

RADCOR 2019

Centre des Congrès du Palais des Papes
Avignon, France

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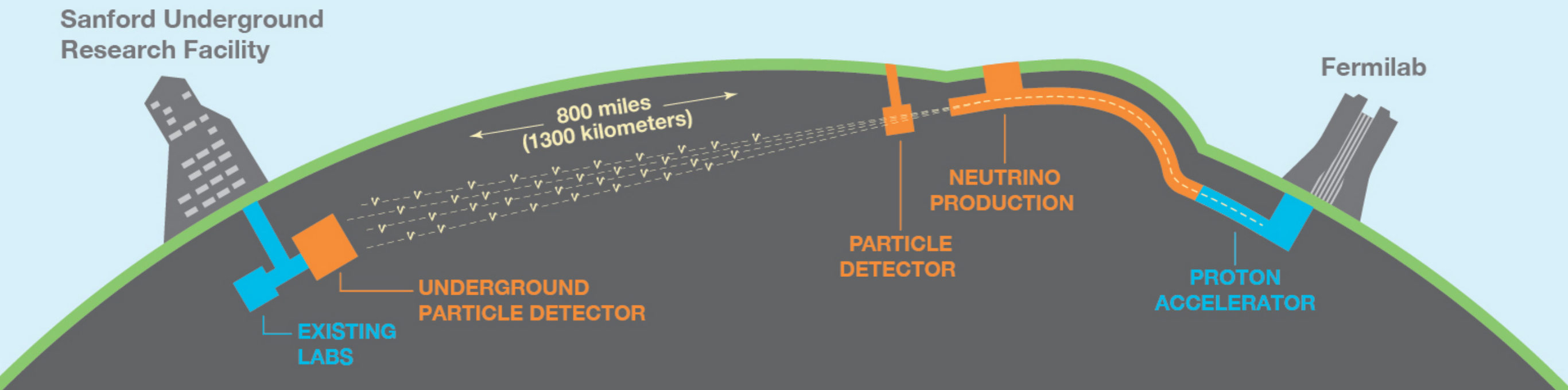
Elastic neutrino-electron scattering in effective field theory



Oleksandr Tomalak

Neutrino experiments

- **DUNE** and Hyper-K: leading-edge ν science experiments



- measurement of ν_μ disappearance and ν_e appearance from count rates

$$N_\nu \sim \int d\omega \Phi_\nu(\omega) \times \sigma(\omega) \times R(\omega, \omega_{\text{rec}})$$

- determination of neutrino fluxes:
one of the main goals of Near Detector

Elastic neutrino-electron scattering

- historic channel in discovery of weak neutral currents
Gargamelle (1973)
- measurement of total cross section and Weinberg angle
CHARM (1988), CHARM-II (1994), BNL-E734 (1990), LAMPF (1993), LSND (2001)
- solar neutrino studies
Kamiokande, SNO, Super-Kamiokande I-IV, Borexino, Hyper-Kamiokande
- reactor antineutrino studies
Savannah river (1976), Krasnoyarsk (1990), Rovno (1993), MUNU (2005), TEXONO (2010), GEMMA (2012)

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- small cross section scales as electron mass m :
 10^{-4} - 10^{-3} of cross section on nucleons and nuclei
- scattering on atomic electrons: standard candle to constrain flux
uncertainty from 9% to 6% and from 7.5% to 4%
MINERvA (2016, 2019), NOvA analysis is ongoing

- relatively clean tool to constrain neutrino flux in DUNE
- goal: EFT-based calculation with accuracy below %

Neutrino scattering in EFT. Matching

- tree-level matching to low-energy EFT:

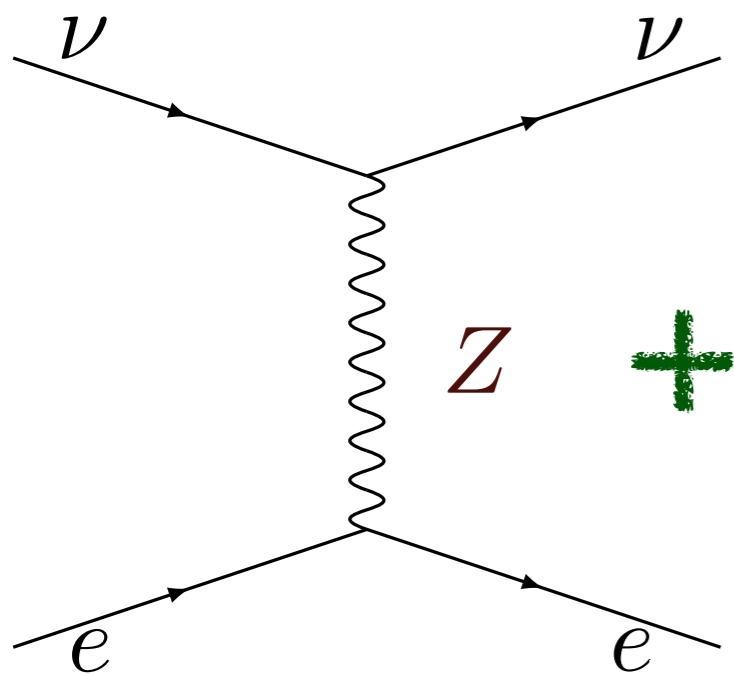
$$\mathcal{L}_{\text{eff}} = -\bar{\nu}\gamma_{\mu}P_L\nu \cdot \bar{e}\gamma^{\mu}(c_L P_L + c_R P_R)e$$

$$c_R = 2\sqrt{2}G_F \sin^2 \theta_W \quad c_L = 2\sqrt{2}G_F (\sin^2 \theta_W - 0.5 + \delta_{\nu,\nu_e})$$

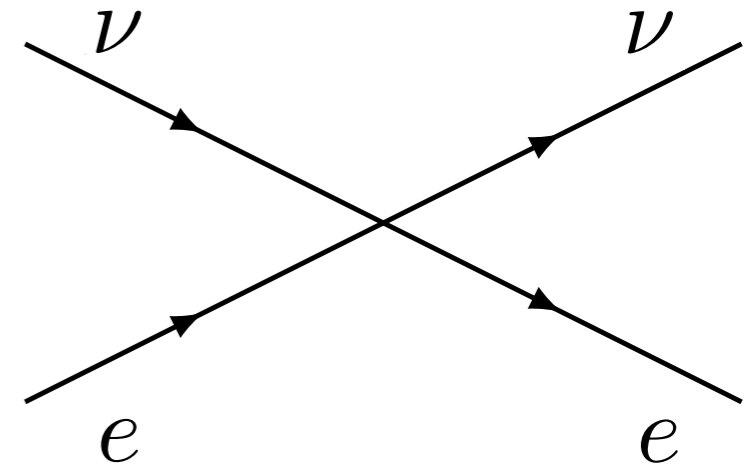
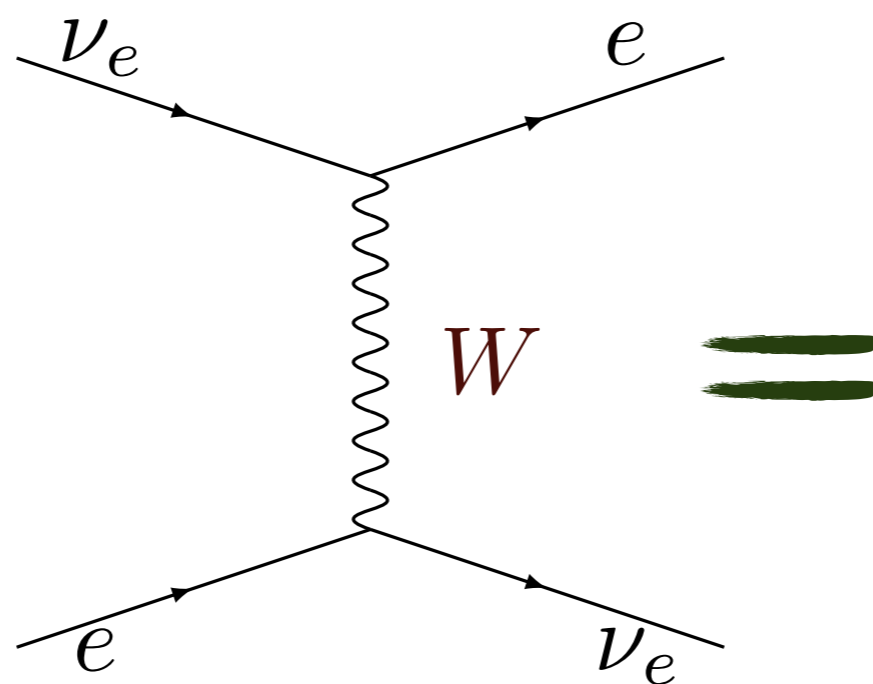
Weinberg (1967), 't Hooft (1971)

- projectors on chiral states: $P_L = \frac{1 - \gamma_5}{2}$ $P_R = \frac{1 + \gamma_5}{2}$

neutral current



charged current



Neutrino scattering in EFT. Matching

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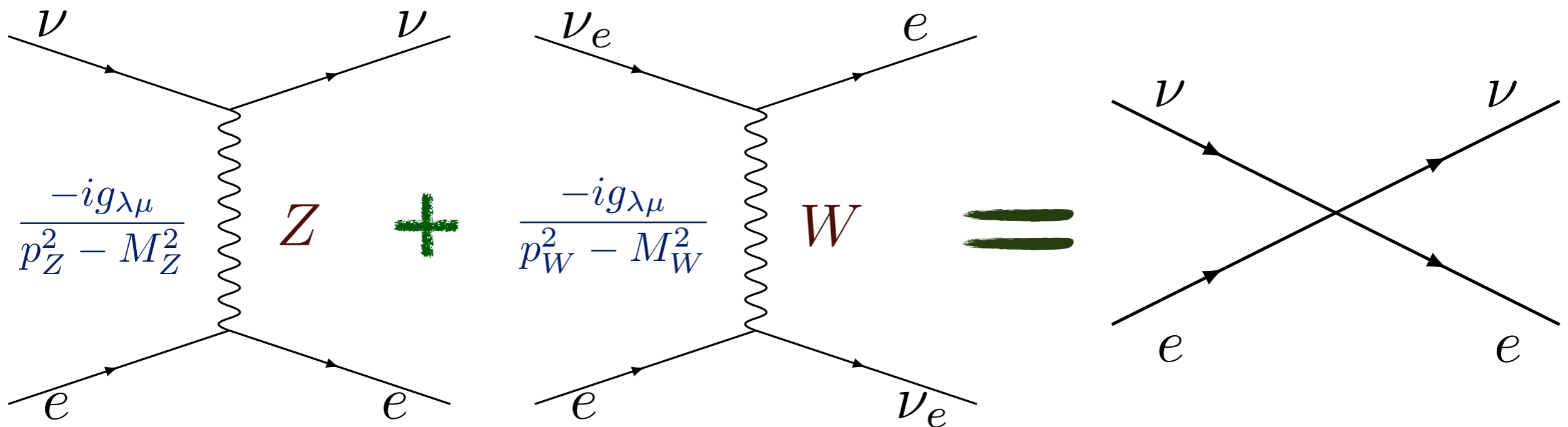
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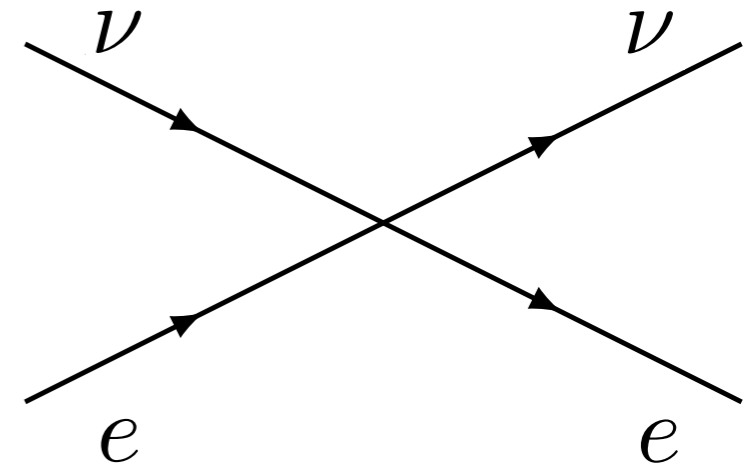
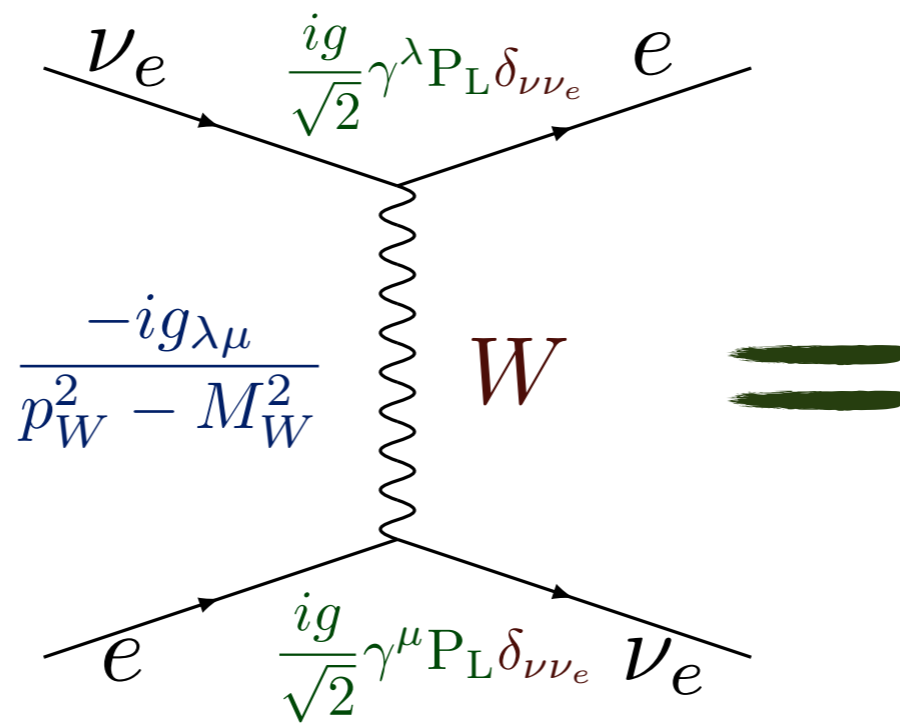
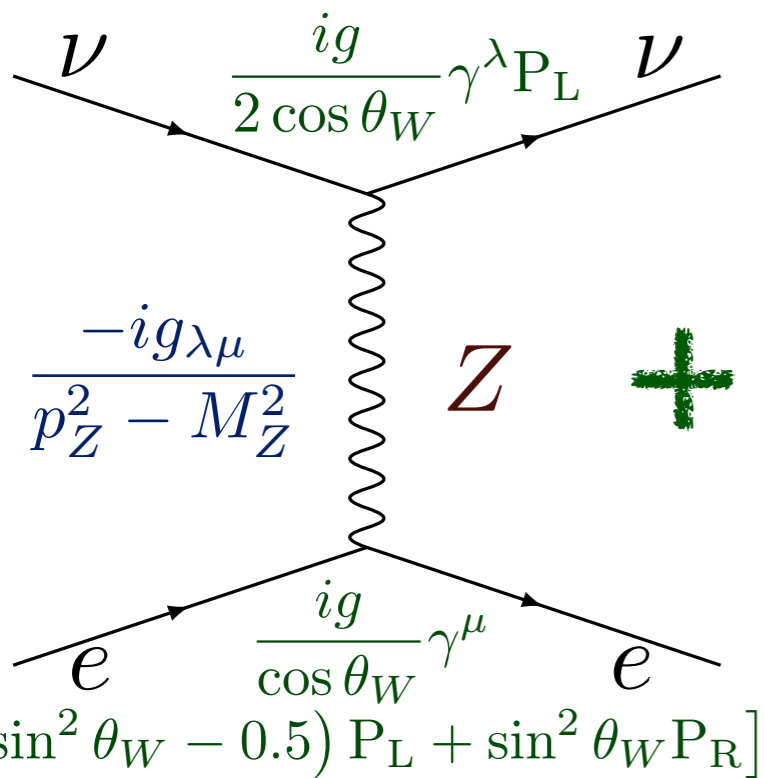
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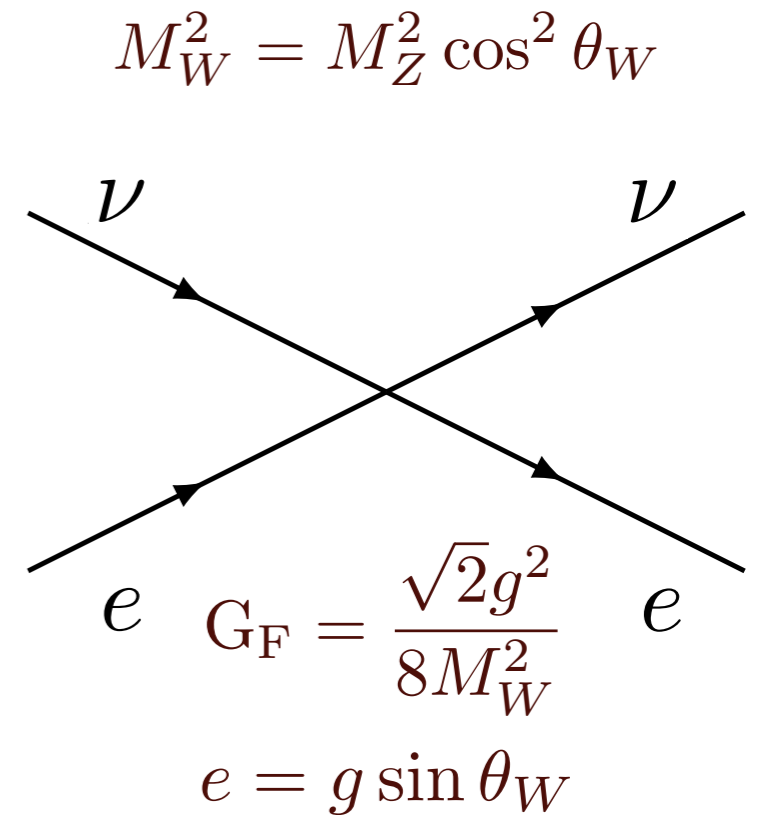
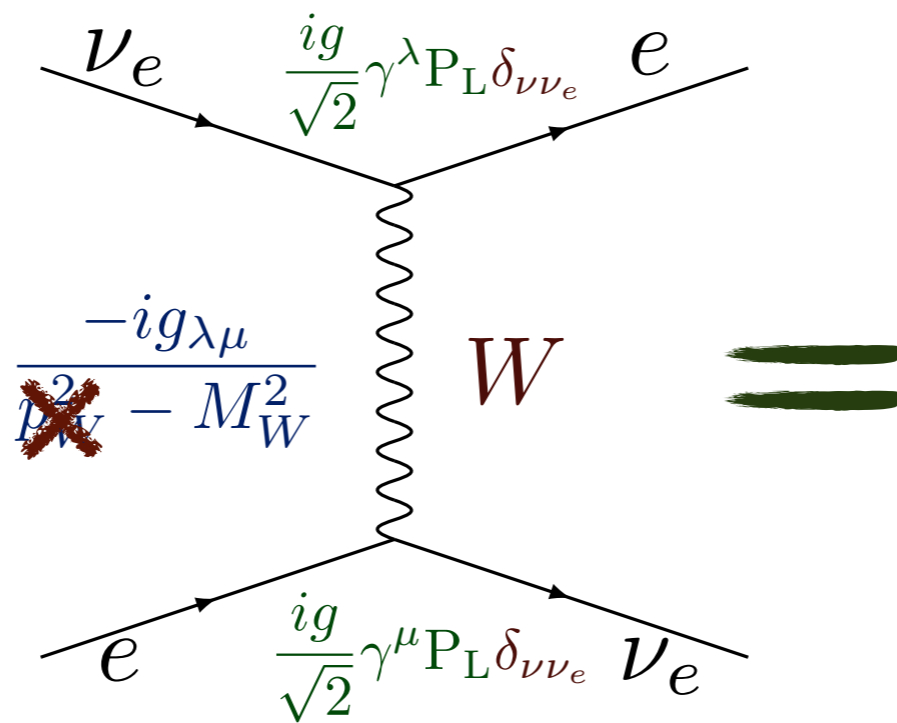
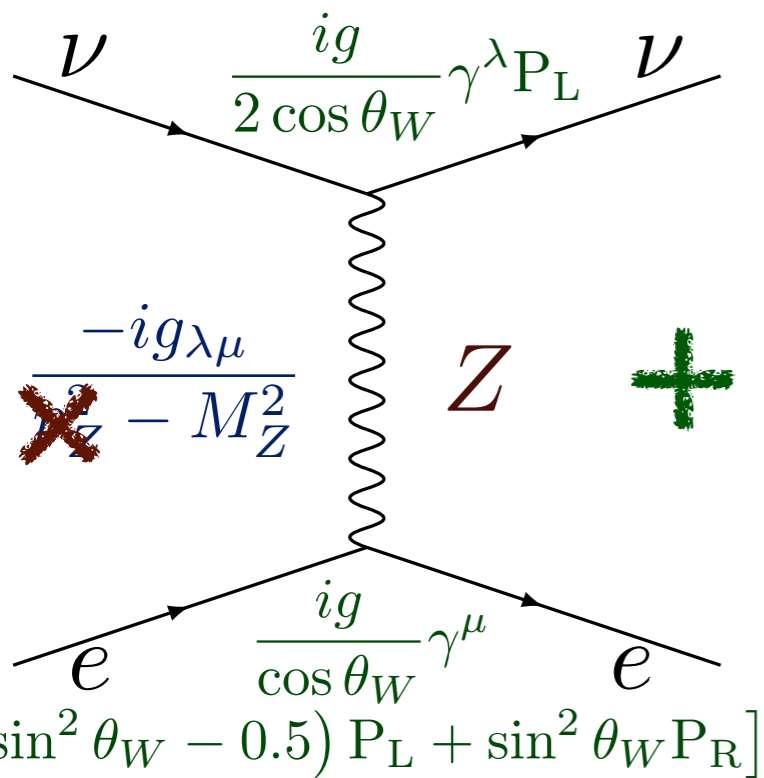
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- projectors on chiral states: $P_L = \frac{1 - \gamma_5}{2}$ $P_R = \frac{1 + \gamma_5}{2}$

