Off-Shell Renormalization of Spontaneously Broken Effective Field Theories

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Outline

Renormalization of gauge EFTs

Power-counting (p.c).-renormalizable Lagrangian plus higher dim. ops. arranged in powers of a large inverse energy scale Λ

$$\mathscr{L}_{(SB)EFTs} = \mathscr{L}_4 + \frac{1}{\Lambda} \sum_i c_i^5 \mathscr{O}_i^5 + \frac{1}{\Lambda^2} \sum_i c_i^6 \mathscr{O}_i^6 + \dots$$

compatible with the low-energy symmetry pattern

Renormalizability in the modern sense

Gomis and Weinberg, Nucl.Phys. B469 (1996) 473-487

- P.c. renormalizabilty is lost (more and more UV divergences at each loop order)
- Locality of the counter-terms (in the sense of formal power series in the fields, the external sources and the momenta) still holds provided that:
    1. generalized field redefinitons are first appropriately taken into account
    2. the renormalization of the gauge-invariant operators is carried out order by order in the perturbative expansion

Derivative interactions

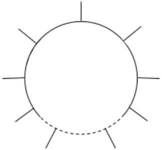
Prototype dim.6 operator

$$\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi\,, \qquad \phi = \frac{1}{\sqrt{2}}(\sigma + v + i\chi)\,,$$

- Power-counting maximally violated

$$\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi \supset \sigma^2\partial^\mu\sigma\partial_\mu\sigma$$

Infinite number of UV divergent amplitudes
already at one loop order



$$\delta = 4 + (2-2) + (2-2) + \cdots = 4$$

Which UV divergences are reabsorbed by (generalized) field redefinitions (GFRs) and which represent genuine physical renormalizations of gauge inv. ops?

- One needs to stay off-shell, for several reasons:
  - Overlapping divergences
  - GFRs' contributions hard (impossible?) to identify by looking only at on-shell amplitudes
  - Symmetries hold off-shell
- Guiding principle needed to discriminate between the UV divergences accounted for by GFRs and by gauge-invariant ops. $\Rightarrow$ the Slavnov-Taylor identities (equivalently the Batalin-Vilkovisky master equation) provide the solution

- There is one further complication specific to SBGTs: due to the non-vanishing v.e.v., gauge-invariant ops. contribute to amplitudes of different dimension

$$D_\mu = \partial_\mu - iA_\mu, \qquad \mathcal{O} = (D^2\phi)^\dagger D^2\phi$$

$$\begin{aligned}(D^2\phi)^\dagger D^2\phi \supset & \frac{v^2}{2}[(A^2)^2 + (\partial A)^2] \\ & - v\Big[A^2(\Box\sigma + \dots) + \frac{1}{2}\partial A(\Box\chi + \dots)\Big] \\ & + (\Box\sigma + \dots)^2 + (\Box\chi + \dots)^2\end{aligned}$$

Slavnov-Taylor Identities

The Slavnov-Taylor (ST) identities provide a way to uniquely determine the GFRs:

$$\mathcal{S}(\Gamma) = \sum_i \int \mathrm{d}^4x \, \frac{\delta\Gamma}{\delta\Phi_i^*} \frac{\delta\Gamma}{\delta\Phi_i} = 0$$

They encode at the quantum level the BRST invariance of the gauge-fixed action and ensure the fulfillment of physical unitarity.

Loop expansion

- Order n in the loop expansion

$$\mathcal{S}_0(\Gamma^{(n)}) + \sum_{j=1}^{n-1} \sum_i \int \mathrm{d}^4x \, \frac{\delta\Gamma^{(j)}}{\delta\Phi_i^*} \frac{\delta\Gamma^{(n-j)}}{\delta\Phi_i} = 0$$

- $\mathcal{S}_0$ is the linearized ST operator

$$\mathcal{S}_0(X) = \sum_i \left(\frac{\delta\Gamma^{(0)}}{\delta\Phi_i^*} \frac{\delta X}{\delta\Phi_i} + \frac{\delta\Gamma^{(0)}}{\delta\Phi_i} \frac{\delta X}{\delta\Phi_i^*} \right)$$

$$s\Phi_i = \frac{\delta\Gamma^{(0)}}{\delta\Phi_i^*} \,, \qquad \mathcal{S}_0(\Phi_i^*) = \frac{\delta\Gamma^{(0)}}{\delta\Phi_i}$$

