Four-Loop Higgs- and Z-Decays and the Five-Loop QCD Beta-Function

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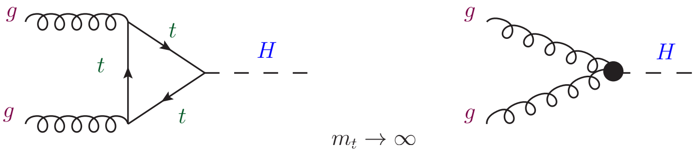
1. Higgs Decay to gg and $b\bar{b}$
2. Z Decay to $\mathcal{O}\left(\alpha_s^4\right)$
3. Beta Function to Five Loops

I. Higgs decay to gg and $b\bar{b}$: the two most important modes

1) $H \to gg$



$$\mathcal{L}_{eff} = -2^{1/4} G_F^{1/2} H\, C_1\left(\alpha_s, \ln\frac{\mu^2}{M_t^2}\right) \left[G^a_{\mu\nu} G^{a\mu\nu}\right]$$

$\mathcal{L}_{eff}$ counts number of heavy quark species

$[GG]$ is the renormalized counterpart of the bare operator

Born approximation $\sim \alpha_s^2$

$$\Gamma_{Born}(H \to gg) = \frac{G_F M_H^3}{36\pi\sqrt{2}} \left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

→ strong scale dependence

→ important role of higher orders

→ corrections for decay

NLO: Inami, Kugota, Okada (1983)

NNLO: Chetyrkin, Kniehl, Steinhauser (1997)

→ corrections for production (gluon fusion)

NLO: Inami, Kubota, Okada (1983); Dawson (1991);
Djouadi, Spira, Zerwas (1991)

NNLO: Harlander, Kilgore (2002); Anastasiou, Melnikov (2002);
Ravindran, Smith, van Neerven (2003)

Important effects of higher orders

- NLO increases both $\sigma\,(pp \to H + X)$ and $\Gamma\,(H \to gg)$ by 60-70% (!!)
- NNLO adds about 20% (for both production and decay)
- residual scale dependence of NNLO result: 15-20%
- similarity of corrections for production and decay $\Rightarrow \sigma_{gg}^{SM} / \Gamma_{gg}^{SM}$ is significantly more stable (Melnikov and Petriello)

N^3LO correction

Optical theorem

$$\Gamma\left(H \to gg\right) = \frac{\sqrt{2}G_F}{M_H} C_1^2 Im\Pi^{GG}\left(q^2 = M_H^2\right)$$

where

$$\Pi^{GG}\left(q^2\right) = \int \mathrm{d}x e^{iqx} \langle 0| T\left(\left[O_1'\right](x)\left[O_1'\right](0)\right)|0\rangle$$

$[O_1'] =$ renormalized counterpart of bare operator $O_1' = G^{0'}_{a\mu\nu} G_a^{0'\,\mu\nu}$

C_1 from massive tadpoles in order α_s^3 (4 loops):
Chetyrkin, Kniehl, Steinhauser (1997)
C_1 in order α_s^4 for $N_c = 3$:
K. Chetyrkin, P. Baikov, JK, PoSLL2016(2016)010
for arbitrary N_c:
M. Gerlach, F. Herren, M. Steinhauser, arXiv:1809.06787

Result: (Baikov, Chetyrkin)

$$
\begin{aligned}
Im\Pi^{gg} &= \frac{2q^4}{\pi}\left(1 + 12.4167a_s + 68.6482a_s^2 - 212.447a_s^3\right)\\
\Gamma\left(H \to gg\right) &= \Gamma_{Born}\left(H \to gg\right) \cdot K; \text{with } \mu = M_H\\
K &= 1 + 17.9167a_s' + 152.5{a_s'}^2 + 381.5{a_s'}^3\\
&= 1 + 0.65575 + 0.2043 + 0.0187\\
&= 1.87875
\end{aligned}
$$

with $a_s = \alpha_s/\pi$
Residual scale dependence $\delta\Gamma/\Gamma$
LO: $\pm 24\%$, NLO: $\pm 22\%$, NNLO: $\pm 10\%$, NNNLO: $\pm 3\%$
result in NNNLO $\approx$ 1.9 result in LO

recent further improvement: N^4LO, less than 1%
F. Herzog, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, arXiv:1707.01404