neutral current

charged current



- integrate out W and Z at tree level

Neutrino scattering in EFT. Matching

- tree-level matching to low-energy EFT:

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Weinberg (1967), 't Hooft (1971)

- consider only leading in G_F terms: loop corrections in α , α_s
- matching of amplitudes, renormalized in $\overline{\text{MS}}$ scheme:

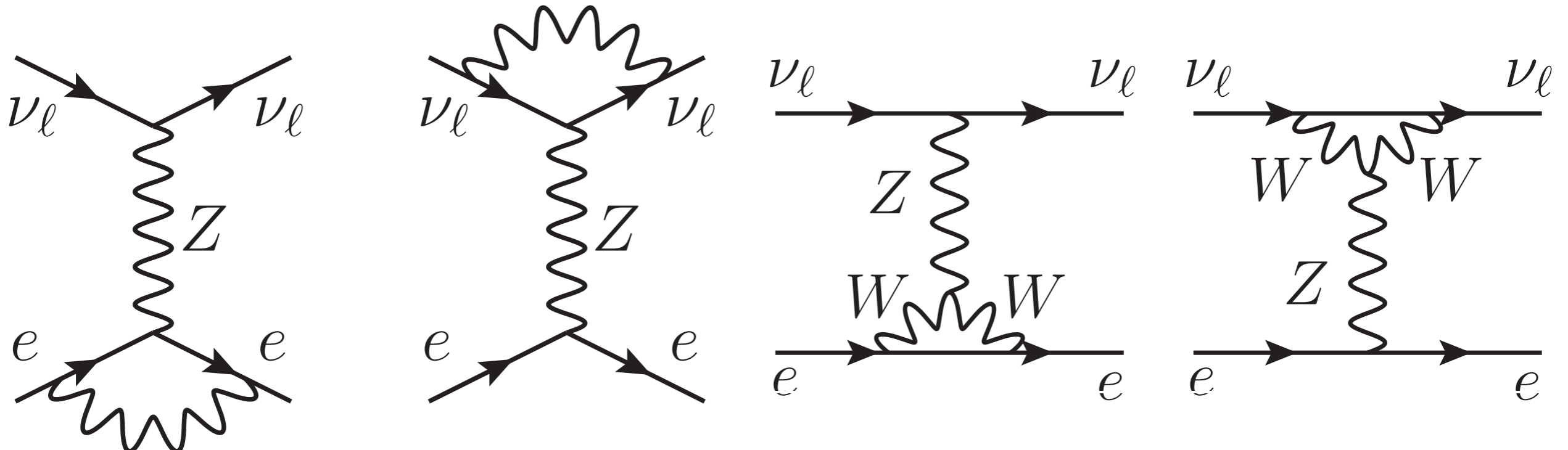
$$\mathcal{M}^{\text{SM}}(\mu) = \mathcal{M}^{\text{EFT}}(\mu) \quad \mu = M_Z$$