- A GFR of the form (ζ BRST invariant sources coupled to gauge inv. ops.)

$$\Phi_i \to P_{ij}(\Phi,\zeta)\Phi_j$$

implemented via the $\mathcal{S}_0$-invariant

$$\mathcal{S}_0 \int \mathrm{d}^4x\, P_{ij}(\Phi,\zeta)\Phi_j\, \Phi^*_i = \int \mathrm{d}^4x \left[s(P_{ij}\Phi_j)\Phi^*_i + P_{ij}(\Phi,\zeta)\Phi_j \frac{\delta\Gamma^{(0)}}{\delta\Phi_i} \right]$$

- The first term in the r.h.s. contains antifield-ghost amplitudes that allow to fix (uniquely) the coefficient of the $\mathcal{S}_0$-invariant (and hence the GFR)

Power-counting and the X-formalism

- Parameterizing in a gauge-invariant way the physical scalar displays a more regular UV behaviour of the amplitudes.
- With dim.6 ops. a weak p.c. emerges in the fields sector (only a finite number of UV divergent amplitudes at each loop order).
- Decorate with insertions of gauge-invariant external sources (resummation).

Describe the physical scalar mode (of mass M) with the variable X_2 :

$$X_2 \sim \frac{1}{v}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)$$

Requirements:

1. Without higher dim. ops the X-representation should give back the p.c.-renormalizable theory (after going on-shell with X_2 and the Lagrange multiplier enforcing the above constraint)
2. The X-theory without higher dim. ops. should stay p.c.-renormalizable.

The X-formalism

Introduce the Lagrange multiplier X_1 (m^2 is an additional unphysical parameter that cancels out when going on-shell with $X_{1,2}$):

$$\int \mathrm{d}^4x \, \frac{1}{v} X_1 (\Box + m^2) \Big(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \Big)$$

The X_1-equation is radiatively stable:

$$\frac{\delta \Gamma}{\delta X_1} = \frac{1}{v} (\Box + m^2) \frac{\delta \Gamma}{\delta \bar{c}^*} \, ,$$

$\bar{c}^*$ being an external source coupled in the classical action to

$$\bar{c}^* \Big(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \Big)$$

For instance in the X-representation

$$\frac{1}{v}\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi \sim X_2(D^\mu\phi)^\dagger D_\mu\phi$$

Since there is also a X_2-equation, amplitudes with external X_2-legs are fixed in terms of amplitudes with other fields and external sources insertions, enjoying either a better UV behaviour or resumming.

The X-formalism

The classical action for the Abelian Higgs-Kibble model in the X-theory with the dim.6 op. $X_2(D^\mu\phi)^\dagger D_\mu\phi$ (D.Binosi and A.Q., arXiv:1904.06692, to appear in JHEP)

$$\Gamma^{(0)} = \int d^4x \Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) - \frac{M^2-m^2}{2}X_2^2 - \frac{m^2}{2v^2}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)^2 \\ - \bar{c}(\Box + m^2)c + \frac{1}{v}(X_1 + X_2)(\Box + m^2)\Big(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\Big) \\ + \frac{g}{\Lambda}X_2(D^\mu\phi)^\dagger(D_\mu\phi) + T_1(D^\mu\phi)^\dagger(D_\mu\phi) \\ + \frac{b^2}{2\xi} - b\Big(\partial A + \frac{ev}{\xi}\chi\Big) + \bar{\omega}\Big(\Box\omega + \frac{e^2v}{\xi}(\sigma+v)\omega\Big) \\ + \bar{c}^*\Big(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\Big) + \sigma^*(-e\omega\chi) + \chi^* e\omega(\sigma+v)\Big].$$

with the X_2-equation:

$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v}(\Box + m^2)\frac{\delta\Gamma}{\delta\bar{c}^*} + \frac{g}{\Lambda}\frac{\delta\Gamma}{\delta T_1} - (\Box + m^2)X_1 - (\Box + M^2)X_2 - v\bar{c}^* .$$

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Going on-shell with X_1 gives the condition

$$\frac{1}{v}(\Box + m^2)\Big(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\Big) = 0 \quad \Rightarrow \phi^\dagger\phi - \frac{v^2}{2} = vX_2 + \eta$$

However, the correlators of η vanish in perturbation theory (a consequence of a further U(1) BRST symmetry associated with the X_1-constraint).
So one can safely set $\eta = 0$.

