\mu_R^2 \frac{d}{d\mu_R^2} K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon \right) = -\mu_R^2 \frac{d}{d\mu_R^2} G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) \equiv -A^I \left(a_s \left(\mu_R^2 \right) \right)$$

Soft plus Jet Function

- Soft plus Jet function satisfy the following Sudakov-type diff. equation similar to FF.

$$Q^2 \frac{d}{dQ^2} \Phi^I_{\rm SJ} = \frac{1}{2}\Big[\bar{K}^I\Big(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \epsilon\Big) + \bar{G}^I_{\rm SJ}\Big(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \epsilon\Big)\Big]$$

Finite

Poles in regularisation parameter

Solution

[Ravindran (0512249, 0603041)]

- The solution is found to be

Spherical factor: $\exp[\frac{\epsilon}{2}(\gamma_E - \ln 4\pi)]$

$$\Phi^I_{\rm SJ} = \sum_{i=1}^{\infty} \hat{a}_s^i S_\epsilon^i \left(\frac{Q^2(1-z)}{\mu^2}\right)^{i\frac{\epsilon}{2}} \frac{i\epsilon}{2(1-z)} \hat{\phi}^{I,(i)}_{\rm SJ}(\epsilon)$$

where $$\hat{\phi}^{I,(i)}_{\rm SJ}(\epsilon) = \frac{1}{i\epsilon}\left[\bar{K}^{I,(i)}(\epsilon) + \bar{G}^{I,(i)}_{\rm SJ}(\epsilon)\right]$$

$$\bar{K}^I = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \bar{K}^{I,(i)} \qquad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q_z^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \bar{G}^{I,(i)}_{\rm SJ}(\epsilon) = \sum_{i=1}^{\infty} a_s^i(Q_z^2) \bar{\mathcal{G}}^I_{i,{\rm SJ}}(\epsilon)$$

with $Q_z^2 = Q^2(1-z)$

Solution

[Ravindran (0512249, 0603041)]

* And the coefficient $\bar{\mathcal{G}}^I_{i,\mathrm{SJ}}(\epsilon)$ is given by

Coming from lower orders through UV renorm.

Collinear ano. dim.

$$\bar{\mathcal{G}}^I_{i,\mathrm{SJ}} = -(B^I_i + f^I_i) + C^I_i + \sum_{k=1}^{\infty} \epsilon^k \bar{\mathcal{G}}^{I,k}_{i,\mathrm{SJ}}$$

Soft ano. dim.

Process dependent

* We have extracted these process dependent coefficients from already computed coefficient function for DIS processes which implies the result of Φ^I_{SJ}.

[Vermaseren et al. (0504242), Soar et al. (0912.0369)]

Connection

[Banerjee, PKD, Ravindran (1805.02637)]

- One can factor all the singular terms from Soft plus Jet function in following way:

Finite to all orders as $\epsilon \to 0$

Contains all singular terms order by order in α_s

$$\mathcal{C}e^{2\Phi^{I}_{\rm SJ}} = \mathcal{Z}^{I} \otimes \mathcal{C}e^{2\Phi^{I,\rm fin}_{\rm SJ}}$$

with $\mathcal{Z}^{I} = \delta(1-z) + \sum_{i=1}^{n}\sum_{j=1}^{2\,i} a_s^i \frac{\mathcal{Z}^{I}_{i\,j}}{\epsilon^j}$ and identification of SCET Jet function as

$$\mathcal{C}e^{2\Phi^{I,\rm fin}_{\rm SJ}} = \delta(1-z) + \sum_{i=1}^{\infty} a_s^i J_i^I\Big|_k$$

Denotes the coefficient of $\delta(1-z)$ or $\mathcal{D}_i(z) = \left[\frac{\log^i(1-z)}{1-z}\right]_+$

Result

[Banerjee, PKD, Ravindran (1805.02637)]

* Delta function coefficient of the gluon jet function is given by

$$J_3^g\big|_\delta = C_A^3\left[\frac{55853711}{26244} - 44\zeta_5 - \frac{452770}{243}\zeta_3 + \frac{1600}{9}\zeta_3^2 - \frac{2055109}{4374}\pi^2 + \frac{1364}{9}\pi^2\zeta_3 + \frac{53633}{1620}\pi^4 - \frac{16309}{20412}\pi^6\right]$$

$$+C_A^2 n_f\left[-\frac{17323633}{26244} + \frac{208}{9}\zeta_5 + \frac{2734}{9}\zeta_3 + \frac{330062}{2187}\pi^2 - \frac{88}{9}\pi^2\zeta_3 - \frac{18727}{2430}\pi^4\right] + C_F^2 n_f\left[\frac{143}{9} - 80\zeta_5 + \frac{148}{3}\zeta_3\right]$$