- neglect fermion masses besides top quark

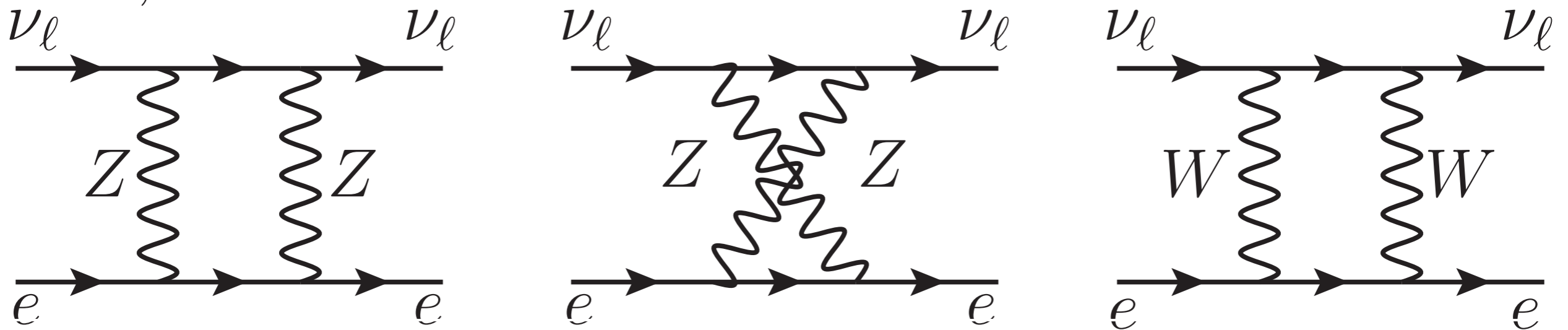
$$G_F = \frac{\sqrt{2}g^2}{8M_W^2}$$

Neutral current in SM

Z, W

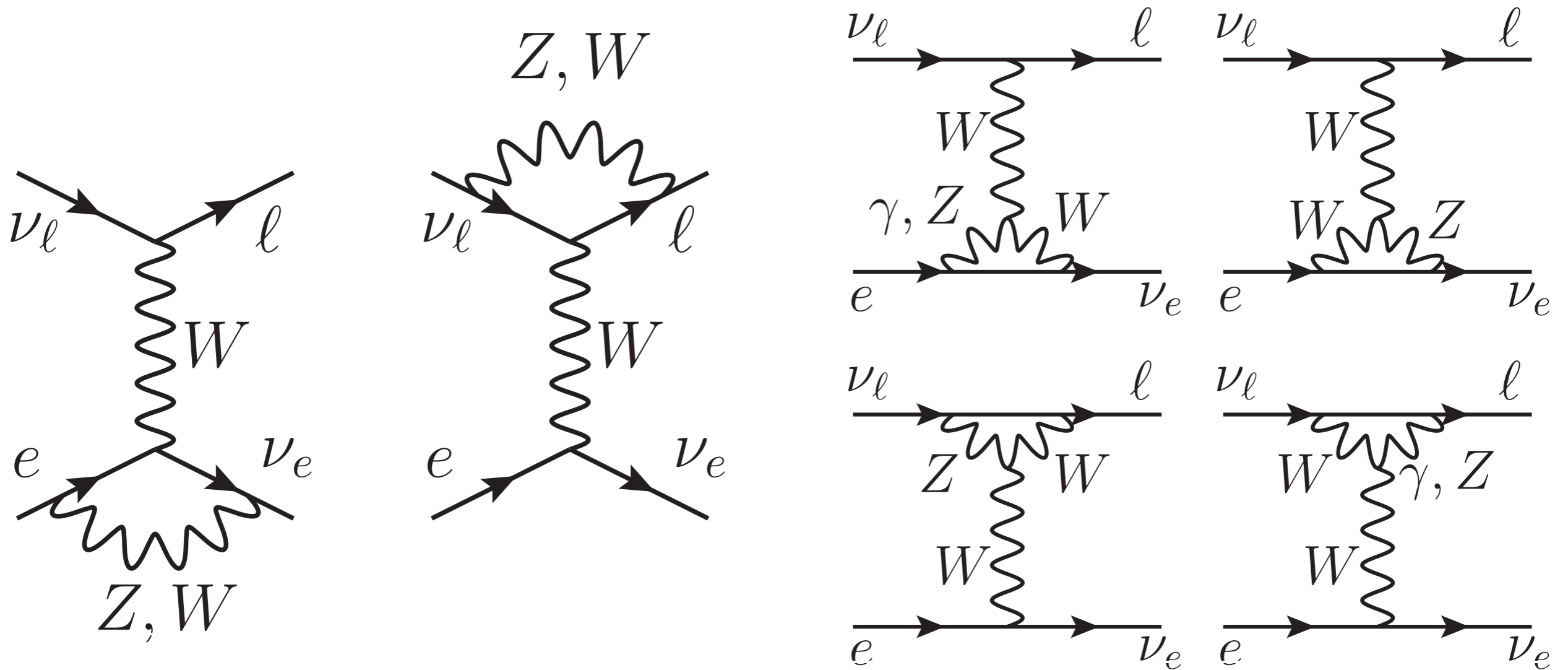


Z, W



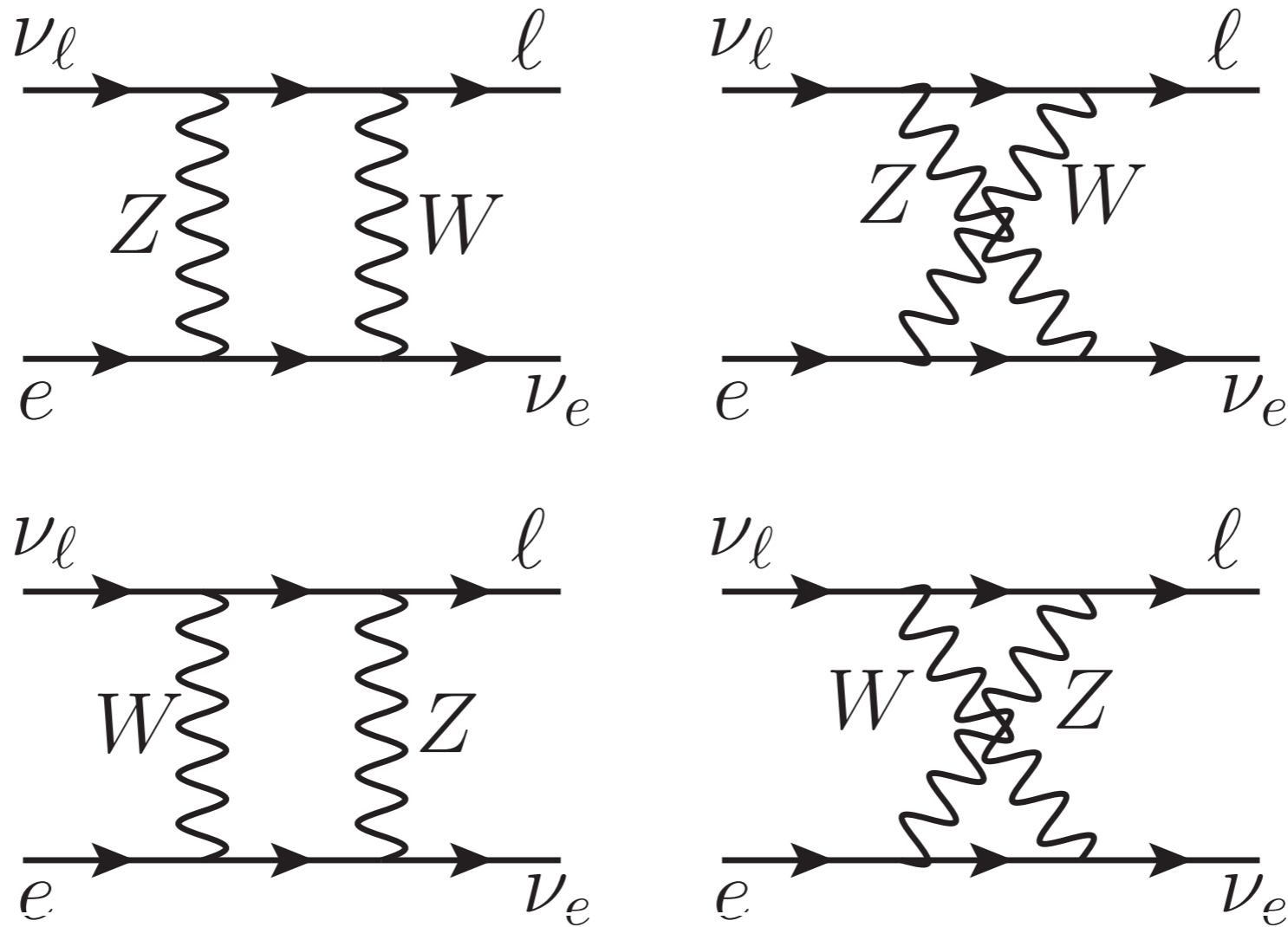
- contribution to effective couplings

Charged current in SM. Vertices



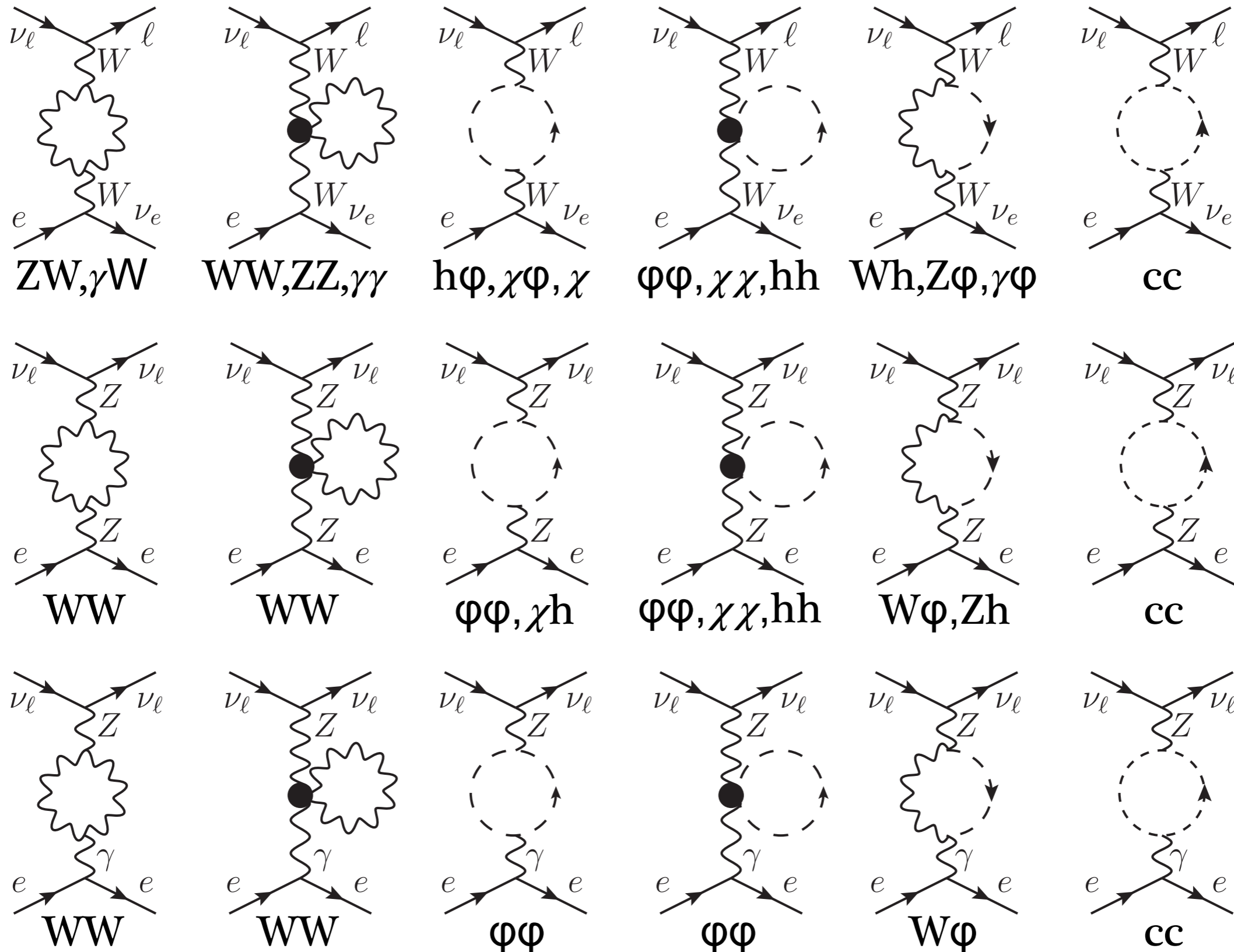
- contribution to effective couplings

Charged current in SM. Boxes

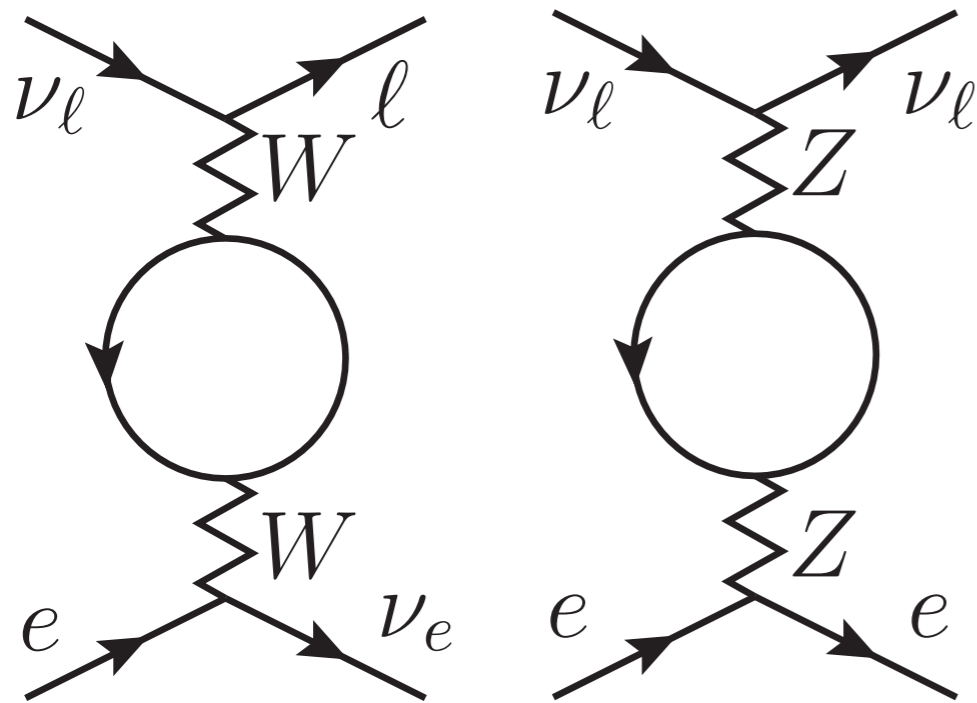


- contribution to effective couplings

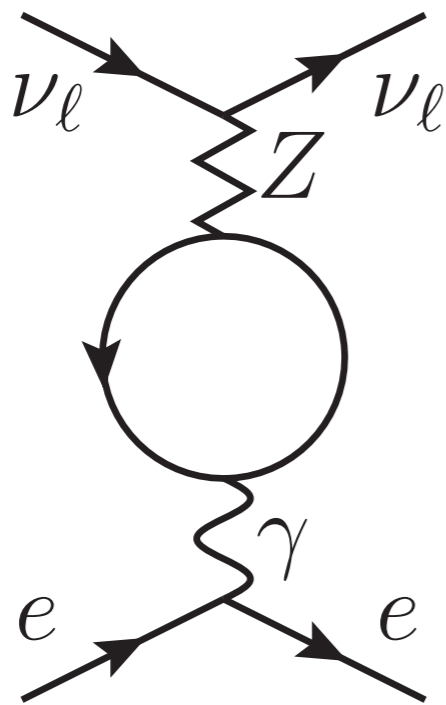
Self energy and γZ mixing. Boson loops



Self energy and γZ mixing. Fermion loops

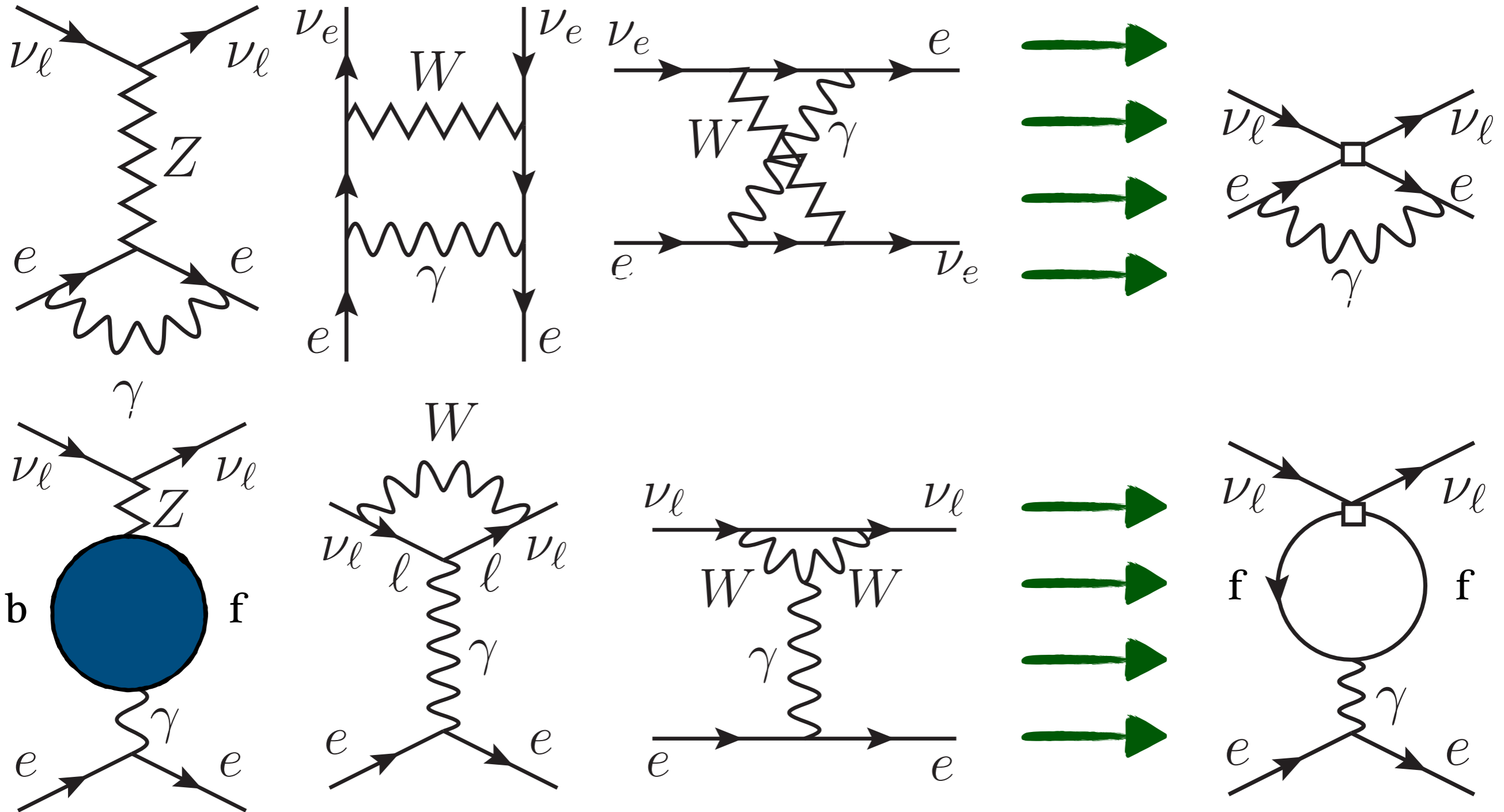


- vanishing contribution to matching besides loops with t quark
- finite contribution to self energy pole vs $\overline{\text{MS}}$ masses
- do not consider Higgs tadpoles (hVV): matching vs self energy cancellation



- gauge-independent contribution
- cancels in matching conditions

Matching at one loop \longrightarrow EFT side



- cancellation of infrared and collinear singularities
- Z and W Z-factors are absent in EFT

Neutrino scattering in EFT. Matching

- tree-level matching to low-energy EFT:

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$$c_R = 2\sqrt{2}G_F \sin^2 \theta_W \quad c_L = 2\sqrt{2}G_F (\sin^2 \theta_W - 0.5 + \delta_{\nu,\nu_e})$$

Weinberg (1967), 't Hooft (1971)

- consider only leading in G_F terms: loop corrections in α , α_s
- gauge-invariant matching of amplitudes, renormalized in $\overline{\text{MS}}$ scheme:

$$\mathcal{M}^{\text{SM}}(\mu) = \mathcal{M}^{\text{EFT}}(\mu) \quad \mu = M_Z$$

- the same tree-level operators after one-loop matching
- exploit G_F for one combination of electroweak parameters

$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \quad G_F = \frac{\sqrt{2}g^2}{8M_W^2}$$

MULAN (2012)

- matching at order $\alpha\alpha_s$: left- and right-handed couplings
- muon lifetime measurement improves precision

Running to low scales

M_Z - integrate out top at Z scale

- PDG running for α , α_s

- only one EFT coupling changes with scale

m_b

m_τ - integrate out GeV particles

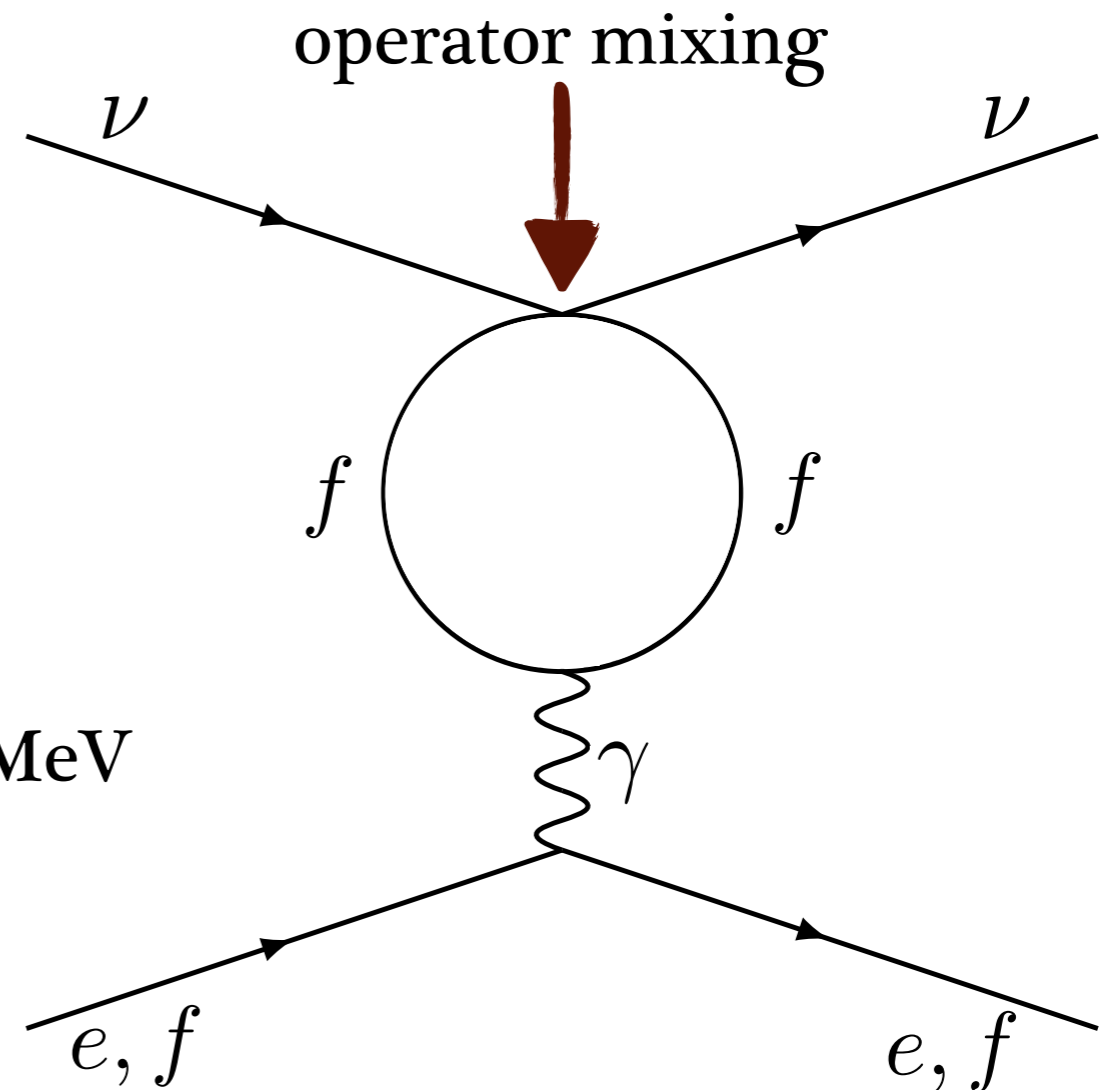
m_c

- α_s becomes too strong

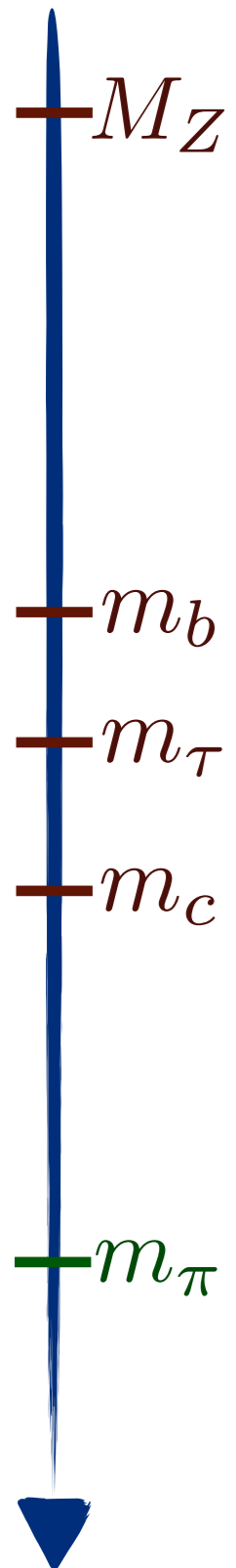
- hadronic physics down to 140 MeV

m_π

- theory with leptons



Scale-independent combinations



$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_l \gamma_\mu P_L \nu_l \cdot \bar{f} \gamma^\mu (c_L^{\nu_l f} P_L + c_R^{\nu_l f} P_R) f$$

- chirally-symmetric universal running + flavor symmetry provide 9 constraints on running in the theory:

$$c_R^b(\mu) - c_R^d(\mu) = 0,$$

$$c_L^{\nu_e e}(\mu) - c_L^{\nu_\mu e}(\mu) = 2\sqrt{2}G_F,$$

$$c_L^{\nu_\mu e}(\mu) - c_R(\mu) = -\sqrt{2}\tilde{G}_F,$$

$$c_L^u(\mu) - c_R^u(\mu) = \sqrt{2}\tilde{G}_F^u,$$

$$c_L^d(\mu) - c_R^d(\mu) = -\sqrt{2}\tilde{G}_F^d,$$

$$c_L^b(\mu) - c_R^b(\mu) = -\sqrt{2}\tilde{G}_F^b,$$

$$c_L^d(\mu) - c_L^b(\mu) + c_R^d(\mu) - c_R^b(\mu) = \sqrt{2}\tilde{G}_F^b - \sqrt{2}\tilde{G}_F^d,$$