Mapping

Going on-shell with the $X_{1,2}$-equations in the X-theory yields a mapping of the X-theory amplitudes into the original ϕ-representation.

The m^2-dependence disappears (a very strong check of the computations) and one recovers the UV divergences of the original theory.

One can also read off directly the genuine physical renormalizations of gauge invariant operators.

Solutions of the $X_{1,2}$-equations

In the X-theory the $X_{1,2}$-equations are solved by the replacements (valid to all orders in perturbation theory):

$$\bar{c}^* = \bar{c}^* + \frac{1}{v}(\Box + m^2)(X_1 + X_2); \qquad \mathscr{T}_1 = T_1 + \frac{g}{\Lambda} X_2 .$$

Go on-shell with $X_{1,2}$ at zero external sources (in the one loop approximation tree-level e.o.m.'s for $X_{1,2}$ are needed):

$$\frac{\delta \Gamma^{(0)}}{\delta X_1} = 0 \Rightarrow X_2 = \frac{1}{v}\left(\phi^\dagger \phi - \frac{v^2}{2}\right)$$

$$\frac{\delta \Gamma^{(0)}}{\delta X_2} = 0 \Rightarrow (\Box + m^2)(X_1 + X_2) = -(M^2 - m^2) X_2 + \frac{g}{\Lambda} (D^\mu \phi)^\dagger D_\mu \phi .$$

At the end of the day the mapping boils down to the following very simple replacement rules of the external sources:

$$\bar{c}^* \to -\frac{(M^2 - m^2)}{v^2}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big) + \frac{g}{v\Lambda}(D^\mu\phi)^\dagger D_\mu\phi;$$

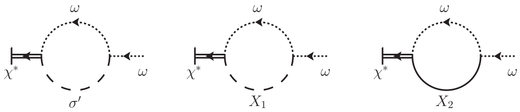
$$\mathscr{T}_1 \to \frac{g}{v\Lambda}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big).$$

For instance the reconstruction of the 2-point σ amplitude works as follows (tilded 1-PI amplitudes are in the ϕ-rep.):

$$\int \Gamma^{(1)}_{T_1} T_1 \underset{\sigma\sigma \text{ term}}{\longrightarrow} \int \frac{g}{2\Lambda v} \Gamma^{(1)}_{T_1} \sigma^2, \qquad \ldots\ldots$$

$$\widetilde{\Gamma}^{(1)}_{\sigma\sigma}(p^2) = \Gamma^{(1)}_{\sigma'\sigma'} + 2\frac{g}{\Lambda}\Gamma^{(1)}_{T_1\sigma'} - 2\frac{M^2 - m^2}{v}\Gamma^{(1)}_{\bar{c}^*\sigma'}$$
$$+ \frac{g}{\Lambda v}\Gamma^{(1)}_{T_1} + \Big(- \frac{M^2 - m^2}{v^2} + \frac{g}{\Lambda v}p^2\Big)\Gamma^{(1)}_{\bar{c}^*}$$
$$+ \frac{g^2}{\Lambda^2}\Gamma^{(1)}_{T_1 T_1} - 2\frac{g}{\Lambda}\frac{M^2 - m^2}{v}\Gamma^{(1)}_{T_1\bar{c}^*} + \frac{(M^2 - m^2)^2}{v^2}\Gamma^{(1)}_{\bar{c}^*\bar{c}^*}$$

- At $T_1 = 0$ there is just one UV divergent antifield-dep. amplitude, $\Gamma^{(1)}_{\chi^*\omega}$:



$$\overline{\Gamma}^{(1)}_{\chi^*\omega}(x,y) = \frac{e^2 M_A}{16\pi^2}\frac{2}{4-D}(1-\delta_{\xi,0})\delta^{(4)}(x-y)$$