2) Scalar Correlator in 5 Loops and Higgs Decay into b-quarks

Higgs boson decays into quark-antiquark pair ($f\bar{f}$)

$$\Gamma\left(H \to f\bar{f}\right) = \frac{G_F M_H}{4\sqrt{2}\pi} m_f^2 \widetilde{R}\left(s = M_H^2\right)$$

where $\widetilde{R}(s) = Im\widetilde{\Pi}(-s - i\epsilon)/(2\pi s)$ is the absorptive part of the scalar two-point correlator

$$\widetilde{\Pi}\left(Q^2\right) = (4\pi)^2 i \int \mathrm{d}x e^{iqx} \left\langle 0 \right| T\left(\left[J_f^S(x)\right]\left[J_f^S(0)\right]\right)\left|0\right\rangle$$

$$\widetilde{R}(s) = 1 + \sum \widetilde{r}_i a_s^i(s)$$

Strong cancellations between ”kinematical” and ”dynamical” terms

$$\widetilde{R} = 1 + ... + a_s^4 \big[(9470.8 - \underline{9431.4}) - n_f(1454.3 - \underline{1233.4}) \\ + n_f^2(54.78 - \underline{45.10}) - n_f^3(0.454 - \underline{0.433})\big]$$

Underlined terms from analytic continuation from spacelike to timelike region!

Remarkable cancellations in all n_f powers, nice ”convergence”:

$$\begin{aligned}\widetilde{R} &= 1 + 5.6667 a_s + 29.147 a_s^2 + 41.76 a_s^3 - 825.7 a_s^4 \\ &= 1 + 0.2075 + 0.0391 + 0.0020 - 0.0018 \\ &= 1.2504\end{aligned}$$

(with $a_s = \frac{\alpha_s}{\pi} = 0.0366$)

dominant contribution to Higgs decay from $H \to b\bar{b}$!
$\text{Br}\big(H \to b\bar{b}\big) = 58.4 \pm 3.3\%$

$m_b =$ running mass at scale m_H
$m_b(m_H) = 2771 \pm 8\big|_{m_b} \pm 15\big|_{\alpha_s}$ MeV
(Chetyrkin et al.; F. Herren, M. Steinhauser: arXiv: 1703.03751)

Complete hadronic Higgs boson decay at order α_s^4:
mixed terms between $H \to gg$ and $H \to b\bar{b}$
$\Rightarrow$ J. Davies, M. Steinhauser, D. Wellmann: arXiv:1703.02988

$\Rightarrow$ dominant Higgs decay modes very well under control

II. Z Decay to $\mathcal{O}(\alpha_s^4)$

drastic variation of QCD coupling α_s between M_τ and M_Z or M_H
example:

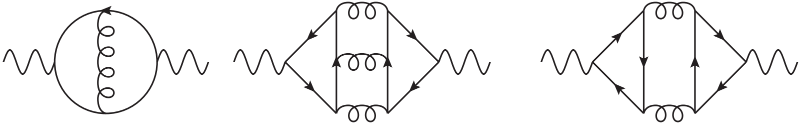
$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{exp} \pm 0.015_{th}$$

four loop running and matching:

$$\Rightarrow \alpha_s(M_Z) = 0.1202 \pm 0.0006_{exp} \pm 0.0018_{th} \pm 0.0003_{evol}$$

(evolution error receives contributions from c and b mass, matching scale, four loop truncation of RG equation) (P. Baikov, K. Chetyrkin, JK, arXiv:0801.1821)