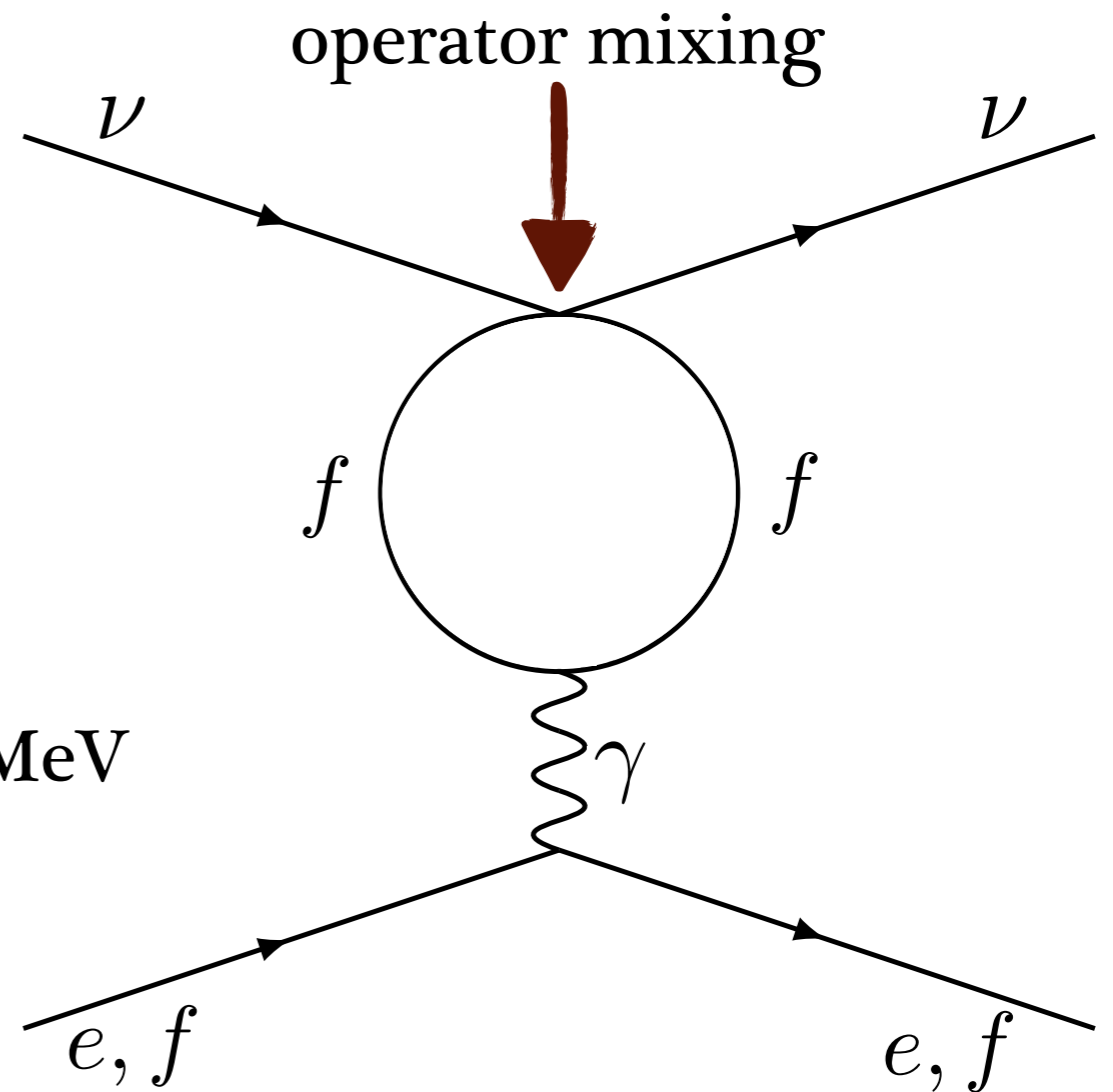
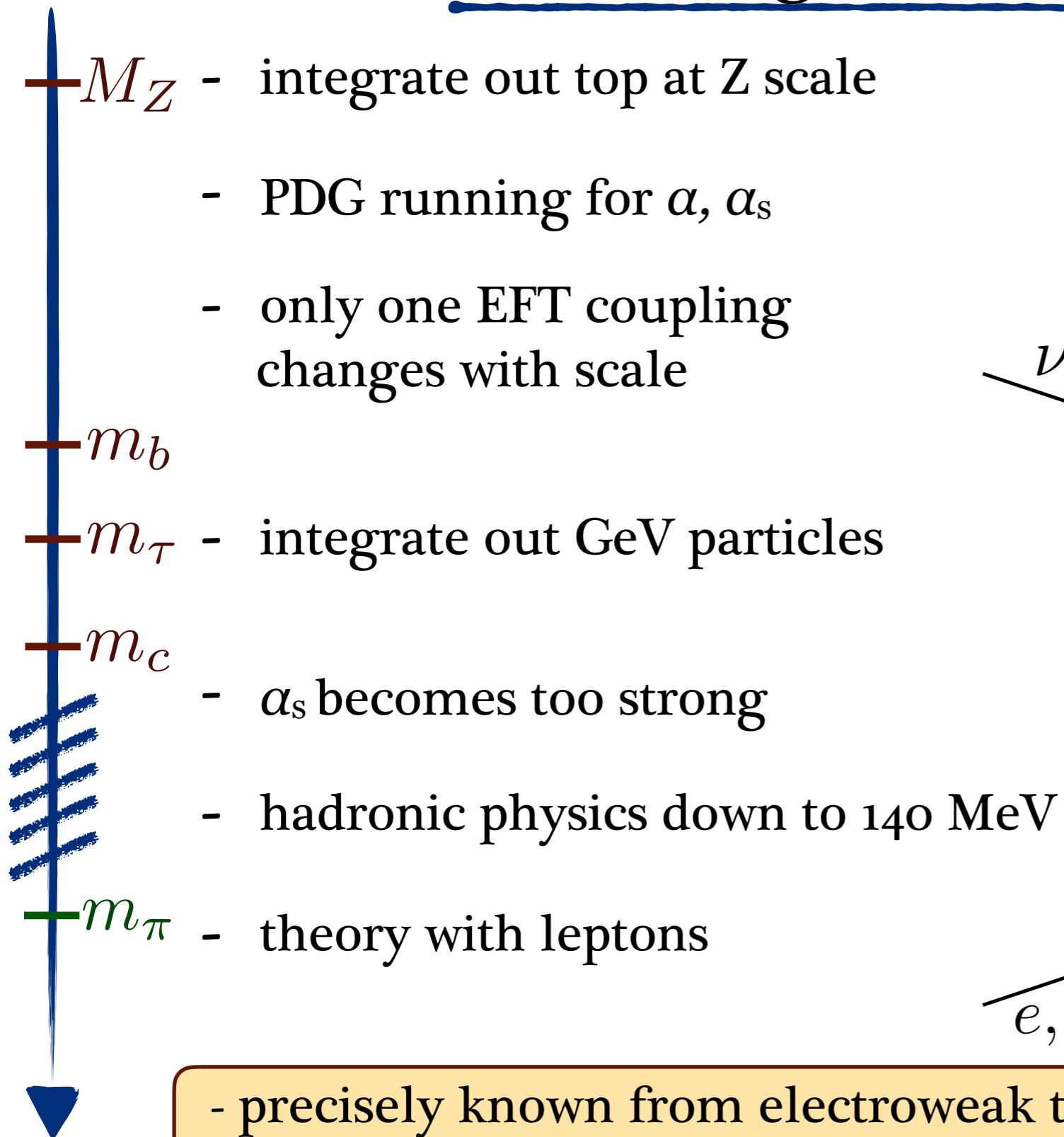
$$4c_R^d(\mu) + 2c_R^u(\mu) = 2\sqrt{2}\tilde{G}_F^d - \sqrt{2}\tilde{G}_F^u - \sqrt{2}\tilde{G}_F^q,$$

$$6c_R^d(\mu) - 2c_R(\mu) = 3\sqrt{2}\tilde{G}_F^d - \sqrt{2}\tilde{G}_F - 2\sqrt{2}\tilde{G}_F^l.$$

- one Fermi coupling and one c_R at leading order

- only 1 effective coupling changes with scale

Running to low scales



- precisely known from electroweak to hadronic scales
 - only 1 effective coupling changes with scale

Running to low scales

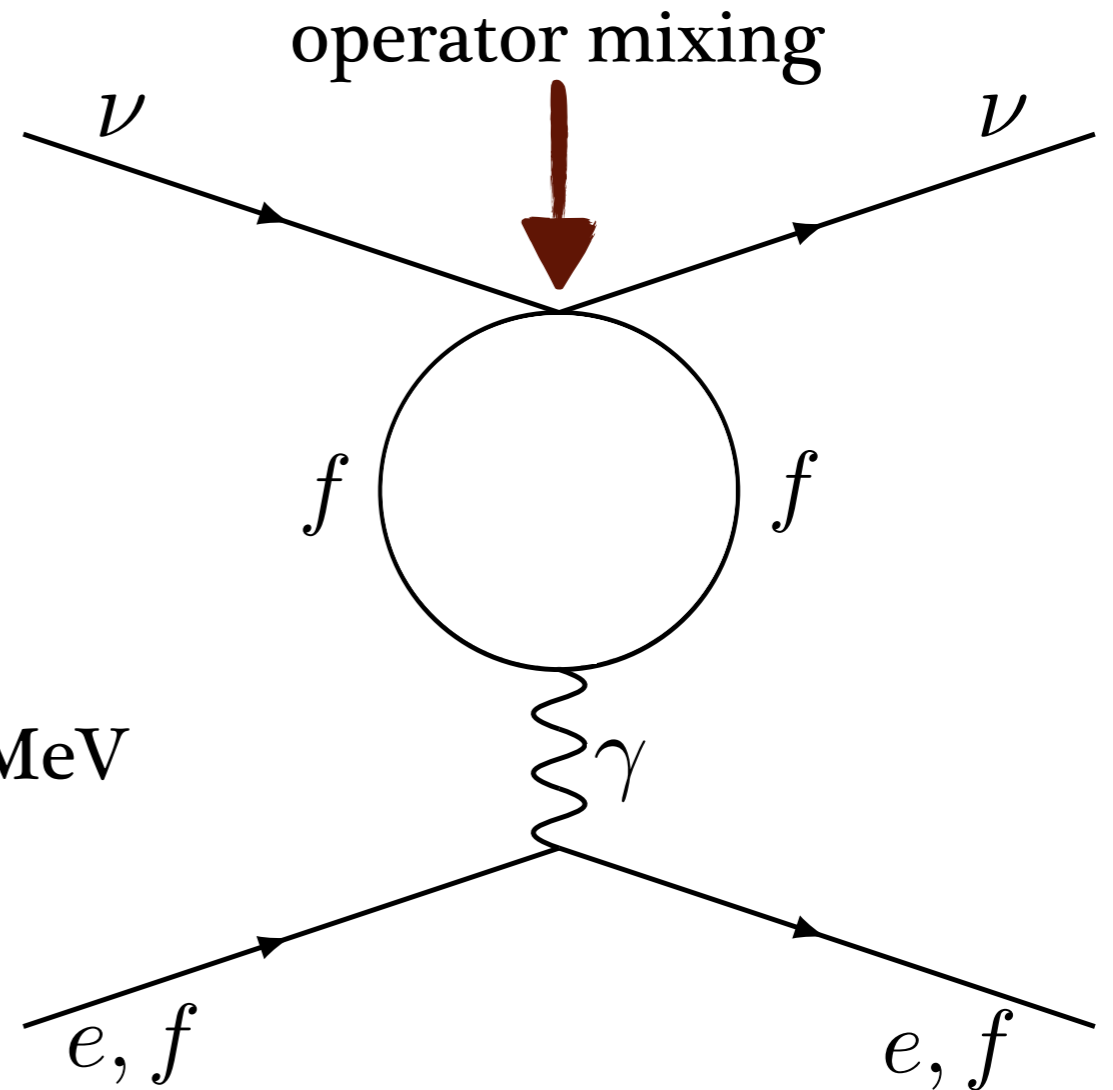
- M_Z - integrate out top at Z scale
- PDG running for α , α_s
- only one EFT coupling changes with scale

$$c_L^{\nu e e} : 2.388 \rightarrow 2.398$$

$$c_L^{\nu \mu e} : -0.911 \rightarrow -0.901 \quad \% \text{ effect}$$

$$c_R : 0.759 \rightarrow 0.769$$

- m_b
- m_τ - integrate out GeV particles
- m_c
- α_s becomes too strong
- hadronic physics down to 140 MeV
- m_π - theory with leptons

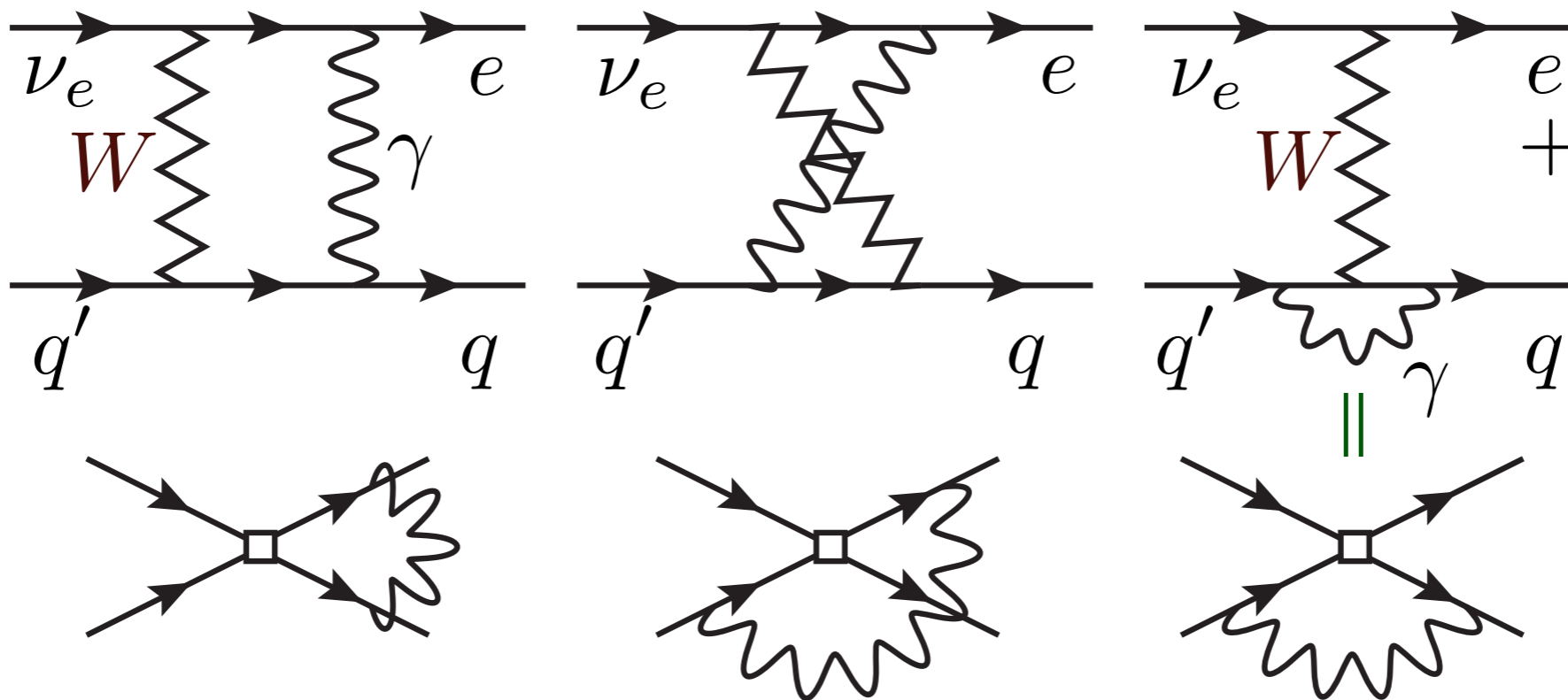


- precisely known from electroweak to hadronic scales
- only 1 effective coupling changes with scale

Semileptonic operators and muon decay

$$M_Z \quad \mathcal{L}_{\text{eff}} = -2\sqrt{2}G_F \sum_{l \neq l'} \bar{\nu}_{l'} \gamma^\mu P_L \nu_l \bar{l} \gamma_\mu P_L l' - c^{qq'} \sum_{q \neq q'} \bar{l} \gamma^\mu P_L \nu_l \bar{q} \gamma_\mu P_L q'$$

- Fermi coupling is scale independent



scheme independent

scheme dependent for quarks

- NDR scheme for γ_5 with $a=-1$ for evanescent operators E

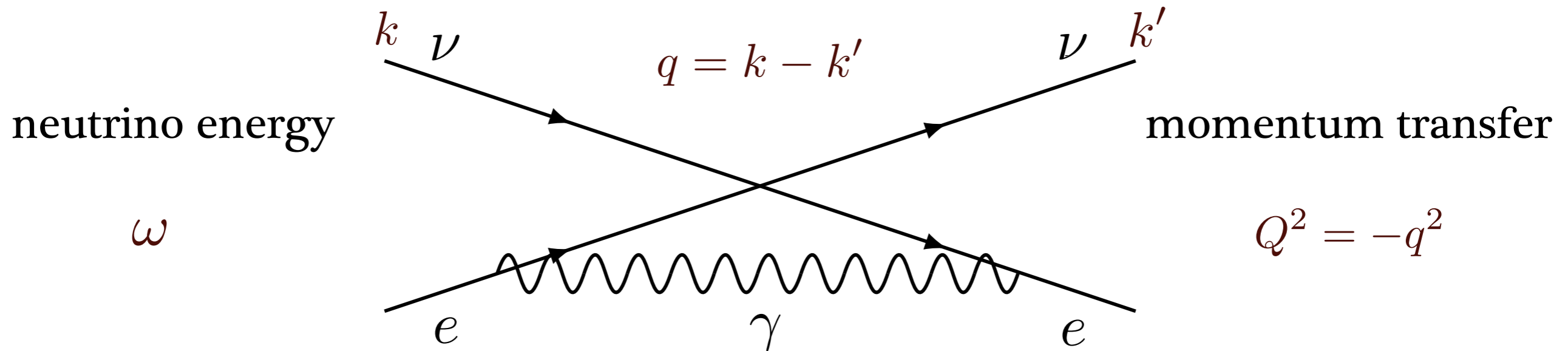
$$\gamma^\alpha \gamma^\beta \gamma^\mu P_L \otimes \gamma_\mu \gamma_\beta \gamma_\alpha P_L = 4(1 + a(4 - d)) \gamma^\mu P_L \otimes \gamma_\mu P_L + E(a)$$

Buras and Weisz (1990)

- Wilson coefficient of semileptonic operator depends on scale

- scheme dependence of 1-loop matching and 2-loop running

Virtual QED corrections. Vertex



- up to suppressed by neutrino mass terms:

$$\bar{e}\gamma^\mu e \rightarrow \frac{\alpha}{\pi} \bar{e} \left[f_1(Q^2) \gamma^\mu + f_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m} \right] e$$

$$\bar{e}\gamma^\mu \gamma_5 e \rightarrow \frac{\alpha}{\pi} [f_1(Q^2) - f_2(Q^2)] \bar{e}\gamma^\mu \gamma_5 e$$

- infrared divergence cancels with radiation of real soft photon
- factorizable in limit of small electron mass: $f_2 = 0$

- given in terms of QED form factors at one loop

Virtual QED corrections. Fermion loop

- charged fermions contribute to elastic scattering at one loop:

$$\mathcal{L}_{\text{eff}}^f = -\bar{\nu}\gamma_\mu P_L \nu \cdot \bar{f}\gamma^\mu (c_L^f P_L + c_R^f P_R) f$$

- adds vector contribution:

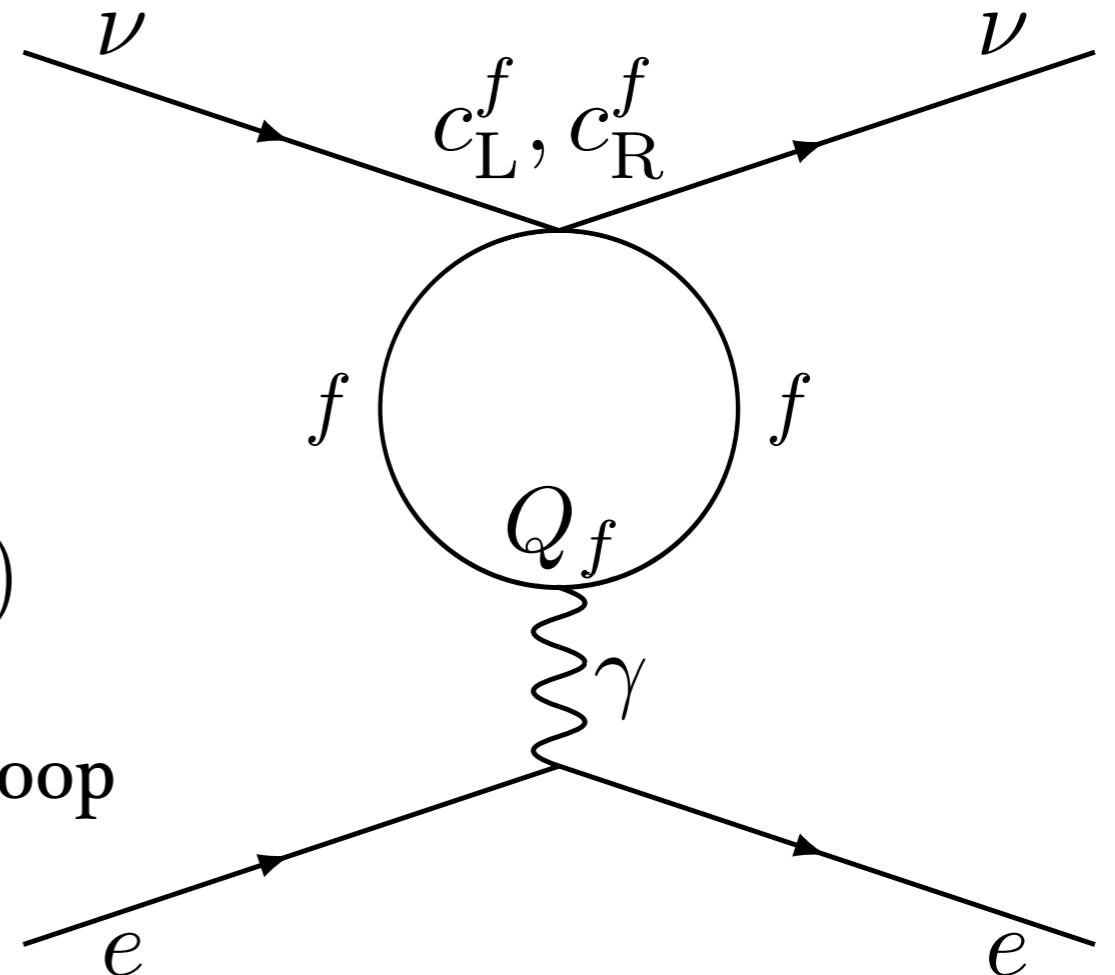
$$c^f \bar{\nu}\gamma_\mu P_L \nu \cdot \bar{e}\gamma^\mu e$$

- fermion-dependent correction:

$$c^f = -\frac{\alpha}{2\pi} Q_f (c_L^f + c_R^f) \Pi(Q^2, m_f)$$

account for one gluon in quark loop

use MS masses for heavy quarks



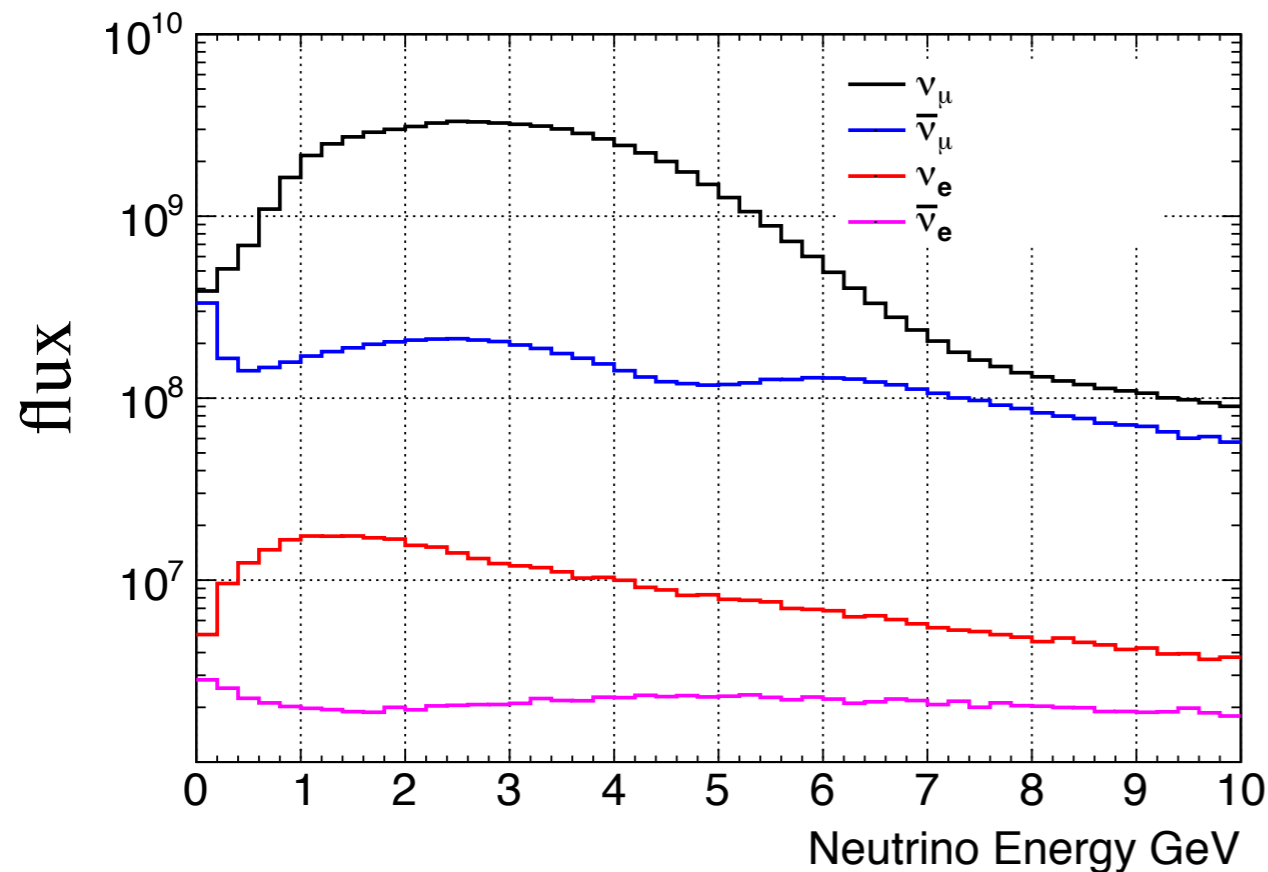
- given in terms of photon vacuum polarization
- sum over all particles with **corresponding couplings**

Main theoretical uncertainty

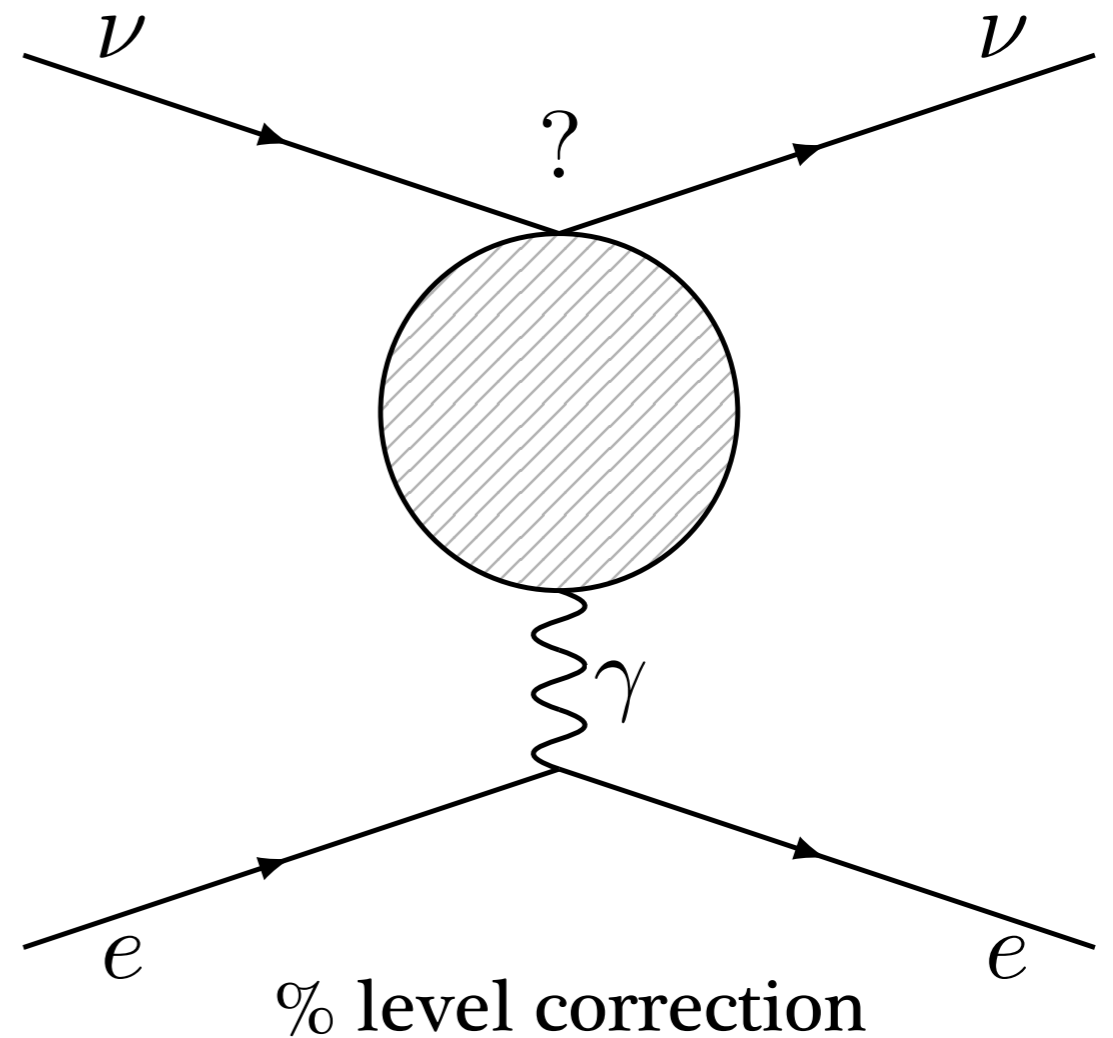
- momentum transfer is suppressed by electron mass:

$$Q^2 < 2m\omega \ll \Lambda_{\text{QCD}}^2$$

- description in terms of quarks is questionable:



DUNE, CDR (2015)



- hadronic correction is the main error in theory

Light-quark contribution

- virtual-loop scale is well below muon and hadron masses:

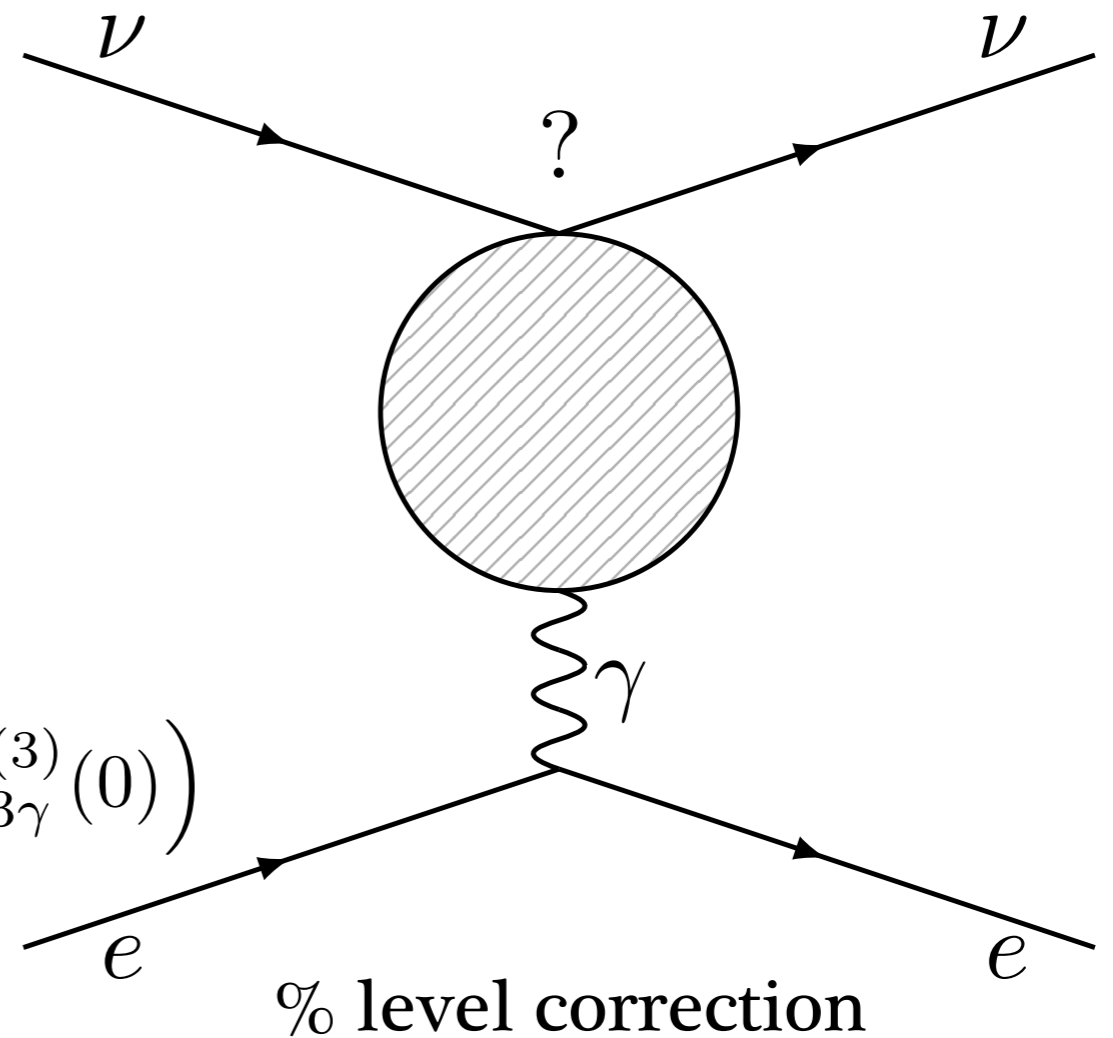
$$Q^2 < 2m\omega \ll \Lambda_{\text{QCD}}^2$$

- adds vector contribution:

$$c^f \bar{\nu} \gamma_\mu P_L \nu \cdot \bar{e} \gamma^\mu e$$

- fermion-dependent correction:

$$c^f = \frac{\sqrt{2}\alpha G_F}{\pi} \left(2 \sin^2 \theta_W \hat{\Pi}_{\gamma\gamma}^{(3)}(0) - \hat{\Pi}_{3\gamma}^{(3)}(0) \right)$$



- hadronic correlators at zero momentum transfer

Light-quark contribution

- vector-vector correlation functions in terms of quark contributions

$$\hat{\Pi}_{\gamma\gamma}^{(3)} = \sum_{i,j} Q_i Q_j \Pi^{ij}$$

$$\hat{\Pi}_{3\gamma}^{(3)} = \sum_{i,j} T_i^3 Q_j \Pi^{ij}$$

- $SU(3)_f$ $\hat{\Pi}^{uu} = \hat{\Pi}^{dd} = \hat{\Pi}^{ss}$

$$\hat{\Pi}_{3\gamma}^{(3)} = \hat{\Pi}_{\gamma\gamma}^{(3)}$$

- $SU(2)_f + \text{OZI}$ $\hat{\Pi}^{ud} = \hat{\Pi}^{us} = \hat{\Pi}^{sd} = \hat{\Pi}^{ss} = 0$

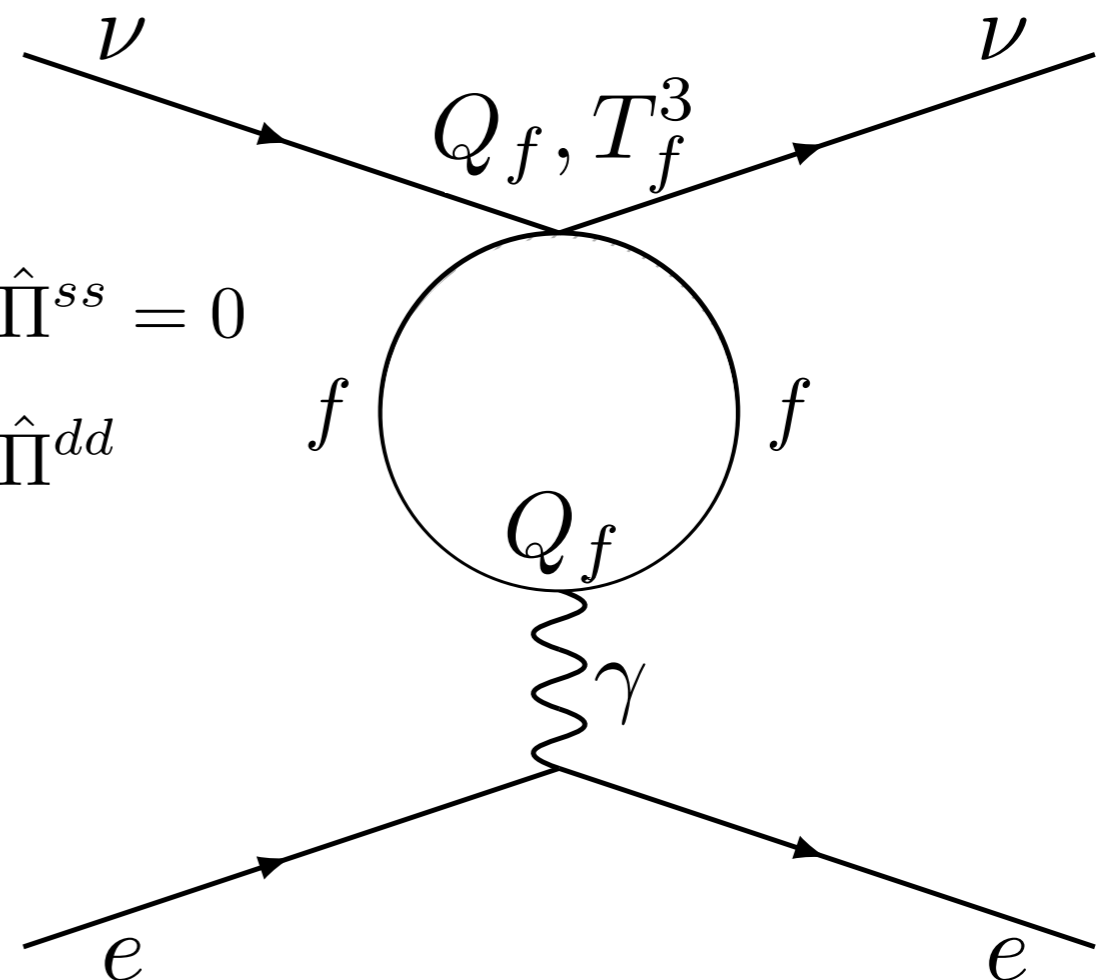
$$\hat{\Pi}_{3\gamma}^{(3)} = \frac{9}{10} \hat{\Pi}_{\gamma\gamma}^{(3)} \quad \hat{\Pi}^{uu} = \hat{\Pi}^{dd}$$