A bar denotes the UV divergent part of the amplitude

- The whole dependence on T_1 arises from repeated insertions of T_1 on the σ-line (Feynman gauge):

$$\overline{\Gamma}^{(1)}_{\chi^*\omega}[T_1](x,y) = \frac{1}{1+T_1(y)} \frac{e^2 M_A}{16\pi^2} \frac{2}{4-D} \delta^{(4)}(x-y)$$

- From the 1-PI amplitude $\overline{\Gamma}^{(1)}_{\chi^*\omega}[T_1]$ we reconstruct the $\mathcal{S}_0$-invariant reproducing the amplitude at hand:

$$\int \mathrm{d}^4x \, \frac{eM_A}{16v\pi^2} \frac{2}{4-D} \mathcal{S}_0\Big(\frac{1}{1+T_1} (\sigma^*\sigma + \chi^*\chi) \Big) \supset$$
$$- \int \mathrm{d}^4x \, \frac{e^2 M_A}{16\pi^2} \frac{2}{4-D} \frac{1}{1+T_1} \chi^* \omega$$

Now we read off the GFRs:

- in the X-theory:

$$\sigma \to \sigma + \frac{c^{(1)}}{1+T_1}\sigma; \quad \chi \to \chi + \frac{c^{(1)}}{1+T_1}\chi; \quad c^{(1)} = \frac{eM_A}{16v\pi^2}\frac{2}{4-D}$$

- in the ϕ-theory:

$$\sigma \to \sigma + \frac{c^{(1)}}{1+\frac{g}{\Lambda v}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)}\sigma; \quad \chi \to \chi + \frac{c^{(1)}}{1+\frac{g}{\Lambda v}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)}\chi$$

i.e. a non-polynomial field redefinition.

Dim.6 bosonic ops in the Abelian HK model

Bosonic dim.6 P-preserving operators (Abelian case)

B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, JHEP 1010 (2010) 085

Warsaw basis	ϕ-theory	X-theory	Coupling
Q_ϕ	$\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^3$	X_2^3	$\frac{v^2 g_1}{3!\Lambda}$
$Q_{\phi\Box}$	$\phi^\dagger\phi\Box\phi^\dagger\phi$	$X_2\Box X_2$	z
$Q_{\phi D}$	$\left(\phi^\dagger\phi - \frac{v^2}{2}\right)(D^\mu\phi)^\dagger D_\mu\phi$	$X_2(D^\mu\phi)^\dagger D_\mu\phi$	$\frac{g}{\Lambda}$
$Q_{\phi G}$	$\left(\phi^\dagger\phi - \frac{v^2}{2}\right)F_{\mu\nu}^2$	$X_2 F_{\mu\nu}^2$	$\frac{g_2}{\Lambda}$

D. Binosi, A.Q., arXiv:1904.06693; in preparation

More external sources are needed to formulate the X_2-equation:

$$T_1(D^\mu\phi)^\dagger D_\mu\phi + RX_2^2 + UF_{\mu\nu}^2$$

and the X_2-equation becomes:

$$\begin{aligned}\frac{\delta\Gamma}{\delta X_2} =& \frac{1}{v}(\Box + m^2)\frac{\delta\Gamma}{\delta\bar{c}^*} + \frac{g}{\Lambda}\frac{\delta\Gamma}{\delta T_1} + \frac{g_1 v^2}{2\Lambda}\frac{\delta\Gamma}{\delta R} + \frac{g_2}{\Lambda}\frac{\delta\Gamma}{\delta U}\\ & - (\Box + m^2)X_1 - (\Box + M^2)X_2 - z\Box X_2 - v\bar{c}^*\end{aligned}$$

The mapping takes the form $\left(\Sigma \equiv \phi^\dagger\phi - \frac{v^2}{2}\right)$

$$\bar{c}^* \to -\frac{M^2 - m^2}{v^2}\Sigma - \frac{z}{v^2}\Box\Sigma + \frac{g}{\Lambda v}(D^\mu\phi)^\dagger D_\mu\phi + \frac{g_2}{\Lambda v}F^2_{\mu\nu} + \frac{g_1}{2\Lambda v}\Sigma^2$$

$$T_1 \to \frac{g}{\Lambda v}\Sigma\,, \qquad R \to \frac{g_1 v}{2\Lambda}\Sigma\,, \qquad U \to \frac{g_2}{\Lambda v}\Sigma$$