$$+C_A n_f^2\left[\frac{1613639}{26244} - \frac{1004}{243}\zeta_3 - \frac{3656}{243}\pi^2 + \frac{506}{1215}\pi^4\right] + C_F n_f^2\left[\frac{7001}{162} - \frac{104}{3}\zeta_3 - \frac{10}{9}\pi^2\right]$$

$$+C_A C_F n_f\left[-\frac{389369}{972} + \frac{584}{9}\zeta_5 + \frac{21200}{81}\zeta_3 + \frac{712}{27}\pi^2 - \frac{160}{9}\pi^2\zeta_3 + \frac{76}{405}\pi^4\right] + n_f^3\left[-\frac{1000}{729} + \frac{40}{81}\pi^2\right]$$

Result

[Banerjee, PKD, Ravindran (1805.02637)]

* Delta function coefficient of the quark jet function is given by

$$J_3^q|_\delta = C_F^3\left[274\zeta_3 + \frac{22}{3}\pi^2\zeta_3 - \frac{400}{3}\zeta_3^2 - 88\zeta_5 + \frac{1173}{8} - \frac{3505}{72}\pi^2 + \frac{622}{45}\pi^4 - \frac{9871}{8505}\pi^6\right]$$
$$+C_AC_F^2\left[-\frac{28241}{27}\zeta_3 + \frac{2200}{27}\pi^2\zeta_3 + \frac{424}{3}\zeta_3^2 + \frac{560}{9}\zeta_5 + \frac{206197}{324} - \frac{17585}{72}\pi^2 + \frac{18703}{1215}\pi^4 + \frac{1547}{4860}\pi^6\right]$$
$$+C_A^2C_F\left[-\frac{187951}{243}\zeta_3 + \frac{394}{9}\pi^2\zeta_3 + \frac{1528}{9}\zeta_3^2 - \frac{380}{9}\zeta_5 + \frac{50602039}{52488} - \frac{464665}{4374}\pi^2 + \frac{1009}{1620}\pi^4 + \frac{221}{5103}\pi^6\right]$$
$$+C_AC_Fn_f\left[\frac{7414}{81}\zeta_3 - \frac{32}{9}\pi^2\zeta_3 + \frac{16}{3}\zeta_5 - \frac{2942843}{13122} + \frac{68324}{2187}\pi^2 - \frac{209}{405}\pi^4\right]$$
$$+C_F^2n_f\left[\frac{11216}{81}\zeta_3 - \frac{136}{27}\pi^2\zeta_3 + \frac{80}{3}\zeta_5 - \frac{261587}{972} + \frac{4853}{108}\pi^2 - \frac{2938}{1215}\pi^4\right]$$
$$+C_Fn_f^2\left[\frac{376}{243}\zeta_3 + \frac{124903}{13122} - \frac{466}{243}\pi^2 + \frac{2}{45}\pi^4\right]$$

In agreement with 1804.09722

Result

[Banerjee, PKD, Ravindran (1805.02637)]

- The coefficients of plus distribution (scale dependent) terms can be obtained through the RGE satisfied by the jet function

$$\mu_R^2 \frac{d}{d\mu_R^2} J^I = \gamma_J^I \otimes J^I$$

Jet anomalous dimension

$$\gamma_J^I = \left[B^I + f^I - A^I \log\left(\frac{Q^2}{\mu_R^2}\right)\right] \delta(1-z) - A^I \mathcal{D}_0$$

Comments From Past

* Previously computed results for the soft function in the context of Higgs & DY agrees with soft functions in SCET framework.

* Several results at N3LO in recent past has been obtained using the above information.

  [C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, B. Mistlberger (1403.4616), T. Ahmed, M. Mahakhud, M. K. Mandal, N. Rana, V. Ravindran (1404.0366, 1404.6504, 1408.0787, 1411.5301)]

* These results were verified from the explicit computation.

  [S. Catani, L. Cieri, D. de Florian, G, Ferrera, M. Grazzini (1404.5839), Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu(1405.4827)]

Summary

- Using the explicit result for DIS coefficient function, we have extracted both quark and gluon jet functions at 3-loops.

- This is achieved through finding a novel connection between soft plus jet functions of DIS and jet functions in the SCET framework.

- These results will contribute in precise theoretical prediction of observables through resummation probing the invariant masses of jets at N3LL accuracy.

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Thank You For Your Attention