- our choice:

$$\hat{\Pi}_{3\gamma}^{(3)} = (1 \pm 0.2) \hat{\Pi}_{\gamma\gamma}^{(3)}$$

- with $\hat{\Pi}_{\gamma\gamma}^{(3)}(0)$ from

Erlar et al (2018)



- non-perturbative light-quark contribution

EFT with leptons

- virtual-loop scale is well below muon and hadron masses:

$$Q^2 < 2m\omega \ll \Lambda_{\text{QCD}}^2$$

- QCD degrees of freedom: vector contribution to effective couplings

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_l \gamma_\mu P_L \nu_l \cdot \bar{l}' \gamma^\mu (c_L^{\nu_l l'} P_L + c_R^{\nu_l l'} P_R) l'$$

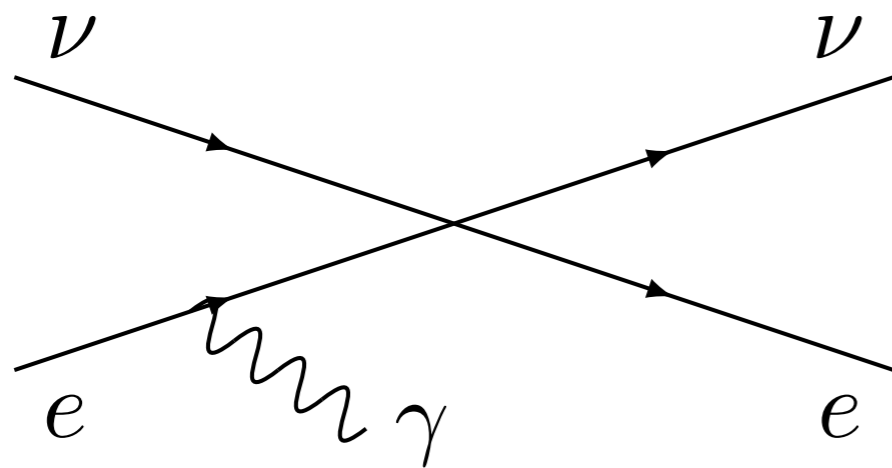
- theory with electron, muon and neutrinos only

	$c_L^{\nu_e e}$	$c_L^{\nu_\mu e}$	$c_R^{\nu_e e}$	$c_R^{\nu_\mu e}$
$\mu = 2 \text{ GeV}$	2.4064(28)	-0.8926(28)	0.7773(28)	0.7773(28)
$\mu = m_\mu$	2.3997(29)	-0.8994(29)	0.7706(29)	0.7706(29)
$\mu = m_e$	2.3865(29)	-0.8988(29)	0.7575(29)	0.7711(29)

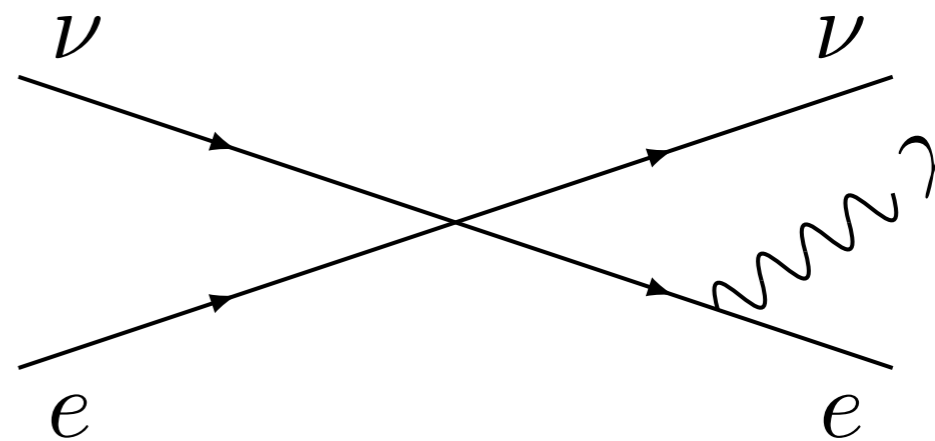
- three right-handed couplings below the muon mass !!!

- neutrino-electron scattering is described by EFT with leptons only

Bremsstrahlung



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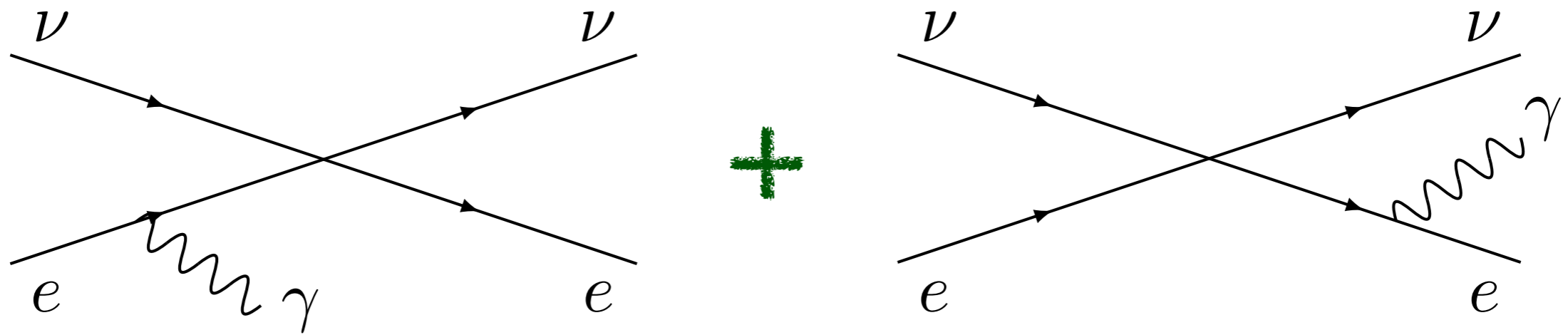


- soft Bremsstrahlung:

$$E_\gamma < \varepsilon$$

$$d\sigma_{\text{LO}}^{\nu e \rightarrow \nu e \gamma} = \delta_s d\sigma_{\text{LO}}^{\nu e \rightarrow \nu e}$$

Bremsstrahlung



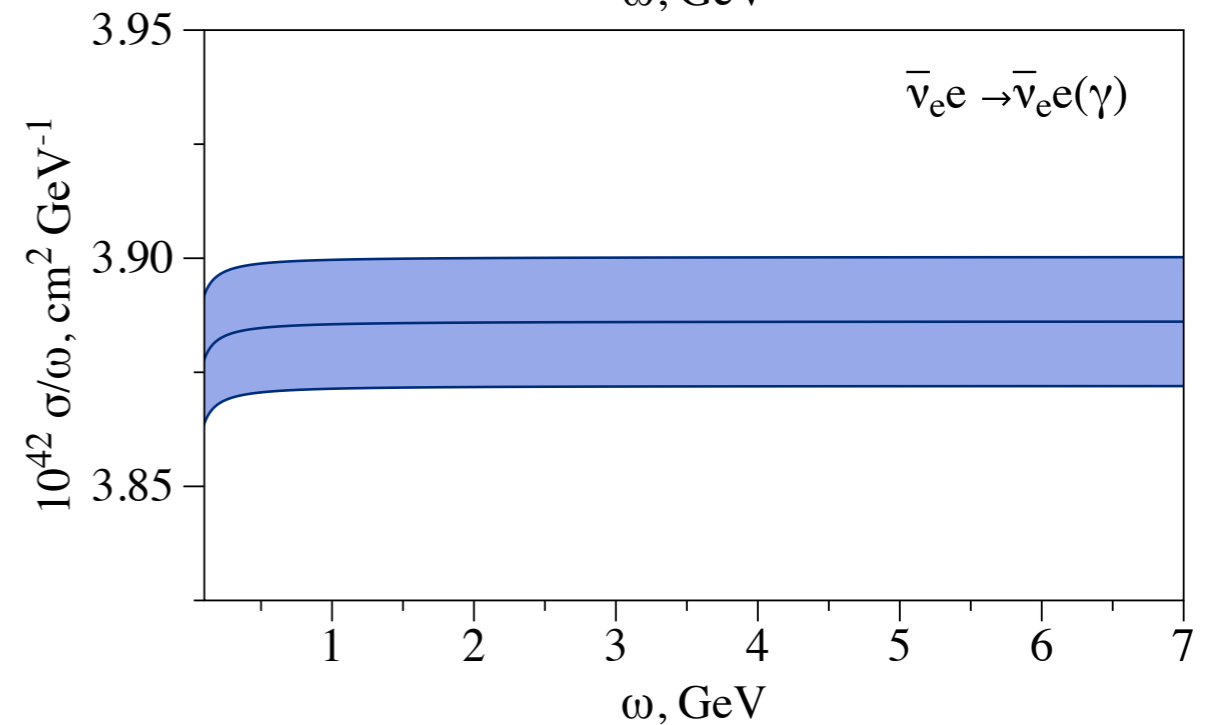
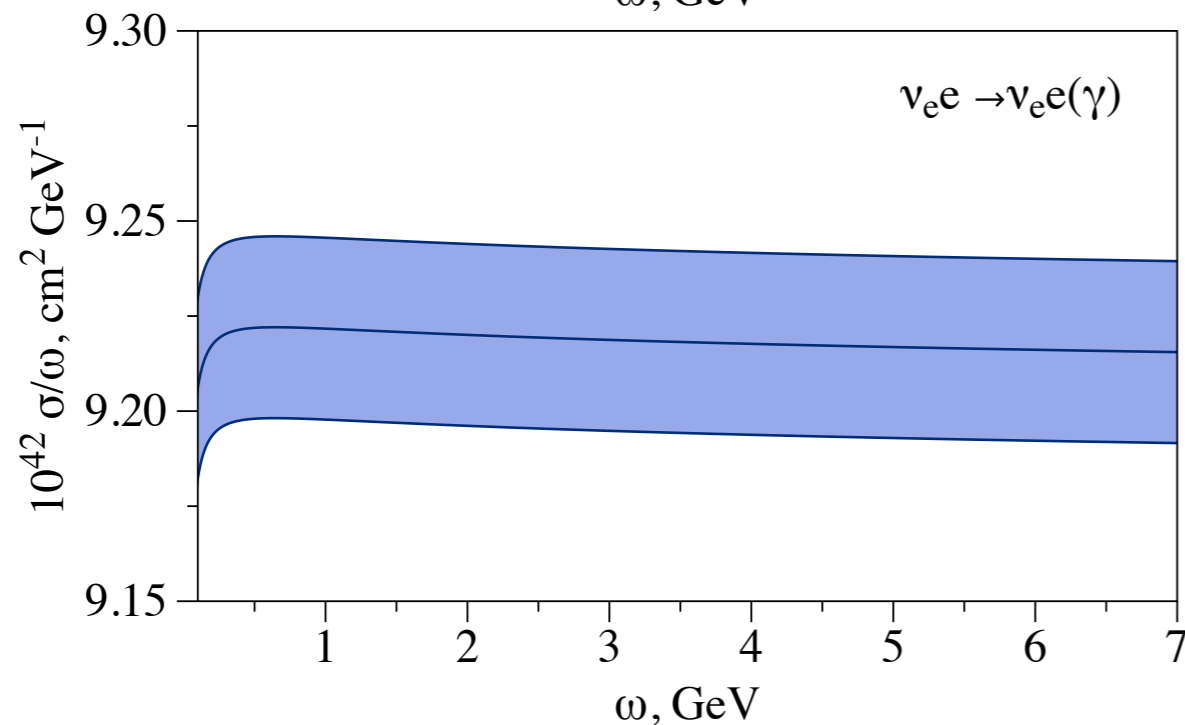
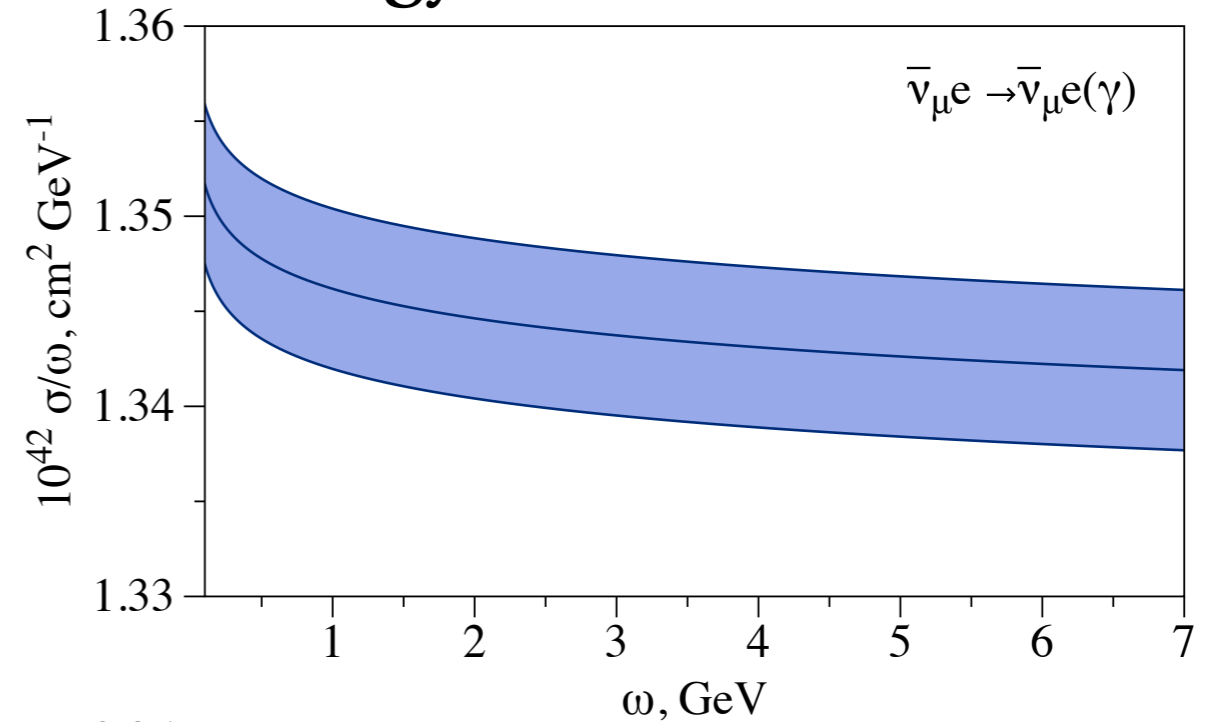
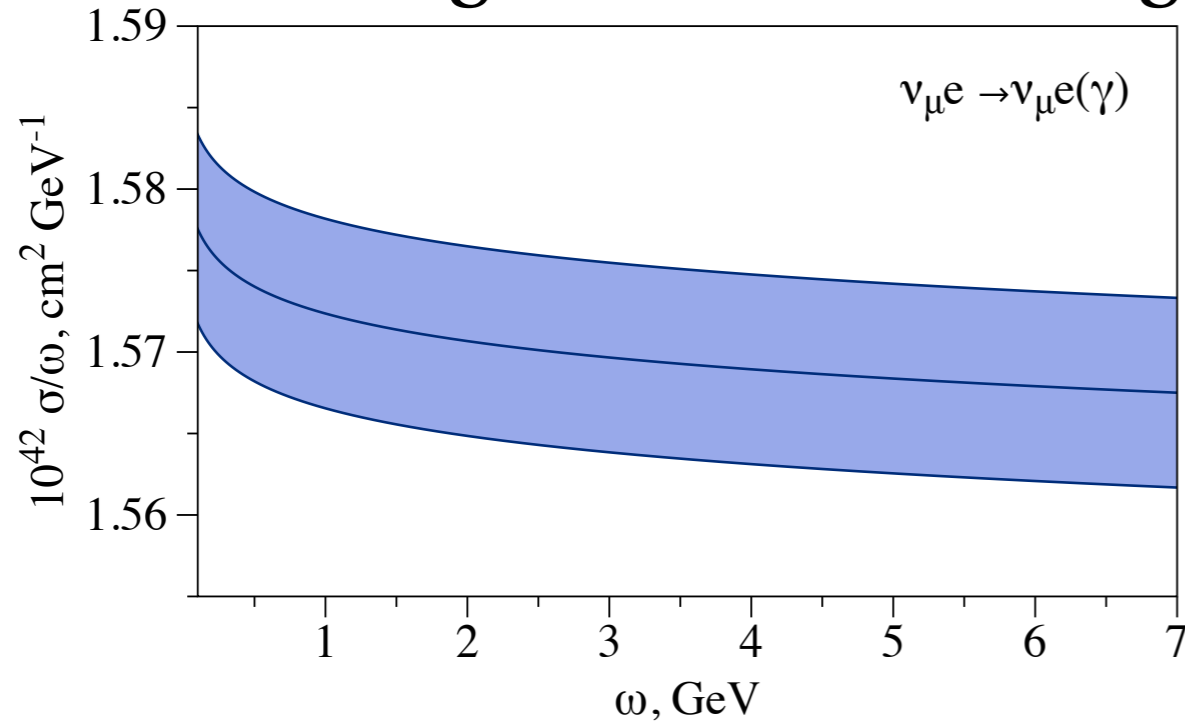
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- soft Bremsstrahlung: $E_\gamma < \varepsilon$ $d\sigma_{LO}^{\nu e \rightarrow \nu e \gamma} = \delta_s d\sigma_{LO}^{\nu e \rightarrow \nu e}$
- soft-photon correction Lee and Sirlin (1964)
- integration technique Ram (1967)
- EW correction Aoiki, Hioki, Kawabe, Konuma and Muta (1980)
- electron energy spectrum and numerically total Aoiki and Hioki (1981)
- electron energy spectrum and EW, small m Sarantakos, Sirlin and Marciano (1982)
- electromagnetic energy spectrum and total Bardin and Dokuchaeva (1983-1985)
- numerically electron and electromagnetic spectra Passera (2000)