At the relevant order (up to dim.6 amplitudes in the ϕ-theory) the GFRs are parameterized by 14 parameters

$$\mathscr{J}_1 = \sigma\sigma^* + \chi\chi^*\,, \qquad \mathscr{J}_2 = (\sigma + v)\sigma^* + \chi\chi^*\,,$$

$$\begin{aligned} S_0\Big[&(\rho_0 + \rho_1\sigma + \rho_2\sigma^2 + \rho_3\chi^2 + \rho_{0T}T_1)\mathscr{J}_1 \\ &+ (\rho'_0 + \rho'_1\sigma + \rho'_2\sigma^2 + \rho'_3\chi^2 + \rho'_4\sigma\chi^2)\mathscr{J}_2 \\ &+ (\rho'_{0T}T_1 + \rho'_{1T}T_1\sigma + \rho'_{3T}T_1\chi^2 + \rho'_{0TT}T_1^2)\mathscr{J}_2\Big] \end{aligned}$$

GFRs in the Abelian HK model

UV divergent contributions in units of $1/4-D$

Coefficient	Feynman	Landau	lin.Feynman	lin.Landau
ρ_0	$\frac{1}{1+z}\frac{M_A^2}{8\pi^2 v^2}$	0	$-z\frac{M_A^2}{8\pi^2 v^2}$	0
ρ_1	$-\frac{z}{(1+z)^2}\frac{M_A^2}{4\pi^2 v^3}$	0	$-z\frac{M_A^2}{4\pi^2 v^3}$	0
ρ_2	$\frac{z(3z-1)}{(1+z)^3}\frac{M_A^2}{8\pi^2 v^4}$	0	$-z\frac{M_A^2}{8\pi^2 v^4}$	0
ρ_3	$-\frac{z}{(1+z)^2}\frac{M_A^2}{8\pi^2 v^4}$	0	$-z\frac{M_A^2}{8\pi^2 v^4}$	0
ρ_{0T}	$-\frac{1}{(1+z)^2}\frac{M_A^2}{8\pi^2 v^2}$	0	$z\frac{M_A^2}{4\pi^2 v^2}$	0
${\rho'}_0$	0	$\frac{M_A^2}{16\pi^2 v^2}$	0	0
${\rho'}_1$	0	$-\frac{z}{1+z}\frac{M_A^2}{8\pi^2 v^3}$	0	$-z\frac{M_A^2}{8\pi^2 v^3}$
${\rho'}_2$	0	$\frac{z(z-1)}{(1+z)^2}\frac{M_A^2}{8\pi^2 v^4}$	0	$-z\frac{M_A^2}{8\pi^2 v^4}$
${\rho'}_3$	$\frac{z}{1+z}\frac{M_A^2}{16\pi^2 v^4}$	$-\frac{z}{1+z}\frac{M_A^2}{16\pi^2 v^4}$	$z\frac{M_A^2}{16\pi^2 v^4}$	$-z\frac{M_A^2}{16\pi^2 v^4}$
${\rho'}_4$	$-\frac{z(1+3z)}{(1+z)^2}\frac{M_A^2}{16\pi^2 v^5}$	$\frac{z(3z-1)}{(1+z)^2}\frac{M_A^2}{16\pi^2 v^5}$	$-z\frac{M_A^2}{16\pi^2 v^5}$	$-z\frac{M_A^2}{16\pi^2 v^5}$

UV divergent contributions in units of $1/4 - D$ (cont'd)

Coefficient	Feynman	Landau	lin.Feynman	lin.Landau
${\rho'}_{0T}$	0	$-\frac{M_A^2}{8\pi^2 v^2}$	0	0
${\rho'}_{1T}$	$\frac{z(z+2)}{(1+z)^2}\frac{M_A^2}{8\pi^2 v^3}$	0	$\frac{zM_A^2}{4\pi^2 v^3}$	0
${\rho'}_{3T}$	0	0	0	0
${\rho'}_{0TT}$	0	$\frac{M_A^2}{8\pi^2 v^2}$	0	0