$\alpha_s(M_Z)$ **from τ decay** in excellent agreement with direct determination in Z decays (P. Baikov, K. Chetyrkin, JK, J. Rittinger: arXiv:1201.5804)



non-singlet & singlet, vector & axial correlators

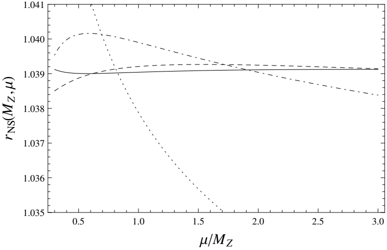
$$R^{nc} = 3\left[\sum_f v_f^2 r_{NS} + \left(\sum_f v_f\right)^2 r_S^V + \sum_f a_f^2 r_{NS} + r^A_{S;t,b}\right] ,$$

with

$$
\begin{aligned}
r_{NS} &= 1 + a_s + 1.40923 a_s^2 - 12.7671 a_s^3 - 79.9806 a_s^4 \, , \\
r_S^V &= -0.41318 a_s^3 - 4.9841 a_s^4 \, , \\
r_{S;t,b}^A &= (-3.08333 + l_t) a_s^2 + (-15.9877 + 3.72222 l_t + 1.91667 l_t^2) a_s^3 \\
&\quad + (49.162 - 17.6822 l_t + 14.7153 l_t^2 + 3.67361 l_t^3) a_s^4 \, ,
\end{aligned}
$$

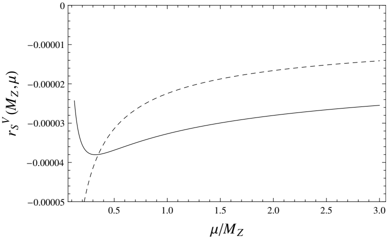
where $l_t = \ln \frac{M_Z^2}{M_t^2}$ and $\Gamma_Z = \Gamma_0 R^{nc} = \frac{G_F M_Z^3}{24\pi\sqrt{2}} R^{nc}$

$\Rightarrow \alpha_s(M_Z) = 0.1190 \pm 0.0026_{exp}$ and small theory error!



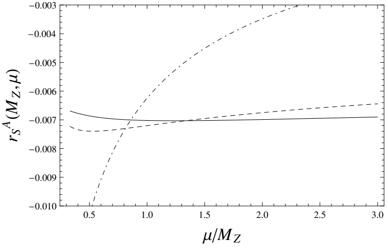
1.041
1.040
1.039
1.038
1.037
1.036
1.035
0.5
1.0
1.5
2.0
2.5
3.0
$r_{\rm NS}(M_Z,\mu)$
μ/M_Z

(a)



0
−0.00001
−0.00002
−0.00003
−0.00004
−0.00005
0.5
1.0
1.5
2.0
2.5
3.0
$r_S{}^V(M_Z,\mu)$
μ/M_Z

(b)



−0.003
−0.004
−0.005
−0.006
−0.007
−0.008
−0.009
−0.010
0.5
1.0
1.5
2.0
2.5
3.0
$r_S{}^A(M_Z,\mu)$
μ/M_Z

(c)

- non-singlet term starts in order α_s^0 (Born) and is identical for τ-decay, $\sigma(e^+e^- \to \textit{hadrons})$ through vector current (= virtual photon) and $\Gamma(Z \to \textit{hadrons})$ through vector and axial correlator
- singlet axial term starts in order α_s^2, is present in $Z \to \textit{hadrons}$ and depends on $\ln \frac{M_Z^2}{M_t^2}$ (origin: imbalance between top and bottom quark)
- singlet vector term is present in $\gamma^* \to \textit{hadrons}$ and $Z \to \textit{hadrons}$ and starts in order α_s^3
- all three terms are known up to order α_s^4

recent confirmation:
F. Herzog, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, arXiv:1707.01044

III. Five-loop β-function

$$\beta(a_s) = \mu^2 \frac{d}{d\mu^2} a_s(\mu) = -\sum_{i\geq 0} \beta_i a_s^{i+2}$$

$$\beta_0 = \frac{1}{4}\left\{11 - \frac{2}{3} n_f\right\},$$

Gross + Wilczek, Politzer

$$\beta_1 = \frac{1}{4^2}\left\{102 - \frac{38}{3} n_f\right\},$$

Caswell, Jones

$$\beta_2 = \frac{1}{4^3}\left\{\frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2\right\},$$