- exactly calculable radiation

Absolute cross section

- linear growth with incoming neutrino energy ω

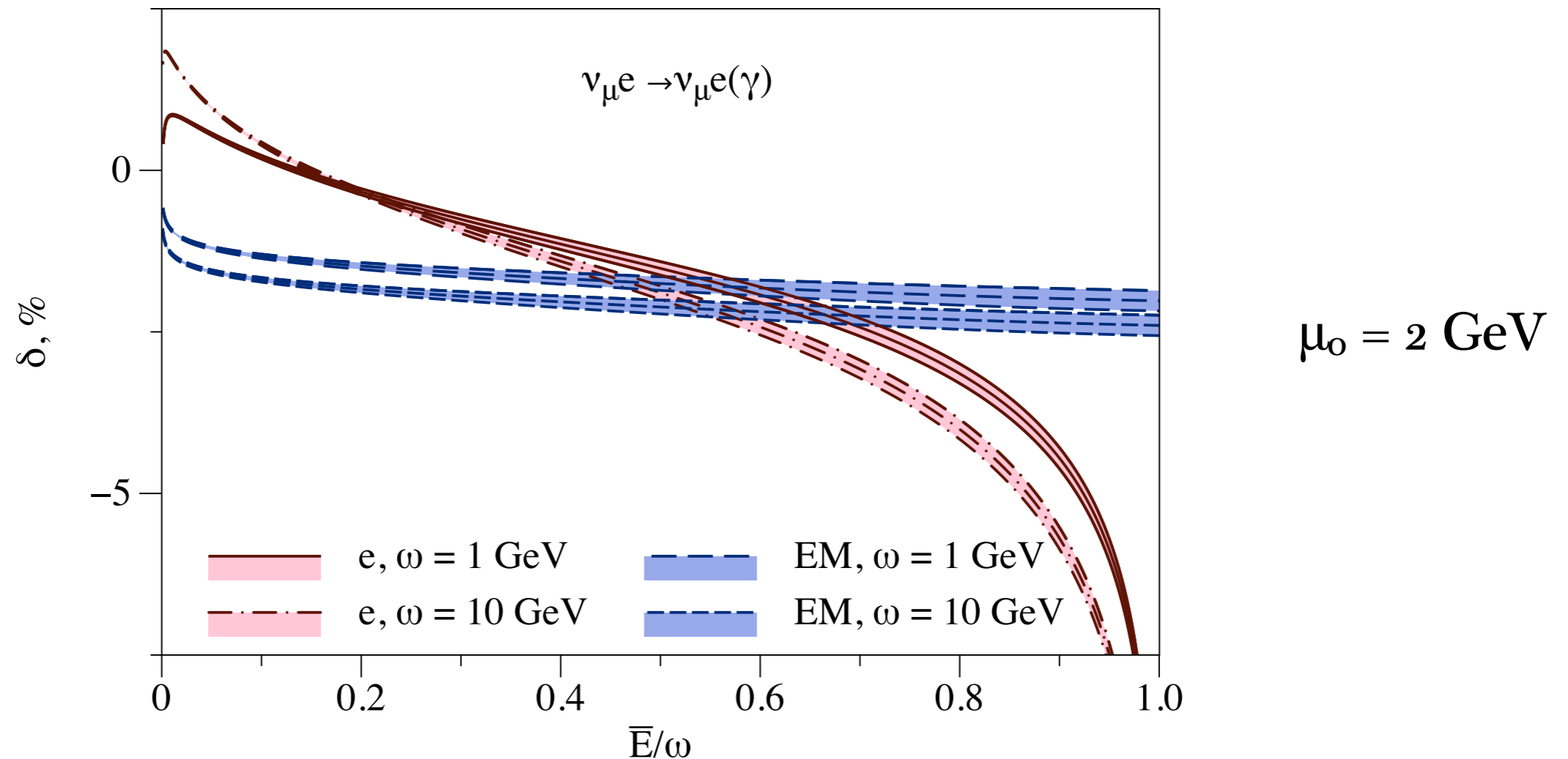


- analytic results and first error estimate at level 0.2-0.4%

Electron vs electromagnetic (EM) spectra

- relative correction depends on $\overline{\text{MS}}$ scale μ : $\mu_0/\sqrt{2} \leq \mu \leq \sqrt{2} \mu_0$

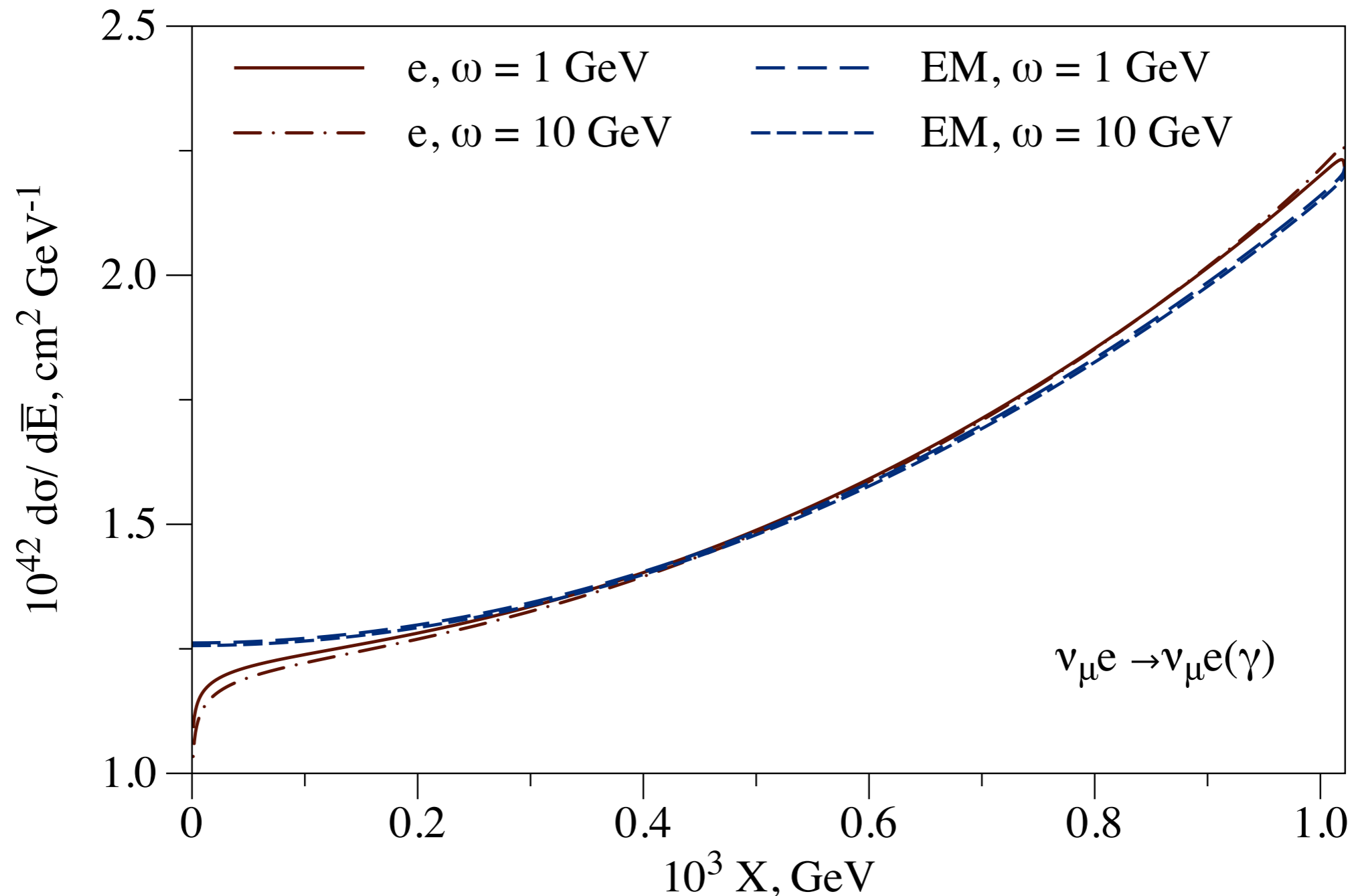
$$\delta = \frac{d\sigma_{\text{LO}}^{\nu e \rightarrow \nu e \gamma} + d\sigma_{\text{NLO}}^{\nu e \rightarrow \nu e} - d\sigma_{\text{LO}}^{\nu e \rightarrow \nu e}}{d\sigma_{\text{LO}}^{\nu e \rightarrow \nu e}}$$



- total correction: Sudakov logarithms cancel
- dramatic difference in radiative corrections

Electron vs electromagnetic (EM) spectra

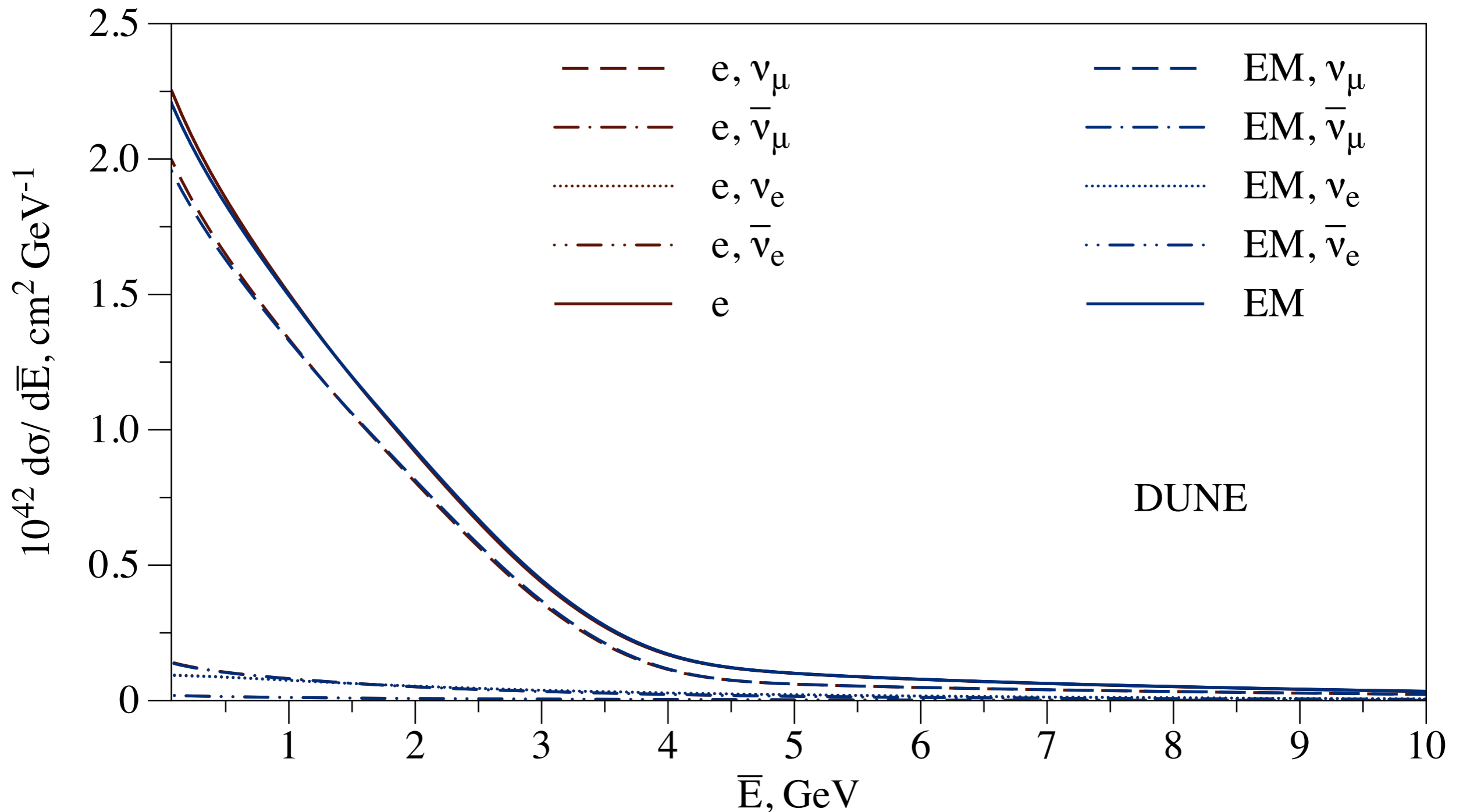
- resulting spectrum: $X = 2m \left(1 - \frac{\bar{E}}{\omega} \right) \Big|_{\bar{E}=E_e} \approx E_e \theta_e^2$



- dramatic difference in radiative corrections: cut dependence

Experimental energy spectrum

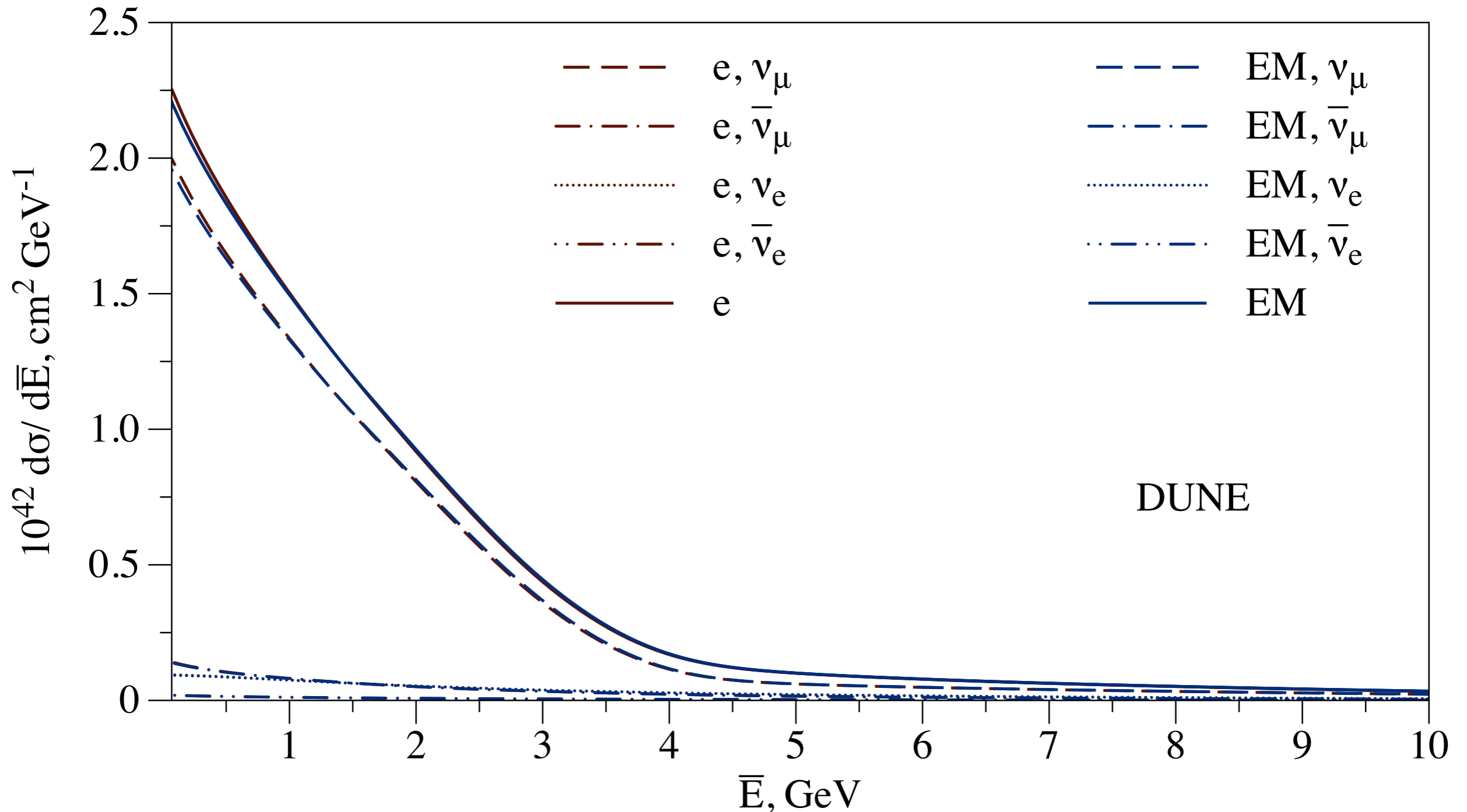
- average over beam flux and sum over flavors:



- 1-3 % difference at low recoil energy

Comparison to MINERvA and GENIE

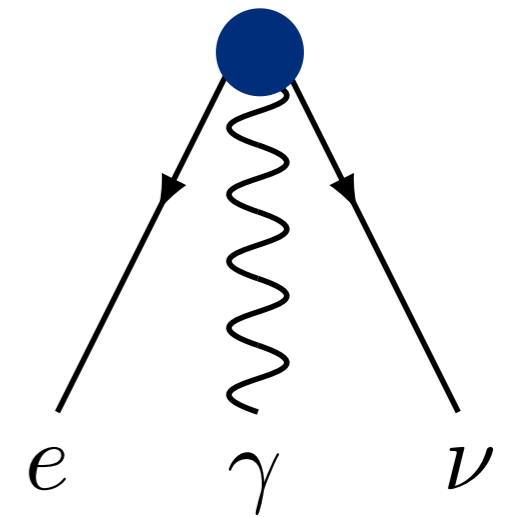
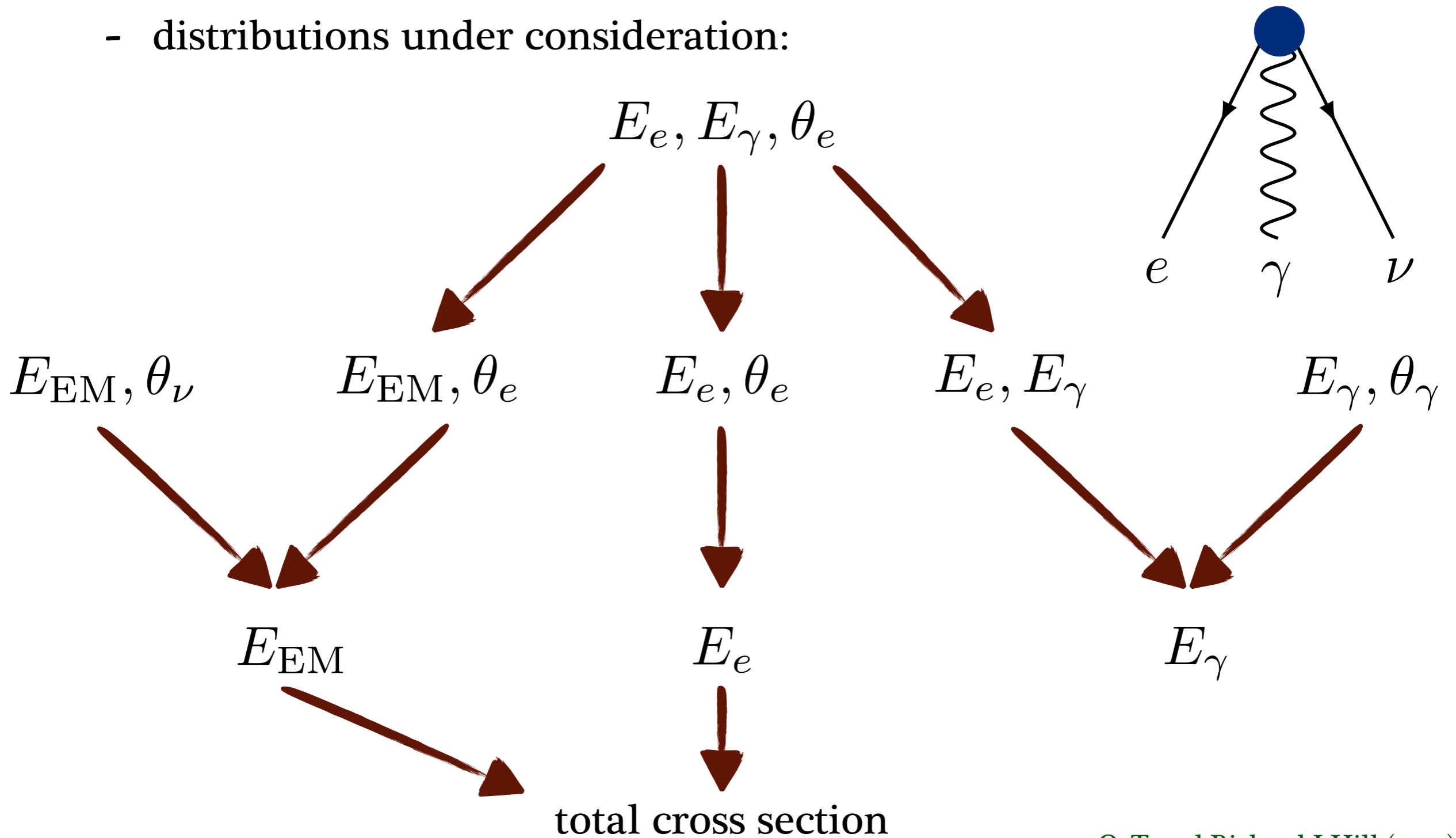
- average over beam flux and sum over flavors:



- 1-3 % difference at low recoil energy

Bremsstrahlung cross sections

- distributions under consideration:

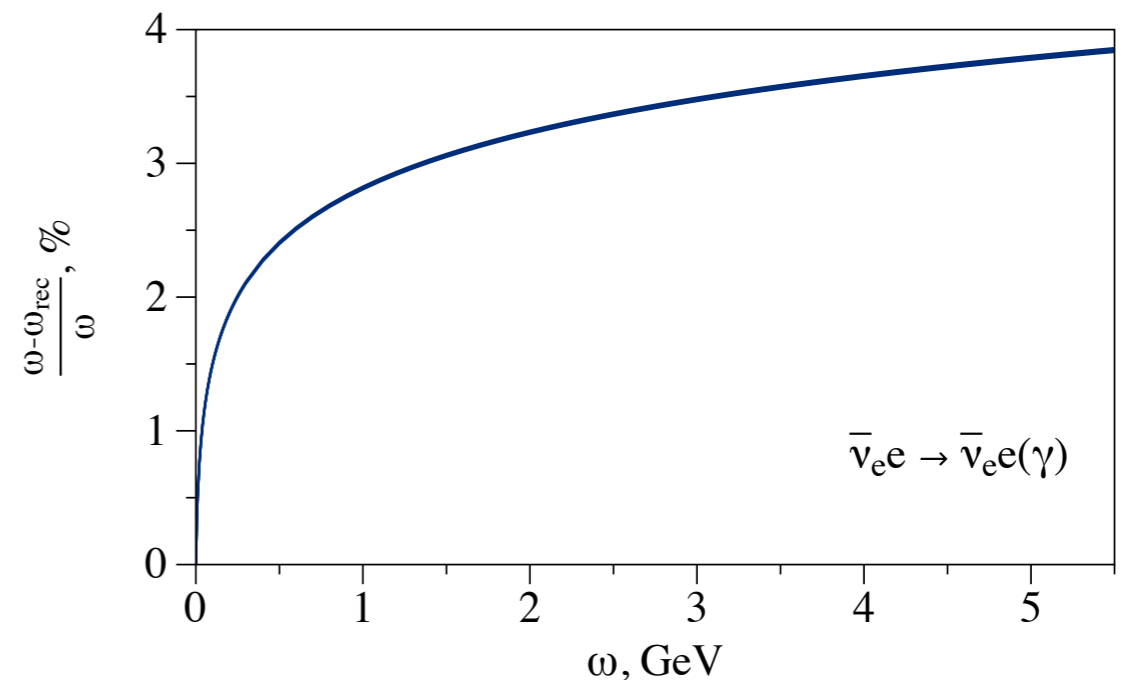
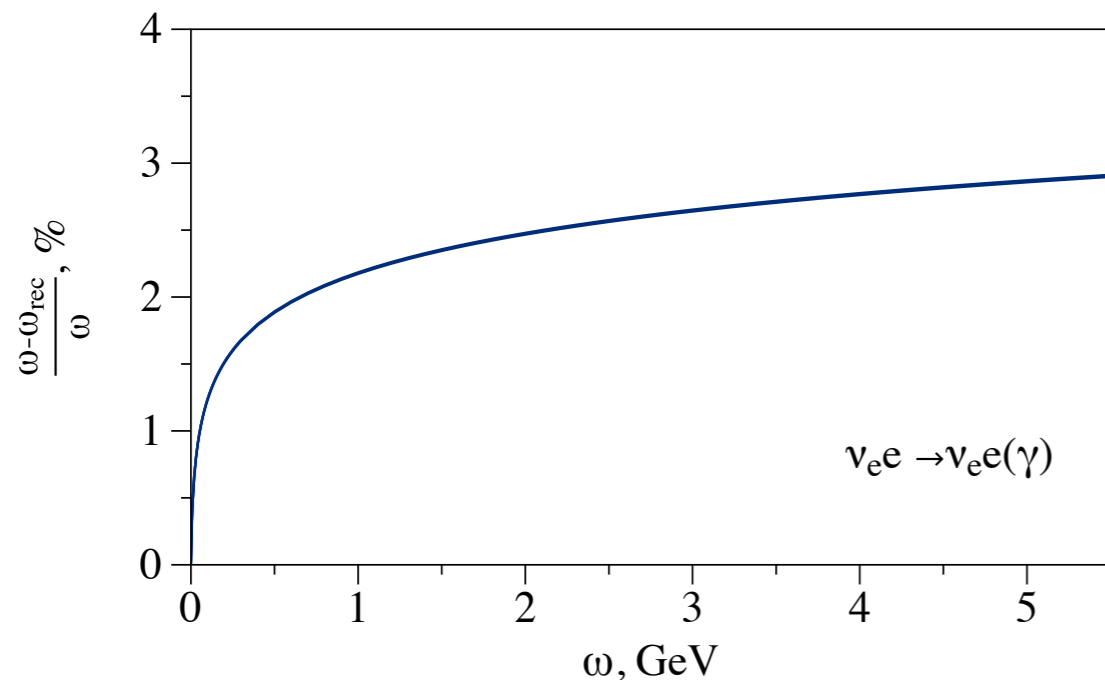
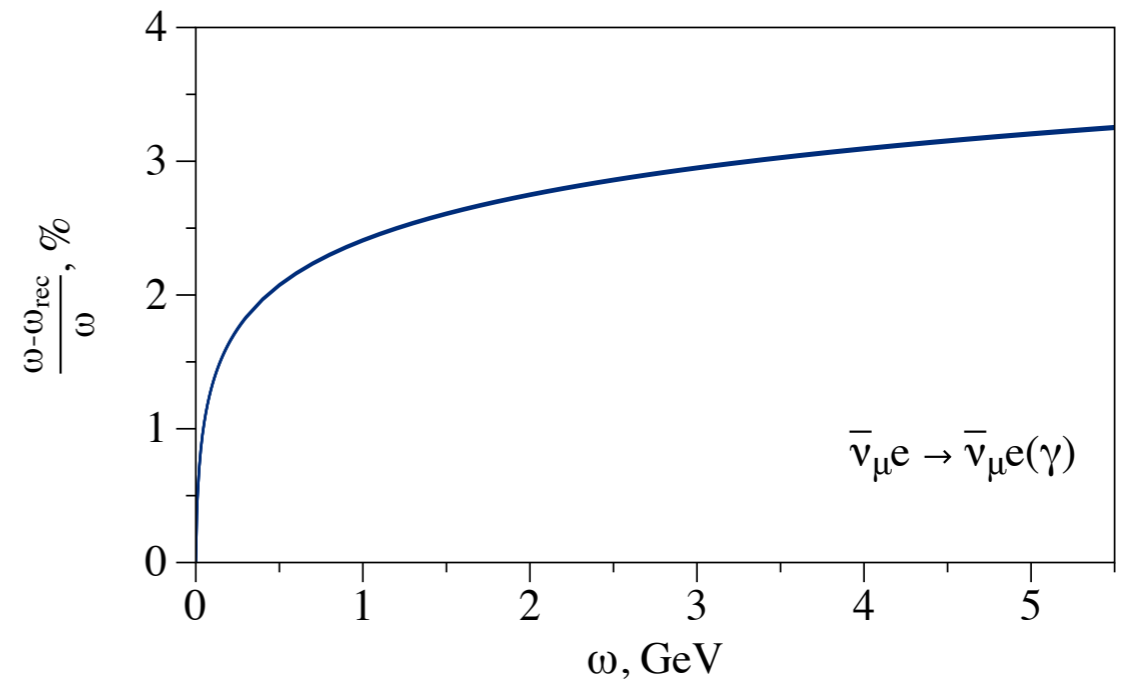
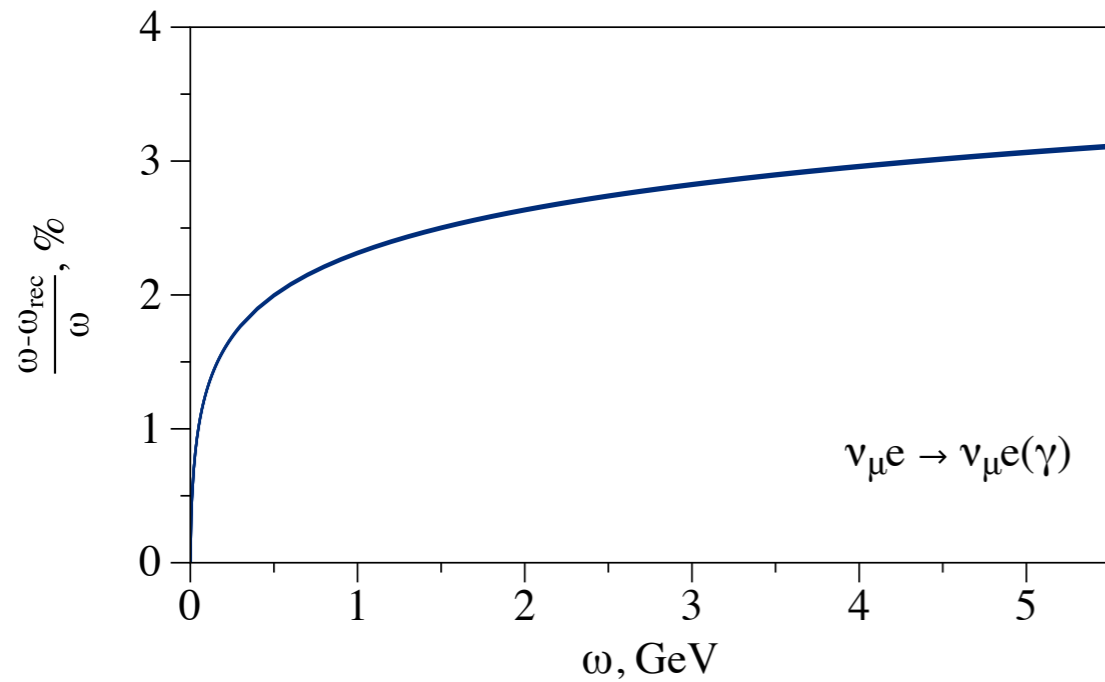


O. T. and Richard J Hill (2019)

- analytical form for finite and small electron mass

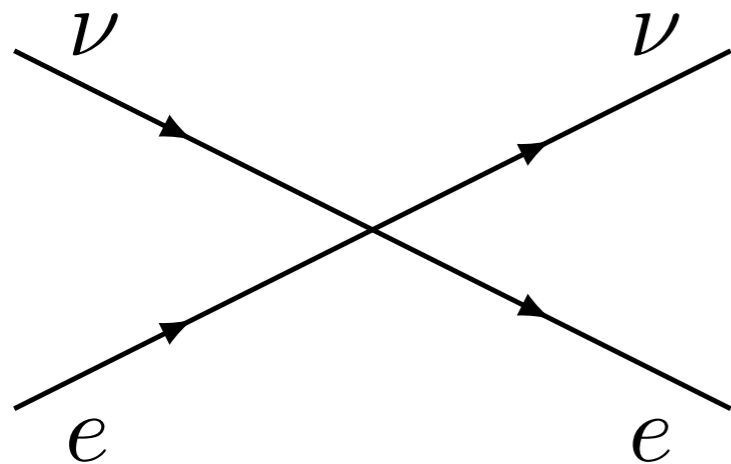
Neutrino energy reconstruction

- reconstruct from elastic kinematics:
$$\omega_{\text{rec}} = \frac{m|\vec{p}_e|}{(E_e + m) \cos \theta_e - |\vec{p}_e|}$$



- radiative corrections are crucial in energy reconstruction

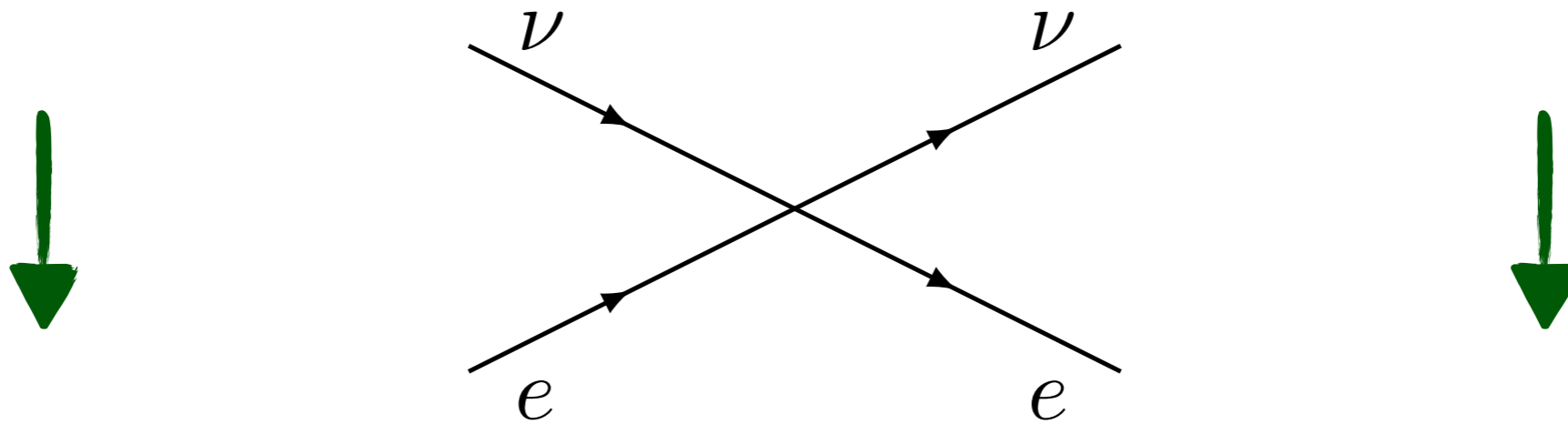
Conclusions



best tool to constrain
neutrino flux

- EFT of neutrino-electron and neutrino-quark scattering:
left- and right-handed couplings at subpermille level
- absolute cross section at permille level and first error analysis
main source of uncertainty: loops with hadrons
- energy spectra and bremsstrahlung cross sections:
new and known results in analytical form
- application to neutrino energy reconstruction

Outlook



- implement framework and results in modern event generators
- study hadronic uncertainty with dispersive methods
- pin down the uncertainty on lattice; connection to running α , $g-2$
- constrain light-quark contribution at DUNE
- application to solar neutrinos and reactor antineutrinos

Thanks for your attention !!!

Proposal for flux determination in DUNE

- determination of neutrino fluxes:

one of the main goals of Near Detector

	relative	normalization
ν_μ mode	$\nu_\mu p \rightarrow \mu^- p \pi^+$	$\nu_\mu e^- \rightarrow \nu_\mu e^-$
$\bar{\nu}_\mu$ mode	$\bar{\nu}_\mu p \rightarrow \mu^+ p \pi^-$	$\bar{\nu}_\mu p \rightarrow \mu^+ n$

according to H. Duyang et al. (2019)

- neutrino-electron scattering plays a role of additional constraint

- ve is required for absolute normalization of ν_μ component

Leading unaccounted correction

- sum of possible permutations of gluon lines
has zero anomalous dimension
contributes with α_s^2 contribution to vacuum polarization