The gauge-invariant sector (X-theory)

The maximal decoupling of layers of different dimensions in the linear system from Lorentz-covariant monomials in the fields, ext. sources and their derivatives to $\mathcal{S}_0$-invariants happens if one uses the so-called contractible pairs:

$$\partial_{(\nu_1\ldots\nu_\ell}A_{\mu)}; \quad \partial_{(\nu_1\ldots\nu_\ell}\partial_{\mu)}\omega; \quad \omega; \quad \partial_{(\nu_1\ldots}\partial_{\nu_{\ell-1}}F_{\nu_\ell)\mu}; \qquad D_{(\nu_1\ldots}D_{\nu_\ell)}\phi$$

$$\partial_{\nu_1\ldots\nu_\ell}A_\mu = \partial_{(\nu_1\ldots\nu_\ell}A_{\mu)} + \frac{\ell}{\ell+1}\partial_{(\nu_1\ldots\nu_{\ell-1}}F_{\nu_\ell)\mu},$$

G.Barnich, F. Brandt, M. Henneaux, Phys.Rept. 338 (2000) 439-569

It is convenient to distinguish between:

- gauge-inv. ops. depending only on the fields (with coefficients λ)
- mixed field-ext.sources operators (with coefficients θ)
- pure ext.sources operators (with coefficients ϑ)

Pure external sources operators

$$\vartheta_1 \int \mathrm{d}^4x\, \bar{c}^*; \qquad \vartheta_2 \int \mathrm{d}^4x\, T_1,$$

$$\vartheta_3 \int \mathrm{d}^4x\, \frac{1}{2}(\bar{c}^*)^2; \qquad \vartheta_4 \int \mathrm{d}^4x\, \frac{1}{2}T_1^2,$$

$$\vartheta_5 \int \mathrm{d}^4x\, \frac{1}{2}T_1\Box T_1; \qquad \vartheta_6 \int \mathrm{d}^4x\, \frac{1}{2}T_1\Box^2 T_1$$

$$\vartheta_7 \int \mathrm{d}^4x\, \bar{c}^* T_1; \qquad \vartheta_8 \int \mathrm{d}^4x\, \bar{c}^*\Box T_1,$$

$$\vartheta_9 \int \mathrm{d}^4x\, \frac{1}{3!}(\bar{c}^*)^3; \qquad \vartheta_{10} \int \mathrm{d}^4x\, \frac{1}{2}(\bar{c}^*)^2 T_1,$$

$$\vartheta_{11} \int \mathrm{d}^4x\, \frac{1}{2}(\bar{c}^*)T_1^2; \qquad \vartheta_{12} \int \mathrm{d}^4x\, \frac{1}{3!}T_1^3$$

$\ldots$

Mixed fields-external sources operators

$$\theta_1 \int d^4x\, \bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2}\right); \qquad \theta_2 \int d^4x\, T_1 \left(\phi^\dagger \phi - \frac{v^2}{2}\right),$$

$$\theta_3 \int d^4x\, \bar{c}^* (D^\mu \phi)^\dagger D_\mu \phi; \qquad \theta_4 \int d^4x\, T_1 (D^\mu \phi)^\dagger D_\mu \phi,$$

$$\theta_5 \int d^4x\, \bar{c}^* \left[(D^2 \phi)^\dagger \phi + \text{h.c.}\right]; \qquad \theta_6 \int d^4x\, T_1 \left[(D^2 \phi)^\dagger \phi + \text{h.c.}\right],$$

$$\theta_7 \int d^4x\, \bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2}\right)^2; \qquad \theta_8 \int d^4x\, T_1 \left(\phi^\dagger \phi - \frac{v^2}{2}\right)^2,$$

$$\theta_9 \int d^4x\, \bar{c}^* F_{\mu\nu}^2; \qquad \theta_{10} \int d^4x\, T_1 F_{\mu\nu}^2,$$

$$\theta_{11} \int d^4x\, \bar{c}^* T_1 \left(\phi^\dagger \phi - \frac{v^2}{2}\right); \qquad \theta_{12} \int d^4x\, T_1^2 \left(\phi^\dagger \phi - \frac{v^2}{2}\right),$$

$$\theta_{13} \int d^4x\, (\bar{c}^*)^2 \left(\phi^\dagger \phi - \frac{v^2}{2}\right),$$

...