Tarasov + Vladimirov + Zharkov, Larin + Vermaseren

$$\beta_3 = \frac{1}{4^4}\left\{\frac{149753}{6} + 3564\zeta_3 - \left[\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right] n_f + \left[\frac{50065}{162} + \frac{6472}{81}\zeta_3\right] n_f^2 + \frac{1093}{729} n_f^3\right\},$$

van Ritbergen + Vermaseren + Larin, Czakon

$$\beta_4 = \frac{1}{4^5}\Big\{\frac{8157455}{16} + \frac{621885}{2}\zeta_3 - \frac{88209}{2}\zeta_4 - 288090\zeta_5\Big]$$
$$+ n_f\Big[-\frac{336460813}{1944} - \frac{4811164}{81}\zeta_3 + \frac{33935}{6}\zeta_4 + \frac{1358995}{27}\zeta_5\Big]$$
$$+ n_f^2\Big[\frac{25960913}{1944} + \frac{698531}{81}\zeta_3 - \frac{10526}{9}\zeta_4 - \frac{381760}{81}\zeta_5\Big]$$
$$+ n_f^3\Big[-\frac{630559}{5832} - \frac{48722}{243}\zeta_3 + \frac{1618}{27}\zeta_4 + \frac{460}{9}\zeta_5\Big]$$
$$+ n_f^4\Big[\frac{1205}{2916} - \frac{152}{81}\zeta_3\Big]\Big\}$$

Baikov, Chetyrkin, JK: PRL 118 (2017) no.8, 082002; arXiv:1606.08659

Absence of ζ_4 and ζ_5 in β_3 term!
Absence of ζ_6 and ζ_7 in β_4 term!
Numerically the terms are surprisingly small!
Consider $\overline{\beta} \equiv \frac{\beta}{-\beta_0 a_s^2} = 1 + \sum_{i\geq 1} \overline{\beta_i} a_s^i$

$$\overline{\beta}(n_f = 3) = 1 + 1.78 a_s + 4.47 a_s^2 + 20.99 a_s^3 + 56.59 a_s^4 \,,$$
$$\overline{\beta}(n_f = 4) = 1 + 1.54 a_s + 3.05 a_s^2 + 15.07 a_s^3 + 27.33 a_s^4 \,,$$
$$\overline{\beta}(n_f = 5) = 1 + 1.26 a_s + 1.47 a_s^2 + 9.83 a_s^3 + 7.88 a_s^4 \,,$$
$$\overline{\beta}(n_f = 6) = 1 + 0.93 a_s - 0.29 a_s^2 + 5.52 a_s^3 + 0.15 a_s^4$$

very modest growth of coefficients!

qualitative agreement with β_4, as calculated with Asymptotic Páde Approximant (APAP)

$$\beta_4^{APAP} = 740 - 213 n_f + 20 n_f^2 - 0.0486 n_f^3 - \boxed{0.0017993 n_f^4} \leftarrow \text{input}$$

$$\beta_4^{exact} = 524.56 - 181.8 n_f + 17.16 n_f^2 - 0.22586 n_f^3 - 0.0017993 n_f^4$$

but large cancellations $\Rightarrow$ numerical disagreement

n_f	0	1	2	3	4	5	6
$(\beta)_4^{exact}$	525	360	228	127	57	15	0.27
$(\beta)_4^{APAP}$	741	548	395	281	205	169	170

excellent agreement between $\alpha_s(M_Z)$ from τ decays (+ running and matching) and direct measurement

$$\alpha_s(m_\tau) = 0.33 \pm 0.014 \Rightarrow \alpha_s(M_Z) = 0.1198 \pm 0.0015$$

$$\textbf{vs } \alpha_s(M_Z) = 0.1197 \pm 0.0028 \textbf{ directly from Z} - \textbf{decay}$$

result confirmed and generalized to arbitrary gauge group:

F. Herzog, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, arXiv:1701.01404

T. Luthe, A. Maier, P. Marquard, Y. Schroeder, arXiv:1709.07718

K. Chetyrkin, G. Falcioni, F. Herzog, J. Vermaseren, arXiv:1709.08541

Summary

- QCD corrections for
  Higgs decay to $f\bar{f}$, Higgs decay to gluons, τ decay to $\nu + had$,
  $\mathbf{R} = \frac{\sigma_{tot}(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$, Z decay to $f\bar{f}$. All are availabe with
  corrections to $\mathcal{O}(\alpha_s^4)$ corresponding to 4 loops
- matched by QCD β-function in 5-loops
- excellent agreement between theory and experiment
- theory prediction significantly ahead of experiment