Pure fields operators

$$\lambda_1 \int \mathrm{d}^4x \left(\phi^\dagger\phi - \frac{v^2}{2}\right); \qquad \lambda_2 \int \mathrm{d}^4x \left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2;$$

$$\lambda_3 \int \mathrm{d}^4x \left(\phi^\dagger\phi - \frac{v^2}{2}\right)^3; \qquad \lambda_4 \int \mathrm{d}^4x \,(D^\mu\phi)^\dagger D_\mu\phi;$$

$$\lambda_5 \int \mathrm{d}^4x \,\phi^\dagger[(D^2)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu]\phi;$$

$$\lambda_6 \int \mathrm{d}^4x \left(\phi^\dagger\phi - \frac{v^2}{2}\right)\left(\phi^\dagger D^2\phi + (D^2\phi)^\dagger\phi\right);$$

$$\lambda_7 \int \mathrm{d}^4x \left(\phi^\dagger\phi - \frac{v^2}{2}\right)(D^\mu\phi)^\dagger D_\mu\phi; \qquad \frac{\lambda_8}{2} \int \mathrm{d}^4x \,F_{\mu\nu}^2\,;$$

$$\lambda_9 \int \mathrm{d}\partial^\mu F_{\mu\nu}\partial^\rho F_{\rho\nu}; \qquad \lambda_{10} \int \mathrm{d}^4x \,F_{\mu\nu}^2\left(\phi^\dagger\phi - \frac{v^2}{2}\right).$$

Fixing the coefficients

Example: two point Goldstone function

$$
\begin{aligned}
&{\rho'}_0\mathscr{S}_0 \int \mathrm{d}^4x\,[\sigma^*(\sigma+v)+\chi^*\chi] + \rho_0\mathscr{S}_0 \int \mathrm{d}^4x\,(\sigma^*\sigma+\chi^*\chi) + \lambda_1 \int \mathrm{d}^4x\,\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)\\
&+\lambda_4 \int \mathrm{d}^4x\,(D^\mu\phi)^\dagger D_\mu\phi + \lambda_5 \int \mathrm{d}^4x\,\phi^\dagger[(D^2)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu]\phi\\
&\supset \int \mathrm{d}^4x\,\Big[\frac{1}{2}\Big(\lambda_1 - m^2{\rho'}_0\Big)\chi^2 + \Big({\rho'}_0 + \rho_0 + \frac{\lambda_4}{2}\Big)\partial^\mu\chi\partial_\mu\chi + \frac{3}{2}\lambda_5\chi\Box^2\chi\Big],
\end{aligned}
$$

$$
\lambda_1 - m^2\rho_0 = \overline{\Gamma}^{(1)}_{\chi\chi}\Big|_{p^2=0}; \qquad 2(\rho_0+\rho_1)+\lambda_4 = \frac{\partial\overline{\Gamma}^{(1)}_{\chi\chi}}{\partial p^2}\Bigg|_{p^2=0}; \qquad 3\lambda_5 = \frac{\partial\overline{\Gamma}^{(1)}_{\chi\chi}}{\partial(p^2)^2}\Bigg|_{p^2=0}.
$$

$$
\begin{aligned}
\lambda_4 &= -\frac{1}{32\pi^2v^2}\Big[\frac{gv}{\Lambda}\Big(4-\frac{gv}{\Lambda}\Big)M^2 + M_A^2\Big(16+12\frac{gv}{\Lambda}+3\frac{g^2v^2}{\Lambda^2}\Big)\Big]\frac{2}{4-D},\\
\lambda_5 &= \frac{g^2}{96\pi^2\Lambda^2}\frac{2}{4-D}.
\end{aligned}
$$

Example: two point gauge function

$$
\begin{aligned}
&{\rho'}_0 \mathcal{S}_0 \int d^4x \, [\sigma^*(\sigma + v) + \chi^* \chi] \\
&+ \lambda_5 \int d^4x \, \phi^\dagger [(D^2)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu]\phi + \frac{\lambda_8}{2} \int d^4x \, F^2_{\mu\nu} + \lambda_9 \int d^4x \, \partial^\mu F_{\mu\nu} \partial_\rho F^{\rho\nu} \\
&\supset \int d^4x \, \Big[\Big(\rho_0 + \frac{\lambda_4}{2}\Big) e^2 v^2 A^2 - \frac{\lambda_5}{2} e^2 v^2 (2A^\mu \partial_\mu \partial A + A^\mu \Box A_\mu) + \frac{\lambda_8}{2} (\partial^\mu A^\nu - \partial^\nu A^\mu)^2 \\
&+ \lambda_9 (\Box A^\mu - \partial^\mu (\partial A))^2 \Big]
\end{aligned}
$$

$\lambda_9 = 0$ since there are no p^4-terms in $\overline{\Gamma}_{A_\mu A_\nu}$. Moreover

$$
\Big[e^2 v^2 (2\rho_0 + \lambda_4) + (2\lambda_8 + e^2 v^2 \lambda_5) p^2 \Big] g^{\mu\nu} + 2\Big(e^2 v^2 \lambda_5 - \lambda_8 \Big) p^\mu p^\nu = \overline{\Gamma}^{(1)}_{A^\mu A^\nu}(p) \,,
$$

so that

$$
\lambda_8 = -\frac{M_A^2}{96\pi^2 v^2} \Big(2 + 2\frac{gv}{\Lambda} + \frac{g^2 v^2}{\Lambda^2} \Big) \frac{2}{4 - D} \,,
$$

Example: full one-loop expression (beyond lin.approx.) for

$$\tilde{\lambda}_7 \Big(\phi^\dagger \phi - \frac{v^2}{2}\Big)(D^\mu \phi)^\dagger D_\mu \phi$$

$$\tilde{\lambda}_7 = \frac{g(m^2 - M^2)}{\Lambda v^3}\vartheta_3 + \frac{g^2}{\Lambda^2 v^2}(\vartheta_5 + \vartheta_7) + \frac{2g}{\Lambda v^3}(m^2 - M^2)\vartheta_8 + \frac{g}{\Lambda v}(\theta_1 + \theta_4) + \frac{m^2 - M^2}{v^2}\theta_3 + \lambda_7$$
$$= -\mu^{-\epsilon}\frac{1}{32\pi^2 v^3}\frac{g}{\Lambda}\Big[M_A^2\Big(36 + 8\frac{gv}{\Lambda} - 3\frac{g^2 v^2}{\Lambda^2}\Big) + M^2\Big(16 - 14\frac{gv}{\Lambda} + 3\frac{g^2 v^2}{\Lambda^2}\Big)\Big]\frac{2}{4 - D}.$$

Notice that the m^2-dependence has disappeared ($\epsilon = 4 - D$).
The β function is finally

$$\beta_7 = (4\pi)^2 \frac{\mathrm{d}}{d \log \mu}\widetilde{\lambda}_7 \supseteq \frac{2}{v^3}(4M^2 + 9M_A^2)\widetilde{\lambda}_7$$

General solution to the ST identity

Contractible pairs are appropriate variables in the jet space of fields, antifields, external sources and their derivatives.

In spontaneously broken theories the complete solution to the ST identity in ghost number zero (without locality approximation) can be obtained by homotopy operators and bleaching variables.

The ST identity can be seen as a first-order functional differential equation, fixing the dependence of Γ on the Goldstone fields in terms of boundary conditions given by the Goldstone-independent amplitudes.

Outlook and Conclusions

- A constructive approach to Weinberg's renormalizability in the modern sense for SB gauge EFTs
- Generalized field redefinitions are present and not even polynomial. They matter also in the linearized approximation
- Cohomological tools provide a way to handle this problem and identify the genuine physical renormalizations of gauge invariant operators
- Systematic recursive approach
- Further steps: $SU(2) \times U(1)$; background field method and GFRs; Zimmermann's reduction of couplings.

A.Q., arXiv:1610.00150; D.Binosi, A.Q., arXiv:1709.09937,arXiv:1904.06693,arXiv:1904.06692